IN-18
28222
P-10

NASA Technical Memorandum 109150

Three-Axis Stabilization of Spacecraft Using Parameter-Independent Nonlinear Quaternion Feedback

Suresh M. Joshi
Langley Research Center, Hampton, Virginia

Atul G. Kelkar
NRC Fellow, Langley Research Center, Hampton, Virginia

September 1994

National Aeronautics and
Space Administration
Langley Research Center
Hampton, Virginia 23681-0001

Summary

This paper considers the problem of three-axis attitude stabilization of rigid spacecraft. A nonlinear control law which uses the feedback of the unit quaternion and the measured angular velocities is proposed and is shown to provide global asymptotic stability. The control law does not require the knowledge of the system parameters, and is therefore robust to modeling errors. The significance of the control law is that it can be used for large-angle maneuvers with guaranteed stability.

Introduction

Attitude control of a free-flying spacecraft has long been known as an important problem, and has been the subject of many papers since the fifties and sixties [1,2,3]. It is also a unique problem in dynamics because of the fact that the finite rotation of a rigid body does not obey the laws of vector addition (in particular, commutativity) and, as a result, the angular velocity of the body can not be integrated to give the attitude of the body. The most widely used method of defining the rotation of a body between two different orientations is an Euler angle description. A 3×3 direction cosine matrix (of Euler rotations) is used to describe the orientation of the body (achieved by three successive rotations) with respect to some fixed frame of reference. However, there is an inherent geometric singularity in the Euler representation. This problem can be avoided by using a 4-parameter description of the orientation [2,3,4], known as 'quaternions', which can be used to describe all possible orientations. The quaternion approach uses Euler's theorem which states that any rotation of a rigid body can be described by a single rotation about a fixed axis. The advantage of using quaternions is that successive rotations result in successive quaternion multiplications which are commutative. Some early results on the use of quaternion feedback for attitude error representation and automatic control of the attitude can be found in [1]. Quaternions were used for the simulations of the rotational motion of rigid bodies as early as the 1950's [5]. The use of quaternion feedback for controlling robotic manipulators can be found in [6,7], and for spacecraft control can be found in [8-11].

Various linear and nonlinear quaternion-based control laws have been recently proposed [10, 11] for the attitude control of a single-body rigid spacecraft. However, the control laws proposed in [10] require the knowledge of the system's moments of inertia and also constrain the choice of the gain matrices. In [11], both model-dependent and model-independent control laws were presented; however, the control laws used scalar gains.

In this note, a model-independent, nonlinear control law is presented which uses quaternion feedback and symmetric and positive definite gain matrices. Global asymptotic stability of the proposed control law is shown by using Lyapunov analysis. The Lyapunov function used here for proving asymptotic stability does not need a cross term similar to the one used in [11], and the proof is made much simpler.

Quaternion Feedback Control

The rotational equations of motion of a rigid spacecraft are given by:

$$J\dot{\omega} + \omega \times (J\omega) = u \tag{1}$$

where J is the 3×3 inertia matrix; ω is the 3×1 angular velocity vector; and u is the 3×1 vector of actuator torques. The objective of the control system is to bring the spacecraft to the desired attitude (orientation) starting from any initial condition.

The orientation of a free-floating body can be minimally represented by a 3-dimensional orientation vector. However, as stated previously, this representation is not unique. One minimal representation that is commonly used to represent the attitude is Euler angles. The 3×1 Euler angle vector η is given by : $E(\eta)\dot{\eta} = \omega$, where $E(\eta)$ is a 3×3 transformation matrix. $E(\eta)$ becomes singular for certain values of η; however, it should be noted that the limitations imposed on the allowable orientations due to this singularity are purely mathematical in nature and do not represent physical restrictions. The problem of singularity in a 3-parameter representation of attitude has been studied in detail in the literature [2,3,8,10,11]. An effective way of overcoming the singularity problem is to use the quaternion formulation (see [1]- [3]). The unit quaternion α is defined as follows.

$$\alpha = \{\overline{\alpha}^T, \alpha_4\}^T, \qquad \overline{\alpha} = \begin{bmatrix} \hat{\alpha}_1 \\ \hat{\alpha}_2 \\ \hat{\alpha}_3 \end{bmatrix} sin(\frac{\phi}{2}), \qquad \alpha_4 = cos(\frac{\phi}{2}) \tag{2}$$

$\hat{\alpha} = (\hat{\alpha}_1, \hat{\alpha}_2, \hat{\alpha}_3)^T$ is the unit vector along the eigen-axis of rotation and ϕ is the magnitude of rotation. The quaternion is also subjected to the norm constraint:

$$\overline{\alpha}^T\overline{\alpha} + \alpha_4^2 = 1 \tag{3}$$

It can be also shown [12] that the quaternion obeys the following kinematic differential equations.

$$\dot{\overline{\alpha}} = \frac{1}{2}(\omega \times \overline{\alpha} + \alpha_4 \omega) \tag{4}$$

$$\dot{\alpha}_4 = -\frac{1}{2}\omega^T\overline{\alpha} \tag{5}$$

The quaternion can be computed [12] using Euler angle measurements (Eq. 4). The equilibrium solutions of the open-loop system, given by equations (1), (4), and (5), can be obtained by setting all the derivatives to zero. That is,

$$\omega \times (J\omega) = 0 \tag{6}$$

$$\omega \times \overline{\alpha} + \alpha_4 \omega = 0 \tag{7}$$

$$\omega^T\overline{\alpha} = 0 \tag{8}$$

Taking the dot product of ω with both sides of Eq. (7),

$$\alpha_4(\omega \cdot \omega) = 0$$

That is, either $\alpha_4 = 0$, or $\omega = 0$, or both are 0. If $\alpha_4 \neq 0$, then $\omega = 0$. If $\alpha_4 = 0$, then from (7) and (8), $\omega \times \overline{\alpha} = 0$, and $\omega \cdot \overline{\alpha} = 0$, i.e., $\omega = 0$, or $\overline{\alpha} = 0$, or both are zero. However, from (3), $\alpha_4 = 0 \Rightarrow \overline{\alpha} \neq 0$; therefore, $\omega = 0$ when the system is in equilibrium. The system has multiple equilibrium solutions: $(\overline{\alpha}_{ss}{}^T, \alpha_{4ss})$, where, the subscript '$ss$' denotes the constant steady-state value.

Consider the control law u, given by:

$$u = -\frac{1}{2}[(\tilde{\overline{\alpha}} + \alpha_4 I)G_p + \gamma(1-\alpha_4)I]\overline{\alpha} - G_r\omega \tag{9}$$

where G_p and G_r are symmetric positive definite (3×3) matrices; γ is a positive scalar; and $\tilde{\overline{\alpha}}$ represents the 3×3 cross product matrix of the vector $\overline{\alpha}$. Equation (9) represents a nonlinear control law. The following result gives the closed-loop equilibrium solutions.

Lemma 1. Suppose G_p is symmetric and positive definite and $0 < \lambda_M(G_p) < 2\gamma$, where $\lambda_M(\cdot)$ denotes the largest eigenvalue. Then the closed-loop system given by (1), (4), (5) and (9) has exactly two equlibrium solutions: $[\overline{\alpha} = \omega = 0, \alpha_4 = 1]$ and $[\overline{\alpha} = \omega = 0, \alpha_4 = -1]$.

Proof.- The closed-loop system is in equilibrium when the derivatives in equations (1), (4), and (5) are zero. Proceeding as in the open-loop case, the closed-loop equilibrium solution is given by: $\omega = 0, \overline{\alpha} = \overline{\alpha}_{ss}, \alpha_4 = \alpha_{4ss}$. From equation (1), $\omega = 0 \Rightarrow u = 0$. From equation (9), we have

$$[(\tilde{\overline{\alpha}} + \alpha_4 I)G_p + \gamma(1-\alpha_4)I]\overline{\alpha} = 0 \tag{10}$$

Pre-multiplying the above equation by $\overline{\alpha}^T$ and noting that the first term vanishes, we have the following:

$$u = 0 \Rightarrow \overline{\alpha}^T M \overline{\alpha} = 0$$

where

$$M = \alpha_4 G_p + \gamma(1-\alpha_4)I \tag{11}$$

The eigenvalues of M are given by: $\lambda_i(M) = \alpha_4\lambda_i(G_p) + \gamma(1-\alpha_4) = \alpha_4(\lambda_i(G_p) - \gamma) + \gamma$. M is singular when $\lambda_i(M) = 0$, i.e., when $\alpha_4 = \frac{-\gamma}{\lambda_i(G_p)-\gamma}$. There are three different subcases that need to be examined: (a) $0 < \lambda_i(G_p) < \gamma$, (b) $\lambda_i(G_p) = \gamma$, and (c) $\gamma < \lambda_i(G_p) < 2\gamma$. In subcases (a) and (c), $\lambda_i(M) = 0$ only if $|\alpha_4| > 1$, which is not feasible, since $-1 \leq \alpha_4 \leq 1$. That means, for subcases (a) and (c), $\lambda_i(M) \neq 0$ for any feasible values of α_4. In subcase (b), i.e., when $\lambda_i(G_p) = \gamma$, $\lambda_i(M) = \gamma > 0$. Therefore, M is nonsingular, and $\overline{\alpha} = 0$. Then, from equation (3), we have: $\alpha_4 = 1$ or -1. (Note that, if $\lambda_i(G_p) \geq 2\gamma$ then there are feasible values of α_4 for which $\lambda_i(M) = 0$). ∎

(The symbol ■ denotes the end of the proof.)

From Lemma 1, there appear to be two closed-loop equilibrium points corresponding to $\alpha_4 = 1$ and $\alpha_4 = -1$ (all other state variables being zero). However, from equation (2), $\alpha_4 = 1 \Rightarrow \phi = 0$, and $\alpha_4 = -1 \Rightarrow \phi = 2\pi$ (or more generally $2n\pi$), i.e., there is only one equilibrium point in the physical space. We shall define the desired equilibrium state as: $\overline{\alpha} = \omega = 0, \alpha_4 = 1$. In order to make the origin of the state space the desired state, define $\beta = (\alpha_4 - 1)$. Equations (4) and (5) can then be rewritten as:

$$\dot{\overline{\alpha}} = \frac{1}{2}(\omega \times \overline{\alpha} + (\beta + 1)\omega) \tag{12}$$

$$\dot{\beta} = -\frac{1}{2}\omega^T \overline{\alpha} \tag{13}$$

The system represented by equations (1), (12) and (13) can be expressed in the state-space form as follows:

$$\dot{x} = f(x, u) \tag{14}$$

where $x = (\overline{\alpha}^T, \beta, \omega^T)^T$. Note that the dimension of x is 7, which is one more than the dimension of the system. However, one constraint (Eq. 3) is now present. It can be easily verified from (4) and (5) that the constraint (3) is satisfied for all $t > 0$ if it is satisfied at $t = 0$.

If the objective of the control law is to transfer the state of the system from one orientation (equilibrium) position to another orientation (i.e., a rest-to-rest maneuver), then without loss of generality, the target orientation can be defined to be zero. The initial orientation, given by $(\overline{\alpha}(0), \beta(0))$ can always be defined in such a way that $-1 \leq \beta \leq 0$ (i.e., $0 \leq \alpha_4(0) \leq 1$), corresponding to $|\phi| \leq \pi$.

Consider the control law given by:

$$u = -\frac{1}{2}[(\tilde{\overline{\alpha}} + (\beta + 1)I)G_p - \gamma\beta I]\alpha - G_r\omega \tag{15}$$

The control law given by (15) was stated in [11]; however, conditions for the existence of the closed-loop equilibrium solutions were not investigated, and stability proof was not given.

The following theorem establishes the global asymptotic stability of the physical equilibrium state (the origin of the state-space) of the system.

Theorem 1. Suppose G_p and G_r are symmetric and positive definite, and $0 < \lambda_M(G_p) < 2\gamma$. Then, the closed-loop system given by equations (1), (12), (13), and (15) is globally asymptotically stable (g.a.s.).

Proof. Consider the candidate Lyapunov function

$$V = \omega^T J\omega + \overline{\alpha}^T G_p \overline{\alpha} + \gamma\beta^2 \tag{16}$$

V is clearly positive definite and radially unbounded with respect to the state vector $x = \{\overline{\alpha}^T, \beta, \omega^T\}^T$. Taking the time derivative of V, we have:

$$\dot{V} = 2\omega^T[-\omega \times (J\omega) + u] + 2\overline{\alpha}^T G_p(\omega \times \alpha + (\beta + 1)\omega) - \gamma\beta\omega^T\overline{\alpha} \tag{17}$$

Noting that the first term in $\dot{V}$ is zero, and substituting for u from (15), after simplification, we get: $\dot{V} = -\omega^T G_r \omega$, i.e., $\dot{V}$ is negative semidefinite. $\dot{V} = 0$ only when $\omega = 0$. Following the same procedure as Lemma 1, it can be shown that $\dot{V} = 0$ only at the two equilibrium points, $\overline{\alpha} = \omega = 0, \beta = 0$ (corresponding to $\alpha_4 = 1$) and $\overline{\alpha} = \omega = 0, \beta = -2$ (corresponding to $\alpha_4 = -1$).

Consistent with the previous discussion, these values correspond to two equilibrium points representing the same physical equilibrium state. It can be easily verified, from equation (16), that any small perturbation ϵ in β from the equilibrium point corresponding to $\beta = -2$ will cause a decrease in the value of V (ϵ has to be > 0 because $-2 \leq \beta \leq 0$). Thus, in the mathematical sense, $\beta = -2$ corresponds to an isolated equilibrium point such that $\dot{V} = 0$ at that point, and $\dot{V} < 0$ in a neighborhood of that point, i.e., $\beta = -2$ is a 'repeller' and not an 'attractor'. It has been already shown that $\dot{V}$ is negative everywhere in the feasible state space except at the two equilibrium points. That is, if the system's initial condition lies anywhere in the state space except at the equilibrium point corresponding to $\beta = -2$, then the system will asymptotically approach the origin ($x = 0$); and if the system is at the equilibrium point corresponding to $\beta = -2$ at $t = 0$ then it will stay there for all $t > 0$.

However, this is the same equilibrium point in the physical space; hence, it can be concluded by LaSalle's theorem that the system is globally asymptotically stable. ■

Concluding Remarks

The problem of three-axis attitude stabilization of a spacecraft was considered. A nonlinear quaternion-based feedback control law was given, and was shown to provide global asymptotic stability. The control law does not depend on the knowledge of the system parameters (i.e., moments of inertia), and is therefore robust to modelling errors and parametric uncertainties.

References

1. Mortesen, R. E.: A Globally Stable Linear Attitude Regulator, International Journal of Control, Vol. 8, No. 3, 1968, pp. 297-302.

2. Ickes, B. P.: A New Method for Performing Control System Attitude Computation Using Quaternions, AIAA Journal, Vol. 8, pp. 13-17, January 1970.

3. Margulies, G.: On Real Four-Parameter Representations of Satellite Attitude Motions, Mathematical Analysis Department, Philco Western Lab., Palo Alto, California, Report 52, September 1963.

4. Kane, T. R.: Solution of Kinematical Differential Equations for a Rigid Body, Journal of Applied Mechanics, pp. 109-113, March 1973.

5. Robinson, A. C.: On the Use of Quaternions in Simulation of a Rigid Body Motion, December 1958, U.S. Air Force, Wright Air Development Center, Wright-Patterson Air Force Base, Ohio.

6. Gu, Y. L.: Analysis of Orientation Representations by Lie Algebra in Robotics, Proc. 1988 IEEE Int. Conf. Robotics and Automation, Philadelphia, Pennsylvania, April 1988, pp. 874-879.

7. Wampler, C. W. III: Manipulator Inverse Kinematics Solutions Based on Vector Formulation and Damped Least-Squares Methods, IEEE Trans. Systems, Man, Cybernetics, Vol. 15, No. 1, 1986, pp. 93-101.

8. Meyer, G.: Design and Global Analysis of Spacecraft Attitude Control Systems, NASA Tech. Rep. R-361, Mar. 1971.

9. Wie, B. and Barba, P. M.: Quaternion Feedback for Spacecraft Large Angle Maneuvers, Journal of Guidance and Control, Vol. 8, No. 3, 1985, pp. 360-365.

10. Wie, B., Weiss, H., and Arapostathis, A.: Quaternion Feedback for Spacecraft Eigenaxis Rotations, Journal of Guidance and Control, Vol. 12, No. 3, 1989, pp. 375-380.

11. Wen, J. T. and Kreutz-Delgado, K.: The Attitude Control Problem, IEEE Transactions on Automatic Control, Vol. 36, No. 10, pp. 1148-1163, 1991.

12. Haug, E. G.: *Computer-Aided Kinematics and Dynamics of Mechanical Systems*, Allyn and Bacon Series in Engineering, 1989.

REPORT DOCUMENTATION PAGE			Form Approved OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*

2. REPORT DATE
September 1994

3. REPORT TYPE AND DATES COVERED
Technical Memorandum

4. TITLE AND SUBTITLE
Three-Axis Stabilization of Spacecraft Using Parameter-Independent Nonlinear Quaternion Feedback

5. FUNDING NUMBERS
WU 233-01-01-05

6. AUTHOR(S)
Suresh M. Joshi and Atul G. Kelkar

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
NASA Langley Research Center
Hampton, VA 23681-0001

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)
National Aeronautics and Space Administration
Washington, DC 20546-0001

10. SPONSORING / MONITORING AGENCY REPORT NUMBER
NASA TM-109150

11. SUPPLEMENTARY NOTES
A. G. Kelkar (NRC Fellow, NASA LaRC)

12a. DISTRIBUTION / AVAILABILITY STATEMENT
Unclassified - Unlimited

Subject Category 18

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

This paper considers the problem of three-axis attitude stabilization of rigid spacecraft. A nonlinear control law which uses the feedback of the unit quaternion and the measured angular velocities is proposed and is shown to provide global asymptotic stability. The control law does not require the knowledge of the system parameters, and is therefore robust to modeling errors. The signficance of the control law is that it can be used for large-angle maneuvers with guaranteed stability.

14. SUBJECT TERMS
Nonlinear Control, Attitude Control, Quaternion Feedback Control

15. NUMBER OF PAGES
9

16. PRICE CODE
A02

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified		