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Failure Tolerant Operation of Kinematically Redundant Manipulators

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Abstract— **The high cost involved in the retrieval and repair of robotic manipulators used for remediating nuclear waste, processing hazardous chemicals, or for exploring space or the deep sea, places a premium on the reliability of the system as a whole. For such applications, kinematically redundant manipulators are inherently more reliable since the additional degrees of freedom (DOF) may compensate for a failed joint. In this work, a redundant manipulator is considered to be fault tolerant with respect to a given task if it is guaranteed to be capable of performing the task after any one of its joints has failed and is locked in place. A method is developed for insuring the failure tolerance of kinematically redundant manipulators with respect to a given critical task. Techniques are developed for analyzing the manipulator's workspace to find regions which are inherently suitable for critical tasks due to their relatively high level of failure tolerance. Then, constraints are imposed on the range of motion of the manipulator to guarantee that a given task is completable regardless of which joint fails.**

I. Introduction

Kinematically redundant manipulators have been proposed for use in the cleanup and remediation of nuclear and hazardous materials, as well as for remote applications such as deep space or sea exploration, where repair of broken actuators and sensors is impossible and the probability of their failure is increased due to the harsh operating environment [2], [3]. In these situations the extra degrees of freedom of a redundant manipulator may be used to compensate for the failed joints if the manipulator has been properly designed and controlled. The most basic task of a manipulator, i.e. the positioning/orienting the end effector in the workspace, is described by the forward kinematic equation

$$x = f(\theta), \tag{1}$$

where $x \in R^m$ is the generalized vector of the position/orientation of the end effector and $\theta \in R^n$ is the vector of joint variables. In this framework, point to point tasks can be described by a series of end-effector positions to be obtained at desired times, i.e., $x(t_i)$, with a kinematic inverse equation

$$\theta = f^{-1}(x) \tag{2}$$

being solved to determine the corresponding required joint values, $\theta(t_i)$. A kinematically redundant manipulator can, in general, satisfy an end effector positioning constraint, $x(t_i)$, with an infinite family of joint values satisfying (2). The underlying premise for advocating the use of redundant manipulators for critical applications is that if a joint should fail, then the redundancy of the manipulator may permit the completion of the task. Although commercial manipulators currently are not equipped with the necessary circuitry to detect failures and apply the brakes to any failing joint, the need for such a mechanism is well known [12],[13]. If failed joints are locked, then, a single joint failure reduces the number of degrees of freedom (DOF) of the system by one, and the new kinematic functions $\hat{f}()$ and their inverses $\hat{f}^{-1}$ differ markedly from the original ones.

In [12] a method is described for designing manipulators to be fault tolerant with regards to a given point to point task. They assume that any joint may fail anywhere within its entire range of motion. A manipulator is said to be fault tolerant with respect to a given set of task points $x(t_i)$ only if there exist solutions to (2) for every possible failure. With this assumption, the worst case typically occurs when a failing joint is folded in on itself. In the work described here, failure tolerance is achieved by imposing constraints on the motion of all joints prior to a failure. By judiciously selecting the specific solution from the family of solutions to (2), the worst case need not occur. Thus failure tolerance may be achieved with less complex manipulator designs, and for manipulators not originally designed with failure tolerance in mind.

An alternative to defining the manipulator's task as a sequence of end-effector positions is to specify the end-effector velocity profile. At the velocity level, the kinematic equations relating the joint rates $\dot{\theta}$ to the end-effector's velocity $\dot{x}$ are given by

$$\dot{x} = J\dot{\theta} \tag{3}$$

where $J \in R^{m\times n}$ is the manipulator Jacobian matrix which is a function of the manipulator's configuration.

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The solution for all joint rates that satisfy the desired end-effector velocity can be represented by

$$\dot{\theta} = J^+\dot{x} + (I - J^+J)z \tag{4}$$

where $^+$ indicates the pseudoinverse, $(I - J^+J)$ is the projection onto the null space, and z represents an arbitrary vector in the joint velocity space [8]. The second term in this equation clearly indicates that there is a family of joint trajectories that satisfy (3). However, unlike the kinematic function $f()$ relating the joint values to the end-effector's position, the Jacobian for the failed system is easily derived from the original system's Jacobian by zeroing the column of the failed joint. Using this fact it is possible to develop an inverse kinematic function which insures that the manipulator will have some degree of local dexterity after an arbitrary joint failure [7]. The measure of dexterity in this case is defined as the smallest singular value of the Jacobian, σ_m, so that a kinematic failure tolerance measure, kfm, is given by

$$kfm(\theta) = \min_{f=1-n} \sigma_m({}^fJ) \tag{5}$$

where ${}^fJ$ is the manipulator Jacobian matrix for the system with its f'th joint locked. Having a large value for $kfm(\theta)$ insures that after an arbitrary joint failure the manipulator will still be able to satisfy an arbitrary desired end-effector velocity in the vicinity of the failure. Unfortunately, this measure is inherently local in nature and can not guarantee that the complete trajectory remains feasible after the failure. However, it will be shown that the local failure tolerance measure, $kfm(\theta)$, can be used to guide the search for regions within the workspace for which one can insure that the entire desired task can be completed regardless of joint failures.

The remainder of this paper is organized as follows. First, a method for analyzing the fault tolerance of a given location in the workspace is discussed. Second, the constraints necessary to guarantee fault tolerance for a single point are described. Third, a procedure that uses the local measure of fault tolerance to identify candidate regions of the workspace where critical task should be placed is discussed. Then, a method for determining the constraints necessary to guarantee the fault tolerance of the manipulator with respect to the given critical path is outlined.

II. Surfaces of Self-Motion

For a kinematically redundant manipulator the family of joint configurations satisfying (1) forms an $(n-m)$-dimensional hyper-surface in the n-dimensional configuration space of the manipulator [1],[6]. Joint motion constrained to this hyper-surface does not affect the position/orientation of the end effector so that these hyper-surfaces are frequently referred to as self-motion manifolds. The null space of the manipulator's Jacobian given by the set of vectors satisfying (3) with $\dot{x} = 0$ defines the tangent plane to the self-motion manifold. As a simple example, consider the 3 DOF planar manipulator shown

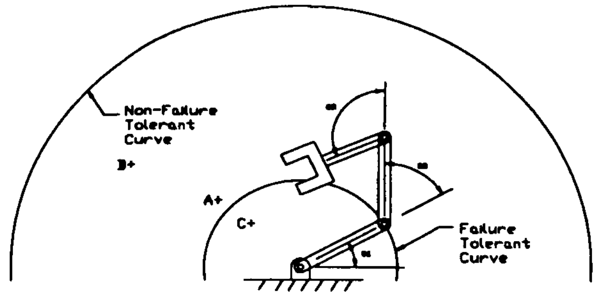

Fig. 1. A three degree of freedom planar manipulator with equal link lengths.

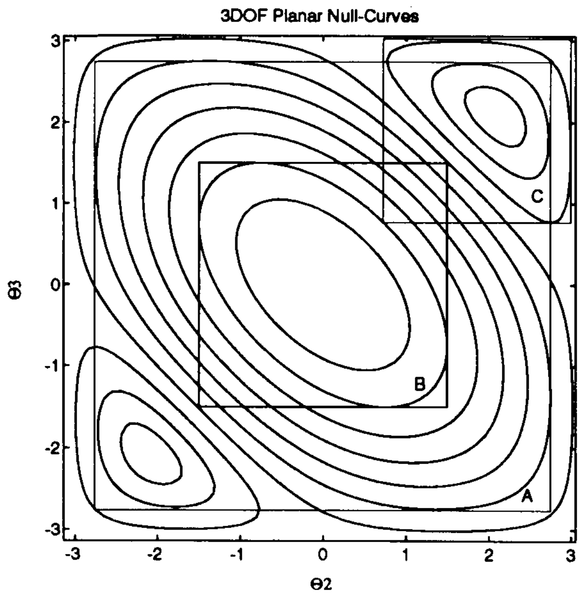

Fig. 2. The set of joint configurations having the manipulator's end-effector at a single location form curves in the configuration space of the manipulator. These curves are the self-motion surfaces for a 3 link planar manipulator. The self-motion surfaces for some regions of the workspace are markedly larger than others. Points with large self-motion surfaces tend to be more failure tolerant.

in Fig. 1 for which the self-motion manifolds are one-dimensional curves. For this manipulator a projection of the self-motion curves onto the $\theta_2 - \theta_3$ plane is shown in Fig. 2. Each curve represents the family of joint variable combinations which place the end-effector at a constant radius from the base. From the figure, it is clear that some regions of the workspace have larger self-motion manifolds than others. The two extremes occur at the boundary of the manipulator's workspace, which corresponds to the point at the origin of the configuration space, and on the circle which is centered at the base and has a radius of 1 meter. In the first case, the self-motion surface vanishes to a point. This fact indicates that the manipulator will not be capable of reaching the original boundary after any joint failure. At the other extreme, the self-motion curve spans the entire range of joint values, even in θ_1 which is not shown. This fact is significant since regardless of which joint fails, or where it fails, the manipulator will always be capable of tracing out the unit circle with it's end-effector. It is interesting to note, that the local failure tolerance measure, (5), reaches its exact theoretically optimal value on the self-motion surface of this globally failure tolerant point. Also note that $kfm(0) = 0$ at the reach singularity. These attributes lead to the use of kfm as a first pass when evaluating the workspace in order to place critical task. Clearly, when a joint fails and is locked, the manipulator is more likely to be able to reach points with large self-motion surfaces than those with small ones.

To guarantee that a manipulator is able to return to a desired workspace location, one must, in general, constrain the motion range for each of the n joints. The minimum and maximum joint values of the ith joint, denoted $\theta_{i_{min}}$ and $\theta_{i_{max}}$, respectively, can be determined from the minimum and maximum values of θ_i over the entire self-motion manifold. This effectively superscribes an n-dimensional box aligned with the joint axes around the self-motion manifold. The size of this bounding box is an indication of the inherent failure tolerance of the workspace point for which it was computed. If the manipulator fails while operating within the bounding box of a given desired end-effector location x, then it will always be able to position its end-effector at that point regardless of where the end effector is located when the failure occurs. For example, consider again the 3DOF manipulator, for which the bounding boxes associated with the self-motion surfaces for the three workspace points labeled A, B, and C in Fig. 1 have been drawn in Fig. 2. Note, that although θ_1 and it's associated boundaries are not shown, they need to be considered. If the manipulator fails while within the boundary of any one of the bounding boxes, then the manipulator will always be able to position its end-effector at those points regardless of which joint fails. The region of the configuration space which lies inside all three bounding boxes is of particular interest. If the manipulator operates within this region, then regardless of which joint fails, it will be able to reach all three points. Unfortunately, it is not possible to reach points B and C and stay within this region of the joint space, however, it should be clear that obtaining the bounding region for self-motion manifolds reveals the potential failure tolerance of various locations within the workspace.

Several iterative methods exist in the literature for characterizing one dimensional self-motion curves [1],[4],[5],[9]. For systems with two or more degrees of redundancy an estimate of the size of the self-surface may be obtained by using a Jacobian iteration of the form

$$\dot{\theta} = \pm(I - J^{+}J)\hat{e}_i + J^{+}(x^* - x) \tag{6}$$

where $\hat{e}_i$ is a unit vector along the ith joint axis and x^* is the workspace end-effector location being evaluated. The first term represents motion along the self-motion manifold until the tangent to the manifold becomes orthogonal to the joint axis direction $\hat{e}_i$. The second term is required to compensate for any errors that are accumulated during the iterative procedure [5]. This method is effective for one-dimensional self-motion curves as in the 3DOF planar case, but may yield an insufficiently low estimate for self-motion manifolds of higher dimensions.

For a two-dimensional self-motion surface, a simple and effective method for estimating the bounds of the self-motion surface is to iteratively trace out a linearly increasing spiral on the self-motion surface. Keeping track of the values obtained by each joint along the spiral provides an estimate of the bounding box containing the self-motion surface. A non-escaping spiral, depicted in Fig. 3, has a parameterized equation of the form

$$\begin{aligned} \dot{\phi} &= \tfrac{v}{r} \\ r &= \gamma\phi \end{aligned} \tag{7}$$

where v is the velocity along the spiral, r and ϕ are the polar coordinates of the spiral, and γ controls the distance between successive rotations. Since this particular spiral passes within a controlled distance from every point in the plane, when it is transformed onto the self-motion surface it will tend to fill the surface. The iterative transformation procedure from parameter to configuration space is given by

$$\dot{\theta} = \sin(\phi)\hat{v}_{n-1} + \cos(\phi)\hat{v}_n + J^{+}(x^* - x) \tag{8}$$

where $\hat{v}_{n-1}$ and $\hat{v}_n$ are orthogonal unit vectors that span the null-space of the manipulator's Jacobian evaluated at the current configuration. The vectors $\hat{v}_n$ and $\hat{v}_{n-1}$ can be computed as the singular vectors from the singular value decomposition of J. Since $\hat{v}_{n-1}$ and $\hat{v}_n$ are not unique, one must be careful to ensure that vectors chosen are the ones nearest to those of the previous iteration. For example, if the current singular vectors are represented by $\hat{v}_{n-1}$ and $\hat{v}_n$ then once (8) is evaluated and used to update the manipulator configuration, the new Jacobian will in general have different singular vectors $\hat{v}'_{n-1}$ and $\hat{v}'_n$. To accurately reflect the continuous rotation of these two vectors as the null space rotates, one can use the following set of equations

$$\begin{aligned} \hat{v}'_{n-1} &= \lambda\hat{w}_1 + (1-\lambda)\hat{w}_2 \\ \hat{v}'_n &= (1-\lambda)\hat{w}_1 - \lambda\hat{w}_2 \end{aligned} \tag{9}$$

where

$$\lambda = \frac{(\hat{w}_1^T\hat{v}_{n-1})^2}{(\hat{w}_1^T\hat{v}_{n-1})^2 + (\hat{w}_2^T\hat{v}_{n-1})^2} \tag{10}$$

where $\hat{w}_1$ and $\hat{w}_2$ are any unit vectors that span the new null space. Note, that the sign should be examined to select the smallest resulting rotation. An ideal algorithm for computing the SVD that automatically calculates the continuous rotation of the null space is presented in [11]. An illustration of this technique for mapping out a two-dimensional self-motion surface is presented in Fig. 4. This figure shows a three-dimensional projection of the five-dimensional configuration space for a PUMA used in three-dimensional positioning tasks.

III. Joint Constraints to Guarantee Fault Tolerance

As was indicated in the previous section, a workspace location, x^*, may be guaranteed to be reachable regardless of joint failures if the manipulator is constrained to operate within the associated self-motion manifold's bounding

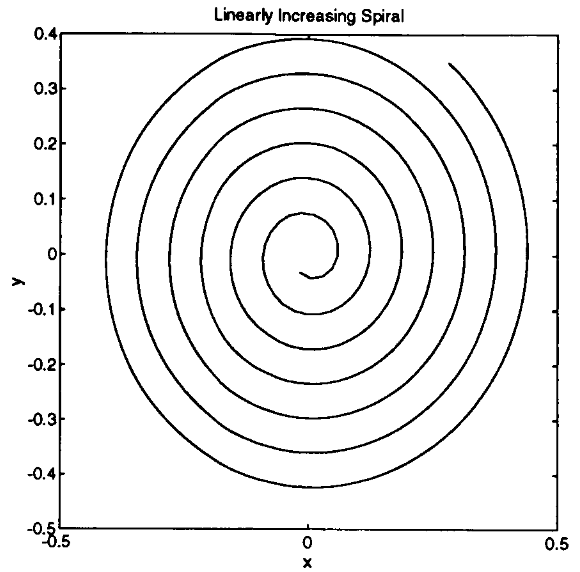

Fig. 3. A linearly increasing spiral passes within a controlled distance from every point in the plane, and thus it may be used to estimate the bounds of a 2D surface in an n-dimensional space.

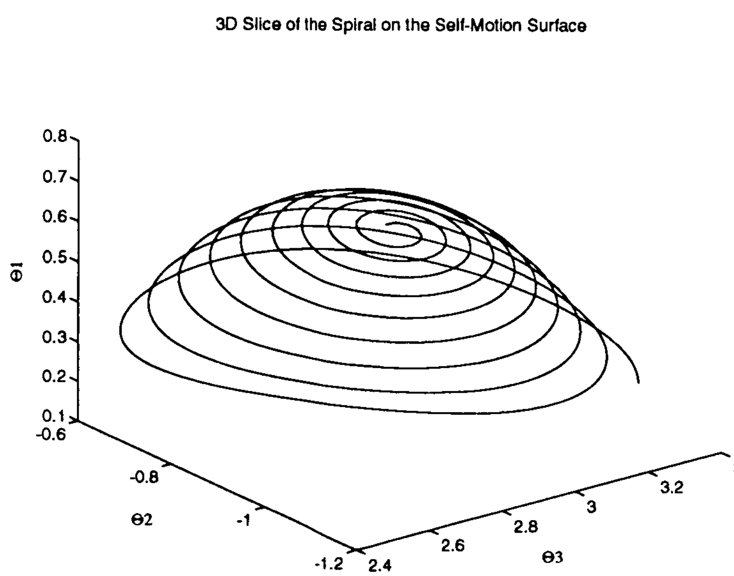

Fig. 4. 3D Slice of the spiral traced out on the self-motion surface in joint space for a PUMA 560 robot used only for positioning.

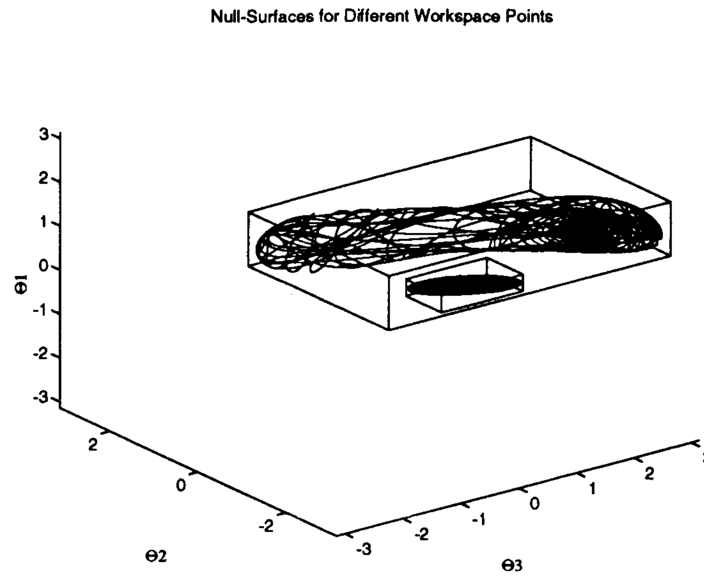

Fig. 5. The set of joint configurations for a kinematically redundant manipulator that yield identical end-effector positions is a surface in an n-dimensional space. A spiral traced out on the tangent plane defined by the null-vectors of the manipulator's Jacobian reveals the shape of the self-motion surface for any given point. Here, two distinct points are shown, one having a large self-motion surface, and one with a small one as indicated by their bounding boxes.

box. This is evident since regardless of which joint fails, by definition there exists at least one configuration on the self-motion manifold associated with x^* that corresponds to the joint value at which the joint failed. Therefore, the problem of maintaining the fault tolerance of a given critical location reduces to that of maintaining joint limits specified by the bounding box of the self-motion manifold for that location. This problem was first solved in [8] by using (4) and selecting z to result in motion away from the joint limits. The vector z may be computed by combining smooth functions so that the joint limits only effect the manipulators motion when it is near the constraint boundaries [10].

To maintain a high degree of fault tolerance, one would like to locate critical task points in locations where the self-motion manifold bounds are large. For instance jigs and fixtures in general should not be placed near the workspace boundaries since joint failures will render such regions unreachable. Although the tedious chore of measuring the size of the self-motion manifolds throughout the workspace could be done off-line, it has been found that the local measure of fault tolerance, $kfm(\theta)$, is a good indicator of size of the self-motion manifolds.

To insure that a task defined by a sequence of critical points may be performed regardless of joint failures, each point must be analyzed, the associated range of its self-motion surface determined, and then the intersection of the ranges for each point computed to determine the required joint constraints (see Fig. 5). Finally, it must be verified that the manipulator is able to reach each critical point while maintaining these constraints.

In summary, the following procedure is used to guarantee the failure tolerance of a redundant manipulator with respect to a critical task. First, the workspace is analyzed using the local failure tolerance measure (5). Second, critical task are placed in regions of the workspace that have high values of local failure tolerance. Third, the bounding boxes for the self-motion surfaces associated with each critical location are determined using the procedures outlined in section II. Fourth, the intersection of the bounding boxes is calculated to determine the required constraints. Fifth, each critical workspace point is checked to determine if the manipulator is capable of positioning it's end-effector at the desired location while maintaining the constraints imposed by the intersection of all bounding boxes. Finally, (4) is used with the joint limit constraints to insure the failure tolerance of the manipulator for the specified task.

IV. Conclusions

This paper has developed a method for insuring the failure tolerance of kinematically redundant manipulators. In this work, a redundant manipulator is considered to be fault tolerant with respect to a given task if it is guaranteed to be capable of performing the task after any one of its joints has failed and is locked in place. Methods were developed for analyzing the manipulator's workspace to find regions which are inherently suitable for critical task due to their relatively high level of failure tolerance. Then, the required constraints were imposed on the range of motion of the manipulator to guarantee that a given task is completable regardless of arbitrary joint failures.

References

[1] J. W. Burdick, "On the inverse kinematics of redundant manipulators: Characterization of the self-motion manifolds", *in Proc. 1989 IEEE International Conference on Robotics and Automation*, pp. 264–270, (Scottsdale, AZ), May 14-18 1989.

[2] B. Christensen, W. Drotning, and S. Thunborg, "Model-based, sensor-directed remediation of underground storage tanks", *Journal of Robotic Systems*, vol. 9, pp. 145–159, 1992.

[3] R. Colbaugh and M. Jamshidi, "Robot manipulator control for hazardous waste-handling applications", *Journal of Robotic Systems*, vol. 9, pp. 215–250, 1992.

[4] D. DeMers and K. Kreutz-Delgado, "Issues in learning global properties of the robot kinematic mapping", *in Proc. 1993 IEEE International Conference on Robotics and Automation*, pp. 205–212, (Atlanta, Georgia), May 2-6 1993.

[5] C. A. Klein and B. E. Blaho, "Dexterity measures for the design and control of kinematically redundant manipulators", *International Journal of Robotics Research*, vol. 6, pp. 72–83, Summer 1987.

[6] C. A. Klein and C. H. Huang, "Review of pseudoinverse control for use with kinematically redundant manipulators", *IEEE Transactions on Systems, Man, and Cybernetics*, vol. SMC-13, pp. 245–250, March/April 1983.

[7] C. L. Lewis and A. A. Maciejewski, "Dexterity optimization of kinematically redundant robots in the presence of faults", to appear in *Computers and Electrical Engineering*, 1993.

[8] A. Liégeois, "Automatic supervisory control of the configuration and behavior of multibody mechanisms", *IEEE Transactions on Systems, Man, and Cybernetics*, vol. SMC-7, pp. 868–871, December 1977.

[9] A. A. Maciejewski, "Kinetic limitations on the use of redundancy in robotic manipulators", *IEEE Transactions on Robotics and Automation*, vol. 7, pp. 205–210, April 1991.

[10] A. A. Maciejewski and C. A. Klein, "Obstacle avoidance for kinematically redundant manipulators in dynamically varying environments", *International Journal of Robotics Research*, vol. 4, pp. 109–117, Fall 1985.

[11] A. A. Maciejewski and C. A. Klein, "The singular value decomposition: Computation and applications to robotics", *International Journal of Robotics Research*, vol. 8, pp. 63–79, December 1989.

[12] C. J. J. Paredis and P. K. Khosla, "Kinematic design of fault tolerant manipulators", to appear in *Computers and Electrical Engineering*, 1993.

[13] M. L. Visinsky, I. D. Walker, and J. R. Cavallaro, "Layered dynamic fault detection and tolerance for robots", *in Proc. 1993 IEEE International Conference on Robotics and Automation*, pp. 180–187, (Atlanta, Georgia), May 2-6 1993.