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MULTIMODE SQUEEZING, BIPHOTONS AND UNCERTAINTY RELATIONS IN POLARIZATION QUANTUM OPTICS

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Abstract

The concept of squeezing and uncertainty relations are discussed for multimode quantum light with the consideration of polarization. Using the polarization gauge $SU(2)$ invariance of free electromagnetic fields, we separate the polarization and biphoton degrees of freedom from other ones, and consider uncertainty relations characterizing polarization and biphoton observables. As a consequence, we obtain a new classification of states of unpolarized (and partially polarized) light within quantum optics. We also discuss briefly some interrelations of our analysis with experiments connected with solving some fundamental problems of physics.

1 Introduction

Polarization properties of light were widely investigated long ago when examining some fundamental problems of quantum mechanics including "hidden" variable theories, Bell's inequalities and Einstein-Podolsky-Rosen (EPR) paradox, different topological phases etc. (see, e.g., [1-8] and references therein). Herewith, as a rule, the polarization structure of light has been described in terms of the field correlation functions, associated Stokes parameters and the Poincare sphere which are well adapted to classical optics experiments [7,9] but are not quite adequate to specific quantum ones (photon counting)[3]. Such a description also ignores a polarization $SU(2)$ symmetry[10-12] of light fields though it has been widely used implicitly - through the Stokes parameters $s_{\tilde{a}}$ which determine, in particular, the polarization degre $degP = [s_1^2 + s_2^2 + s_3^2]^{1/2}/s_0$ of monochromatic plane wave light beams[1,3,9,13].

But recently a new formalism[10-12] was proposed for a description of polarization structure of multimode quantum light fields using the polarization $SU(2)$ symmetry and a related concept of the P-quasispin which generalizes the Stokes vector notion at the quantum level and is closely related to the Stokes operators defined in [13]. This approach enabled us to gain a new insight into the polarization structure of light and quantum mechanisms of its depolarization[12,13].

At the same time, so-called squeezed states of light are intensively examined now within quantum optics (see, e.g.,[10,14-17] and references therein) since these states have attractive properties of the "noise reduction" in measurements of some quantum mechanical observables. However, we note that squeezed states have been studied sufficiently well only for the single-mode fields[15,14] whereas for multimode fields it is not the case since even the definition of the concept of multimode squeezing is not unique that is due to a lot of the choices of measurable quantities[16,17].

The aim of this report is to give an analysis of the concept of squeezing of the multimode light related to polarization degrees of freedom by using the above mentioned formalism of P-quasispin. Specifically, we will show that there exist new quantum states of light beams exhibiting , in a sense, an absolute squeezing in polarization degrees of freedom. Such states are generated by specific unpolarized biphoton clusters and have all characteristics of usual unpolarized light, but unlike the latter, new quantum states of unpolarized light are "polarizationally noiseless" [10-12,18]. Besides we discuss briefly some generalizations and applications of new non-classical states of light to setting up new optical experiments related to some fundamental problems of physics.

2 Polarization P-quasispin of electromagnetic fields and unpolarized biphotons

In quantum optics the free transverse electromagnetic(em) field with "m" spatiotemporal modes is described by the vector potential[1,3,12,13]

$$\vec{A}(\vec{r},t) = \vec{A}^{(-)}(\vec{r},t) + \vec{A}^{(+)}(\vec{r},t) = c\sum_{j=1}^{m}(\frac{2\pi\hbar}{\omega_j V})^{1/2}\{\vec{A}^{(-)}(j)\exp[i(\vec{k}_j\vec{r} - \omega_j t)] + h.c.\},$$

$$\vec{A}^{(-)}(j) = \sum_{\alpha=+,-,3} \vec{e}_\alpha(j)a_\alpha^+(j), \vec{A}^{(+)} = (\vec{A}^{(-)})^+ \qquad (2.1)$$

where $a_\alpha(j)/a_\alpha^+(j)$ are destruction/creation operators for j-th spatiotemporal and α-th polarization modes of the field, $\vec{e}_\alpha(j)$ are the polarization unit vectors adapted to the helicity basis, $\vec{e}_3(j) = \vec{k}_j/\omega_j$, V is a quantization volume, etc. With the help of Eq. (2.1) one determines correlation tensors[3]

$$G^{(s,p)}_{i_1...i_s;j_1...j_p}(\{\vec{r}_a,t_a\};\{\vec{r}_b',t_b'\}) = Tr[\rho E^{(-)}_{i_1}(\vec{r}_1,t_1)...E^{(-)}_{i_s}(\vec{r}_s,t_s)E^{(+)}_{j_1}(\vec{r}_1',t_1')...E^{(+)}_{j_p}(\vec{r}_p',t_p')],$$

$$\vec{E}^{(\pm)} = c^{-1}\partial\vec{A}^{(\pm)}/\partial t \qquad (2.2)$$

which correspond to different physical quantities, measurable in optical experiments, and are expressed in terms of quantum expectations of ordered polynomials in operators $a_\alpha(j)$ and $a_\alpha^+(j)$[6]. We note that quantum expectations of any physical quantities are calculated by averaging on the space $L_{phys} = L_F(m)$ spanned by basis vectors

$$|\{n_i^\sigma\} >= N(\{n_i^\sigma\}) \prod_{i=1}^{m} \prod_{\sigma=-,+} [n_i^\sigma!]^{-1/2} (a_\sigma^+(i))^{n_i^\sigma} |0> \qquad (2.3)$$

which are generated by the creation operators $a_\alpha^+(i)$ of photons with transverse ($\alpha = +,-$) polarizations (helicities) only (that corresponds to a standard form of the gauge condition for transverse radiation fields in quantum electrodynamics[12,13]).

The most important of such measurable quantities is the field Hamiltonian

$$H_f = \sum_{i=1}^{m}\omega_i \sum_{\alpha=+,-,3} a_\alpha^+(i)a_\alpha(i) \qquad (2.4)$$

which determines the time-evolution of other field observables[3]. But in polarization quantum optics there are specific observables which characterize proper polarization properties of light beams and correspond to to the group $U(2)$ of a specific polarization gauge invariance of the Hamiltonian (2.4)[10-12]. This continious polarization group $U(2)$ is closely related to discrete symmetries of em fields(mirror reflection $\hat{\sigma}: a^+_\pm(j) \to a^+_\mp(\hat{j}), \vec{k}_{\hat{j}} = -\vec{k}_j$ and the spatial inversion $\hat{P}$) since all these (chiral) symmetries act in a natural manner on a 2-dimensional "polarization spinor" $\{\vec{e}_\alpha(i), \alpha = \pm\}$ spaces[11,13].

The generators of the polarization group $U(2)$ are of the form

$$P_0 = \frac{1}{2}\sum_{i=1}^{m}[a^+_+(i)a_+(i) - a^+_-(i)a_-(i)] = \sum_i P_0(i),$$

$$P_\pm = \sum_{i=1}^{m} a^+_\mp((i)a_\pm(i) = \sum_i P_\pm(i), \quad N = \sum_{i=1}^{m}\sum_{\alpha=+,-} a^+_\alpha(i)a_\alpha(i) = \sum_i N(i) \tag{2.5}$$

where N is the total photon number operator and operators P_α are generators of the $SU(2)$ subgroup defining the polarization (P) (quasi)spin [10-12]. The operators P_β and N satisfy commutation relations

$$[N, P_\alpha] = 0, \quad [P_0, P_\pm] = \pm P_\pm, \quad [P_+, P_-] = 2P_0 \tag{2.6}$$

and in the case $m = 1$ coincide up to the factor 1/2 with Stokes operators $\Sigma_\alpha : \Sigma_1 = 2P_2, \Sigma_2 = -2P_0, \Sigma_3 = -2P_1$ [13]. As is clear from Eqs (2.5) the total P-quasispin of the em field is obtained by adding of the appropriate quasispin quantities for single spatiotemporal modes. However, from the experimental viewpoint the total P-quasispin of the em field enable us to examine new interesting physical phenomena connected with correlations of different modes, in particular, with so-called "entangled states" which are widely discussed in multiparticle interferometry [2,5,19].

Note that the operators P_α do not commute with components S_α of the gauge non-invariant (and hence locally non-observable) ordinary spin $\vec{S} = (S_1, S_2, S_3)$ of the em field wich define the field transformations with respect to the $SO(3) \subset SL(2C)$ group of rotations in the usual 3-dimensional space and are expressed in terms of the $\vec{A}(\vec{r}, t)$ Fourier components $A^{(\pm)}_a(j)$ as follows [13,12]

$$S_a = -i\sum_j\sum_{b,c}\epsilon_{abc}A^{(-)}_b(j)A^{(+)}_c(j) \tag{2.7}$$

where ϵ_{abc} is the fully antisymmetric tensor ($\epsilon_{123} = 1$). Specifically, from Eqs.(2.1),(2.7) one easily finds relations specifying "rotation" properties of different physical operators[12]. For example, in the case of plane wave beams, when in (2.1) $e_{3a}(j) = \delta_{3a}, a = 1,2,3, e_{\pm 3}(j) = 0$ ($e_{\alpha a}(i)$ is the projection (directing cosine) of $\vec{e}_\alpha(i)$ on the "a''-th axis of a fixed spatial frame of reference with the axe $O\vec{X}_3$ being parallel to all $\vec{k}_j$) and $S_3 = 2P_0$, one finds a relation

$$exp(i\phi S_3)P_\alpha exp(-i\phi S_3) = exp(i2\alpha\phi)P_\alpha, \alpha = 0, \pm, \tag{2.8}$$

defining transformations of P-spin components under rotations around the light beam axis.

From Eqs (2.5), (2.7) it follows that the P-quasispin formalism has evident advantages in comparison with the ordinary spin for describing properly polarization properties of light since its components have a clear physical meaning and are measurable in quantum optics polarization experiments related to counting photons with definite polarizations[12]. In particular, the total

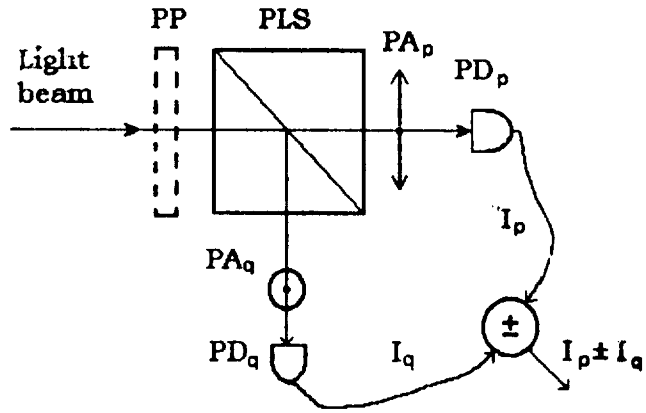

Figure 1: Scheme of the measurement of P-quasispin components

helicity $2P_0$ of the field is the difference $(N_+ - N_-)$ of the right- and left- handed photon numbers and Hermitian operators $2P_1 = (P_+ + P_-)$ and $2P_2 = i(P_+ - P_-)$ determine (cf.[9,6]) differences of photon numbers for two pairs of orthogonal linear polarizations which are connected with the helicity basis by the linear transformations[12]

$$a)\quad a_1^+(j) = \frac{1}{\sqrt{2}}\{a_-^+(j) - a_+^+(j)\},\quad a_2^+(j) = \frac{i}{\sqrt{2}}\{a_+^+(j) + a_-^+(j)\} \tag{2.9a}$$

$$b)\quad \hat{a}_1^+(j) = \frac{1}{\sqrt{2}}\{a_1^+(j) + a_2^+(j)\},\quad \hat{a}_2^+(j) = \frac{1}{\sqrt{2}}\{-a_1^+(j) + a_2^+(j)\} \tag{2.9b}$$

implemented, for example, with the help of phase plates and polarization rotators [9,6,7]. (From the formal viewpoint components P_1 and P_2 correspond to the choice of different subgroups $SO(2) \subset SU(2)$ unlike the helicity subgroup $U(1)$ for P_0. Moreover, basis wave functions with linear polarization defined by Eqs (2.9) are eigenstates of operators describing the abovementioned discrete symmetries $\hat{\sigma}, \hat{P}$ of light fields.)

A typical principal scheme[18] of the measurement of components P_α of P-quasispin is presented on Fig. 1, where we use the following notations: PP denotes phase plates, PLS stands for polarization light beam splitters, PA_a and PD_a are, respectively, polarization analyzers and photodetectors for polarization modes "a". We note that this scheme can be realized in both single-mode ($m = 1$) and multimode ($m > 1$) regimes. However, as it will be seen later, the use of multimode regimes enables us to reveal new interesting physical phenomena, in particular, an absolutely unpolarized quantum light[10-12].

Since in the case of the monochromatic plane waves quantum expectations $< P_\alpha >$ are proportional to the Stokes parameters $s_\alpha =< \Sigma_\alpha >, \alpha = 1,2,3; s_0 =< N >$ [13], then in general cases one can consider that quantities $< P_\alpha >, < N >$ determine the polarization degree $degP$ of light beams with arbitrary wave fronts and frequencies by the relation

$$degP = 2[\sum_{\alpha=0,1,2} (< P_\alpha >)^2]^{1/2} / < N > \tag{2.10}$$

generalizing the appropriate definition for one-mode light beams[3]. At the same time the quantum averages $< |P^2| >= \bar{P}(\bar{P}+1)$ of the $SU(2)_{pol}$ Casimir operator $P^2 = (1/2)(P_+P_- + P_-P_+) + P_0^2$

are connected by the relation

$$< |P^2| >= \bar{P}(\bar{P}+1) = \sum_{\alpha=0,1,2} [\sigma_\alpha + (< |P_\alpha| >)^2] \tag{2.11}$$

with the variances $\sigma_\alpha =< |P_\alpha^2| > -(< |P_\alpha| >)^2$ determining "polarization noises" [3,12,18] and different uncertainty measures for operators P_α(cf. [20-22]).

Therefore, one may use P-spin (P_α) as an adequate tool for studying proper polarization properties of quantum light fields in parallel to the usual apparatus of the correlation functions and Stokes vector $\vec{s} = (s_1/s_0, s_2/s_0, s_3/s_0)$ running on the Poincare sphere[3]. But unlike the latter, the use of the P-spin formalism allows us to gain a more deep insight into the inner nature of the polarization struture of light beams with arbitrary wave fronts.

Indeed,as it was shown in [10-12], one can decompose the Fock space $L_F(m)$ spanned by the vectors (2.3) into the direct sum

$$L_F(m) = \sum_{P,\pi} L(P\pi) \tag{2.12}$$

of infinite-dimensional subspaces $L(P\pi)$ which are specified by eigenvalues P, π of the P-spin and P_0 respectively and spanned by basis vectors $|P\pi; n, \lambda >$ of the form

$$|P\pi; n, \lambda >= \sum C(\{\alpha_i, \beta_{ij}, \gamma_{ij}\}) \prod_i (a_\pm^+(i))^{\alpha_i} \prod_{i\leq j} (Y_{ij}^+)^{\beta_{ij}} (X_{ij}^+)^{\gamma_{ij}} |0 > \tag{2.13}$$

where $\sum \alpha_i = 2|\pi|, \sum \beta_{ij} = (P - |\pi|), \sum \gamma_{ij} = n/2 - P$. For example, in the cases $m = 1$ and $m = 2$ we have the following expressions[23]

$$a)|P\pi >= [(P-\pi)!(P+\pi)!]^{-1/2} (a_+^+(1))^{|\pi|+\pi} (a_-^+(1))^{|\pi|-\pi} (Y_{11}^+)^{P-|\pi|} |0 >, \tag{2.14a}$$

$$b)|P\pi = \pm P; n, t >= [(n+1)!(n-2P)!(P-t)!(P+t)!/(2P+1)!]^{-1/2}$$
$$(a_\pm^+(1))^{P+t} (a_\pm^+(2))^{P-t} (X_{12}^+)^{n/2-P} |0 >, 2t = n(1) - n(2) \tag{2.14b}$$

for some of such vectors. In general, the coefficients $C(...)$ in (2.13) are determined from the defining equations

$$P^2|P\pi; n, \lambda >= P(P+1)|P\pi; n, \lambda >; \quad P_0|P\pi; n, \lambda >= \pi|P\pi; n, \lambda >,$$

$$N \mid P\pi; n, \lambda >= n \mid P\pi; n, \lambda > \tag{2.15}$$

and some equations for fixing an extra (vector) label λ (see [12,11] and references therein). Operators

$$Y_{ij}^+ = \frac{1}{2}(a_+^+(i)a_-^+(j) + a_-^+(i)a_+^+(j)), X_{ij}^+ = a_+^+(i)a_-^+(j) - a_-^+(i)a_+^+(j) \tag{2.16}$$

in (2.13), (2.14) are the solutions of the operator equations

$$[P_0, Y_{ij}^+] = 0; \quad [P_\alpha, X_{ij}^+] = 0, \quad \alpha = 0, +, - \tag{2.17}$$

and may be interpreted as creation operators of P_0-scalar and P-scalar biphoton kinematic clusters, respectively.

From Eqs(2.16), (2.17) one easily obtains that $< P_\alpha >= 0, \alpha = 0,1,2$, in states generated by actions on the vacuum vectors $|0>$ of operators $(X^+_{ij})^a (Y^+_{ij})^b$ only (and spanned by vectors (2.13) with $\pi = 0$); these states are examples of entangled states of multiparticle interferometry[5,19]. In general, the states (2.13) describe light beams representing a mixture of both usual (uncoupled) photons $a^+_\alpha(j)$ and unpolarized P- and P_0-scalar biphoton clusters X^+_{ij}, Y^+_{ij} [10-12]. As it follows from (2.13), the total number operators N_{ph}, N_X, N_Y, respectively, of uncoupled photons and X-and Y-type biphotons are given as follows,

$$N_{ph} = 2|P_0| = 2\sqrt{(P_0)^2}, N_X = N/2 - P, N_Y = P - |P_0|, P = -1/2 + \sqrt{1/4 + P^2} \qquad (2.18)$$

We, however, note that biphotons Y^+_{ij} exist for any number "m" of spatiotemporalal modes whereas $X^+_{ij} \neq 0$ only for "m" ≥ 2. We also emphasize that in contrast to the usual photon operators $a^+_\alpha(j), a_\alpha(j)$ the operators $X_{ij} = (X^+_{ij})^+, X^+_{ij}, Y_{ij} = (Y^+_{ij})^+, Y^+_{ij}$ satisfy not the canonical commutation relations but trilinear commutation relations for quanta of generalized parastatistical fields (these operators, however, can be transformed in some "particle-like" quanta ones)[11].

Further, the decomposition (2.12) is invariant with respect to the Lie algebra $so^*(2m)$ generated by biphoton operators X_{ij}, X^+_{ij} and commuting with the polarization invariance algebra $su(2) = Span\{P_\alpha\}$[12,11]. Therefore, states $|\psi>$ belonging to a subspace $L(P\pi)$ with given P, π at initial time will be in it for the time evolution governed by the interaction Hamiltonians $H_{int} = H'_i(\{X_{ij}, X^+_{ij}\})$ what is similar to the situation in the theories with spontaneosly broken symmetry; examples of such Hamiltonians are given by those of some parametric processes [10-12]. Extending the algebra $so^*(2m)$ by adding operators Y_{ij}, Y^+_{ij} we get the algebra $u(m,m)$ commuting with the polariztion subalgebra $u(1) = Span\{P_0\} \subset su(2)$ and associated with interaction Hamiltonians $H_{int} = H''_{int}(\{Y_{ij}, Y^+_{ij}; X_{ij}, X^+_{ij}\})$ (describing, for example, light propagation in Kerr media) which keep invariant for time evolution subspaces $L'(\pi) = \sum_{P \geq ||\pi|} L(P\pi)$ (with fixed π)[12]. If we restict ourselves by biphoton operators Y_{ij}, Y^+_{ij} only we obtain the subalgebra $sp(2m, R) \subset u(m,m)$. So, algebras $so^*(2m), sp(2m,R)$ and $u(m,m)$ describe specific P-and P_0-scalar degrees of freedom of light fields which are complementary, in a sense, to polarization ones.

3 Squeezing in polarization quantum optics. A new classification of unpolarized light

The decomposition (2.12) implies a new classification of the polarization states of quantum light fields from the physical viewpoint [11,12]. This classification is closely related to a specific sort of squeezing of multimode light beams with consideration of polarization.

In fact, a definition of squeezing in quantum mechanics is based on an analysis of different uncertainty relations for expectations $< |(A_i)^s| >$ of a set $\{A_i, i = 1, ..., r > 1\}$ of non-commuting Hermitian operators A_i representing some quantum observables[1,14-17,20-25]. These relations are connected with specific measures of admissible quantum fluctuations ("noises") for *joint measurements of all observables* A_i in a state $| >$ which characterize differences betveen quantum observables and their classical analogs($< |A_i| >$) and are displayed with the help of different quasiprobability functions[3,14,25] and generalized coherent states[3,10,20-22]. Specifically, the most widespread uncertainty relation (of the Heisenberg type) has the form[1,14,20-22]

$$\Delta A_i \Delta A_j \geq 1/2| < |[A_i, A_j]| > | \qquad (3.1)$$

where $(\Delta A)^2 \equiv \sigma_A =< |(A)^2| > -(< |A| >)^2$ is a standard quadratic measure (variance) of a deviation of the quantum quantity A from its classical analog. Then the problem of squeezing consists in finding quantum states minimizing both the product $\Delta A_i \Delta A_j$ of two "individual" uncertainty measures (the condition of a *joint* quasiclassical behaviour of A_i and A_j) and one (say, ΔA_i) of them (the condition of properly squeezing).

If the right side of inequality (3.1) is a c-number this problem is easily solved and leads to a definition of the usual concept of squeezing related to generalized coherent states of the group $SU(1,1)$[14-17]. For example, it is the case for single-mode em field when we use as observables A_i two quadrature components $A_1 \equiv X_1 = (a_\alpha^+(j) + a_\alpha(j))/\sqrt{2}, A_2 \equiv X_2 = i(a_\alpha^+(j) - a_\alpha(j))/\sqrt{2}(\alpha, j$ are fixed) [14,15]. However, for multimode em fields the situation becomes more complicated since in this case we have a more vast set of observables which obey non-trivial commutation relations[10,16,17]. Therefore, there are many possibilities of definition of squeezing related to a choice (from physical considerations) of some subsets of observables (and adequate joint uncertainty meausures for them) for which a solution of this problem is comparatively simple. As we established above, in polarization quantum optics it is natural to take as such subsets components P_α of the P-quasispin obeying the commutation relations (2.6) of the $su(2)$ algebra as well as subsets of unpolarized biphoton operators (2.16) of X- and Y- types (related to the "biphoton algebras" $so^*(2m), sp(2m,R)$ and $u(m,m)$). That enables us to define a specific polarization squeezing which is closely related to a new physical phenomenon of biphoton unpolarized light(UL)[12].

Since operators P_α are similar to angular momentum operators $J_\alpha, \alpha = 1,2,3$ obeying the $su(2)$ commutation relations, one can apply analysis[20,21,24] of uncertainty relations and an appropriate concept of squeezing for operatorsJ_α to analysis of those for operators $P_\alpha, \alpha = 1,2,3 (P_3 = P_0)$. As is known[20-22], the Heisenberg uncertainty relations (3.1) for $A_i = J_i (i = 1,2,3)$ are minimized on the $SU(2)$ generalized coherent states[20] $|\zeta; \pm j >= exp(\zeta J_+ - \zeta^* J_-)|j; \pm j >, \zeta = -\frac{\theta}{2} exp(-i\phi)$ where $|j; \pm j >$ is the highest (or lowest) vector of the $SU(2)$ irreducible representation D^j. The states $|\zeta; \pm j >$ are maximally close to classical ones[20] and minimize a $SU(2)$-invariant (cf. (2.11)) "radial" uncertainty meausure $\sum_i \sigma_{J_i}$ ($\sum_i \sigma_{J_i} = min = j$ on the states $|\zeta; \pm j >$) which is an adequate characteristic of quasiclassical behaviour of a whole set $\{J_i\}$[20,21]. Besides, these states are used for a definition of polarization analogs $Q(\theta, \phi; \rho)_{\pm P} = | < \zeta; \pm P|\rho|\zeta; \pm P > |^2 (\rho$ is a density matrix of a light beam) [23] of Q-functions of quasiprobability[3,14] which are well adapted for displaying squeezing properties of oscillator systems. Evidently, for physical systems with a fixed value of j (e.g., for usual spin systems) we obtain an "absolute" squeezing for $\{J_i\}$, characterized by relations

$$\sum_i \sigma_{J_i} = 0, \Delta J_i = 0 =< |J_i| > \forall i, \tag{3.2a}$$

only for the unique vector $|\zeta; 0 >= |0; 0 >$. But for em fields the situation is quite different because of the decomposition (2.12) for $L_F(m)$.

Specifically, as is seen from Eqs (2.13), (2.17), the states $| > \in L(00) = Span\{|00; n, \lambda >\}$ satisfy Eqs (3.2a) and provide an "absolute" minimum of both the aforementioned "radial" uncertainty measure $\sum_i \sigma_{P_i}$ as well as uncertainty relations of the (3.1) type for operators P_i; besides these states form the infinite-dimensional space on which three non-commuting operators P_α behave themselves as c-numbers exhibiting an "absolute squeezing" and totally *classical* behaviour in polarization degrees of freedom (that it is of interest for designing different experiments related to the EPR-paradox and "hidden variable" theories[1,2,5,11]). We note that in $L_F(m)$ there exists another class of quantum states displaying a similar (though more weak, than (3.2a)) property of polarization squeezing. Namely, for states $| > \in L'(\pi = 0)$ we find from (2.12)-(2.14)

$$\Delta P_0 = 0 =< |P_i| >, \sum_i \sigma_{P_i} = \bar{P}(\bar{P}+1) \neq 0 \tag{3.2b}$$

As it follows from Eqs (3.2), states $|P0;... > \in L(P0) \subset L'(\pi = 0)$ and $|00;... > \in L(00)$ possess the characteristic property ($< |P_\alpha| >= 0$) of UL (cf.[3,9]). Besides, the calculations [11] showed that $< |S_\alpha| >= 0$ for all α and correlation tensors $G^{(1,1)}_{ij}(\vec{r}, \vec{t}; \vec{r}, \vec{t})$ have for these states a form corresponding to UL beams with, in general, arbitrary wave fronts. But unlike classical (chaotic) UL, for the states $|00;... >$ and $|P0;... >$ we have additional characteristics of light depolarization which follow from Eqs (2.12)-(2.14), (2.17) and are expressed in terms of higher moments for P_α:

$$< |(P_0)^s| >= 0 \forall s = 1,2,..., | > \in L'(\pi = 0);$$

$$< |(P_\alpha)^s| >= 0 \forall \alpha = 0,+,-, s = 1,2,..., | > \in L(00) \tag{3.3}$$

showing the absence of appropriate polarization "noises" ($< |(P_\alpha)^s| > -(< |(P_\alpha)| >)^s_r$; $\alpha = 0,1,2$ for $| > \in L(00)$ and $\alpha = 0$ for $| > \in L'(\pi = 0)$) of any order measured by appropriate noises of difference photocurrents in schemes of Fig. 1; herewith, as it follows from Eq. (2.8), for axial (plane wave) light beams results of measurements do not depend on rotations of analyzers around beam axis.

So, for states $| > \in L(00)$ all proper polarization properties are identical with those for vacuum state $|0>$, but unlike the latter the light intensity $< |H_f| >$ in these states(with the Hamiltonian H_f from Eq. (2.3)) is not equal to zero. Consequently, they may be recognized as states describing absolutely unpolarized light while the states $| > \in L'(0)$ have a hidden polarization structure revealed in measurements of linear polarization noises. Therefore, states $|\phi > \in L'(0)$ generated by biphotons Y^+_{ij}, X^+_{ij} and $|\psi > \in L(00) \subset L'(0)$ generated only by biphotons X^+_{ij} describe new types of UL due to strong quantum phase correlations rather than random mixing light beams as it is the case for the classical UL [3,9]. Examples of such states are yielded by generalized coherent states of the above biphoton algebras (and appropriate groups) related with interaction Hamiltonians $H_{int} = H^1_{int} + H^2_{int}$ where $H^1_{int} = \sum_{i<j}(g_{ij}X_{ij} + g^*_{ij}X^+_{ij})$, $H^2_{int} = \sum_{i,j}(f_{ij}Y_{ij} + f^*_{ij}Y^+_{ij})$ describing some specific parametric processes[11]. In particular, generalized coherent states of the $SO^*(2m)$ group orbit type

$$|\{\gamma_{ij}\} >_P = S_X(\{\gamma_{ij}\})|0 >= \exp[\sum(\gamma_{ij}X_{ij} - \gamma^*_{ij}X^+_{ij})]|0 > \tag{3.4}$$

discussed together with some related models in [10-11] are generated by H^1_{int} whereas H^2_{int} produces generalized coherent states of the group $Sp(2m, R) \subset U(m,m)$

$$|\{\beta_{ij}\} >_{P_0} = S_Y(\{\beta_{ij}\})|0 >= \exp[\sum(\beta_{ij}Y^+_{ij} - \beta^*_{ij}Y_{ij})]|0 > \tag{3.5}$$

coinciding in the case $m = 1$ with two-mode squeezed states introduced in [15] and related to the $SU(1,1)$ group [11,12]. In general cases states (3.4), (3.5) display some properties of specific multimode squeezing associated with biphoton algebras $so^*(2m), sp(2m,R), u(m,m)$ (cf.[10-12,16]). Therfore, operators S_X, S_Y can be called as biphoton squeezing operators. Without dwelling here

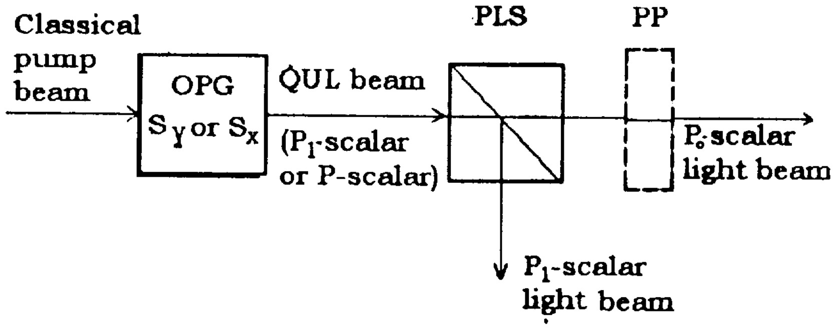

Figure 2: Scheme of production of biphoton unpolarized light

on analysis of all their properties we note that operators S_X commute with the "proper" polarization squeezing operators $S_P(\zeta) = exp(\zeta P_+ - \zeta^* P_-)$ while it is not the case for the operators S_Y.

Physical realizations of such states, connected with actions of P_0- and P-scalar biphoton squeezed operators $S_Y(\{\beta_{ij}\})$ and $S_X(\{\gamma_{ij}\})$ on the vacuum vectors $|0>$, are represented schematically on Fig. 2 where POG stands for parametric oscillator generators corresponding to the operators S_Y, S_X and other notations are the same as on Fig 1. We note that, in practice, it is easier to realize such schemes corresponding to Eq. (3.5) rather than Eq. (3.4) because the latter require parametric oscillator crystals with highly anisotropic properties. Therefore, for production of P-scalar light it is preferable to combine more simple schemes of production of P_0-scalar light together with some interferometric schemes[5,18,19].

Thus, our analysis displays inner mechanisms of the light depolarization at the quantum level by contrast to the generally accepted viewpoint [9] that randomization is the only way of obtaining UL. Besides, the P-spin formalism yields (see (2.17) and (2.18)) some new natural measurable quantitative characteristics of light depolarization, namely, degrees $dep_P = (1 - 2\bar{P}/\bar{N})$ and $dep_{P_0} = (1 - |2\bar{\pi}|/\bar{N})$ of the content of P-scalar and of P_0-scalar biphotons where $\bar{P}, \bar{\pi} = \bar{P}_0, \bar{N}$ denote expectation values of appropriate operators; herewith $\bar{P} = -1/2 + [1/4 + <|P^2|>]^{1/2}$ is determined from Eqs (2.10), (2.11) as a function of $degP$, $\bar{N}$ and variances σ_α. Evidently, dep_{P_0} is connected with thewell-known degree of circular polarization $|<N_+> - <N_->|/<N>$ whereas dep_P provides a new quantitative characteristic of polarization structure of light related to measurements of polarization noises.

We also note that analysis above can be extended by considering modifications of the decomposition (2.12) where any other Hermitian operator $P_{\vec{n}} = S_P(\zeta(\vec{n}))P_0(S_P(\zeta(\vec{n})))^+$ is diagonalized instead of P_0 $(S_P(\zeta(\vec{n})) = exp(\zeta(\vec{n})P_+ - \zeta^*(\vec{n})P_-)$, $\zeta(\vec{n}) = -\frac{\theta}{2}exp(-i\phi)$ and vector $\vec{n} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$ corresponds to a position of the Stokes vector $\vec{s}$ on the Poincare sphere). Specifically, one can diagonalize Hermitian operators $P_\alpha, \alpha = 1, 2$ corresponding to a linear polarization basis of light beams[12]. Such extensions lead to new states of quantum UL generated by $P_{\vec{n}}$(e.g., P_1- or P_2) - scalar biphotons $Y_{ij}^+(\vec{n}) = (S_P(\zeta(\vec{n})))^+ Y_{ij}^+ S_P(\zeta(\vec{n}))$ of the (2.16) type and having characteristics similar to those described by Eqs (3.2)-(3.3) but with some peculiarities concerning their "rotation" properties determined by Eqs (2.8); for example, the condition $\Delta P_{1(2)} = 0$ is valid only for quite definite angle positions of polarization analyzers. We also note that usual multimode Glauber coherent states $|\{\alpha_i^+, \alpha_j^-\}> = \prod_i exp(\alpha_i^+ a_+^+(i) + \alpha_i^- a_-^+(i) - \alpha_i^{+*} a_+(i) - \alpha_i^{-*} a_-(i))|0>, \alpha_i^\pm \neq 0$, which are in general cases states of partially polarized light, contain (for special values of parameters $\alpha_i^{\pm}$) a subclass of states corresponding to UL. In particular, all such states display properties($<|P_\alpha|>=0, \sigma_{P_\alpha}\neq 0 \forall\alpha$) of usual UL, when the condition $|\alpha_i^+|=|\alpha_i^-|$ is fulfilled[8,18].

All this leads to a new classification of states of UL within quantum optics which can be represented by a chain of embedded subsets

$$UL^0 \supset UL^c \supset UL^{bp} \supset UL^{P_0} \supset UL^P \tag{3.6}$$

with the following typical density matrices for each subset:

$$a) UL^0 \rightarrow \rho_{th}, \tag{3.7a}$$

$$b) UL^c \rightarrow \rho_c = |\{\alpha_\pm\}><\{\alpha_\pm\}|, |\alpha_+| = |\alpha_-|, \tag{3.7b}$$

$$c) UL^{bp} \rightarrow \rho_{bp} = |\lambda><\lambda|, |\lambda> = exp(\lambda P_+ - \lambda P_-)|>, |> \in L'(\pi=0), \tag{3.7c}$$

$$d) UL^{P_0} \rightarrow \rho_{P_0} = |><|, |> = S_Y(\{\beta_{ij}\})|0> \in L'(\pi=0), \tag{3.7d}$$

$$e) UL^P \rightarrow \rho_P = |><|, |> = S_X(\{\gamma_{ij}\})|0> \in L(00) \tag{3.7e}$$

where ρ_{th} is a density matrix for the thermal radiation[3], ρ_c describes coherent UL whereas $\rho_{bp}, \rho_{P_0}, \rho_P$ correspond to different kinds of biphoton UL[12]. We note that all these classes of UL are distinguished by values of $dep_P = (1 - 2\bar{P}/\bar{N})$ and $dep_{P_0} = (1 - |2\bar{\pi}|/\bar{N})$.

4 Generalizations and conclusion

Thus, in the previous sections we have shown that in the Fock space $L_F(m)$ of multimode light with consideration of polarization one can pick out with the help of Eq. (2.12) subspaces ($L(P=0\pi=0), L'(\pi=0)$ and someones related to them) of quantum states describing different new types of UL light and, simultaneously, manifesting specific forms of squeezing in polarization optics. All other subspaces $L(P\pi), L'(\pi), \pi>0$, in the decomposition (2.12) describe, generally speaking, states of partially depolarized quantum light (see [10,11] where we also examined various types of polarization generalized coherent states of light).

However, in real physical experimental situations states of light beams do not belong to a single subspace $L(P\pi)$ but are superpositions of states from different subspaces $L(P\pi)$. Therefore, it is of interest to study polarization squeezing properties (with using measurement devices of schemes on Fig. 1) of partially polarized light beams obtained by actions of the biphoton squeezing operators S_Y, S_X together with the "proper" polarization squeezing operators $S_P(\zeta)$ on states $|in>_{phys}$ of some physical input light beams

$$|PPSL>_{X/Y} = S_P(\zeta)(S_X(\{\gamma_{ij}\})/S_Y(\{\beta_{ij}\}))|in>_{phys} \tag{4.1}$$

that is presented schematically on Fig 3. As a result we can obtain new (non-classical) sets of states of partially polarized light which can be called as partially polarized squeezed light(PPSL). Specifically, taking as $|in>_{phys}$ usual multimode Glauber coherent states $|\{\alpha_i^+, \alpha_j^-\}>, \alpha_i^{\pm}\neq 0$, we get in such a manner states of PPSL which contain (at the condition $|\alpha_i^+|=|\alpha_i^-|$) a subclass of states corresponding to UL^c in the classification above. In general, transmitting different input

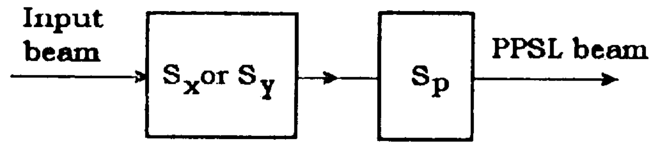

Figure 3: Scheme of production of partially polarized squeezed light

beams through physical devices corresponding to different combination of the above squeezing operators in (4.1), one can obtain new classes of partially polarized light distinguished by values of $dep_P = (1 - 2\bar{P}/\bar{N})$ and $dep_{P_0} = (1 - |2\bar{\pi}|/\bar{N})$ by analogy with UL.

In conclusion we emphasize that the above results give a more deep insight into polarization structure of light beams enabling to determine new nonusual states in quantum optics. In a sense, the results of section 2,3 and those of papers [10-12] yield all necessary prerequisites for developing a quantum description of unpolarized light waves whose existence has not yet an adequate solution within the classical optics[26]. All this opens some possibilities in setting new optical experiments related, in particular, to "hidden" variables, "entangled states" and EPR paradox [1,2,5,6,19], polarization chaos, spontaneous symmetry breaking and bistability [8,11,12], "optical atoms" and reduction of quantum noises [4,6,11,12,19] etc. From other lines of possible applications of the results above we point out precise measurements in spectroscopy of anisotropic media[18] and studies of interaction of light in different new polarization states with optically active biological macromolecules (using the interrelations between the above chiral symmetries $su(2), \hat{\sigma}, \hat{P}$ of em fields and chiral properties of such molecules)[27].

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