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# EFFECT OF DISPERSION FORCES ON SQUEEZING WITH RYDBERG ATOMS

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**Abstract**

We report exact results concerning the effect of dipole-dipole interaction (dispersion forces) on dynamic and steady-state characteristics of squeezing in the emitted fluorescent field from two identical coherently driven two-level atoms. The atomic system is subjected to three different damping baths in particular the normal vacuum, a broad band thermal field and a broad band squeezed vacuum. The atomic model is the Dicke model, hence possible experiments are most likely to agree with theory when performed on systems of Rydberg atoms making microwave transitions. The presence of dipole-dipole interaction can enhance squeezing for realisable values of the various parameters involved.

# 1 Introduction

The fundamental importance of squeezed light and its interaction with matter as well as potential applications have become the focus of considerable attention in the last few years both theoretically and experimentally. Indeed, the coupling of a two-level atomic system to squeezed light for example can lead to interesting new phenomena as reported by a number of authors [1,2,3]. In this paper we consider the possibility of generating squeezing in the emitted fluorescent field from two identical coherently driven two-level atoms with dipole-dipole interaction (dispersion forces). The atomic system is subjected to three different damping baths in particular, the normal vacuum, a broad band squeezed vacuum and a broad band thermal bath. We report exact results concerning the time evolution as well as steady-state emitted field quadratures which may lead to squeezing. It is shown that squeezing may be enhanced for certain realisable values of the system parameters.

The dipole-dipole coupling is expected to play a significant role especially in the high atomic density limit [4]. It is especially important because it generally breaks the permutation symmetry of the atom-field coupling [5] which is the basic assumption of the collective spontaneous emission process (superradiant) involving $N_A$-atom system. As a consequence, the $S^2$ conservation (where $S$ is the total 'spin' of $N_A$ two-level atoms) which appear inherently in the point like Dicke model [6] is affected significantly. Since the atomic model is the coherently driven two-atom Dicke model, possible experiments are most likely to agree with theory when performed on systems of Rydberg atoms making microwave transitions - see e.g. [7] and references therein.

## 2 Description of the model

Our starting point is the master equation for the reduced atomic density operator in a frame rotating at $\omega_L$, the laser frequency - see equation (5) of ref. [8]:

$$\begin{aligned}
\frac{\partial \rho}{\partial t} &= -i\Delta[S^z, \rho] + i\Omega[S^+ + S^-, \rho] - i\alpha_+[S^+S^-, \rho] \\
&+\frac{\gamma}{2}(N+1)(2S^-\rho S^+ - S^+S^-\rho - \rho S^+S^-) \\
&+\frac{\gamma N}{2}(2S^+\rho S^- - S^-S^+\rho - \rho S^-S^+) \\
&-\frac{\gamma|M|\exp(-i\Phi)}{2}(2S^+\rho S^+ - S^+S^+\rho - \rho S^+S^+) \\
&-\frac{\gamma|M|\exp(+i\Phi)}{2}(2S^-\rho S^- - S^-S^-\rho - \rho S^-S^-)
\end{aligned}$$

where $S^{\pm}$, $S^z$ are the usual collective atomic operators; $2\Omega$ is the Rabi frequency characterizing the strength of the driving field, $\gamma$ is the single atom Einstein $A$ coefficient, $\alpha_+$ is the static dipole-dipole interaction potential such that

$$\alpha_+ = \Omega_{ij} = \Omega_{ji} \simeq \frac{3\gamma}{2(kr_{ij})^3}[1 - 3(\hat{\mu}\cdot\hat{r}_{ij})^2],$$

and $\Delta = \omega_A - \omega_L - \alpha_+$ ($\omega_A$ is the atomic transition frequency).

Note that $\Delta = -\alpha_+$ represents the on-resonant case. $N$ and $M$ are parameters of the squeezed vacuum describing squeezing with $|M|^2 = N(N+1)$ holding for minimum uncertainty squeezed state. $\Phi$ is the relative phase between the squeezed vacuum and the laser. In what follows we set $|M|^2 = N(N+1)$. Note that if $N = 0$, then the atoms are damped by the normal vacuum whilst the case $N \neq 0$, $M = 0$ corresponds to the atoms damped by a broad band thermal field such that $N \to \bar{n}$ which is the mean occupation number of the resonant field mode. The master equation is solved using the fourth order Runge-Kutta method.

## 3 Squeezing in resonance fluorescence

We now discuss the possibility of generating squeezing in the emitted fluorescent radiation field and the effect of $\alpha_+$ on it. The "in phase" and "phase quadrature" components of the emitted fluorescence field denoted by $F_x$ and $F_y$ are respectively

$$\begin{aligned}
F_x &= (\Delta S^x)^2 - \frac{1}{2}|\langle S^z\rangle| \\
F_y &= (\Delta S^y)^2 - \frac{1}{2}|\langle S^z\rangle|
\end{aligned}$$

with

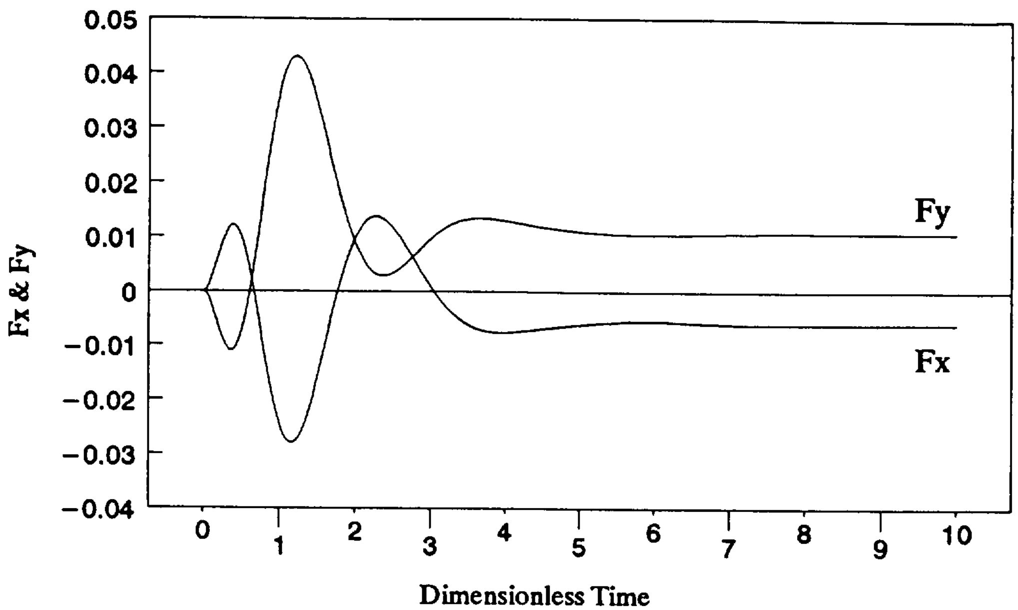


Fig.1 Fx and Fy against $\tau$ for atoms damped by a normal vacuum ; $\frac{\Omega}{\gamma}=0.5$ , $\frac{\alpha_+}{\gamma}=1.0$ and $\frac{\Delta}{\gamma}=0.0$

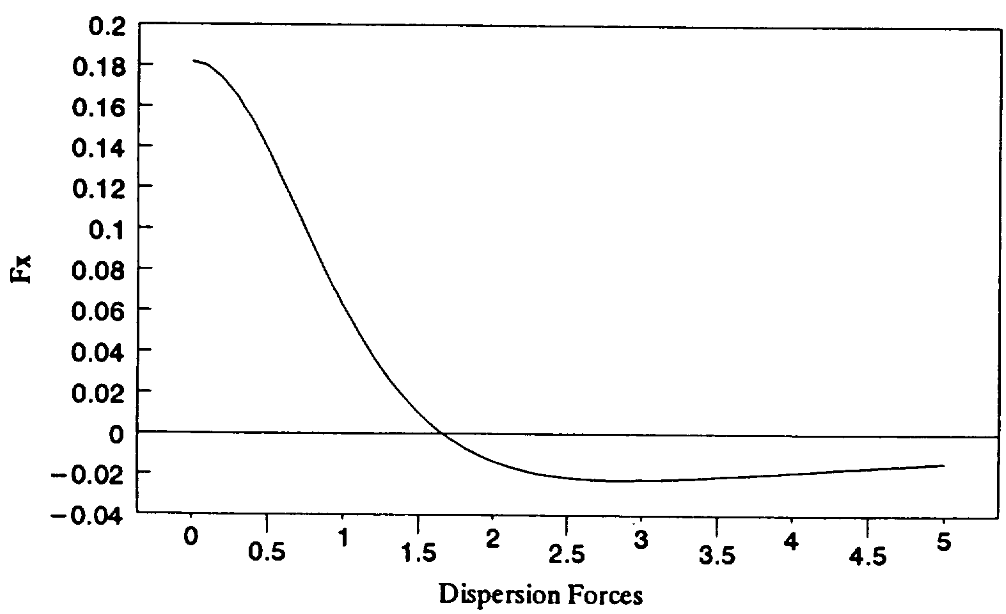


Fig.2 Steady−state Fx versus $\frac{\alpha_+}{\gamma}$ for atoms damped by a normal vacuum ; $\frac{\Omega}{\gamma}=0.5$ and $\frac{\Delta}{\gamma}=-\frac{\alpha_+}{\gamma}$

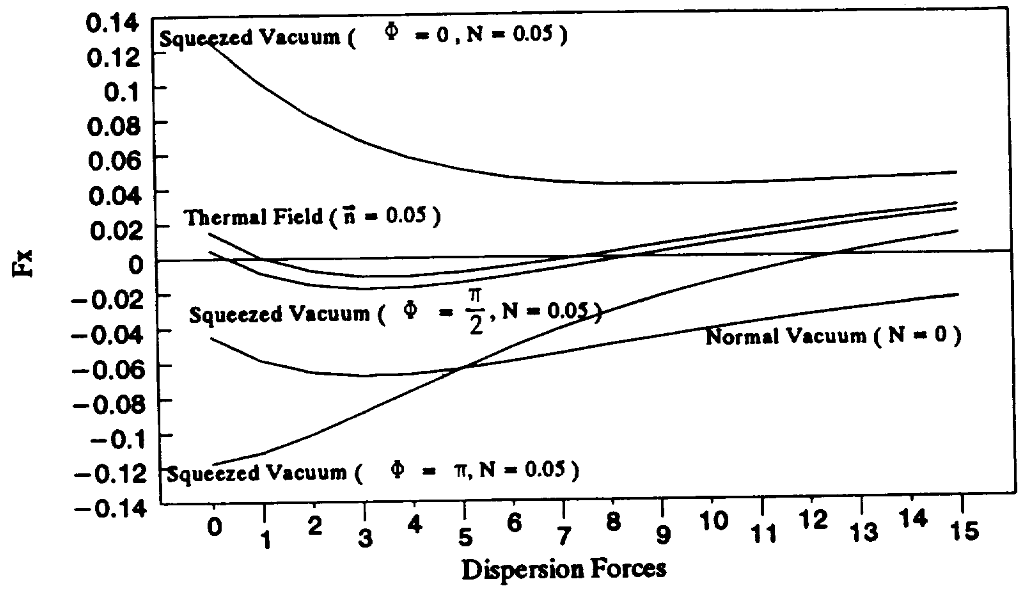


Fig.3 Steady−state Fx versus $\frac{\alpha_+}{\gamma}$ for atoms damped either by a normal vacuum, a broad band thermal field or a broad band squeezed vacuum ; $\frac{\Omega}{\gamma}$ =10 and $\frac{\Delta}{\gamma}$ =15

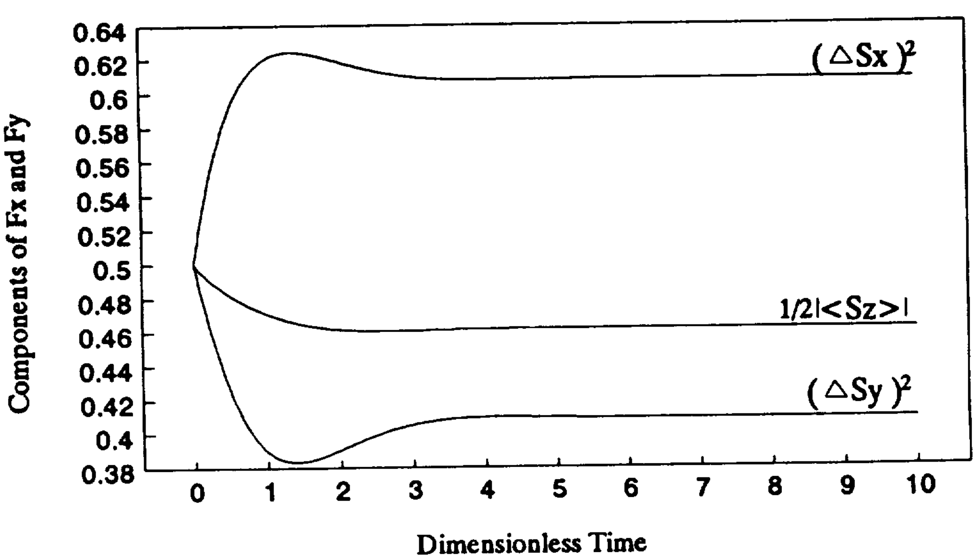


Fig.4 Components of Fx and Fy as functions of $\tau$ for atoms damped by a broad band squeezed vacuum ; $\frac{\Omega}{\gamma}$ =0.1 , $\frac{\alpha_+}{\gamma}$ =0.5 , $\frac{\Delta}{\gamma}$ =0 , N=0.05 and Φ =0

$$
\begin{aligned}
(\Delta S^x)^2 &= \frac{1}{4}\{\langle S^+S^-\rangle + \langle S^-S^+\rangle - \langle S^+\rangle^2 - \langle S^-\rangle^2 - 2\langle S^+\rangle\langle S^-\rangle + \langle S^+S^+\rangle + \langle S^-S^-\rangle\} \\
(\Delta S^y)^2 &= \frac{1}{4}\{\langle S^+S^-\rangle + \langle S^-S^+\rangle + \langle S^+\rangle^2 + \langle S^-\rangle^2 - 2\langle S^+\rangle\langle S^-\rangle - \langle S^+S^+\rangle - \langle S^-S^-\rangle\}
\end{aligned}
$$

as there is a linear functional relation between the atomic operators and the field operators [9]. Squeezing occurs whenever $F_x$ or $F_y < 0$.

Fig. 1 shows the time evolution of $F_x$ and $F_y$ for the atomic system damped by the normal vacuum ($N = 0$) and driven by an off-resonant weak laser with weak dipole-dipole interaction: $\frac{\Omega}{\gamma} = 0.5$; $\frac{\alpha_+}{\gamma} = 1.0$. In transient, both $F_x$ and $F_y$ display squeezing but do not occur at the same time due to Heisenberg Uncertainty relation, with maximum squeezing occurring in $F_x$. In this case steady-state squeezing is only predicted in $F_x$. Increasing $\alpha_+$ further suppresses the squeezing in $F_x$ while $F_y$ is positive in the steady-state. In Fig. 2 all the parameters are the same as in Fig. 1 but now the atoms are driven by a resonant laser ($\Delta = -\alpha_+$). There the steady-state $F_x$ is plotted against $\frac{\alpha_+}{\gamma}$. Clearly, squeezing is obtained when $\frac{\alpha_+}{\gamma} > 1.7$ and the squeezing keep increasing with $\alpha_+$ and reaches its maximum value at $\frac{\alpha_+}{\gamma} \approx 2.5$. But it is reduced when $\alpha_+$ is increased further.

The effects of an off-resonant strong laser ($\frac{\Omega}{\gamma} = 10$) on steady-state squeezing in $F_x$ for the three damping baths are shown in Fig. 3. The role of the dispersion forces (dipole-dipole interaction) to produce optimum squeezing is best seen for the normal vacuum, thermal field and squeezed vacuum ($\Phi = \pi/2$, $N = 0.05$) cases. In Fig. 4 we plot the the components of $F_x$ and $F_y$ as functions of dimensionless time ($\tau = \gamma t$) for a detuned weak laser ($\frac{\Omega}{\gamma} = 0.1$, $\frac{\alpha_+}{\gamma} = 0.5$) driving the atomic system and damped by a broad band squeezed vacuum ($N = 0.05, \Phi = 0$). Here, squeezing is evident in $F_y$ in the transient as well as in the steady-state since it satisfies the squeezing condition $(\Delta S^y)^2 < \frac{1}{2}|\langle S^z\rangle|$. The steady-state squeezing achieved is estimated about 12%.

In conclusion, the dipole-dipole interaction proves to be important in the analysis of squeezing in the emitted fluorescent field from a cooperative system coherently driven and damped either by a normal vacuum, broad band thermal field or a broad band squeezed vacuum.

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