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Further Evidence For the EPNT Assumption

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ABSTRACT

We recently proved a theorem extending the Greenberger-Horne-Zeilinger (GHZ) Theorem from multi-particle systems to two-particle systems. This proof depended upon an auxiliary assumption, the EPNT assumption (Emptiness of Paths Not Taken). According to this assumption, if there exists an Einstein-Rosen-Podolsky (EPR) element of reality that determines that a path is empty, then there can be no entity associated with the wave that travels this path (pilot-waves, empty waves, etc.) and reports information to the amplitude, when the paths recombine. We produce some further evidence in support of this assumption, which is certainly true in quantum theory. The alternative is that such a pilot-wave theory would have to violate EPR locality.

INTRODUCTION

Recently, in trying to extend the GHZ (Greenberger-Horne-Zeilinger) Theorem[1,2] down to two-particle systems[3], we produced a proof that we realized depended on a further assumption, which went beyond the EPR (Einstein-Podolsky-Rosen) assumptions[4]. This assumption was the EPNT assumption-- the Emptiness of the Paths Not Taken. This assumption ruled out the possibility of any kind of information-bearing entity traveling down a path, provided one could produce an EPR element of reality connected with the path being empty. The EPR criterion depends on one being able to perform an experiment far away, without in any way affecting any particle that could possibly be travelling down this path. Then if this experiment shows that the path is empty, the path must be truly empty, according to EPR, since one has not interfered in any way with anything along the path. This fact of emptiness is then an "element of reality", because it is true independently of anything an experimenter might later do that might interfere with the path or particles along it.

One might be tempted to think that the EPNT assumption rules out any kind of interference at all, as when a particle passes through a beam splitter, and the two paths are later recombined and interfere. But for a single particle, one cannot produce an element of reality connected with the path, because any measurement on the particle to determine which path it takes will necessarily disturb it. So the EPNT assumption does not apply to one-particle systems. But for a two-particle system, one may make a measurement on one particle that determines which path the second particle takes, and here the EPNT assumption does apply, and it gives results that accord with quantum theory. Further questions concerning the applicability and plausibility of the EPNT assumption are dealt with in Ref. (3). The reason that the assumption is worth exploring in detail is that there are other theories, such as pilot wave theories, that compete with quantum theory and that do depend on inforation-bearing empty waves for their effects. It should also be pointed out that an alternative, complementary approach to this problem, not along the lines of GHZ, has been taken by L. Hardy, who can prove that for a certain percentage of particles in the beam, the GHZ theorem must be true for two particles[5].

THE SITUATION IN QUANTUM THEORY

One can easily show that within quantum theory if a path is empty, it is truly empty, which means that if the amplitude for a particle to be travelling along a path is zero, then no information can be transmitted along that path. For example, in Fig. (1) we depict a unitary device which takes an incoming wave function that can be along either of the paths 1 or

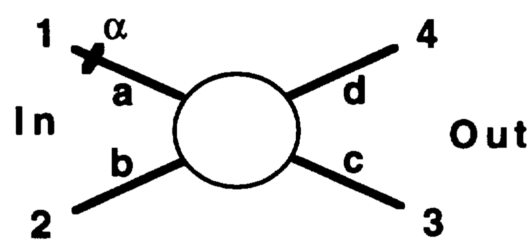

Fig. (1). Unitary Device for a 2-path Amplitude.
The particle enters from the left and exits to the right.
There is a phase shifter of angle α located on path 1.

2, with amplitudes a or b respectively, and converts it into a wave function travelling along the paths 3 and 4, with amplitudes c and d respectively.

Since the device is unitary, its most general form is

$$\begin{pmatrix} c \\ d \end{pmatrix} = e^{i\lambda} \begin{pmatrix} \cos\gamma & e^{i\beta}\sin\gamma \\ -e^{i\delta}\sin\gamma & e^{i(\beta+\delta)}\sin\gamma \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix}.$$

If now a phase-shifting device, that shifts the phase by α, is placed into beam 1, the incoming beam will change from amplitude a to $ae^{i\alpha}$. An infinitesimal change in α will produce the result in beams 3 and 4,

$$\delta c = e^{i\lambda}\cos\gamma\, ae^{i\alpha} i\, \delta\alpha,$$

$$\delta d = e^{i((\lambda+\delta)}\sin\gamma\, ae^{i\alpha} i\, \delta\alpha.$$

One sees that both of these terms will be zero if a is zero. This result shows that if the beam 1 is empty (i.e., a=0), there is no way to transmit any change in α to any amplitude downstream of beam 1, even if there is a unitary connection between the beams. This result is easily generalized to any number of particles and amplitudes. It shows that according to quantum mechanics, no information can be transmitted through an empty beam. This of course is the essential content of the EPNT assumption.

We are assuming that quantum theory gives correct results and that it is the burden of any alternative theory to reproduce these results. The reason that the discussion cannot stop here is that one might choose not to believe quantum mechanics and say that there are indeed alternative ways to produce the results of the theory without accepting the unitarity and linearity of the theory. We shall show that multi-particle superpositions place a heavy burden on any such theory.

INFORMATION PASSED ALONG EMPTY BEAMS VIOLATES THE UNCERTAINTY PRINCIPLE

We will now show that if one assumes that a beam is empty, as an EPR element of reality, but one still insists that it can carry information, then if this information can be operationally transmitted to another beam, this information can violate the uncertainty principle.

We shall work out a particular example, but it is obvious that the thrust of the argument is very general. Consider a particle at rest that can decay into two particles, as the one at point O in Fig. (2). The two

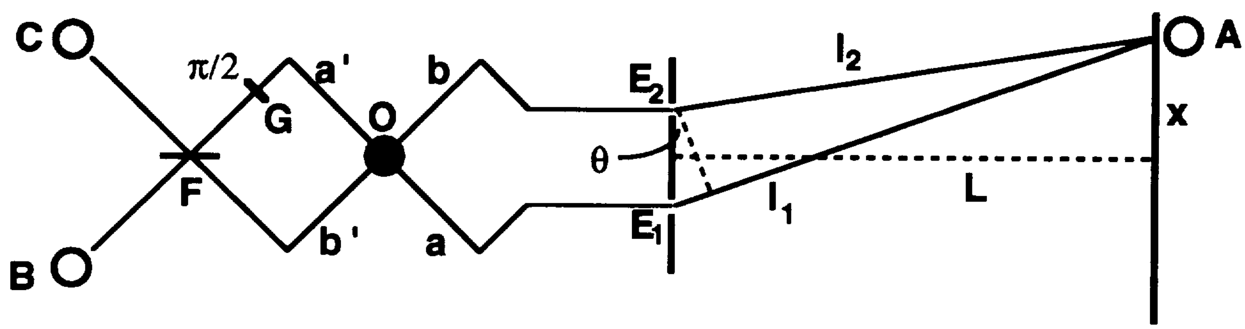

Fig. (2). Two-Slit Experiment With A Two-Particle Interferometer.
An interference pattern is produced by measuring coincidences between detectors A, B, and C.

particles come off in opposite directions, and are restricted by slits to the two sets of directions, *a-a'*, and *b-b'*. So the state of the system after decay is

$$\psi = \frac{1}{\sqrt{2}}(|aa'\rangle + |bb'\rangle).$$

The primed particle paths are directed through a beam-splitter at F toward the detectors B and C. (We assume, for unitarity, that the reflected ray picks up a 90^o phase). The paths of the unprimed particle are directed toward a screen with two slits, such that path *a* leads to one slit, E_1 while path *b* leads to the other slit, E_2. Each of the slits is of width d, and they are separated by a distance D, such that D » d. The diffraction pattern formed at a distance L from the slits is picked up by the detector A. We also assume that L » D, and that the position of A is given by *x*, as measured at L, perpendicularly from the center of the slits (see Fig. (2)).

An important feature of multi-particle beams that applies here should be noted. If the detector A is moved as a function of the distance x, there will be no diffraction pattern observed, as there will be no single particle interference from this setup. Only if A is monitored in coincidence with the detector B or C will a pattern appear. We shall confirm this below. The reason we have used this two-particle setup to produce diffraction is that we can remove the beam-splitter at F. Then if detector B fires, we know that the particle had taken the path a', and so its partner must have taken the path a. Thus this particle must have entered the slit E_1 and there will be only a one-particle interference pattern at A, of angular width $\theta \approx \lambda / d$, where we assume for convenience that $\lambda \ll d$. Similarly, if detector C fires, we know the particle must have entered the slit E_2 and will also produce a one-particle interference pattern at A. Since we can determine which path and slit the unprimed particle takes, without in any way interfering with the particle, this knowledge is an EPR element of reality. In other words, according to EPR, it is an objective fact. So even if we do not bother to remove the beam-splitter at F, EPR would conclude that the element of reality exists, because we could have removed the beam splitter without affecting the particle, and so the particle actually takes one slit or the other. They would conclude that because the particle takes one path or the other, but quantum theory is powerless to describe this fact, that quantum theory is therefore an incomplete theory.

But of course, according to quantum theory, whether we remove the beam-splitter or not is a crucial fact, one that completely changes the context of the experiment. If we remove it, then indeed the particle is in one path or the other. But if we do not remove it, the particle cannot be described as being in either path.

If we perform the experiment with the beam-splitter in place at F then the wave function ψ becomes (we are also including a $\pi/2$ phase shifter at G, purely for computational convenience)

$$\begin{aligned} \psi &\rightarrow \tfrac{1}{\sqrt{2}}\varphi(k_x)\left(ie^{ik\ell_1}|Aa'\rangle + e^{ik\ell_2}|Ab'\rangle\right) \\ &\rightarrow \tfrac{1}{2}\varphi(k_x)\left(ie^{ik\ell_1}|A\rangle(|B\rangle + i|C\rangle) + e^{ik\ell_2}|A\rangle(|C\rangle + i|B\rangle)\right) \\ &= \tfrac{1}{2}\varphi(k_x)|A\rangle\left(i(e^{ik\ell_1} + e^{ik\ell_2})|B\rangle + (e^{ik\ell_2} - e^{ik\ell_1})|C\rangle\right). \end{aligned}$$

The probability for coincident counts at A and B, or at A and C is

$$P_{AB} = |\varphi(k_x)|^2 \cos^2 \tfrac{k\Delta\ell}{2}, \quad P_{AC} = |\varphi(k_x)|^2 \sin^2 \tfrac{k\Delta\ell}{2},$$
$$\Delta\ell = \ell_1 - \ell_2.$$

In this equation $\varphi(k_x)$ represents the Fourier Transform of the single slit pattern produced by each of the slits, for which $\theta \approx \lambda / d$, and it is much wider than one of the two-particle pattern maxima, whose width is of order $\theta \approx \lambda / D$. So in fact, in the region of the central maximum where the term $k\Delta\ell/2 \approx \pi Dx / L\lambda$ contributes, $\varphi(k_x)$ can be considered to be a constant. (In this region, the minimum x_m of the cosine term occurs when the argument equals $\pi/2$, or $x_m = L\lambda/2D \approx L\theta$, and $\theta \approx \lambda/2D$.) One can experimentally isolate this central maximum from the others, and since both slits contribute to it, one has $\Delta x \approx D$, while $\Delta p_x \approx p\theta \approx p\lambda / D \approx \hbar / D$. Thus this central maximum is of the order of a minimum uncertainty packet. Note also that if one only triggers the detector A, ignoring the detectors B and C, one will get a number of counts independent of the path difference $\Delta\ell$ between the beams, $P_A = P_{AB} + P_{AC} = const.$, which proves our original assertion that one must count coincidences to see the interference pattern in this experiment.

For the case when one removes the beam-splitter, one has only the top equation above for ψ,

$$\psi \to \frac{1}{\sqrt{2}}\varphi(k_x)\left(ie^{ikl_1}|Aa'\rangle + e^{ikl_2}|Ab'\rangle\right)$$

If then counter B fires in coincidence with A, one knows that the unprimed particle is in the beam a' and similarly if counter C fires it is in beam b', and so the coincidence counting rates will be $P_{AB} = \frac{1}{2}|\varphi(k)|^2$, $P_{AC} = \frac{1}{2}|\varphi(k)|^2$, which will be a constant on the scale of the two-particle pattern, but will fall off as $\theta \approx \lambda / d$. This is consistent with the uncertainty principle, since in this case, the particle is going only through one slit, so that $\Delta x \sim d, \Delta p \sim p\theta \sim \hbar / d$.

Now we come to the point of the argument. What if one were to believe that in the case of an ordinary single-particle two-slit experiment, when one can detect into which slit the particle enters one will obtain single slit patterns, as quantum theory predicts? But when one does not know which slit the particle enters there is some kind of information-bearing pilot wave that carries the information about the second slit, so that even though the particle travels through one slit only, nonetheless it is aware of the existence of the second slit through the intermediary of the pilot wave, and so one gets a two-slit diffraction pattern. It is for this case that we have devised our experiment above. For we can produce the EPR element of reality needed to prove the particle takes only one beam, by removing the beam-splitter. However if one believes that the element of reality persists when one does not remove the beam splitter, and also that a pilot wave of some sort carries information about the second slit, so that the diffraction pattern can occur, we believe that this leads to a contradiction in our experiment.

In our experiment, if one accepts another principle of EPR, that of locality, then one must accept the fact that the unprimed particle receives no information that can tell it whether in fact the beam splitter at F has been removed or not. So there is no way for the particle to know how to evaluate the information obtained from the pilot wave. Does it lead to a diffraction pattern or not? In an ordinary one-particle experiment, there is no way to observe that the particle has actually taken one slit or the other. But in our experiment, we can provide that information without disturbing the particle approaching the slit. According to the EPNT assumption, when one of the paths is truly empty in the EPR sense, there can be no information transmitted along the other path. The negation of this assumption implies that some information *can* be carried along this other path. If this is so, are there any experimental implications of this? If not, it is merely an idle statement.

The way to exploit these facts is to assume that the beam-splitter is present, but that there is another detector present, in beam b'. This detector will fire if the particle takes beam b'. If it does *not* fire, then we know that the particle is in beam a-a'. But there will be an empty wave in beam b-b'. Since the detector has not fired, the empty wave will presumably pass along to the beam splitter at F. Does it share any of its information with the other beam? To decide this, we assume that there is some parameter β that determines how much information the pilot wave carries along one beam when it is known that the particle takes the other beam, through the other slit. In that case, the particle will produce a coincidence count probability for the counters A and B to fire,

$$P \propto \frac{1}{4}|\varphi(k_x)|^2\left|e^{ikl_1} + \beta e^{ikl_2}\right|^2,$$

where β varies between 0 and 1.

This would lead to a diffraction pattern with a contrast of $C = 2\beta/(1+\beta^2)$. Not only would this disagree with quantum theory, but also with the uncertainty principle directly, since then $(\Delta p)^2 \sim \left[\beta^2(\hbar / D)^2 + (1-\beta^2)(\hbar / d)^2\right]$, while $(\Delta x)^2 \sim d^2$, since one knows which slit the particle actually took. For finite β, this violates the uncertainty principle, giving in the limit of $\beta \to 1$ the result $\Delta x \Delta p \to \hbar d / D \ll \hbar$. The alternative is that the theory must violate EPR locality.

THE IMPOSSIBILITY OF PILOT WAVES IN A THREE-PARTICLE SYSTEM

The argument supporting the EPNT assumption in a three-particle system is even stronger than for two particles. Again we shall work out a special example, but the results are generalizable. Consider a particle that decays into three particles. If the particles are of the same mass, and when they are counted, it is checked that each has the same energy, then they will come off at 120^o apart. They are now restricted by slits to three sets of directions, *a-a'-a"*, *b-b'-b"*, and *c-c'-c"*. (Particle 1 is unprimed, particle 2 is primed, and particle 3 is double-primed. If particle 1 takes path *a*, then 2 must take path *a'*, and 3 takes path *a"*, etc.) (See Fig. (3)). This is a simple generalization of the two-particle process in

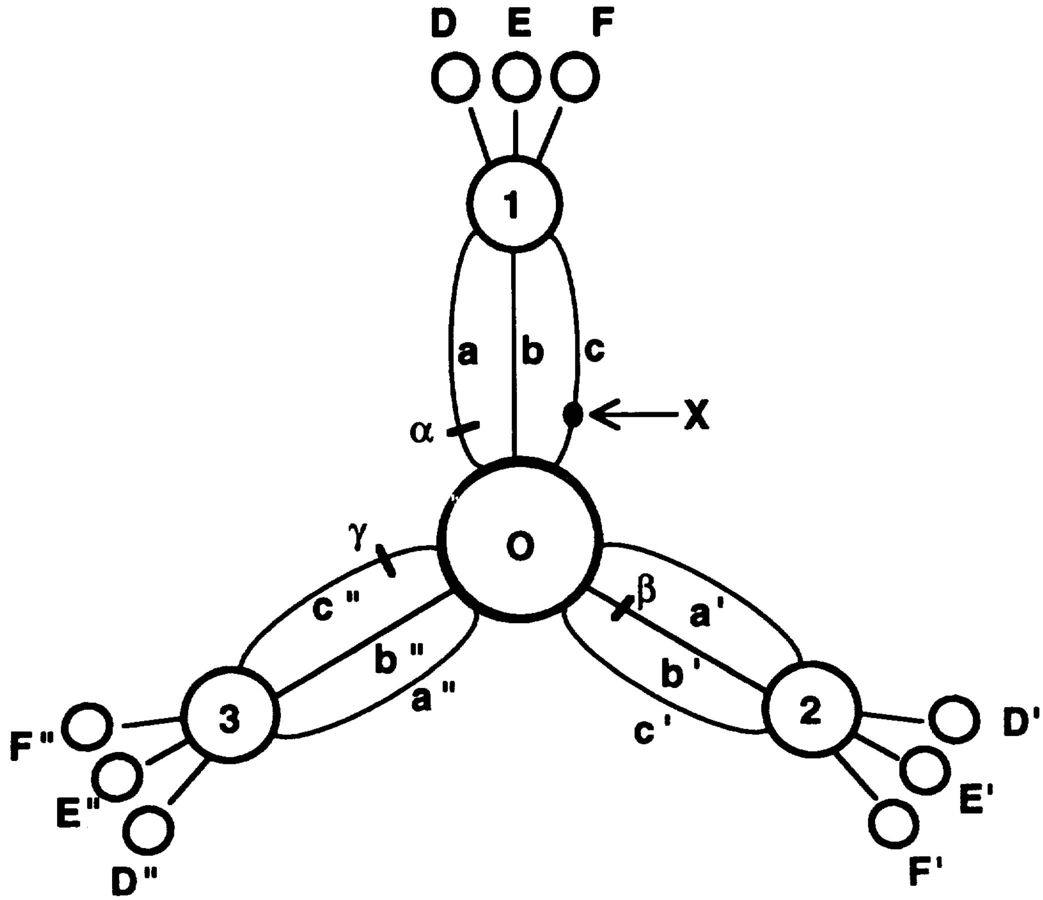

Fig. (3). A Three-Particle Interferometer With Three Tritters.
A particle at O decays into three particles which take the possible paths *a-a'-a"*, *b-b'-b"*, or *c-c'-c"*. The three paths for each particle converge at a tritter, and then pass to one of three detectors. Each particle has one phase shifter in one path, α, β, or γ. In the second part of the experiment, a detector is placed at X to determine whether the particles have taken the paths *c-c'-c"*.

the previous example. Each particle is now refocussed into a unitary 3-way beam-splitter. (We have previously called such devices "multi-ports", or "critters", and in particular, a 3-way device is a "tritter"[6]. Such devices can emulate an arbitrary unitary transformation of the system.) For simplicity, our particular tritter is taken to have a specific unitary transformation (see Fig. (4)). If the beams

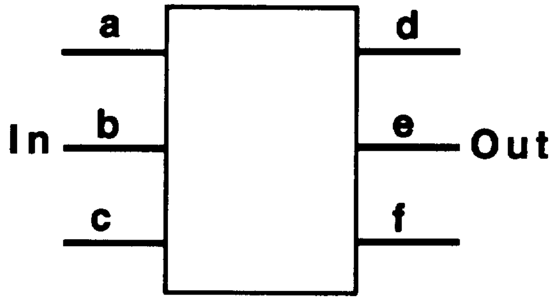

Fig. (4). A Three-Particle Unitary Device.
We call this device a "tritter". The three input beams a, b, c, are transformed unitarily into the output beams d, e, f.

a, b, c, are the incident beams, and d, e, f, are the outgoing beams, they will be related by the relations

$$|a\rangle \to \tfrac{1}{\sqrt{3}}(|d\rangle + |e\rangle + |f\rangle),$$
$$|b\rangle \to \tfrac{1}{\sqrt{3}}(|d\rangle + \lambda|e\rangle + \mu|f\rangle),$$
$$|c\rangle \to \tfrac{1}{\sqrt{3}}(|d\rangle + \mu|e\rangle + \lambda|f\rangle),$$

where 1, $\lambda = e^{2\pi i/3}$, and $\mu = e^{4\pi i/3}$ are the cube roots of unity, and

$$\mu^2 = \lambda,\ \lambda^2 = \mu,\ \lambda\mu = 1,$$
$$\mu^* = \lambda,\ \lambda^* = \mu,\ 1 + \lambda + \mu = 0.$$

The actual setup is as shown in Fig. (3). There is a tritter in the path of each particle. There is also a phase shifter, of phase α in beam a,one of phase β in beam b', and one of phase γ in beam c''. The initial wave function of the three particles is

$$\psi = \tfrac{1}{\sqrt{3}}(e^{i\alpha}|aa'a''\rangle + e^{i\beta}|bb'b''\rangle + e^{i\gamma}|cc'c''\rangle).$$

Each of the tritter outputs goes to a detector, labelled D, E, F, for particle 1, and similarly, with primes, for the other particles. The amplitude after passing through the tritters and reaching the detectors is

$$\psi \to \tfrac{1}{9}\big[e^{i\alpha}(|D\rangle + |E\rangle + |F\rangle)(|D'\rangle + |E'\rangle + |F'\rangle)(|D''\rangle + |E''\rangle + |F''\rangle)$$
$$+e^{i\beta}(|D\rangle + \lambda|E\rangle + \mu|F\rangle)(|D'\rangle + \lambda|E'\rangle + \mu|F'\rangle)(|D''\rangle + \lambda|E''\rangle + \mu|F''\rangle)$$
$$+e^{i\gamma}(|D\rangle + \mu|E\rangle + \lambda|F\rangle)(|D'\rangle + \mu|E'\rangle + \lambda|F'\rangle)(|D''\rangle + \mu|E''\rangle + \lambda|F''\rangle)\big].$$

From this one can calculate the output to any set of detectors.

Rather than write down all possible outputs explicitly, we shall merely as an example write down all those that involve the counters D and D', namely DD'D", DD'E", and DD'F". They are

$$\psi \rightarrow \tfrac{1}{9}\left[(e^{i\alpha}+e^{i\beta}+e^{i\gamma})|DD'D''\rangle + (e^{i\alpha}+\lambda e^{i\beta}+\mu e^{i\gamma})|DD'E''\rangle + (e^{i\alpha}+\mu e^{i\beta}+\lambda e^{i\gamma})|DD'D''\rangle\right]$$
$$+etc.$$

So for example, the probability of counting a coincidence in the detectors DD'D" is

$$P_{DD'D''} = \tfrac{1}{81}(3+2\cos(\alpha-\beta)+2\cos(\beta-\gamma)+2\cos(\gamma-\alpha)).$$

The significant point here is that the counting rate associated with this set of coincidence counts depends symmetrically on the three phase shifts α, β, γ.

As with the two-particle case, only coincidences between all three counters will lead to a diffraction pattern of counting. If one looks at only two detectors (or one), one will find a flat rate. For example, if one adds the rates (amplitudes squared) for the three counts given above, one will get for the probability of a count in DD',

$$P_{DD'} = P_{DD'D''} + P_{DD'E''} + P_{DD'F''} = const.$$

Now assume that a detector is placed into the beam c at point X, so that if it fires, one knows that the first particle has taken this path, and therefore that particle 2 took path c', and particle 3 took path c''.This establishes the path as an EPR element of reality, as one can determine the path of two of the particles by intercepting the third one. Thus in this experiment, the EPNT assumption applies and says that if a particular path is empty, then there is no entity associated with this path that can carry information through the system if the paths later happen to rejoin.

We shall be interested in the case where the detector is installed at X, but it does not fire. In this case, one knows that the particle is *not* located along the paths c-c'-c''. In other words, this set of paths is empty. Because the phase shifter γ is located in the path c'', the EPNT theorem would predict that when the counter X does not fire, the counting rate for coincidences cannot depend upon the angle γ. The counter X will fire 1/3 of the time. If we keep the same normalization as before, so that the total probability for events in which X does not fire is 2/3, then the wave function reaching the set of tritters is

$$\psi_X = \tfrac{1}{\sqrt{3}}(e^{i\alpha}|aa'a''\rangle + e^{i\beta}|bb'b''\rangle).$$

Thus we see already that, quantum mechanically, it cannot depend on γ. The wave function reaching the counters will be

$$\psi_X \rightarrow \tfrac{1}{9}\left[e^{i\alpha}(|D\rangle+|E\rangle+|F\rangle)(|D'\rangle+|E'\rangle+|F'\rangle)(|D''\rangle+|E''\rangle+|F''\rangle)\right.$$
$$\left.+e^{i\beta}(|D\rangle+\lambda|E\rangle+\mu|F\rangle)(|D'\rangle+\lambda|E'\rangle+\mu|F'\rangle)(|D''\rangle+\lambda|E''\rangle+\mu|F''\rangle)\right].$$

Now the probability for a coincidence in DD'D" will be

$$\psi_X \rightarrow \tfrac{1}{9}\left[(e^{i\alpha}+e^{i\beta})|DD'D''\rangle + etc.\right],$$

$$P_{X,DD'D''} = \tfrac{2}{81}(1+\cos(\alpha-\beta)).$$

Not only is this independent of γ but by a suitable choice of angles, one can make the result either greater or less than the result when the detector X was absent. For example, when $\alpha=\beta$, then $P_{X,DD'D''} = 4/81$, while if $\gamma=\alpha=\beta$, then $P_{DD'D''} = 9/81$; but if $\alpha=\beta=\gamma+\pi$, then $P_{DD'D''} = 1/81$.

So, because there exists an element of reality connected with the fact that the path containing γ is empty, this probability no longer depends on γ. However, if one looks at Fig. (4), one sees that the phase shifter γ lies in the beam c'', while the detector X lies in the beam c. So according to EPR locality, there is no way in which particle 3, on one of the double-primed paths, could be made aware of whether the counter X has fired or not, or even whether it is present or not. Thus if there existed a pilot wave that sampled the double-primed paths, there is no way in which it can have been notified to change the nature of the information it passes to the other beams when it reaches the tritter. In fact, the only difference between the cases when the detector X is present or not, is that now there is a label attached to the particle expressing the EPR element that the actual path of the particle excludes c''. And the existence of this element also must be responsible for the fact that the resulting count rate no longer depends upon γ. Similarly, the unprimed particle, one of whose paths contains the detector X, has no knowledge of γ at all, and neither does the primed particle. So everything connected with this experiment can be explained by the EPNT assumption, but appears to be extremely implausible if one accepts EPR locality, and relies on a pilot wave type of explanation. Of course, if one drops EPR locality, one can use the Bohm-Hiley theory[7] to explain these events, as it is equivalent to quantum theory, and non-local.

We believe that if one accepts that pilot waves can exist in a local theory, then one necessarily will produce effects that violate quantum theory. The alternative EPNT assumption rules out such effects, even in two-particle systems, and is consistent with quantum theory.

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