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Higher–order squeezing of the quantum electromagnetic field and the generalized uncertainty relations in two–mode squeezed states

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It is found that the two–mode output quantum electromagnetic field in two–mode squeezed states exhibits higher–order squeezing to all even orders. And the generalized uncertainty relations are also presented for the first time.

The concept of higher–order squeezing of the single–mode quantum electromagnetic field was first introduced and applied to several processes by Hong and Mandel in 1985[1,2]. Lately Li Xizeng and Shan Ying have calculated the higher–order squeezing in the process of degenerate four–wave mixing[3] and presented the higher–order uncertainty relations of the fields in single–mode squeezed states[4]. In this paper we generalize the above work to the higher–order squeezing in two–mode squeezed states. The generalized uncertainty relations are also presented for the first time.

1 Definition of higher–order squeezing in two–mode squeezed states

The definition of two–mode squeezed states was given by Caves and Schumaker[5]:

$$|\alpha_+,\alpha_-;\varsigma> = \hat{S}(\varsigma)|\alpha_+,\alpha_- > . \tag{1}$$

Where $\hat{S}(\varsigma)$ is the two–mode squeezed operator

$$\hat{S}(\varsigma) = exp[\frac{1}{2}(\varsigma^*\hat{a}_+\hat{a}_- - \varsigma\hat{a}_+^+\hat{a}_-^+)], \tag{2}$$

$|\alpha_+,\alpha_- >$ is the two–mode coherent state, $\hat{a}_\pm$ are two–mode annihilation operators, $\alpha_\pm$ are eigenvalues of $\hat{a}_\pm$ in $|\alpha_+,\alpha_- >$.

Define the two–mode squeezed annihilation operators by $\hat{A}_\pm$,

$$\hat{A}_\pm = \hat{S}(\varsigma)\hat{a}_\pm\hat{S}^+(\varsigma) = \mu\hat{a}_\pm + \nu\hat{a}_\mp^+. \tag{3}$$

where

$$\mu = coshr, \qquad \nu = e^{i\theta}sinhr, \tag{4}$$

$\varsigma = re^{i\theta}$ is the squeeze parameter.

Then the two-mode squeezed states are the eigenstates of $\hat{A}_\pm$,

$$\hat{A}_\pm|\alpha_+,\alpha_-;\varsigma>=\alpha_\pm|\alpha_+,\alpha_-;\varsigma>, \tag{5}$$

and $\alpha_\pm$ are the eigenvalues of $\hat{A}_\pm$.

The real two-mode output field $\hat{E}$ can be decomposed into two quadrature components $\hat{E}_1$ and $\hat{E}_2$, which are canonical conjugates. The output field $\hat{E}$ exhibits higher-order squeezing to any higher-order (Nth order) in $\hat{E}_1$, if there exists such a phase angle ϕ that the higher-order moment $<(\Delta\hat{E}_1)^N>$ in a two-mode squeezed state is smaller than its value in a completely tow-mode coherent state, viz.,

$$<(\Delta\hat{E}_1)^N>_{S.S\ two-mode} \ < \ <(\Delta\hat{E}_1)^N>_{C.S\ two-mode}.$$

This is the definition of higher-order squeezing in two-mode squeezed states.

2 The quadrature components of the two-mode output field $\hat{E}$

The electric field operator for the two-mode output field has the form of

$$\hat{E}(x,t)=\hat{E}^{(+)}(x,t)+\hat{E}^{(-)}(x,t). \tag{6}$$

Where

$$\hat{E}^+(x,t) = \sqrt{\frac{\omega_+}{2}}\hat{a}_+e^{-i\omega_+(t-x)}+\sqrt{\frac{\omega_-}{2}}\hat{a}_-e^{-i\omega_-(t-x)}, \tag{7}$$

$$\hat{E}^-(x,t) = \sqrt{\frac{\omega_+}{2}}\hat{a}_+^+e^{i\omega_+(t-x)}+\sqrt{\frac{\omega_-}{2}}\hat{a}_-^+e^{i\omega_-(t-x)}. \tag{8}$$

We now introduce two Hermitian quadrature components $\hat{E}_1$ and $\hat{E}_2$ of the electric field defined by

$$\hat{E}_1(x,t)=\hat{E}^{(+)}e^{i[\Omega(t-x)-\phi]}+\hat{E}^{(-)}e^{-i[\Omega(t-x)-\phi]}, \tag{9}$$

$$\hat{E}_2(x,t)=\hat{E}^{(+)}e^{i[\Omega(t-x)-(\phi+\frac{\pi}{2})]}+\hat{E}^{(-)}e^{-i[\Omega(t-x)-(\phi+\frac{\pi}{2})]}. \tag{10}$$

Then, $\hat{E}(x,t)$ can be decomposed into two quadrature components $\hat{E}_1$ and $\hat{E}_2$, which are canonical comjugates

$$\hat{E}(x,t)=\hat{E}_1cos[\Omega(t-x)-\phi]+\hat{E}_2sin[\Omega(t-x)-\phi], \tag{11}$$

$$[\hat{E}_1,\hat{E}_2]=2iC_0,$$

Where Ω is the carrier frequency

$$\Omega=\frac{\omega_++\omega_-}{2},$$

and ϕ is an arbitrary phase angle that may be chosen at will.

The units are chosen so that $\hbar = c = 1$.
Substituting Eqs.(7) and (8) into (9), we obtain

$$\hat{E}_1(x,t) = g_+\hat{a}_+ + g_-a_- + g_+^*\hat{a}_+^+ + g_-^*\hat{a}_-^+. \tag{12}$$

where

$$g_\pm = \sqrt{\frac{\omega \pm \epsilon}{2}} e^{-i[\phi \pm \epsilon(t-x)]}, \tag{13}$$

and

$$\epsilon = \omega_+ - \Omega = \Omega - \omega_- \tag{14}$$

is the modulation frequency.
From Eq.(3), we get

$$\hat{a}_\pm = \mu^*\hat{A}_\pm - \nu\hat{A}_\mp^+. \tag{15}$$

Substituting (15) to (12), we obtain $\hat{E}_1$ in terms of $\hat{A}_\pm$

$$\hat{E}_1(x,t) = (h_+\hat{A}_+ + h_-\hat{A}_-) + (h_+^*A_+^+ + h_-^*\hat{A}_-^+). \tag{16}$$

Where

$$h_\pm = g_\pm\mu^* - g_\mp^*\nu^*. \tag{17}$$

Define

$$\hat{B} = h_+\hat{A}_+ + h_-\hat{A}_-, \tag{18}$$

Then

$$\hat{E}_1 = \hat{B} + \hat{B}^+. \tag{19}$$

3 Higher−order noise moment $< (\Delta\hat{E}_1)^N >$ and Higher−order squeezing

By using the Campbell−Baker−Hausdorff formula, we get

$$\begin{aligned} < (\Delta\hat{E}_1)^N > \; &= \; <:: (\Delta\hat{E}_1)^N ::> + \frac{N^{(2)}}{1!}(\frac{1}{2}C_0) <:: (\Delta\hat{E}_1)^{N-2} ::> + \frac{N^{(4)}}{2!}(\frac{1}{2}C_0)^2 <:: (\Delta\hat{E}_1)^{N-4} ::> \\ &+ \cdots + (N-1)!!C_0^{N/2}. \qquad \text{(N is even)} \end{aligned} \tag{20}$$

where $N^{(r)} = N(N-1)\cdots(N-r+1), C_0 = \frac{1}{2i}[\hat{E}_1, \hat{E}_2] = [\hat{B}, \hat{B}^+]$, " :: " denotes normal ordering with respect to $\hat{B}$ and $\hat{B}^+$.
Now we take the two−mode squeezed states, then

$$<:: (\Delta\hat{E}_1)^N ::> = < \alpha_+, \alpha_-; \varsigma| :: (\Delta\hat{E}_1)^N :: |\alpha_+, \alpha_-; \varsigma > = \sum_{\gamma=0}^{N} \left[\begin{array}{c} N \\ \gamma \end{array} \right] <:: (\Delta\hat{B}^+)^\gamma (\Delta\hat{B})^{N-\gamma} ::> = 0, \tag{21}$$

and

$$C_0 = [\hat{B}, \hat{B}^+] = |h_1|^2 + |h_2|^2 = (|g_+|^2 + |g_-|^2)(|\mu|^2 + |\nu|^2) - \Omega\sqrt{1 - \frac{\epsilon^2}{\Omega^2}}(\mu^*\nu e^{-2i\phi} + \mu\nu^* e^{2i\phi}). \quad (22)$$

From (20), (4) and (13), we get

$$< (\Delta\hat{E}_1)^N >= (N-1)!!\Omega^{N/2}[cosh(2r) - \sqrt{1 - \frac{\epsilon^2}{\Omega^2}}sinh(2r)cos(\theta - 2\phi)]^{N/2}. \quad (23)$$

If ϕ is chosen to satisfy $cos(\theta - 2\phi) = 1$, then Eq(23) leads to the result

$$< (\Delta\hat{E}_1)^N >= (N-1)!!\Omega^{N/2}[cosh(2r) - \sqrt{1 - \frac{\epsilon^2}{\Omega^2}}sinh(2r)]^{N/2}. \quad (24)$$

when $coshr < \frac{\Omega}{\epsilon}$, the right hand side is smaller than $(N-1)!!\Omega^{N/2}$, which is the corresponding Nth order moment for two−mode coherent states.

It follows that the two−mode output field exhibits higher−order squeezing to all even orders.

4 The generalized uncertainty relations

[A]. Higher−order noise moment $< (\Delta\hat{E}_2)^N >$

$\hat{E}_2$ can be regarded as a special case of $\hat{E}_1$, in which if ϕ is replaced by $\phi + \pi/2$, then from (23) it follows that

$$< (\Delta\hat{E}_2)^N >= (N-1)!!\Omega^{N/2}[cosh(2r) + \sqrt{1 - \frac{\epsilon^2}{\Omega^2}}sinh(2r)cos(\theta - 2\phi)]^{N/2}. \quad (25)$$

If ϕ is chosen to satisfy $cos(\theta - 2\phi) = 1$, then

$$< (\Delta\hat{E}_2)^N >= (N-1)!!\Omega^{N/2}[cosh(2r) + \sqrt{1 - \frac{\epsilon^2}{\Omega^2}}sinh(2r)]^{N/2}. \quad (26)$$

When $coshr < \frac{\Omega}{\epsilon}$, the right hand side is greater than $(N-1)!!\Omega^{N/2}$.

[B]. Generalized uncertainty relations

From (24) and (26), we obtain

$$< (\Delta\hat{E}_1)^N > \cdot < (\Delta\hat{E}_2)^N >= [(N-1)!!]^2\Omega^N[1 + \frac{\epsilon^2}{\Omega^2}sinh^2(2r)]^N. \quad (27)$$

Equation (27) shows that $<(\Delta\hat{E}_1)^N>$ and $<(\Delta\hat{E}_2)^N>$ in two−mode squeezed states can not be made arbitraily small simultaneously. We call Eq.(27) the generalized uncertainty relations in two−mode squeezed states, and the right hand side (constant) is dependent on N, ϵ, Ω, and r.

Since

$$1 + \frac{\epsilon^2}{\Omega^2} sinh^2(2r) > 1$$

so

$$<(\Delta\hat{E}_1)^N> \cdot <(\Delta\hat{E}_2)^N> \; > \; [(N-1)!!]^2\Omega^N. \tag{28}$$

If $r = 0$, the two−mode squeezed states become two−mode coherent states, then

$$<(\Delta\hat{E}_1)^N>_{c,s} \cdot <(\Delta\hat{E}_2)^N>_{c,s} = [(N-1)!!]^2 \cdot \Omega^N. \tag{29}$$

This is the generalized uncertainty relations in two−mode coherent states.

If $\epsilon = 0, N = 2$, we obtain

$$<(\Delta\hat{E}_1)^2> \cdot <(\Delta\hat{E}_2)^2> = \Omega^2. \tag{30}$$

This is just the usual Heisenberg uncertainty relations in relevant references[1,2,4,5].

5 Application

As an application of the above result, we calculate the generation of higher−order squeezing by non−degenerate four−wave mixing (NDFWM). It can be shown that the field of the combined mode of the probe wave and the phase−conjugate wave exhibits higher−order squeezing to all even orders, and the generalzied uncertainty relations still hold in NDFWM process.

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