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A METHOD OF MEASURING THE AMPLITUDE-MODULATED VACUUM FIELD NEAR A CONDUCTING MIRROR

Sun-Hyun Youn, Jai-Hyung Lee, Joon-Sung Chang
Department of Physics
Seoul National University, Seoul 151-742, Korea

Abstract

Electromagnetic fields of the vacuum mode near a conducting mirror are modified with respect to those in free space, with their amplitudes having a sinusoidal spatial dependence from the mirror. Therefore if we combine this spatially amplitude-modulated vacuum field mode and intense corehent light with a beam splitter, we may detect this fluctuation of the vacuum mode in a homodyne detection scheme. It will give new method to produce squeezed states of light with a single mirror placed close to an unused port of a beam splitter. We show that the amplitude fluctuation of the combined light can be reduced by a factor of 2 below that of the coherent light. We also discuss the limitations due to the finite line width of the laser and the effective absorption length of the photodiodes.

1 Squeezed light generation with a conducting mirror

The characteristics of vacuum fluctuations in a confined space have been studied in several cases, for example near an infinite plane conducting mirror [1], between two parallel mirrors and in a spherical cavity [3]. Here we consider a tangential vacuum-field mode with its wave vector k normal to the conducting surface (z-direction) [2], and we choose the polarization and the propagation of the electric field along the x and z-axis, respectively. Then the electric field operator for the vacuum field entering the beam splitter (see Fig. 1) becomes

$$\hat{E}_s = \sum_k \sqrt{\frac{\hbar\omega_k}{\varepsilon_o V}} \sin(kz)(\hat{a}_k e^{-i\omega_k t} - a_k^\dagger e^{i\omega_k t})\vec{x}, \tag{1}$$

where ω_k is the frequency for the mode $k(\omega_k = ck)$, $\hbar$ and ε_o have the usual meanings, and V is the normalization volume [1]. Note that the sinusoidal spatial-dependence $\sin(kz)$ for the field amplitude comes from the boundary condition that the tangential components of the electric field modes on the conducting surface should vanish.

We now consider a homodyne detection scheme to measure the quantum mechanical noise of the signal (i.e. amplitude-modulated vacuum field) as shown in Fig. 1. Let us assume, for simplicity, that the electric field for the local oscillator has a single mode(we will consider the multimode effects in the next section). Then the local oscillator field can be written as

$$\hat{E}_l = \hat{E}_{cl} + \hat{E}_q \tag{2}$$

with

$$\hat{E}_{cl} = i\sqrt{\frac{\hbar\omega}{2\varepsilon_o V}}(\alpha\, e^{-i(\omega t-k_o z)} - \alpha^*\, e^{i(\omega t-k_o z)})\,\vec{x}, \tag{3}$$

$$\hat{E}_q = i\sum_k \sqrt{\frac{\hbar\omega_k}{2\varepsilon_o V}}(\hat{b}_k\, e^{-i(\omega_k t-kz)} - \hat{b}_k^\dagger\, e^{-i(\omega_k t-kz)})\,\vec{x}, \tag{4}$$

where ω is the laser frequency ($\omega = ck_o$). Here we have decomposed the coherent light $\hat{E}_l$ into $\hat{E}_{cl}$ and $\hat{E}_q$ [4]. $\hat{E}_{cl}$ is the classical analogy of the coherent light which has a definite amplitude and phase, whereas $\hat{E}_q$ is the quantum fluctuation in the electric field of the coherent light which is simply equivalent to the vacuum fluctuation.

Considering the vacuum mode relation in the Fig. 2, we can get the electric field in fluctuating vacuum modes at the detector 1

$$\begin{aligned}\hat{E}^{(+)}_{vac,1} &= \sum_k i\sqrt{\frac{\hbar\omega_k}{4\varepsilon_o V}}\,\{\sqrt{T}\hat{b}_k^\dagger e^{i(\omega_k t-kZ_1)} + \hat{a}_{1,k}^\dagger e^{i(\omega_k t+kz_1)} \\ &- R\hat{a}_{1,k}^\dagger e^{i(\omega_k t-kz_1)} - \sqrt{RT}\hat{a}_{2,k}^\dagger e^{i(\omega_k t-kz_1)}\},\end{aligned} \tag{5}$$

where $z_1(Z_1)$ represents the length for the beam path between the single mirror (laser) and the detector 1. We have separated the electric field into positive- $\hat{E}^{(+)}$ ($\sim e^{i\omega t}$) and a negative-frequency components $\hat{E}^{(-)}$ ($\sim e^{-i\omega t}$), and note the propagation-direction of traveling wave. We also add the factor $\frac{1}{\sqrt{2}}$ for the normalization of the vacuum fluctuation.

Using photodetection theory [5], we obtain the photocurrent $\hat{I}$ as

$$\hat{I} = \int_{-\infty}^{t} dt' h(t-t')\hat{E}^{(+)}(z,t')\,\hat{E}^{(-)}(z,t'), \tag{6}$$

where $h(t-t')$ is the photodetector response function [6]. Assuming instantaneous response of the detector, we can approximate $h(t-t')$ as $h\delta(t-t')$. Then the photocurrent induced on the detector 1 is

$$\hat{I}_1(z,t) = h\{\sqrt{T}\hat{E}_{cl}^{(+)}(Z_1,t) + \hat{E}^{(+)}_{vac,1}\} \times \{\sqrt{T}\hat{E}_{cl}^{(-)}(Z_1,t) + \hat{E}^{(-)}_{vac,1}\}. \tag{7}$$

Since the expectation value of $\alpha^*\alpha$ is much greater than that of $\hat{a}^\dagger\hat{a}$ and $\hat{b}^\dagger\hat{b}$, we keep only the terms which contain α or α^*. If the reflectivity (R) of the beam splitter approaches to 1, the Eq. (5) surely represents the positive frequency part of the Eq. (1). In that case, $\hat{a}_1$ and $\hat{a}_2$ are totally decoupled and we can find the standing vacuum mode in the port 1.

Using the condition in Eq. (5), $\hat{I}_1$ is obtained as

$$\begin{aligned}\hat{I}_1^o(z_1,Z_1) = \frac{|\,\alpha\,|}{\sqrt{2}}\{&\sqrt{T}e^{i\phi}(T + e^{-ik_o(Z_1+z_1)} - Re^{-ik_o(Z_1-z_1)})\hat{a}_1 - T\sqrt{R}e^{i\phi}(1 + e^{-ik_o(Z_1-z_1)})\hat{a}_2 \\ &+\sqrt{T}e^{-i\phi}(T + e^{ik_o(Z_1+z_1)} - Re^{ik_o(Z_1-z_1)})\hat{a}_1^\dagger - T\sqrt{R}e^{-i\phi}(1 + e^{ik_o(Z_1-z_1)})\hat{a}_2^\dagger\},\end{aligned} \tag{8}$$

where $\hat{I}_1^o$ is the time-averaged photocurrent which is normalized with respect to h, and we have replaced α with $|\,\alpha\,|\,e^{i\phi}$. Note that in Eq. (8) we have included only one vacuum field mode identical to the laser light with its wavevector k and frequency $\omega = ck_o$ since only that mode survives as a result of the time average effect on the detector. We have neglected the $\alpha^*\alpha$ term in $\hat{I}_1^o$, since it does not include any fluctuation and corresponds to a constant dc current which can be filtered out by ac coupling. And the average values of $\hat{I}_1^o$ are expected to be zero, since the operators $\hat{a}_1$ and $\hat{a}_2$ both represent the fluctuating vacuum field modes.

The sum and difference of these photocurrents I_1^o, I_2^o are obtained from the Eq. (8). It becomes

$$\begin{aligned}\hat{I}_1^o \pm \hat{I}_2^o &= \frac{|\alpha|}{\sqrt{2}}[\sqrt{T}e^{i\phi}\{T + e^{-ik_o(Z_1+z_1)} - Re^{-ik_o(Z_1-z_1)} \pm R \pm Re^{-ik_o(Z_2-z_2)}\}\hat{a}_1 \\ &+ \sqrt{T}e^{-i\phi}\{T + e^{ik_o(Z_1+z_1)} - Re^{ik_o(Z_1-z_1)} \pm R \pm Re^{ik_o(Z_2-z_2)}\}\hat{a}_1^\dagger \\ &- \sqrt{R}e^{i\phi}\{T + Te^{-ik_o(Z_1-z_1)} \pm R \pm e^{-ik_o(Z_2+z_2)} \mp Te^{-ik_o(Z_2-z_2)}\}\hat{a}_2 \\ &- \sqrt{R}e^{-i\phi}\{T + Te^{ik_o(Z_1-z_1)} \pm R \pm e^{ik_o(Z_2+z_2)} \mp Te^{ik_o(Z_2-z_2)}\}\hat{a}_2^\dagger]. \end{aligned} \tag{9}$$

We now evaluate the square of each quantity to find its fluctuations. Squaring the Eq. (8), we gather the terms $(\hat{a}_i\hat{a}_i^\dagger)$ which contains the nonzero vacuum expectation value. Then the results become

$$[I_1^o(z_1, Z_1)]^2 = \frac{T\,|\,\alpha\,|^2}{2}\{T^2 + 2T\cos[k_o(Z_1 + z_1)] + 1 + R^2 - 2R\cos(2k_oz_1) + 2RT\}, \tag{10}$$

and the modulation effect due to the $\cos(Z_i + z_i)$ term will vanish , since the origins of the Z_1 and Z_2 are not an absolute one for the traveling local oscillator mode. However, the origin of the z_1 and z_2 is absolutely fixed to the mirror position so we will keep only this modulation effect due to $\cos(2z_{1,2})$ term. Together with the relation $R + T = 1$, Eq. (10) finally become

$$\overline{I_1^2(z_1)} = T\,|\,\alpha\,|^2\,\{1 - R\cos(2k_oz_1)\}, \tag{11}$$

These results , Eq. (11), clearly show that the intensity fluctuations measured at each photodetector exhibit a sinusoidal spatial dependence. The photocurrent fluctuations for each detector comes partly from the vacuum field in the local oscillator itself, and partly from the vacuum field modified by the perfect mirror. Note that the sinusoidal modulation in Eq. (11), which is responsible for amplitude squeezing, is totally due to the vacuum field mode that has the same frequency as the laser oscillator but is altered by the perfect mirror.

For the balanced homodyne detection, assuming $T = R = 1/2$, the resulting quantum fluctuation of the *modulated* light may fall below that of the coherent state i.e. become squeezed as shown in Fig. 2. The fluctuation of the coherent state without the perfect mirror can be calculated by replacing $\cos(k_oz_{1,2})$ by its average value zero so that Eq. (11) simply becomes a constant value $\frac{|\alpha|^2}{2}$ as the result of the usual beam splitter . On the other hand, if we consider the situation such that the distance $z_i(i = 1, 2)$ between the perfect mirror and each detector is well resolved and satisfies $k_oz_i = n\pi(n$: positive integer), then Eq. (11) reduces to $\frac{|\alpha|^2}{4}$. In other words, the measured quantum fluctuations may fall below that of the coherent vacuum state by as much as 50%. This squeezing limit comes from the intrinsic fluctuations of the coherent state of the laser itself, which is combined with the modified vacuum field with its amplitude suppressed at the detector position such that $k_oz_i = n\pi(i = 1, 2)$.

The quantum fluctuations of the square of the sum and difference of the photocurrents I_1^o and I_2^o are similarly obtained from Eq. (9). Squaring this Eq. (9) and keeping the non vanishing terms, we can obtain

$$
\begin{aligned}
&< (I_1^o \pm I_2^o)^2 > \equiv \overline{(I_1 \pm I_2)^2} \\
= \; & \frac{T \mid \alpha \mid^2}{2}[T^2 \pm 2TR + 3R^2 + 1 - 2R\cos(2k_o z_1) \pm 2R\cos(2k_o z_1) \mp 2R^2] \\
+ \; & \frac{R \mid \alpha \mid^2}{2}[R^2 \pm 2TR + 3T^2 + 1 - 2T\cos(2k_o z_2) \pm 2T\cos(2k_o z_2) \mp 2T^2],
\end{aligned}
\tag{12}
$$

where we have used the relation $Z_1 - z_1 = Z_2 - z_2$. Using the relation $T + R = 1$, we simplify this equation (12) like this,

$$\overline{(I_1 \pm I_2)^2} = \mid \alpha \mid^2 [1 - (1 \mp 1)TR\{\cos(2k_o z_1) + cos(2k_o z_2)\}]. \tag{13}$$

We find that fluctuations of the sum of I_1^o and I_2^o in Eq. (13) is not dependent on z_1, z_2, since this fluctuation has come from the local oscillator. On the other hand, as in the case of $\overline{I_1^2}$ and $\overline{I_2^2}$, the fluctuation of $\overline{(I_1 - I_2)^2}$ which comes from the modulated vacuum field contains the important spatial modulation with a period of π/k_o. The spatially-averaged fluctuation of $\overline{(I_1 - I_2)^2}$ is $\mid \alpha \mid^2$, which is just the quantum fluctuation of the free-space vacuum field without the mirror. However, the fluctuation of the difference at the detector position such that $k_o z_i = n\pi (i = 1,2)$ becomes zero. The resulting fluctuations of $\overline{(I_1 - I_2)^2}$ in Eq. (13) are also plotted in Fig. 2. In this Fig. 2, we have replaced the $\cos(k_o z_2)$ with zero, which represents the average value, and have plotted $\overline{(I_1 - I_2)^2}$ as a function of z_1.

2 Practical limits: finite linewidth and absorption length

So far, we have considered a monochromatic coherent light for the local oscillator. The laser light, however, always has a finite linewidth, so that the modulation depth in Eqs.(11) and (13) may be decreased as the linewidth increases. In this section, we will consider the practical limits to squeezing due to the finite active layer depth of the photodetector as well as the finite laser linewidth.

The effects of the line broadening can be calculated from the Gaussian probability density function [7]

$$P(k) = \frac{1}{(\sqrt{\pi}\Delta k)} e^{[-(k-k_o)^2/\Delta k^2]}. \tag{14}$$

Another practical limit comes from the finite thickness of effective absorption-layer of the photodetectors. The period of the spatial modulation of the quantum fluctuations is of the order of the optical wavelength, whereas the effective depth of the detectors is typically much larger than the wavelength. The probability that a photon is converted into an electron-hole pair at a distance ξ from the surface of the detector's active region can be written as [8]

$$P_D(\xi) = \kappa e^{-\kappa\xi}, \tag{15}$$

where κ the absorption coefficient of the detector material.

We can evaluate the expectation values of the fluctuations in Eqs. (11) and (13), with respect to this probability function and obtain

$$\overline{I_1^2} = T \mid \alpha \mid^2 [1 - Re^{-z_1^2 \Delta k^2} \times \frac{\kappa\{\cos(2k_o z_1 + \phi_o) - e^{-\kappa D}\cos[2k_o(z_1 + D) + \phi_o]\}}{\sqrt{4k_o^2 + \kappa^2}}] \tag{16}$$

and

$$\begin{aligned} \overline{(I_1 - I_2)^2} = & \mid \alpha \mid^2 (1 - 2TR \times [\\ & \frac{\kappa e^{-z_1^2 \Delta k^2}\{cos(2k_o z_1 + \phi_o) - e^{-\kappa D}\cos[2k_o(z_1 + D) + \phi_o]\}}{\sqrt{4k_o^2 + \kappa^2}} \\ + & \frac{\kappa e^{-z_2^2 \Delta k^2}\{cos(2k_o z_2 + \phi_o) - e^{-\kappa D}\cos[2k_o(z_2 + D) + \phi_o]\}}{\sqrt{4k_o^2 + \kappa^2}}]). \end{aligned} \tag{17}$$

where $\phi_o = \arctan(\frac{2k_o}{\kappa})$ and D is the thickness of the depletion layer of the photo detector. Note that we also have used the fact that the variation of $e^{-(Z+\xi)^2 \cdot \Delta k^2}$ in the region [0,D] is so small that we can extract this term as a constant out of the integrand in the above equations.

We can easily see from Eqs. (16) and (17) that all the modulation terms which have z_1, z_2 dependence are reduced by a scale factor $e^{-z_i^2 \Delta k^2}\kappa/\sqrt{4k_o^2 + \kappa_2}$. In Figure 3, we plot the fluctuations of $\overline{I_1^2}$ and $\overline{(I_1 - I_2)^2}$ in Eqs.(16) and (17) with respect to the absorption coefficient κ and Δz_1. To observe deep modulations more than anything else, we need a large absorption coefficient κ for the detector. For an Ar^+ laser light ($\lambda = 514.5nm$ or $k_o = 122\,122cm^{-1}$), the absorption coefficients of the Si and Ge are about $10^4 cm^{-1}$ and $4 \times 10^5 cm^{-1}$, respectively. The scale factors are thus 0.04 and 0.85 for Si and Ge detectors. Therefore it would be possible to observe the squeezing effects with a Ge type detector. However, it might be hard to measure a modulation of the quantum fluctuations for the Ar^+ ion laser with a silicon detector . If we make a very thin depletion layer for the silicon detector in order not to wash out the spatial modulation, the quantum efficiency η will decrease, which implies the fact that all the photons are not converted into electron-hole pairs. Some extra noise will then add up with a relative amplitude $(1 - \eta)$, and this extra noise will also degrade the modulation of the quantum fluctuations.

The linewidth of a single-mode frequency-stabilized Ar^+ laser is better than 1 MHz , which is equivalent $\Delta k = 0.0002cm^{-1}$. The scale factor for this linewidth $e^{-\Delta k^2 z^2}$ is 0.9996 for $z \approx 100cm$. Therefore the practical limit to the modulation or squeezing effects is mainly due to the characteristics of the photodetectors in use.

3 Conclusion

We have proposed the possibility to produce squeezed state of light simply with a single conducting mirror without elaborate experimentation to produce squeezed vacuum. The electromagnetic field modes near a perfect mirror are modified with respect to those in free space due to the cavity QED effects: the modes of the vacuum fluctuations have sinusoidal spatial dependence from the mirror. These modified vacuum-field modes, when combined with a coherent light, may produce spatially modulated squeezing effects for the coherent light, which can be measured in a balanced homodyne detection scheme. If we divide a coherent local oscillator with a 50:50 beam splitter and combine with the modulated vacuum field, we estimate that the quantum fluctuations of the combined light are reduced by as much as 50% below the intrinsic fluctuations of the coherent light at distances $z = n\lambda$ (n : positive integer) between the photodetectors and the perfect mirror. Moreover, at those position, we reduced the fluctuation of the difference of two beams which are come from the unused beam splitter. In other words, we can obtain the highly sub-Poissonian fluctuation of the difference which may come from the totally correlated beams such as photon twins generated by the parametric amplification [9].

The finite laser linewidth degrades the spatial modulation by a factor $e^{-Z_2 \Delta k^2}$, but this factor can be neglected if we use a narrow linewidth laser. Decreasing the distance between the perfect mirror and the detector will also help. The imperfect reflectivity of the mirror may slightly decrease the modulation of the vacuum field. But, since the reflectivity of a metal-coated mirror at the optical frequency is about 97% , this also gives negligible effects. The most important practical limit comes from the characteristics of the photodetector. We need a detector whose quantum efficiency is close to unity and whose absorption coefficient is large enough that the depth of the effective absorption region is smaller than the wavelength (e.g., Ge type photo detector). Including all these limits we have calculated the quantum fluctuation of the light intensity in the last section and shown in Fig. 3. The results indicate that we may increase the ratio of the average intensity to the intensity fluctuation using this homodyne detector with a good conducting mirror.

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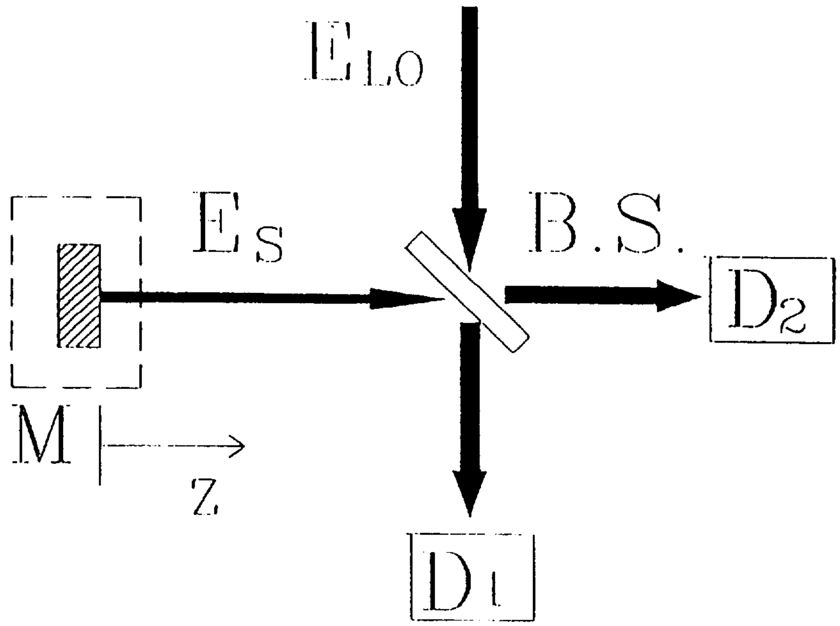

Fig. 1 Homodyne detector. D_1, D_2 : photo detector, B.S. : beam splitter. E_s and E_L represent the signal and local oscillator, respectively.

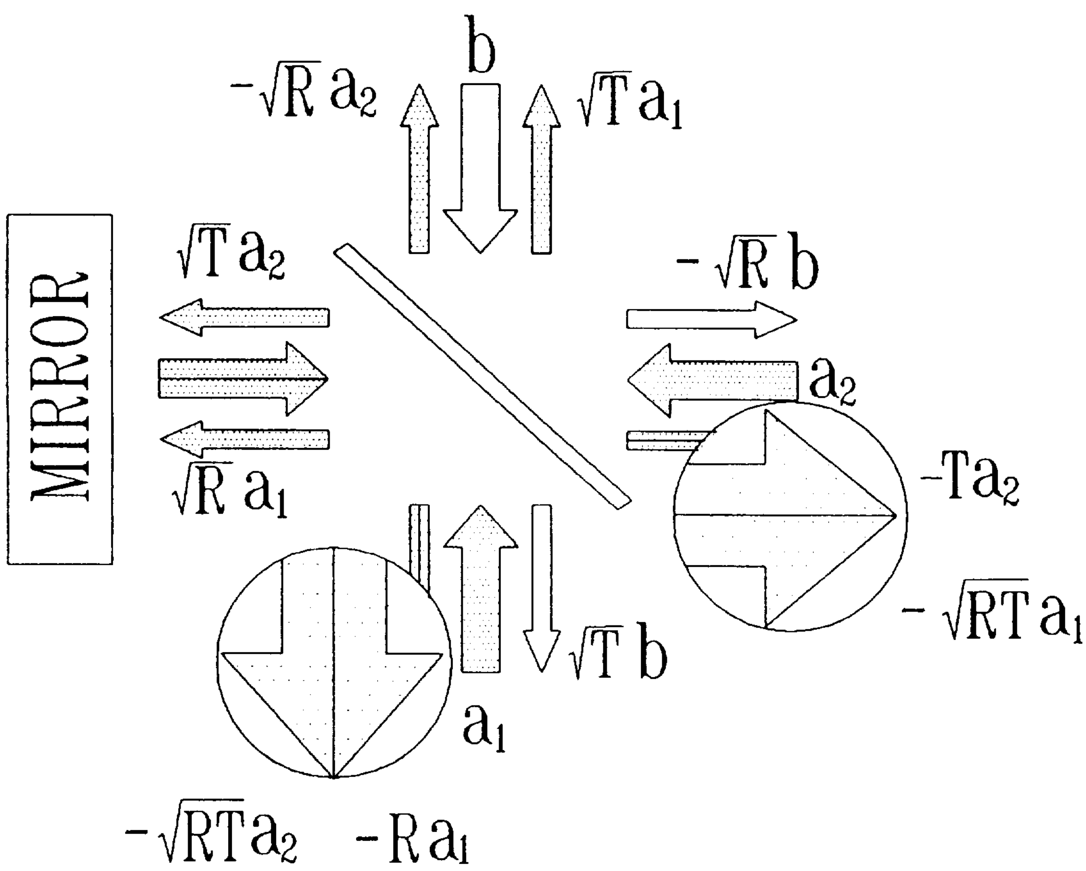

Fig. 2 The vacuum mode relations in the beam splitter with a conducting mirror.

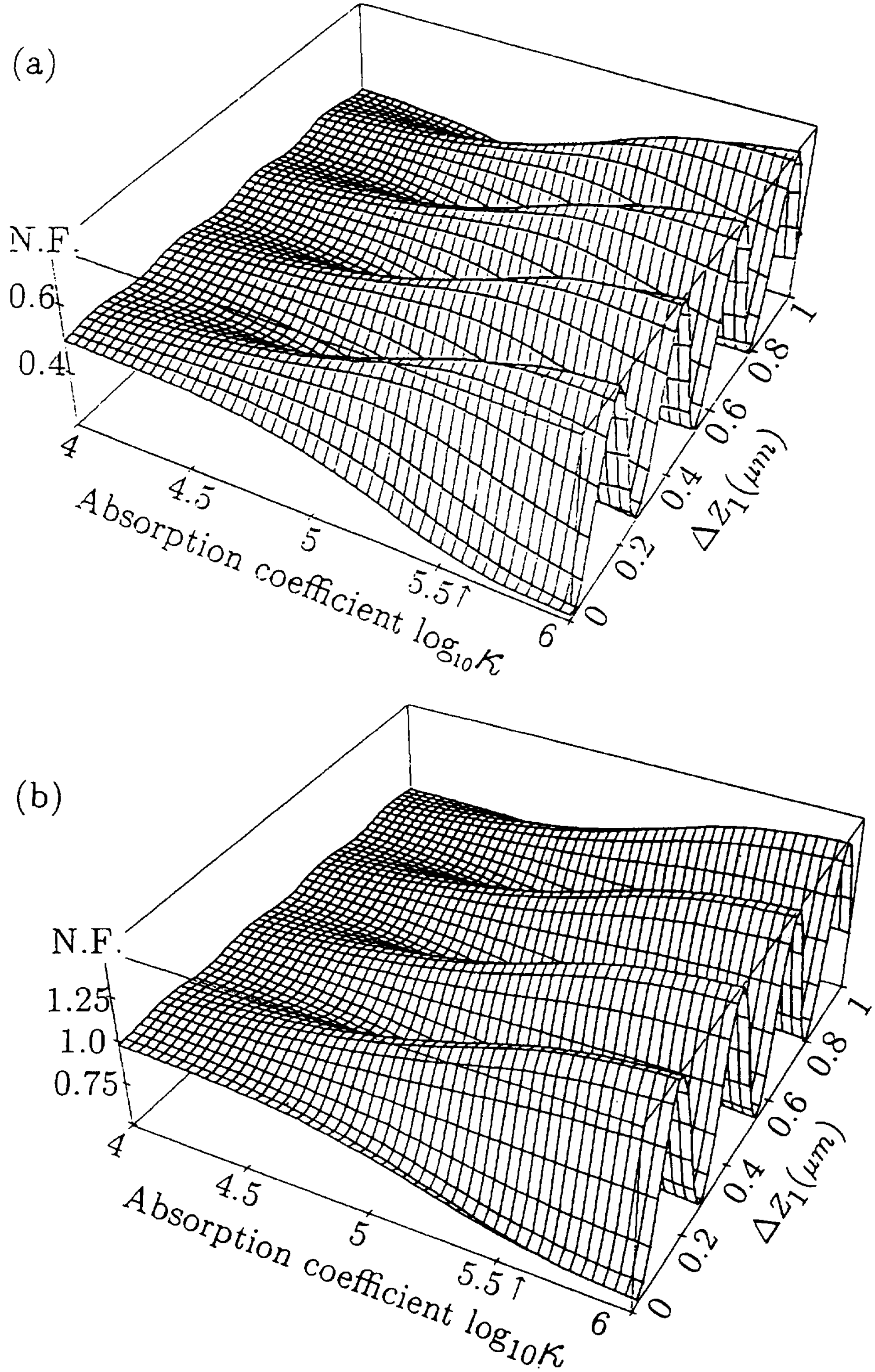

Fig. 3 The amplitude fluctuation of (a)$\overline{I_1^2}$ and (b) $\overline{(I_1 - I_2)^2}$, ,normalized to $|\ \alpha\ |^2$, as a function of $\log \kappa$ (κ : absorption coefficient in units of cm^{-1} and Δz_1 is displacemen.) Note that the intrinsic quantum fluctuation without the mirror correspond to 0.5 in (a) and 1.0 in (b) on the vertical axis. The arrow indicates for the Ge detector which has $\kappa = 4 \times 10^5 cm^{-1}$ (N.F. : Normalized Fluctuation)