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Annular Suspension and Pointing System with Controlled DC Electromagnets: ECE 485 Senior Design Project Report

by

Josephine Lynn, Vu, and Kwok Hung, Tam,

Student, Old Dominion University.

Submitted to

Dr. Vahala and Dr. Britcher,

Instructors, Old Dominion University.

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Pledge:

Tam, Kwok Hung

Josephine L. Vu

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Illustrations

List of Symbols

A: cross-sectional area of one side of the actuator

a: assigned constant

B: magnetic flux density in the gap

Bm: magnetic flux density inside the actuator

b: assigned constant

c^2: assigned constant

F(x,i):attractive force between the actuator and the rim(suspended mass)

$G_p(s)$: plant dynamic equation or open loop transfer function

g_1, g_2, g: gap distance

g_o: equilibrium distance

I: input current of the actuator

I_b, I_o: equivalent bias current

i: controlled current

L: inductance of the actuator

L_c: inductance of the coil

L_1: inductance of the gap

l: the length of actuator

m: mass of the rim(suspended mass)

N: number of turns of he coil

R: actuator resistance

s: Laplace transform variable

V: input voltage of the actuator

V_b: equivalent bias voltage

v: controlled voltage

x: controlled distance or small perturbation distance from

equilibrium

μ : permeability of the actuator material

μ_c :permeability of free space

ϕ : magnetic flux

L.H.P : left half plane

ASPS : Annular Suspension and Pointing System

I. Introduction

A. Problem Definition

Frictionless electromagnetic suspension and levitation has attracted much attention since 1970. Applications include high-speed machine tool spindles, ultra-centrifuges, high vacuum pumps, and fly-wheels for energy storage. Methods of producing electromagnetic suspension and levitation include controlled DC electromagnets, diamagnetic materials, superconductors, hybrid systems, and tuned LCR circuits. A comprehensive review lecture of electromagnetic suspension and levitation techniques can be found in reference 1.

The technique of suspension and levitation with controlled DC electromagnets is the most advanced and successful at this time. Many investigations are underway worldwide. Advanced ground transportation schemes, contactless bearings for ultra-high speed, and gyroscopes have been successfully demonstrated by many groups of researchers.[1]

The Annular Suspension and Pointing System (ASPS) developed by the Flight System division of Sperry Corporation[1] is a six-degree of freedom payload pointing system designed for use with the space shuttle. This magnetic suspension and pointing system provides precise controlled pointing in six-degrees of freedom, isolation of payload-carrier disturbances, and end mount controlled pointing. Those are great advantages over the traditional mechanical joints for space applications. More detail discussions of the magnetic suspension joints and mechanical joints can be found in reference

2.

Figure 1 and 2 show the ASPS designed by Sperry Corporation. It consists of six actuators, three for vertical movements, two for radial movements, and one for tangential movements. By the coupling and decoupling matrices (figure 3) [2], we can carefully decompose the command signal of each degree of freedom to each actuator individually. In other words, the coupling and decoupling matrices change the six-degree of freedom ASPS control system to six single-degree of freedom ALPS control systems. Hence, we can design each control loop separately.

figure 1.

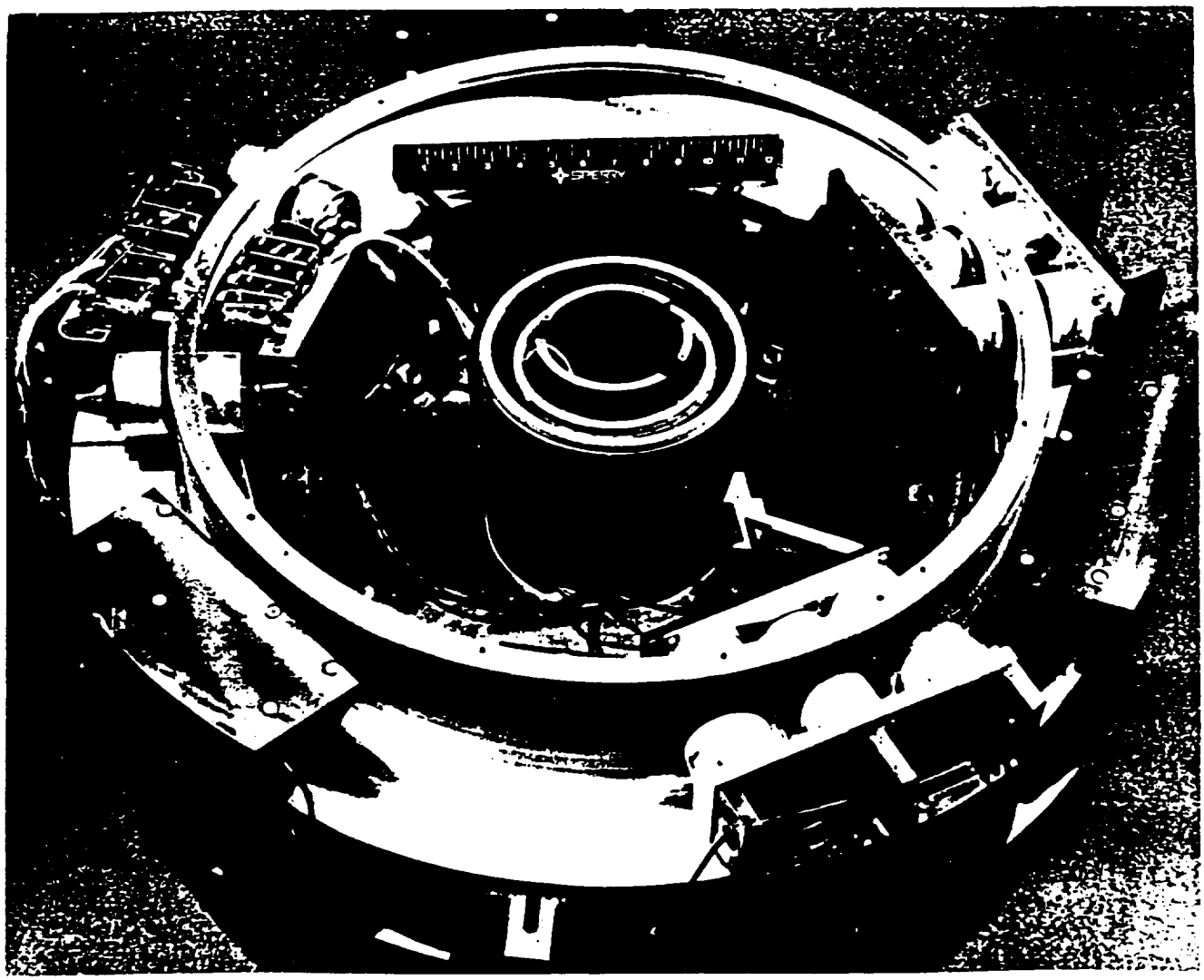
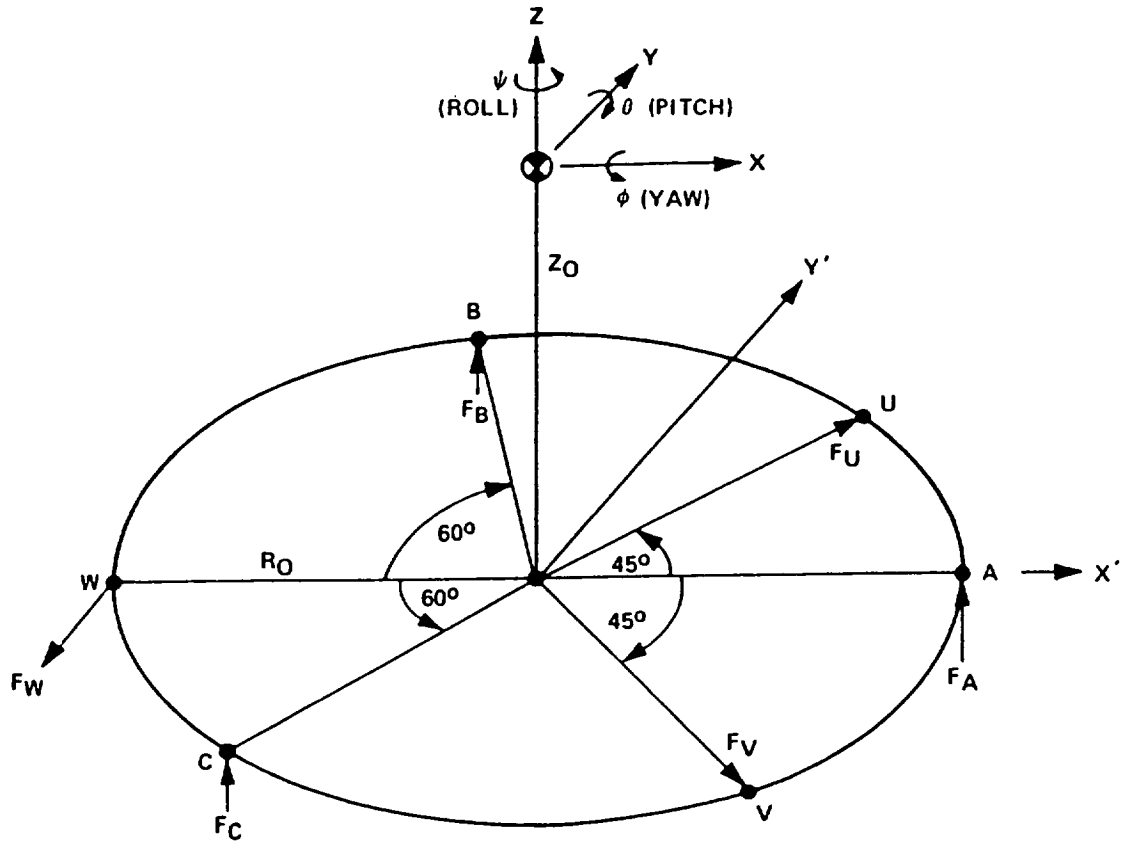


figure 2.

VERNIER ACTUATOR FORCES AND MOMENTS



$F_X = F_U \cos 45^\circ + F_V \cos 45^\circ$

$F_Y = F_U \sin 45^\circ - F_V \sin 45^\circ - F_W$

$F_Z = F_A + F_B + F_C$

$T\phi = F_B R_O \sin 60^\circ - F_C R_O \sin 60^\circ + F_U Z_O \sin 45^\circ - F_V Z_O \sin 45^\circ - F_W Z_O$

$T\theta = -F_A R_O + F_B R_O \cos 60^\circ + F_C R_O \cos 60^\circ - F_U Z_O \cos 45^\circ - F_V Z_O \cos 45^\circ$

$T\psi = F_W R_O$

VERNIER ACTUATOR DECOUPLING

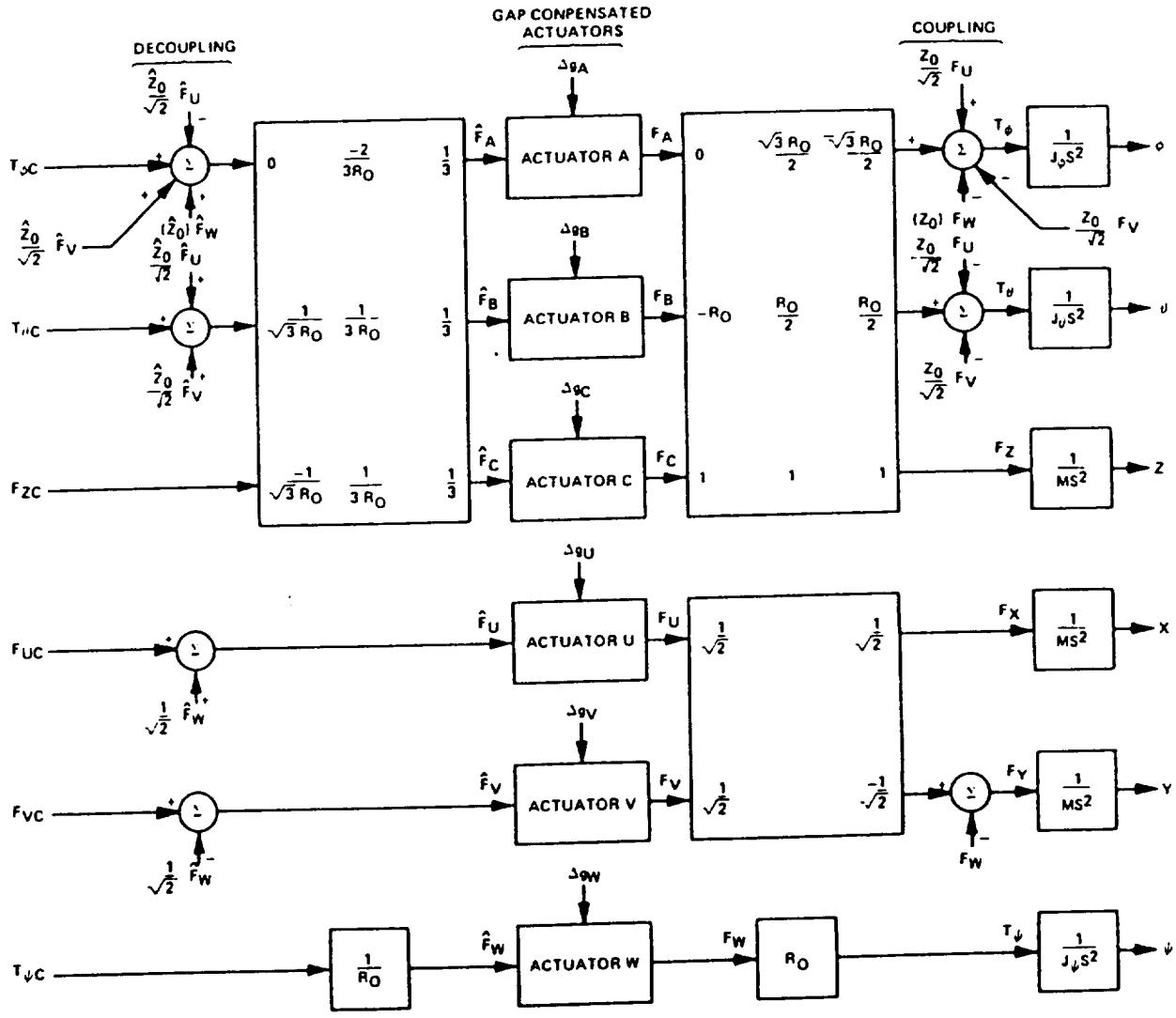


FIGURE 3

B. Project Objectives

(i) Understanding the existing ASPS in the lab.

(ii) Model the dynamics of a single DC controlled ALPS actuator as accurately as possible.

(iii) Re-design a controller for the single degree of freedom ALPS control system to achieve the highest stiffness as possible. {Highest stiffness will have the lowest motion in response to external forces}

C. Summary

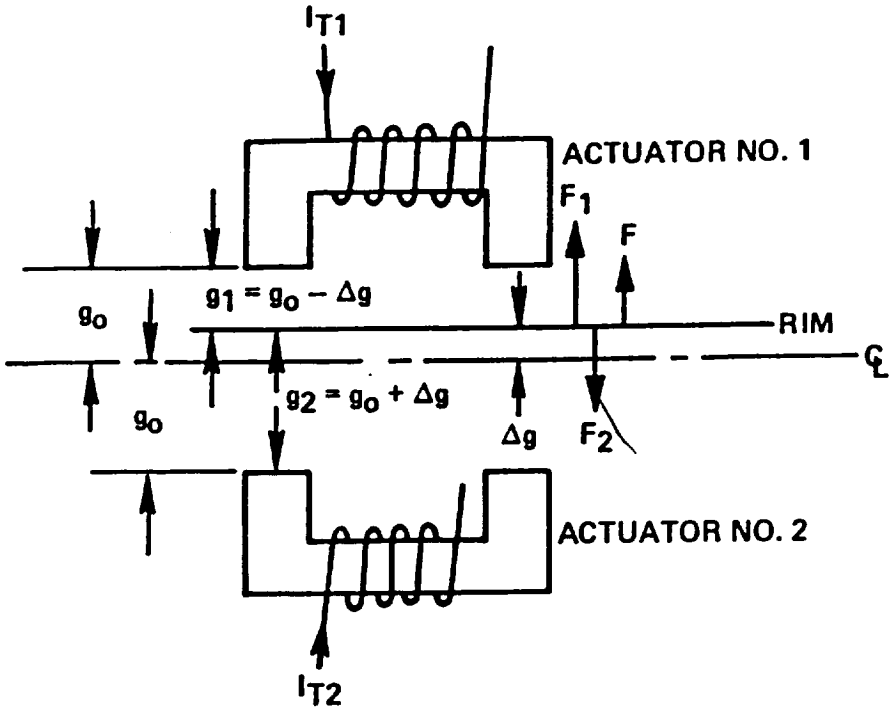
In this design, we first analyzed the assumed model of the single degree ASPS bearing actuator, and obtained the plant dynamics equations. By linearizing the plant dynamics equations, we designed the cascade and feedback compensators such that a stable and satisfied result was obtained. The specified feedback compensator was computer simulated with the nonlinearized plant dynamics equations. The results indicated that an unstable output occurred. In other words, the designed feedback compensator is fail. The failure of the design is due to the Taylor's series expansion does not converge.

II. Modelling of the single degree ASPS bearing actuator

A. Assumptions of the model

The single degree ASPS bearing actuator consists of two pairs of magnetic coil elements, mounted in opposition, to control the rim (suspended mass) along a single axis. Figure 4 shows the configuration of the actuator. The magnetic coil elements have current biasing superimposed by a controlled DC voltage source to produce a force to suspend and point the rim. For fine pointing application, the gap distances between the rim and actuators are kept to a predetermined value(<0.3").

figure 4.



We derived the equation for the single ALPS bearing actuator with the following assumptions:

(i) The force of attraction between magnetized bodies is given by $F = (1/2u) * B^2 * \text{area}$.

(ii) The magnetic flux density is uniform between the gap, or gap distance << size of actuator.

(iii) The rim is a perfect conductor. That is, it does not support any magnetic field strength (H) inside the rim.

(iv) There is no coupling effect between each coil elements.

(v) The controlled electromagnet behaves linearly, and there is no loss or fringing of magnetic field.

(vi) The rim has a plane area over the magnetic coil assemblies.

B. Derivation of ASPS Dynamics

a) B-field in the air-gap.

From magnetic circuit theory, the total current linked by the path of an N-turn coil is given by

$$\sum_i H_i l_i = N\, i \qquad (4)$$

Assume

(i) the magnetic material is approximated by $B_m = \mu_m H_m + B_o$ where B_o is a constant.

(ii) gap distant at path (1) = gap distance at path (2)

ie. $g_1 = g_2 = g$

(iii) from boundary condition, B inside the coil ≈ B in

the gap.

Therefore, equation (1) becomes,

$$H_1 g_1 + H_2 g_2 + H_m l = Ni$$

$$\rightarrow 2Hg + \frac{B_m - B_o}{\mu_m} l = Ni \quad \text{(because the rim is a perfect conductor)}$$

$$\rightarrow 2Hg + \frac{\mu_o}{\mu_m} lH = Ni + \frac{B_o l}{\mu_m}$$

As the term $B_o(1/\mu_m)$ can be equivalently assigned to a magnetomotive force (mmf) $(B_o/\mu_m)1 = NI_o$

So

$$2Hg + \frac{\mu_o l}{\mu_m} H = Ni + NI_o$$

Thus the magnetic flux density at the gap is given by

$$B = \mu_o H = \frac{\mu_N (i + I_o)}{2g + \frac{\mu_o}{\mu_m} l}$$

As for the particular material we used for actuator, $\mu_m >> \mu_o$. Hence

$$B \approx \frac{\mu_o N (i + I_o)}{2g} \qquad (2)$$

b) Relationship between F, I and g

By considering the stored magnetic energy, Bohr[5] and Hayt[6] were able to relate the magnetic attraction force to the magnetic flux density and cross-sectional,

$$F_m = \frac{1}{2\mu_o} B^2 * area$$

The geometry we used is similar to Humphris[7] and Groom[8], figure 4, which have two electromagnets positioned opposite the rim. This kind of configuration is more linear if we separate the magnetic flux density into the controlled and bias components.[7]

Assume no coupling effect between the two actuators, by equation (2):

$$B_1 = \frac{\mu_o N_1 (i_1 + I_1)}{2g_1}; \; B_2 = \frac{\mu_o N_2 (i_2 + I_2)}{2g_2}$$

Let $N_1 = N_2 = N$, $I_1 = I_2 = I_o$, $i_1 = -i_2 = i$ = controlled current. Therefore, the total force acting on the rim is given by

$$F = F_1 - F_2 = \frac{(2A)}{2\mu_o}(B_1^2 - B_2^2) = \frac{A}{\mu_o}(B_1 + B_2)(B_1 - B_2)$$

$$\text{Consider } B_1 - B_2 = \frac{\mu_o N}{2}\left[\frac{I_o + i}{g_o - x} - \frac{I_o - i}{g_o + x}\right]$$

$$= \frac{\mu_o N}{2}\left[\frac{(g_o + x)(I_o + i) - (g_o - x)(I_o - i)}{g_o^2 - x^2}\right]$$

$$= \frac{\mu_o N}{2(g_o^2 - x^2)}[g_o(I_o + i - I_o + i) + x(I_o + i + I_o - i)]$$

$$= \frac{\mu_o N}{2(g_o^2 - x^2)}(2g_o i + 2I_o x) = \frac{\mu_o N(g_o i + I_o x)}{g_o^2 - x^2}$$

$$\text{Similiarly, } B_1 + B_2 = \frac{\mu_o N}{2}\left[\frac{I_o + i}{g_o - x} + \frac{I_o - i}{g_o + x}\right] = \frac{\mu_o N}{g_o^2 - x^2}(g_o I_o + xi)$$

$$\text{Thus } F = \frac{A}{\mu_o}(B_1 + B_2)(B_1 - B_2) = \frac{A}{\mu_o}\left[\frac{\mu_o N(g_o i + I_o x)}{g_o^2 - x^2}\right]\left[\frac{\mu_o N(g_o I_o + xi)}{g_o^2 - x^2}\right.$$

$$= \frac{\mu_o N^2 A(g_o i + I_o x)(g_o I_o + xi)}{(g_o^2 - x^2)^2}$$

By the Taylor's Series Expansion at the equilibrium point (x_o, i_o), we get

$$F(x,i) \approx F(x_o,\ i_o) + (x - x_o,\ i - i_o) * \nabla F|_{(x_o,\ i_o)}$$

$$as\ \frac{\partial F}{\partial i} = \frac{\mu_o N^2 A}{(g_o^2 - x^2)^2}[g_o(g_o I_o + xi) + x(g_o i + I_o x)]$$

$$\frac{\partial F}{\partial x} = \frac{\mu_o N^2 A}{(g_o^2 - x^2)^4}[(g_o^2 - x^2)^2[I_o(g_o I_o + xi) + i(g_o i + I_o x)]$$

$$- 2(g_o^2 - x^2)(-2x)(g_o I_o + xi)(g_o i + I_o x)]$$

at equilibrium point $(x_o,\ i_o) = (0,\ 0)$,

$$F(x_o,\ i_o) = 0$$

$$\frac{\partial F}{\partial x}|_{(x_o,\ i_o)} = \frac{\mu_o N^2 A I_o^2}{g_o^3}$$

$$\frac{\partial F}{\partial i}|_{(x_o,\ i_o)} = \frac{\mu_o N^2 A I_o}{g_o^2}$$

So

$$F(x,\ i) \approx x * \frac{\partial F}{\partial x}|_{(x_o,\ i_o)} + i * \frac{\partial F}{\partial i}|_{(x_o,\ i_o)}$$

$$= \left(\frac{\mu_o N^2 A I_o^2}{g_o^3}\right) x + \left(\frac{\mu_o N^2 A I_o}{g_o^2}\right) i \qquad (3)$$

c) V-I relationship of the actuator.

Recall equation (2):

$$B = \frac{\mu_o N(i + I_o)}{2g}$$

By definition, $\Phi=NBA$ and $L=(d\Phi/di)$[5]. Therefore, the inductance of the gap is

$$L_1 = NA\frac{dB}{di} = \frac{N^2A\mu_o}{2g} \tag{4}$$

and the inductance of the whole circuit is

$$L = L_1 + L_2 + L_c =$$

$$= \frac{N^2A\mu_o}{2g_1} + \frac{N^2A\mu_o}{2g_2} + L_c \qquad \text{(by 4)}$$

$$= \frac{N^2A\mu_o g_o}{g_o^2 - x^2} + L_c$$

By Kirchhoff's voltage law, we have

$$V = Ri + \frac{d}{dt}(Li)$$

$$= Ri + L\frac{di}{dt} + i\frac{dl}{dx}*\frac{dx}{dt}$$

We previously separated the voltage, current and gap distance into the bias components and controlled components.

That is Let $V = V_b + v$

$i = I_b + i$

and $x = g_o + x$

Therefore,

$$V_b + v = R(I_b + i) + L\frac{di}{dt} + i\frac{dl}{dx}*\frac{dx}{dt} + I_b\frac{dl}{dx}*\frac{dx}{dt}$$

As, at equilibrium position, $V_b = RI_b$, assume $i \approx 0$, $x \approx 0$

Therefore,

$$i\frac{dL}{dx}*\frac{dx}{dt} \approx 0$$

$$so \quad v = Ri + L\frac{di}{dt} + I_b\frac{dL}{dx}*\frac{dx}{dt}$$

$$Consider \quad \frac{dL}{dx} = \frac{2N^2A\mu_o g_o}{g_o^2 - x^2}x$$

$$\therefore \quad I_b*\frac{dL}{dx}*\frac{dx}{dt} = I_b\frac{2N^2A\mu_o g_o}{(g_o^2 - x^2)^2}*x*\frac{dx}{dt} \approx 0$$

$$Thus \quad v(t) = Ri(t) + L\frac{d}{dt}i(t)$$

taking the Laplace transform on both sides, we get

$$V(s) = RI(s) + LsI(s)$$

$$or \; I(s) = \frac{1}{R + Ls}V(s) \qquad (5)$$

Recall equation (3)

$$F(x,\ i) = \frac{A\mu_o N^2 I_o^2}{g_o^3}x + \frac{A\mu_o N^2 I_o}{g_o^2}i$$

$$By\ Newton\ Second\ Law\ \ F(x,\ i) = m\frac{d^2x}{dt^2}$$

$$So,\ \ m\frac{d^2x}{dt^2} = \frac{A\mu_o N^2 I_o^2}{g_o^3}x + \frac{A\mu_o N^2 I_o}{g_o^2}i$$

Taking the Laplace Transform on both sides, we get

$$ms^2\mathbf{X}(s) = \frac{A\mu_o N^2 I_o^2}{g_o^3}\mathbf{X}(s) + \frac{A\mu_o N^2 I_o}{g_o^2} * \frac{1}{R + Ls}V(s)$$

$$or\ \frac{\mathbf{X}(s)}{V(s)} = \frac{\dfrac{A\mu_o N^2 I_o}{g_o^2}}{(R + Ls)\left(ms^2 - \dfrac{A\mu_o N^2 I_o^2}{g_o^3}\right)} \qquad (6)$$

$$= \frac{\dfrac{A\mu_o N^2 I_o}{g_o^2 mL}}{\left(s + \dfrac{R}{L}\right)\left(s^2 - \dfrac{A\mu_o N^2 I_o^2}{m\, g_o^3}\right)}$$

$$Let\ a = \frac{A\mu_o N^2 I_o}{g_o^2 mL},\quad b = \frac{R}{L},\quad c^2 = \frac{A\mu_o N^2 I_o^2}{m\, g_o^3}$$

Therefore the plant dynamics of the ALPS actuator are

$$G_p(s) \triangleq \frac{\mathbf{X}(s)}{V(s)} = \frac{a}{(s + b)(s^2 - c^2)}$$

Which is similar to the plant dynamic equation obtained by

Kilgore[9] and Jayawant[3].

Referring to Groom[2], the values of those parameters are,

I_o	0.57 Amps
A	$1.1400918*10^{-3}$ m^2
μ_o	$4 \pi *10^{-7}$ H m^{-1}
N	1386 turns per coil
m	7.19712 kg
g_o	0.00762 m
R	8.0 Ω
L	0.1805899 h at g_o

$$\therefore \quad a \approx 20.79$$

$$b = 44.3$$

$$c^2 \approx 280.8$$

Thus, the open loop transfer function is

$$G_p = \frac{20.79}{s^3 + 44.3s^2 - 280.8s - 12439.59}$$

d) Discussion of the plant dynamic of ASPS actuator

The open loop transfer function is a third order, type zero, all poles plant system. The characteristic equation also contains one positive real root, so this plant is not BIBO stable. The pole zero diagram of the plant is shown in fig 5. In order to move the open loop unstable root into the stable region, we need to add zeros in the left half plane so that the locus are pulled into the stable region. In other word, a reshaping of the root locus (compensator) is necessary.

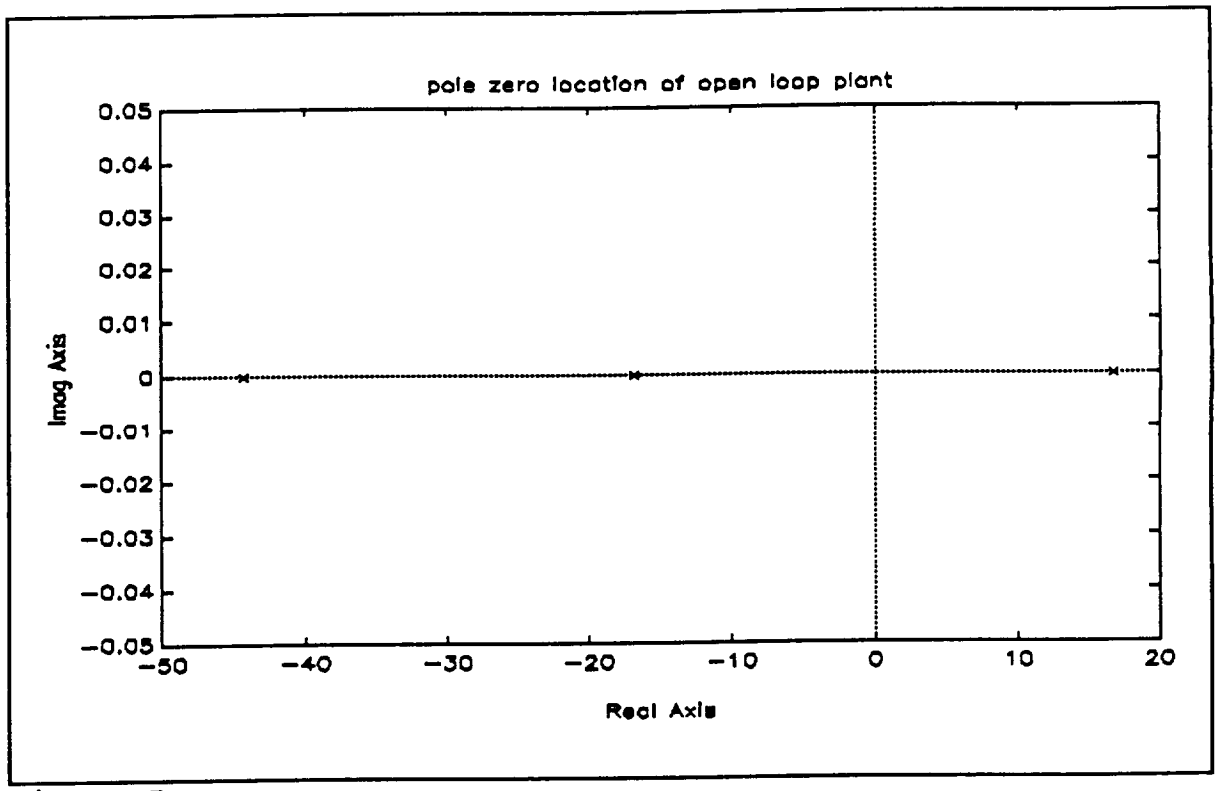


figure 5

III. Compensators Design

A. Cascade Compensator

The approach we used in the cascade compensator is to achieve the goal (shift the root locus to the L.H.P.) with minimum complexity. For the simplest case, by the knowledge that a pole will pull the root locus to the right, a zero will pull the root locus to the left and a pole-zero pair close to the origin will decrease the steady state step error (G_p is type zero), we tried the general lag-lead cascade compensator[10],

$$G_c(s) = A \frac{(s + \frac{1}{T_1})}{(s + \frac{1}{\alpha T_1})} \frac{(s + \frac{1}{T_2})}{(s + \frac{\alpha}{T_2})}$$

with α =10, and gain A. [10] The lag component was fixed at

(s+ 0.05)/(s+ 0.005) and the lead component was moved along the real axis. Some root locus results are shown in appendix I. After studying the results, we decided that we needed to increase the compensator complexity in order to meet the design specifications.

Since the lag component only affected the steady state error, for simplicity, we tried the dual phase advanced compensator with α=10.

$$G_c(s) = \left(\frac{s + \frac{1}{T}}{s + \frac{\alpha}{T}}\right)^2$$

Some root locus results were shown in the appendix II.

Interestingly, when we put the double zero near to the second large negative real pole of the open-loop transfer function, a significant portion of the root locus were pulled into the left half plane, figure 6. This was the result we were looking for.

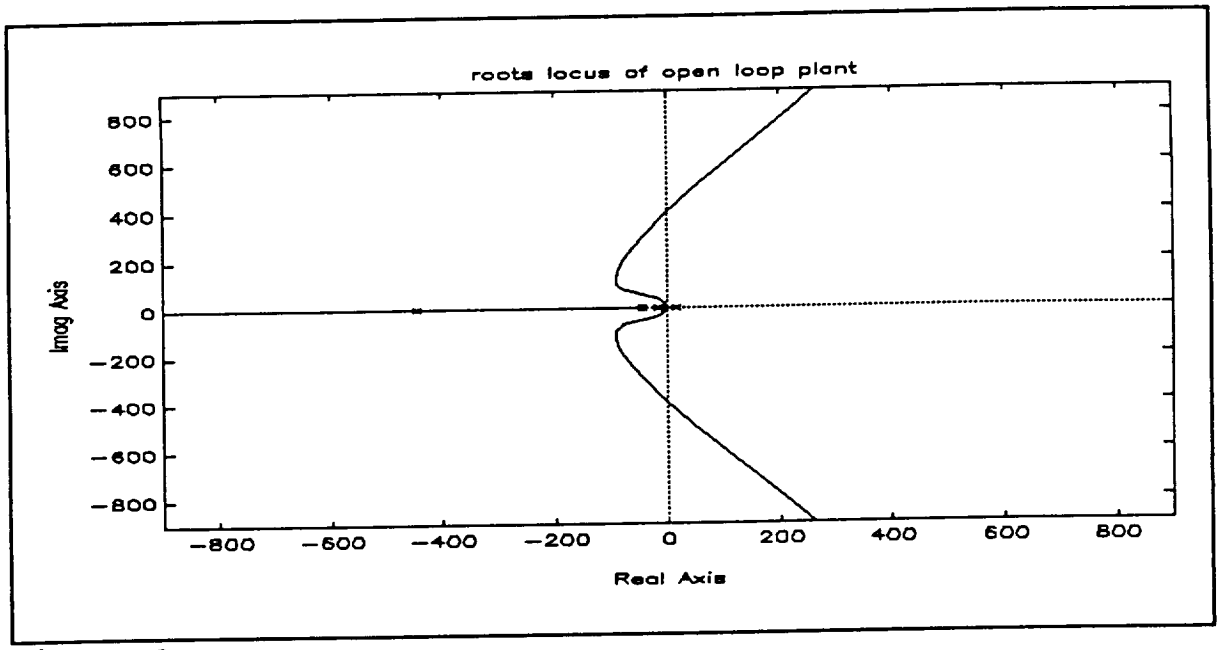


figure 6

Addition of a lag component in the compensator only reduced the steady state step error slightly (0.02), so for simplicity, we used the dual phase advanced compensator

$$G_c(s) = \left(\frac{s + 44.3}{s + 443}\right)^2$$

A block diagram is shown in figure 7. We selected a damping ratio $\xi = 0.7$, and the maximum natural frequency. The figures of merit are,

poles: -86.15 ± 87.89j, -93.24, -620.46

additional gain K = 1.011406 E 6

steady state step error = 0.06

rise time = 0.009 sec.

peak overshoot = 1.42

peak time = 0.025 sec.

settling time = 0.05 sec. (for 5%)

gain margin = 16.75 dB

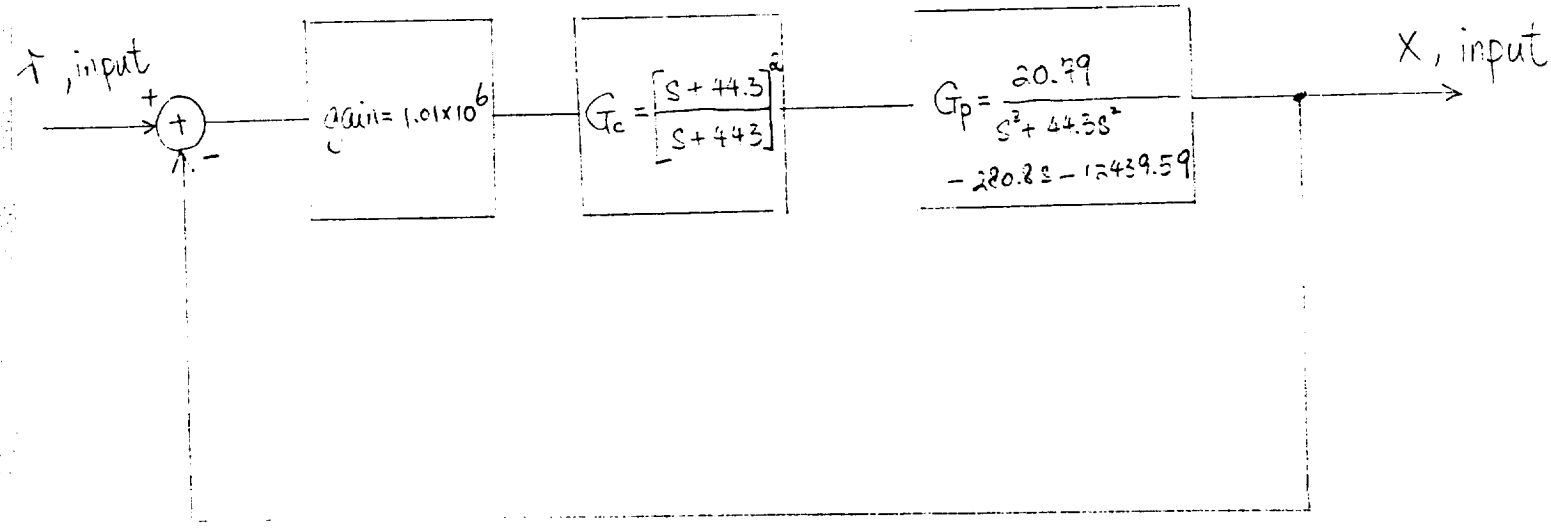
phase margin = 180°

stiffness, ω_n = 15 kN $kg^{-1}m^{-1}$

stability region : 6.0939 E 4 < K < 6.8041 E 6

The step response was shown in figure 8, and the Bode plot was shown in figure 9. Those results are obtained by matlab.

figure 7.



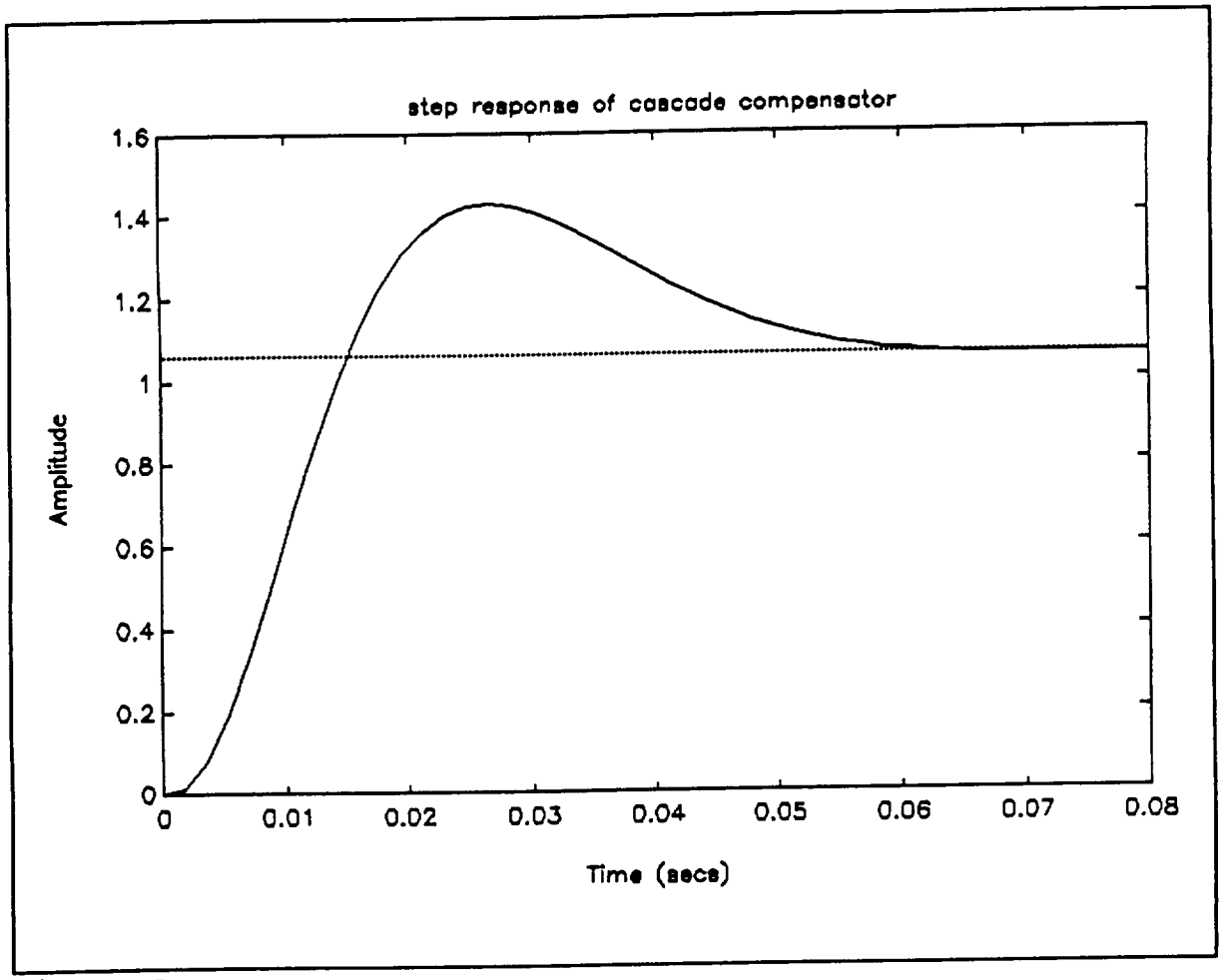

step response of cascade compensator
Amplitude
Time (secs)
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08

figure 8

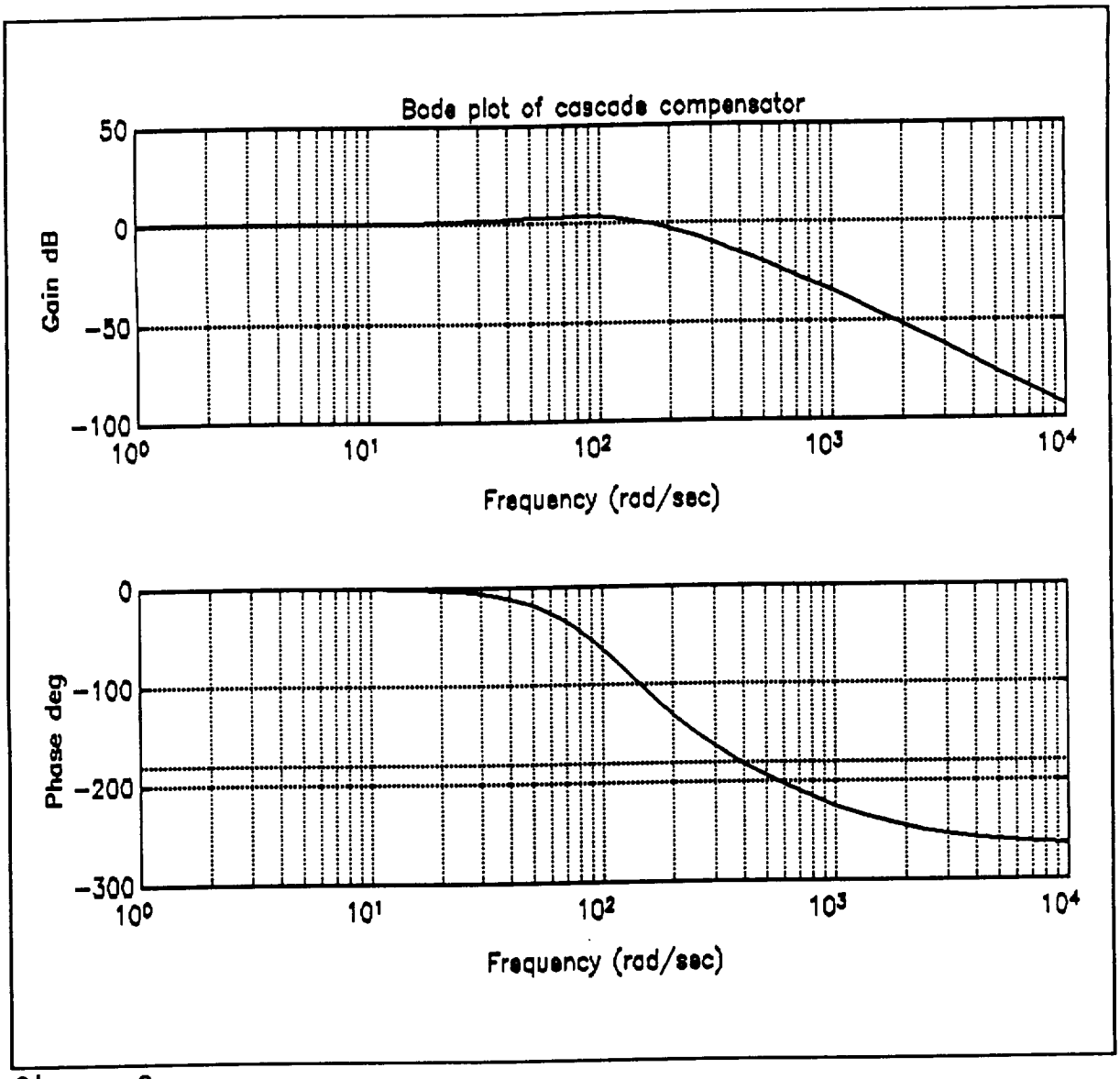

Bode plot of cascade compensator
50
0
−50
−100
Gain dB
10⁰
10¹
10²
10³
10⁴
Frequency (rad/sec)
0
−100
−200
−300
Phase deg
10⁰
10¹
10²
10³
10⁴
Frequency (rad/sec)

figure 9

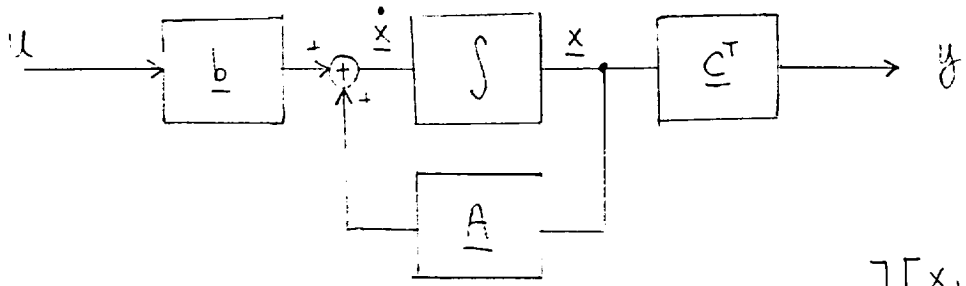
B. Feedback Compensator Design

The cascade compensator will be converted to a digital controller later, and a computer will be involved to control the plant. Therefore, it is natural to design the compensator using the state feedback technique. This technique is flexible and convenient to implement. A brief derivation of the design procedure is shown in Appendix III. In the case where some state variables are not accessible, an observer (estimator) may be used. Observer design procedures are also shown in the Appendix III. This material are come D'Azzo [10]. In this design, we used the full state feedback technique.

A state space representation of the open loop plant is shown in the figure 10.

figure 10

State space representation of the open loop plant



$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -44.3 & 280.8 & 12439.59 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 20.79 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

The controllability matrix is,

$M_C=[b|Ab|A^2b]$

=

$$\left|\begin{array}{c|c|c} 0 & 0 & 20.79 \\ 0 & 20.79 & -920.997 \\ 20.79 & -920.997 & 46637.99 \end{array}\right|$$

as $\det(M_C) \neq 0$, so M_C has full rank.

The observability matrix is

$M_O=[C^T|A^TC^T|(A^T)^2C^T]$

=

$$\left|\begin{array}{c|c|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right|$$

So M_O has full rank. Hence, this system is completely controllable and completely observable.

Motivated by the performance of the cascade compensator, we selected the poles of the control ratio to be $-86.15 \pm 87.89j$, -100, which give us a good step response. A block diagram is shown in the figure 11. The figures of merit are,

gain = 72.8546 E 3

$k_1 = 1.01$

k_2 = 0.021561

k_3 = 0.0001505

poles : -86.15 ± 87.89j, -100

steady state step error = 0

rise time = 0.03 sec.

settling time = 0.045 sec. (for 5 %)

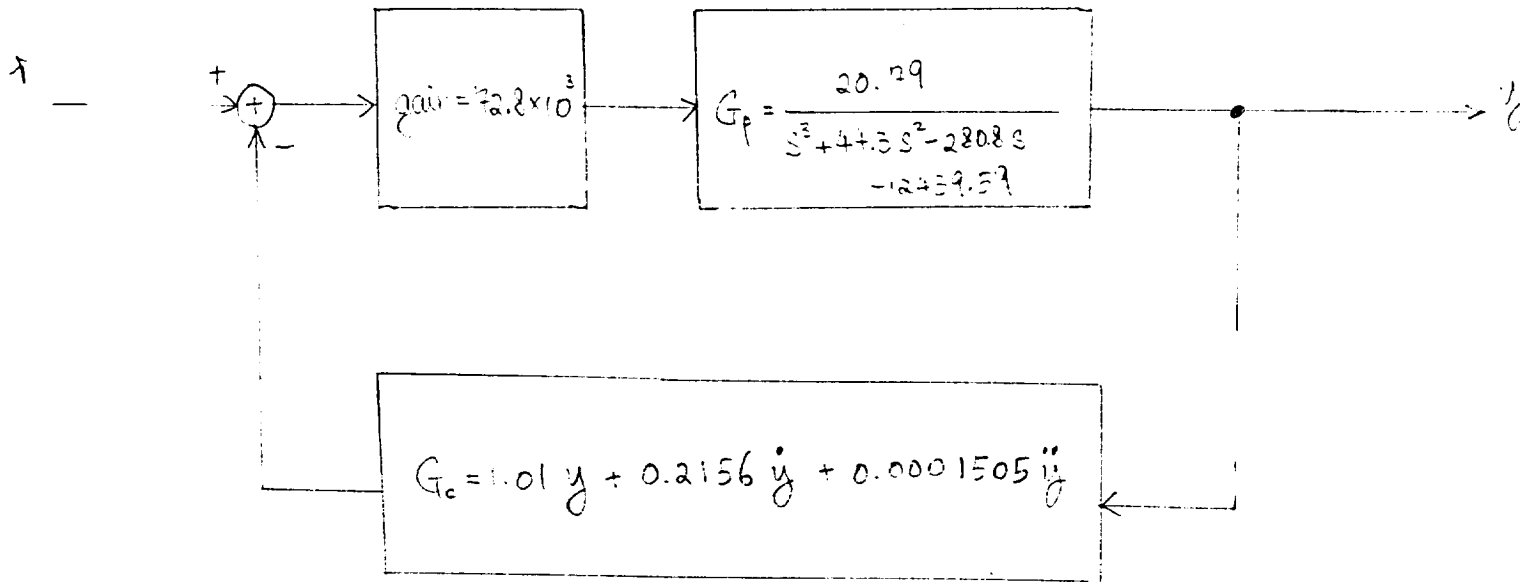
gain margin = 13 dB

phase margin = 160°

stiffness = 15 $kNkg^{-1}$ m^{-1}

A step response and Bode plot were shown in figure 12 and figure 13 respectivity. Those results are obtained by matlab.

figure 11.



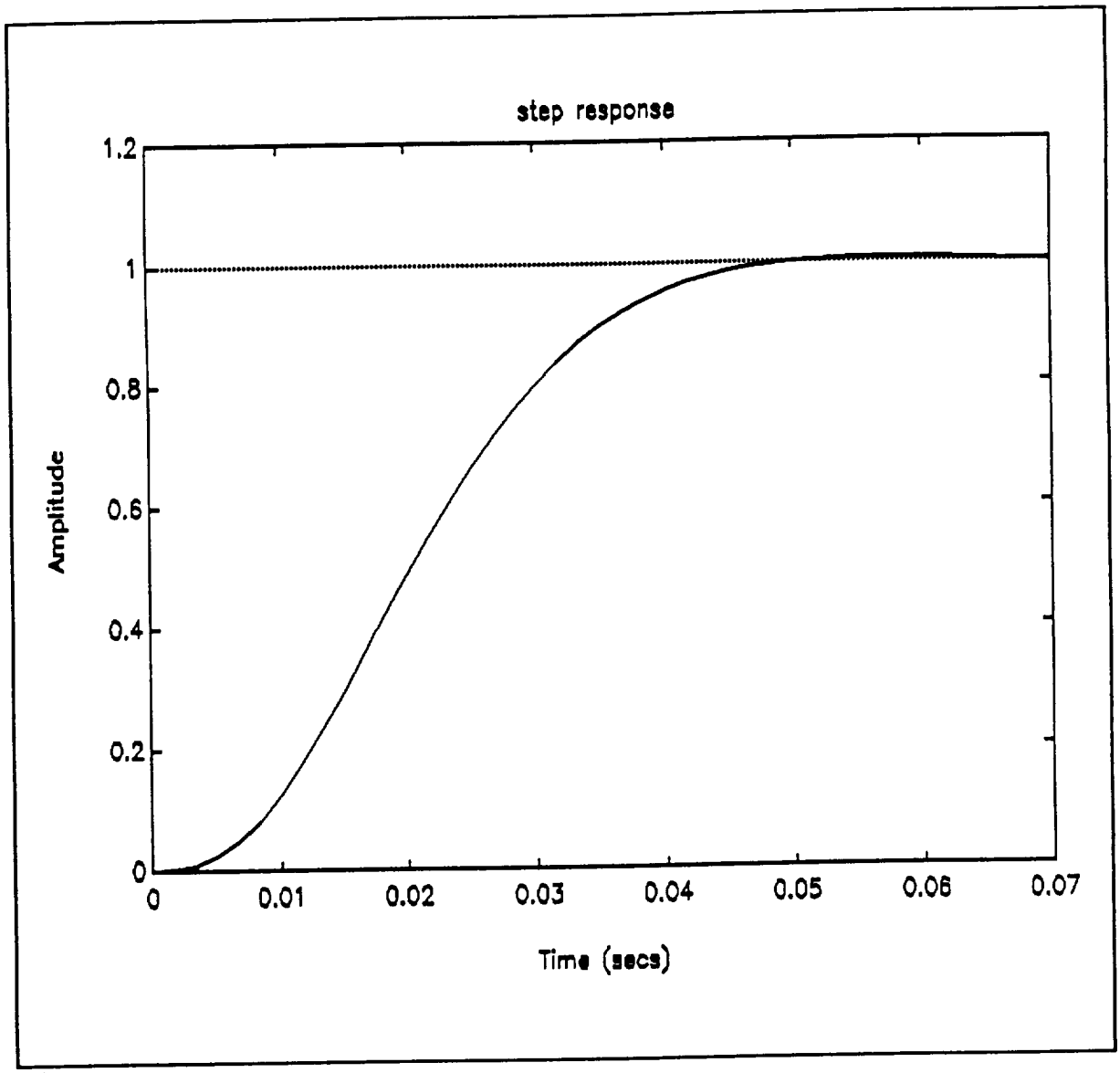

step response
1.2
1
0.8
0.6
0.4
0.2
0
Amplitude
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
Time (secs)

figure 12

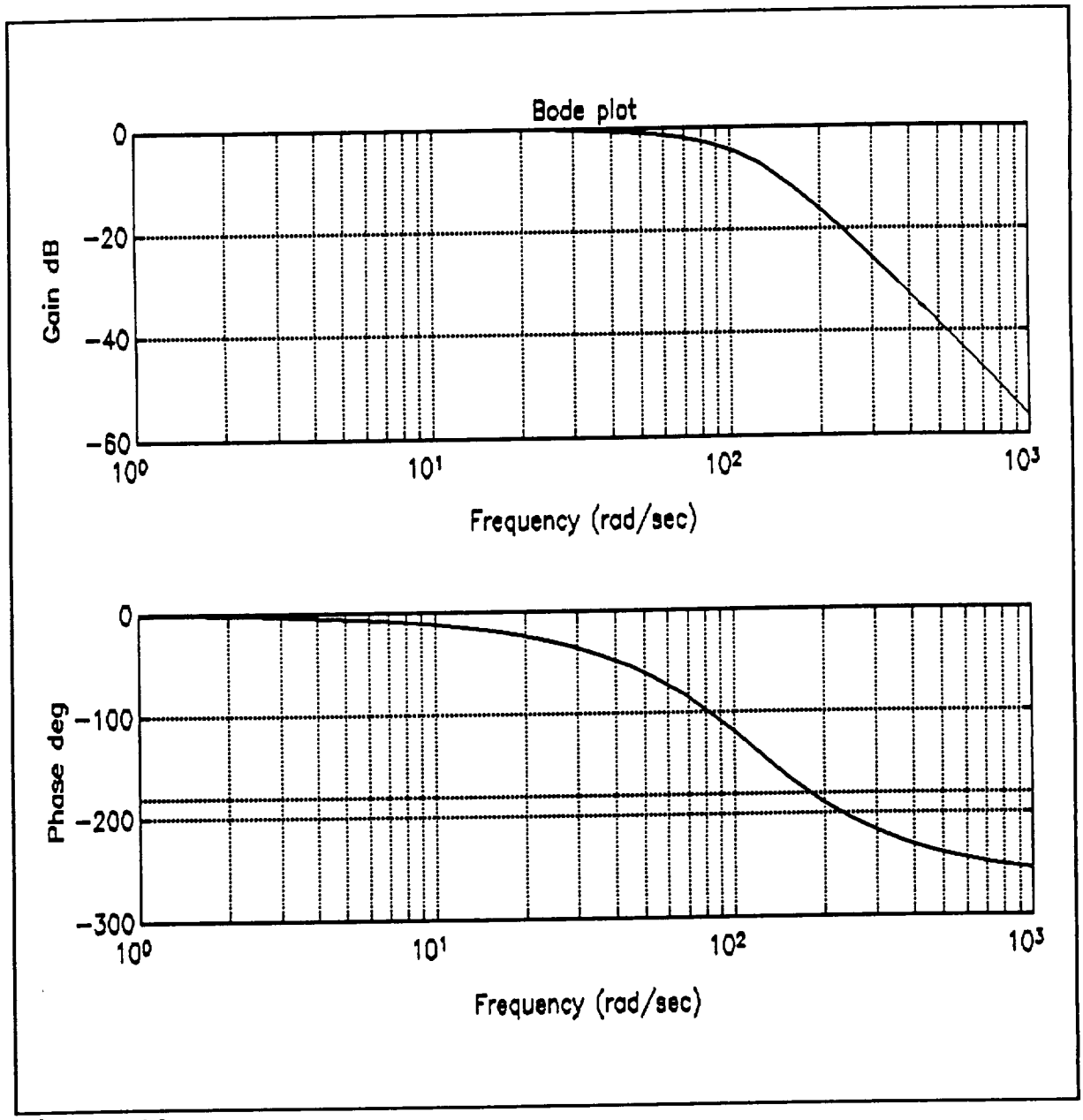

Bode plot
0
-20
-40
-60
Gain dB
10^0
10^1
10^2
10^3
Frequency (rad/sec)
0
-100
-200
-300
Phase deg
10^0
10^1
10^2
10^3
Frequency (rad/sec)

figure 13

IV Computer Simulation

We simulated the state feedback design by a four order Runge Kntta Method in the time domain. The Runge Kntta Method is a special version of the general Taylor's series expansion.

For a general different equation,

$$\frac{d\vec{X}}{dt} = \vec{f}(\vec{X})$$

Let,

$$\vec{f}_1 = \vec{f}(\vec{X})$$

$$\vec{f}_2 = \vec{f}(\vec{X} + \frac{1}{2} h \vec{f}_1)$$

$$\vec{f}_3 = \vec{f}(\vec{X} + \frac{1}{2} h \vec{f}_2)$$

$$\vec{f}_4 = \vec{f}(\vec{X} + h \vec{f}_3)$$

The next $\vec{X}$ can be approximated by

$$\vec{X}_{next} = \vec{X} + \frac{h}{6} [\vec{f}_1 + 2(\vec{f}_2 + \vec{f}_3) + \vec{f}_4]$$

with an error of fifth power term of the Taylor series expansion. The h is the increment step side.

Recall,

$$m\ddot{x} = \frac{A\mu_o N^2 (g_o I_o + x i)(g_o i + I_o x)}{(g_o^2 - x^2)^2}$$

$$v = R i + L\frac{di}{dt}$$

Let

$$x_1 = x = \text{output}$$

$$x_2 = \frac{dx_1}{dt}$$

$$x_3 = i$$

$$x_4 = t$$

$$v = u = \text{input}$$

So

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \dot{x} = \frac{\frac{A\mu_o N^2}{m}(g_o I_o + x_1 x_3)(g_o x_3 + I_o x_1)}{(g_o^2 - x_1^2)^2}$$

$$\dot{x}_3 = -\frac{R}{L} x_3 + \frac{1}{L} u$$

$$\dot{x}_4 = 1$$

Let the control law be

$$u = r - (k_1 x_1 + k_2 x_2 + k_3 \dot{x}_2)$$

where r is the reference input.

In the computer code, we need to estimate the second derivative of x, which is achieved by

$$(x_{2\ present} - x_{2\ previous}) \ / \ \text{time interval}$$

For the linearized plant dynamics equations, slightly modification of the computer codes can do the job. The results of computer simulation are shown in figure 14 to 19. The computer program was written in Pascal language, and is shown in Appendix IV.

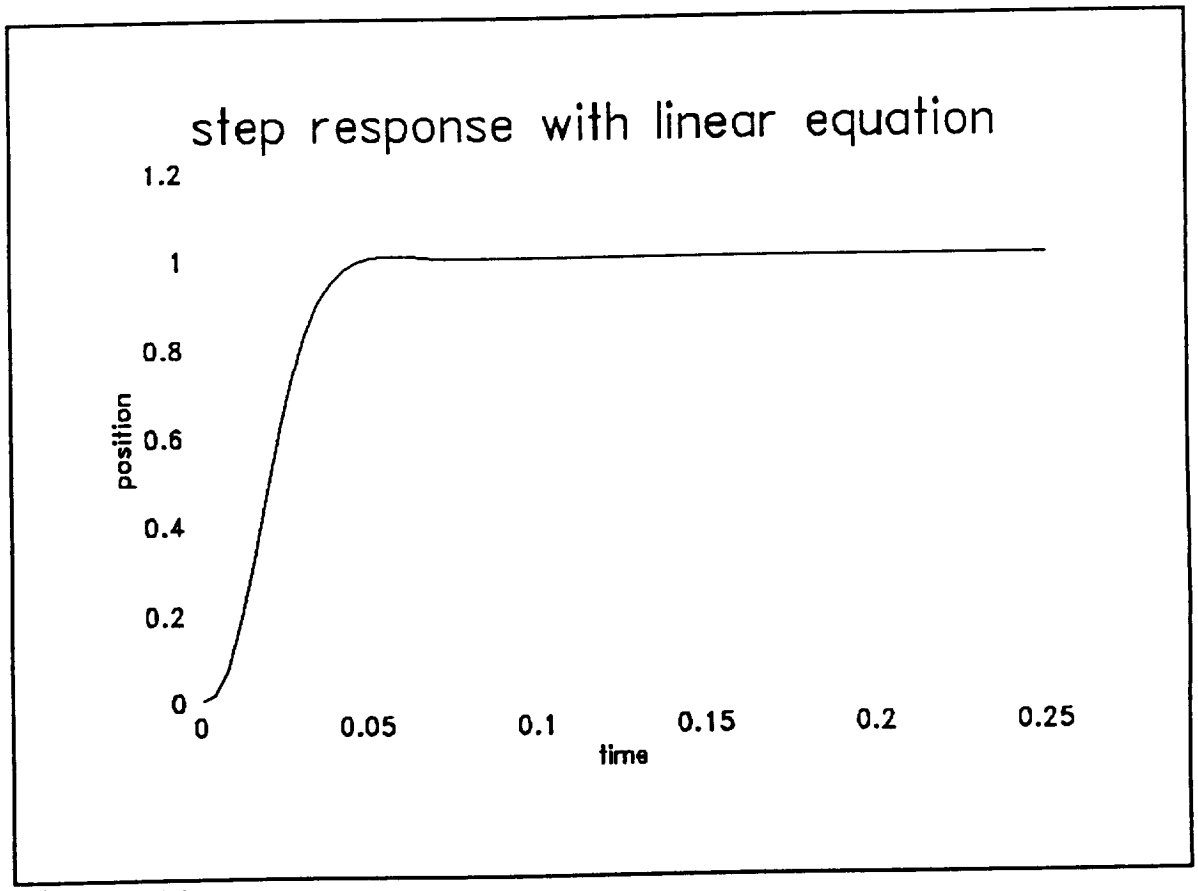


figure 14

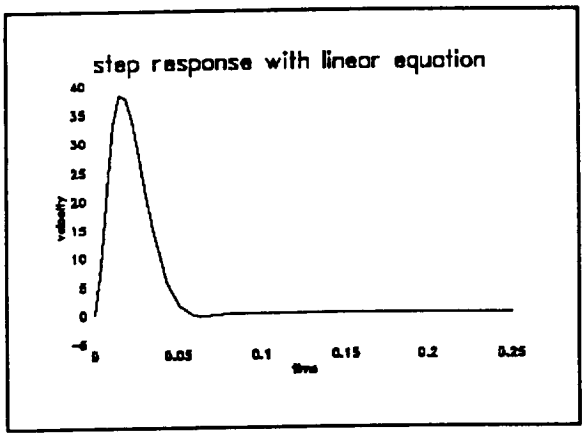


figure 15

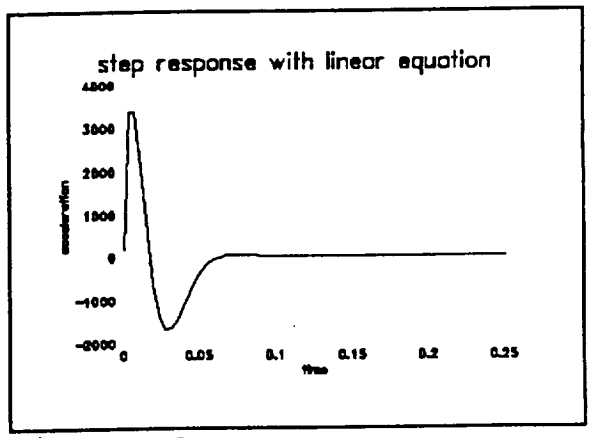


figure 16

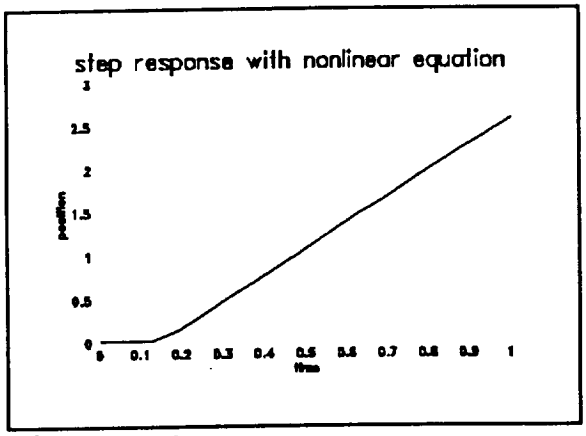


figure 17

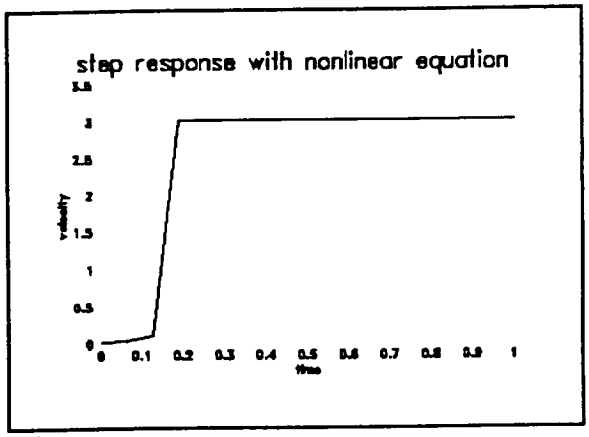


figure 18

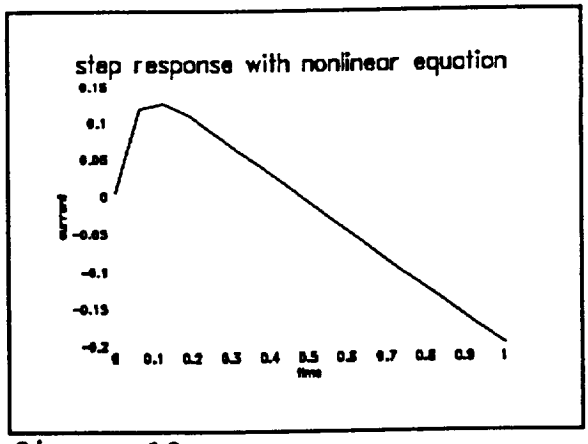


figure 19

V. Discussion of results

The computer simulation showed that the step response increased linearly with time, figure 17. That is, an unstable response occurred.

Consider the plant dynamics equations, recall

$$\ddot{x} = \frac{A\,\mu_o\,N^2\,(\,g_o\,I_o + x\,i\,)\,(\,g_o\,i + I_o\,x\,)}{(\,g_o^2 - x^2)^2}$$

$$As\ for\ \ |\,x\,| < |\,g_o\,|$$

$$\frac{1}{(g_o^2 - x^2)^2} = g_o^{-4}\,\frac{1}{(\,1 - \frac{x^2}{g_o^2})^2}$$

$$= g_o^{-4}\,[\,1 + (\frac{x^2}{g_o^2}) + (\frac{x^2}{g_o^2})^2 + \ldots\,]^2$$

$$= g_o^{-4}\,[\,1 + 2(\frac{x^2}{g_o^2}) + 3(\frac{x^2}{g_o^2})^2 + higher\ terms\,]$$

$$So\ \frac{(g_o\,I_o + x\,i)\ (g_{oi} + I_o\,x)}{(g_o^2 - x^2)^2}$$

$$= g_o^{-4}\,(g_o\,I_o + x\,i)\ (g_o\,i + I_o\,x)$$

$$[\,1 + 2(\frac{x^2}{g_o^2}) + 3(\frac{x^2}{g_o^2})^2 + higher\ terms\,]$$

$$= g_o^{-4}\,(g_o\,I_o^2\,x + g_o^2\,I_o\,i + g_o\,x\,i^2 + I_o\,x^2\,i)$$

$$[\,1 + 2(\frac{x^2}{g_o^2}) + 3(\frac{x^2}{g_o^2})^2 + \ldots]$$

$$= g_o^{-4}\,[(g_o\,I_o^2\,x + g_o^2\,I_o\,i)$$

$$+ (2\,\frac{I_o^2}{g_o}\,x^3 + 3\,I_o\,x^2\,i + g_o\,x\,i^2) + higher\ terms\,]$$

Obviously, the coefficient magnitude of the third terms in the Taylor's series expansion is larger than the coefficient magnitude

of the first term. Thus, at least, we need to include the third terms in the compensator design. However, we cannot use the conventional linear design theory in this situation.

Even thought we include the third terms in the design, the coefficient magnitude of the x^n terms in the series expansion increase without bounded (because $g_o < 1$). So, the Taylor's series expansion does not exist.

VI. Conclusion and recommendation

The plant dynamics equations are nonlinear, and the conventional linearization does not work. We recommend to design the compensator without linearization in the time domain.

As, in general, if we close the loop,

$$\frac{d\vec{x}}{dt} = \vec{f}\ (\ \vec{x}\ ,\ u\ )$$

$$u = g\ (r\ ,\ \vec{x}\ )$$

define PI to be

$$\int_0^{\infty} t\ (x_1 - r)^2\ dt$$

and, minimizing the PI which is subjected to the constraint equation,

$$\frac{d\vec{x}}{dt} = \vec{f}\ (\ \vec{x}\ ,\ g\ (\ r\ ,\ \vec{x}\ )\ )$$

, by the Langrange multiple method. The mathematics is too difficult to carry out, and the analysis is left for interested reader only.

Bibliography

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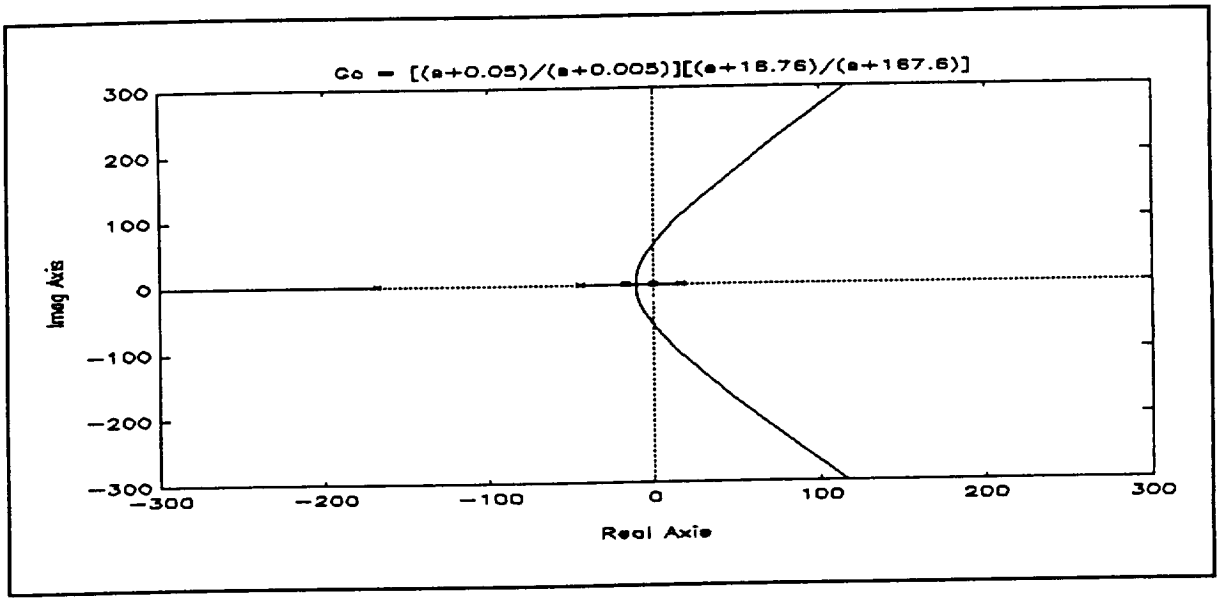
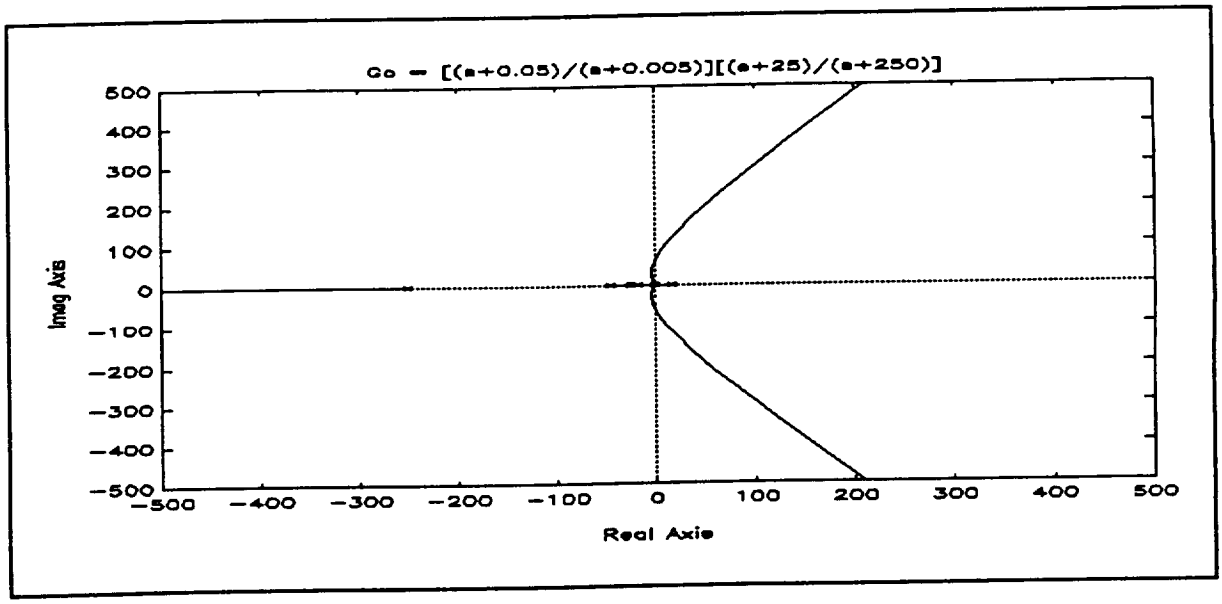
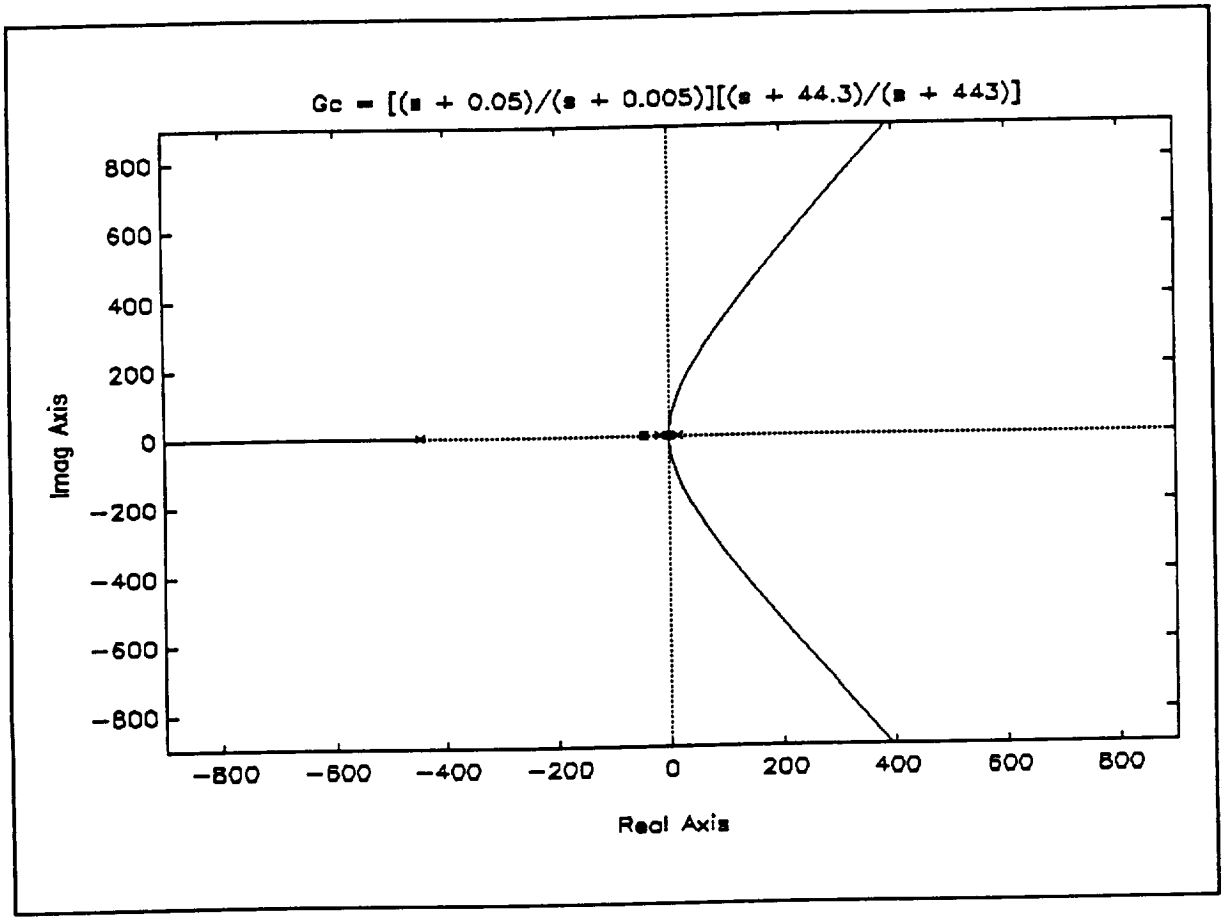
Appendix I

Root locus result of general lead lag cascade compensator

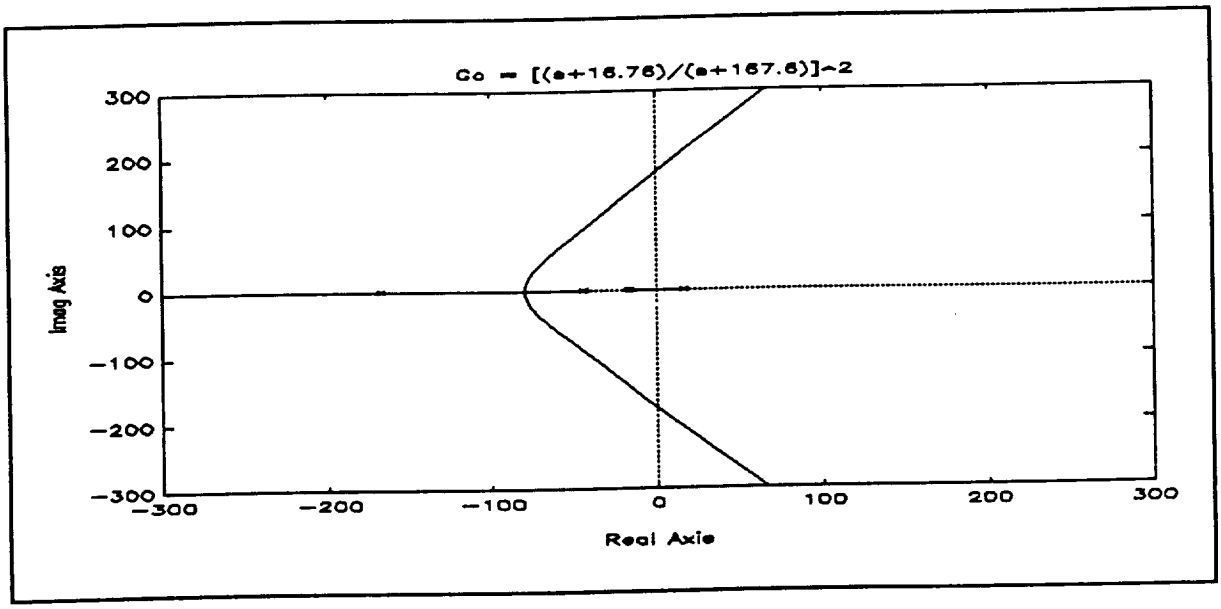




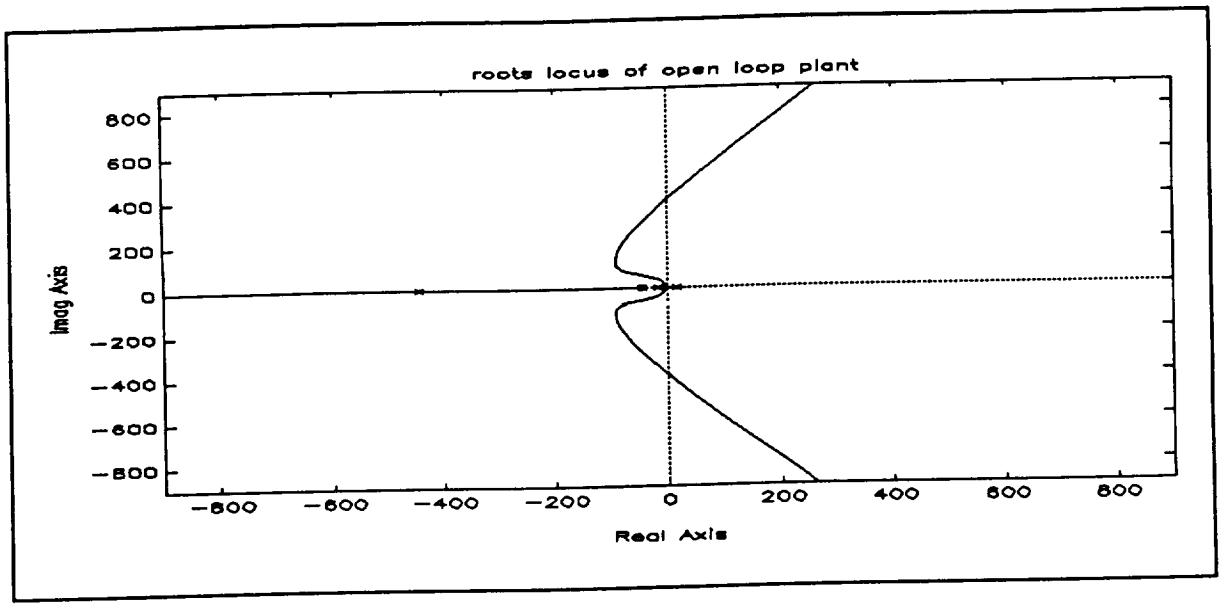


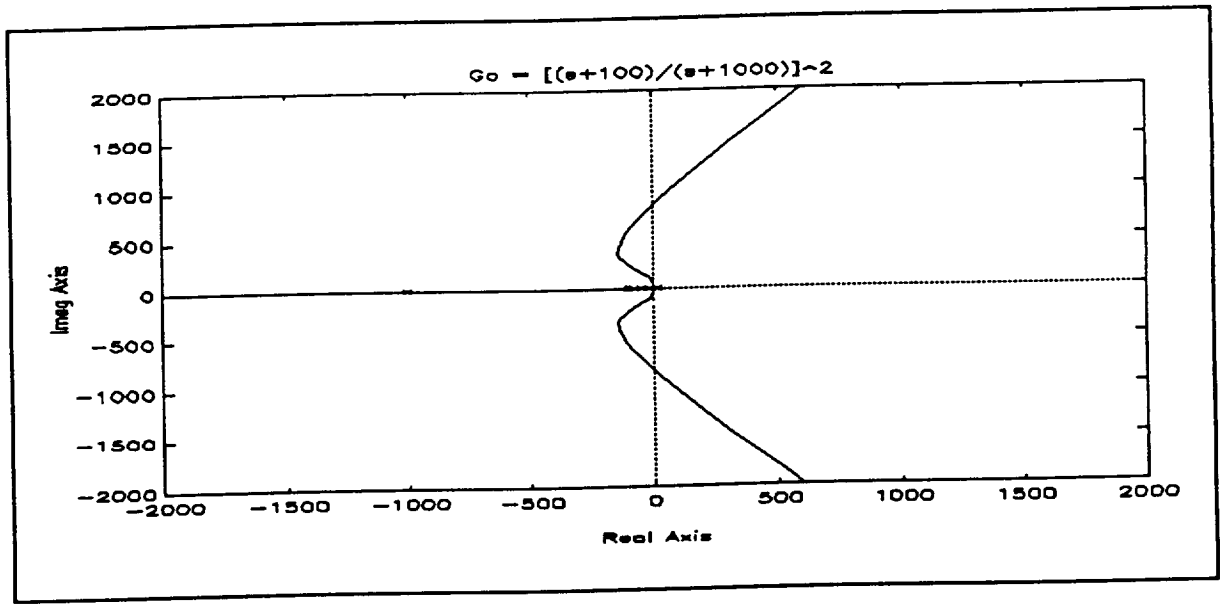
Appendix II

Root locus results of cascade dual phase advance compensator



$$G_c = [(s + 44.3)/(s + 443)]^2$$





Appendix III

State feedback design procedure[10]

Let the open-loop plant transfer function be

$$G_p = \frac{c_0}{s^3 + a_2 s^2 + a_1 s + a_0} = \frac{X}{u}$$

Let, for standard notation,

$$x_1 = y, \quad x_2 = \frac{dy}{dt}, \quad x_3 = \frac{d^2y}{dt^2}$$

$$\therefore \frac{dx_1}{dt} = \frac{dy}{dt} = x_2, \quad \frac{dx_2}{dt} = \frac{d^2y}{dt^2} = x_3, \quad \frac{dx_3}{dt} = \frac{d^3y}{dt^3}$$

and

$$\frac{d\underline{X}}{dt} = \underline{A}\,\underline{X} + \underline{b}\,u$$

$$y = [\,1 \quad 0 \quad 0\,]\,\underline{X} = \underline{c}^T\,\underline{X}$$

Let the control law be

$$u = r - \underline{k}^T\,\underline{X}$$

$$\text{where } \underline{k}^T = [\,k_1 \quad k_2 \quad k_3\,]$$

Consider

$$H(s) = \frac{k^T X(s)}{Y(s)} = \frac{k^T X}{c^T X}$$

$$= \frac{k_1 x_1 + k_2 x_2 + k_3 x_3}{x_1}$$

$$H(s) = \frac{k_1 x_1 + k_2 s x_1 + k_3 s^2 x_1}{x_1}$$

$$= k_1 + s k_2 + s^2 k_3$$

$$\therefore G H = \frac{c_o (k_1 + s k_2 s^2 k_3)}{s^3 + a_2 s^2 + a_1 s + a_0}$$

and the overall transfer function is

$$\frac{Y(s)}{R(s)} = \frac{G}{1 + G H} = \frac{c_0}{(s^3 + a_2 s^2 + a_1 s + a_0) + c_0 (k_3 s^3 + k_2 s + k_1)}$$

$$= \frac{c_0}{s^3 + (a_2 + c_0 k_3) s^2 + (a_1 + c_0 k_2) s + (a_0 + c_0 k_1)}$$

using the final value theorem,

$$\lim_{t \to \infty} y(t) := y_{ss} = \lim_{s \to 0} s\, Y(s)$$

$$\therefore y_{ss}(t) = \lim_{s \to 0} \frac{s\, c_0\, R(s)}{s^3 + (a_2 + c_0 k_3) s^2 + (a_1 + c_0 k_2) + (a_0 + c_0 k_1)}$$

for step input $R(s) = s^{-1}$

$$\therefore \; y_{ss} = \frac{c_0}{a_0 + c_0 k_1} := 1 \; \textit{for zero error}$$

$$ie, \quad k_1 = 1 - \frac{a_0}{c_0}$$

which is fixed. By appropriate selecting the value of k^T, we can implement any desired characteristic equation as we want.

observer design

original plant:

$$\frac{d\underline{X}}{dt} = \underline{A}\,\underline{X} + \underline{b}\,u$$

$$y = \underline{c}^T \underline{X}$$

Let $\hat{\underline{X}}$ be the estimated state vector.

Let

$$\frac{d\hat{X}}{dt} = A\hat{X} + b\,u + L\,(y - \hat{y})$$

$$\hat{y} = c^T \hat{X}$$

where L is the observer matrix

$$L = [\, l_1 \;\; l_2 \;\; l_3 \,]$$

define $e = X - \hat{X}$

$$\rightarrow \frac{de}{dt} = \frac{dX}{dt} - \frac{d\hat{X}}{dt}$$

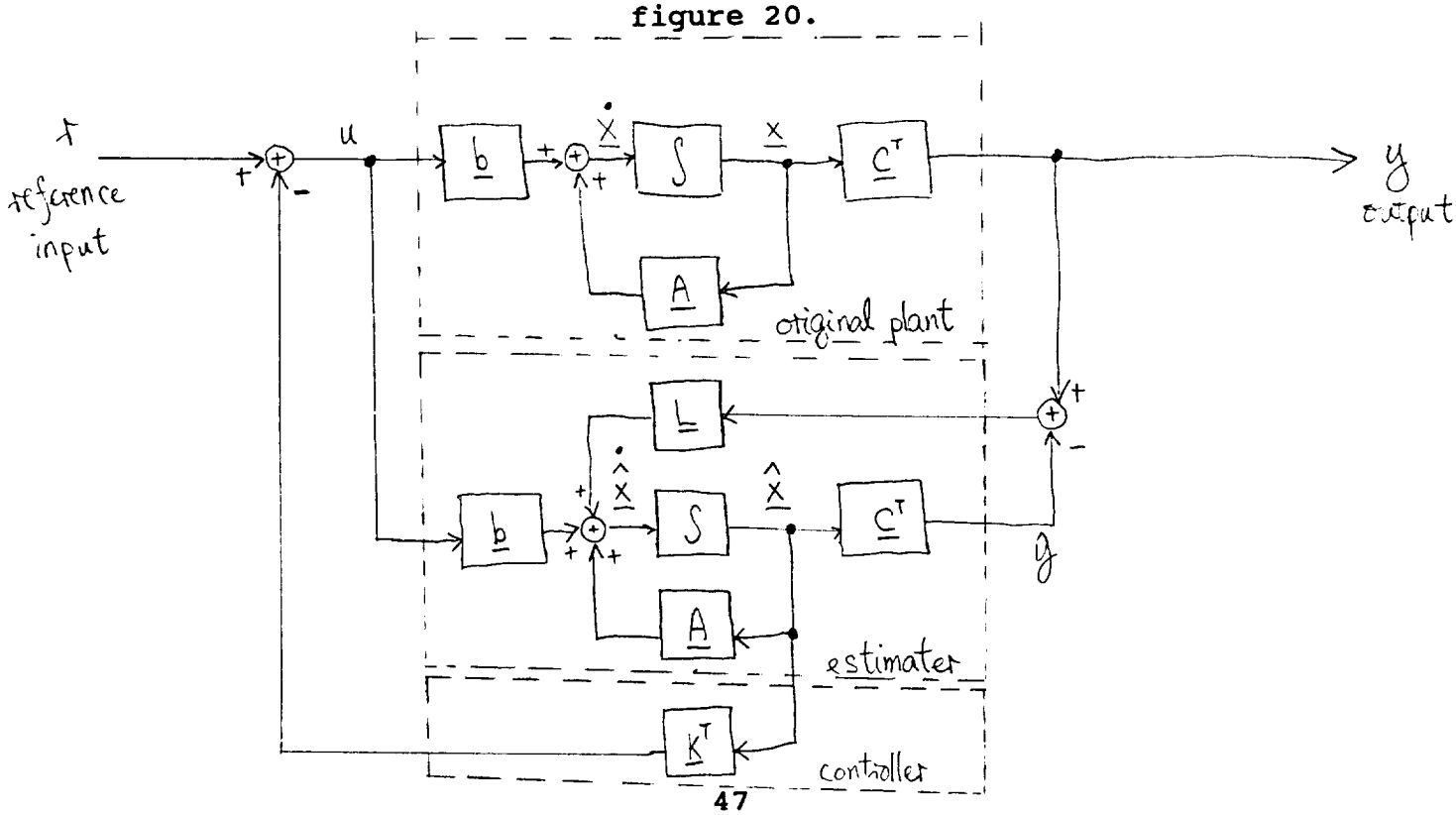
$$= A\,X + b\,u - (\,A\hat{X} + b\,u + L\,(\,c^T X - c^T \hat{X})\,)$$

$$= A(\,X - \hat{X}\,) - L\,c^T(\,X - \hat{X}\,)$$

$$= (\,A - L\,c^T\,)\,e$$

By appropriate selecting the eigenvalues of $\underline{e}$, the error of the estimated state vector will died out very quick. A state diagram is shown in figure 20.

figure 20.



Appendix IV

Computer codes for nonlinear plant dynamics

```
{$N+,E+}
program project(outputo);        {3\21\1993}
  var
    d:char;
    i, j, n, nstep, e, b:integer;
    h, h2, X1max, X4max, t, r, u, dum:real;
    k:array[1..3] of real;
    x,y:array[1..2,1..4] of real;
    f:array[1..4,1..4] of real;
    outputo:text;
  { This is a Runge-Kntta method of order 4. }
  procedure initizing;
    begin
      t:=0;                  {initial time}
      h:=1/1024;             {incremental time step}
      n:=4;                  {number of equations}
      for i:=1 to 2 do       {initial value}
       for j:=1 to 4 do
        begin
        x[i,j]:=0;
        y[i,j]:=0;
        end;
      for i:=1 to 4 do
       for j:=1 to 4 do
```

```
      f[i,j]:=0;
    nstep:=4096;       {number of step}
    r:=1;
    k[1]:=1.01;
    k[2]:=0.021561;
    k[3]:=0.0001505;
  end;

 procedure get_value_f;
  begin
   f[e,1] := x[b,2];
   f[e,2] := (382.3996e-6)*(4.3434e-3 +
             x[b,1]*x[b,3])*(0.00762*x[b,3] +
             0.57*x[b,1])/(sqr(sqr(0.00762) - sqr(x[b,1])));
     dum  := (y[b,2] - x[b,2])/h; {estimated the second derivate}
       u   :=   r  -  (k[1]*x[b,1]  +  k[2]*x[b,2]  +  k[3]*dum);
{control law}
    f[e,3] := -44.2995*x[b,3] + 5.5374*u;
    f[e,4] := 1;        { x[4]= time }
  end;

 procedure RK4SYS;
    begin
       h2:=0.5*h;
       for j:=0 to nstep do
          begin
```

```
for e:=1 to 2 do
begin
  b:=e;
  get_value_f;
  for i:=1 to n do
    x[2,i]:=x[1,i] + h2*f[e,i];
end;
e:=3;
get_value_f;  {get f3}
for i:=1 to n do
  x[2,i]:=x[1,i] +h*f[e,i];
e:=4;
get_value_f;  {get f4}
y[1,2]:=x[1,2];
y[2,2]:=x[2,2];
for i:=1 to n-1 do {compute next x(t+h)}

x[1,i]:=x[1,i]+h*(f[1,i]+2*(f[2,i]+f[3,i])+f[4,i])/6;
x[1,4]:= t+j*h; {advance solution}
if j mod 64 =0 then  {write the result}
begin
  for i:=1 to n do
  write(outputo,x[1,i],',');
  writeln(outputo);
  writeln(x[1,1],',',x[1,4]);
end{if loop}
```

```
    end;{for j loop}
end;{RK4SYS}

begin {main}
   assign(outputo,'a:\p1.dat');
   rewrite(outputo);
   writeln(outputo);
   initizing;
   repeat
   RK4SYS;
   write('want change Y/N ?');
   readln(d);
   if d='y' then
    begin
     writeln('k1=',k[1],' k2=',k[2],' k3=',k[3]);
     write(' enter k1,k2,k3');
     readln(k[1],k[2],k[3]);
    end;
   until (d<>'y');
   writeln(outputo, ' Job completed. ');
   writeln('  Job completed.');
   close(outputo);
end.
```