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THE GENERATION OF ENTANGLED STATES FROM INDEPENDENT PARTICLE SOURCES

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Abstract

The generation of entangled states of two systems from product states is discussed for the case in which the paths of the two systems do not overlap. A particular method of measuring allows one to project out the nonlocal entangled state. An application to the production of four photon entangled states is outlined.

1 Introduction

The importance of non-local entangled states in the study of the Einstein-Podolsky-Rosen paradox (EPR) has been stressed by Horne, Shimony, and Zeilenger (GHZ) [1]. These states are entangled states of two or more particles in which at least one pair of particles has space-like separation. Such states may be produced from independent sources by allowing the particles to scatter from one another and then separate, or by allowing their paths to overlap spatially such as by passing pairs of photons through a single beam splitter. The generation of entangled states from independent particle sources in which the entangled particles overlap has been discussed by Yurke and Stoler [2], and by Tan, Walls and Collett [3]. We consider the case in which the entangled particles are produced from independent sources and never overlap.

2 Independent Sources With Path Overlap

We briefly review the work of Yurke and Stoler. Two independent sources each emit a particle which is passed through a beam splitter and phase shifter. The detection system consists of a pair of detectors, labeled 1 and 2. Each detector is composed of a beam splitter which directs each particle to one of two particle counters denoted by R and G. The initial input state is a product state,

$$|\Psi>=|\alpha>_1|\beta>_2 . \tag{1}$$

After passing through the system this state may be written as

$$\begin{aligned}|\Psi'> &= \frac{1}{4}\{e^{i\theta_1}(|\alpha R1>_1+i|\alpha G1>_1)+i^2|\alpha R2>_1+i|\alpha G2>_1\} \\ & i^2|\beta\ R1>_2+i|\beta G1>_2+e^{i\theta_2}(|\beta R2>_2+i|\beta G2>_2)\}\end{aligned} \tag{2}$$

where the i's come from the reflection off the beam-splitters. If we consider the case in which only the R detectors register, the only part of the state that we look at is

$$e^{i(\theta_1+\theta_2)}|\alpha\, R1>_1 |\beta\, R2>_2 - |\alpha\, R2>_1 |\beta\, R1>_2 \tag{3}$$

which is an entangled state. By varying the phase shifters, interference between the two terms can be detected. This interference occurs because there are two distinct paths from the input state to the detection state that can not be distinguished by the detectors. From this point of view [4] the fact that "vacuum" enters the system through the beam splitters plays no role since the detectors do not see the vacuum states.

3 Independent Sources Without Path Overlap

We now turn to a different method of generating entangled states in which the independent sources are independently detected. The particle paths never cross. The entanglement is obtained by selective detection and the interference is caused by varying the phase shift in each path in a specific way. We shall see below that this allows us to consider the generation of an EPR state of the type envisioned by GHZ. We illustrate this technique for the simplest case, namely, two spin-1/2 particles. The input to the system is two independent particles polarized along the positive z-axis,

$$|\Psi>=|+z>_1|+z>_2\,. \tag{4}$$

Each particle is passed through a separate Stern-Gerlach apparatus which splits each state vector into a superposition of states polarized along the x-axis. After passing through the system we have

$$|\Psi'>=\frac{1}{2}\{|+x>_1+e^{i\theta_1}|-x>_1\}\{|+x>_2+e^{i\theta_1}|-x>_2\}, \tag{5}$$

where the phase shifter effect has been included. Multiplying (5) out gives

$$\begin{aligned}|\Psi'>=\frac{1}{2}\{&|+x>_1|+x>_2+e^{i(\theta_1+\theta_2)}|-x>_1|-x>_2+\\ &e^{i\theta_1}|-x>_1|+x>_2+e^{i\theta_2}|+x>_1|-x>_2\}.\end{aligned} \tag{6}$$

Then we project this onto the detector states

$$|Dk>=|+zk>=\frac{1}{\sqrt{2}}(|+x>_k+|-x>_2),\ \text{k=1,2}. \tag{7}$$

The amplitude for detecting particle 1 at detector 1 and particle 2 at detector 2 is:

$$A=\frac{1}{4}(1+e^{i\theta_1})(1+e^{i\theta_2})=\frac{1}{4}\{(1+e^{i(\theta_1+\theta_2)})+(e^{i\theta_1}+e^{i\theta_2})\}, \tag{8}$$

which is still a product for independent choice of phases. Now if we choose $\theta_2=\theta_1+\pi$, then the second pair of terms in (8) cancel and the first shows interference as θ_1 is varied,

$$A=\frac{1}{4}(1-e^{2i\theta_1}) \tag{9}$$

This is equivalent to producing the entangled state given by the first two terms of (6).

In this case the two independent particles never meet. Here the interference is between the pair of particles that went through the phase shifter and the pair that did not. From the point of view of indistiguishable paths, it is the paths of these *two-particle states* that are indistiguishable. Another way to look at this particular example is to note that the choice of the angles makes the first two term in (6) total spin 1 states and the second two form a total spin 0 state, the detector only detects the spin 1 part of the state.

This argument can be generalized to more than two particles and can be used to form the basis of many-particle interference experiments to test the theorem of Greenberger, Horne and Zeilenger [5]. The use of projection operators to pick off parts of state vectors, which is really what measurements do, is nothing new. It is fundamental to quantum theory. What is different is the means of picking off nonlocal entangled states from independent particles.

4 The Theory of the Four-Photon Experiment

We now apply this to a possible four-photon interference experiment. We wish to show how one can produce the highly correlated state necessary to carry out the experiment envisioned by Greenberger, Horne and Zeilenger. In fig. 1 we show the experiment envisioned. It is a double Franson interferometer [6]. The coincidence counting rate is proportional to

$$P = \int\int\int\int dt_1 dt_2 dt_3 dt_4 S(t_1,t_2,t_3,t_4)|\Phi_I(t_1,t_2)|^2|\Phi_{II}(t_3,t_4)|^2, \tag{10}$$

where the integrals are over the duration of the measurement. The function S expresses the fact that the coincidence time window is one when $t_2 - t_1$, $t_3 - t_1$ and $t_4 - t_1$ are all less than some small coincidence time t_{coin} and rapidly goes to zero when this is not true.

$$\Phi_j(t_1,t_2) = \frac{1}{2}(\Psi_j(L_1,L_2) + \Psi_j(S_1,L_2) + \Psi_j(L_1,S_2) + \Psi_j(S_1,S_2)), \tag{11}$$

where j=I and II . The middle two terms can be discriminated against for $L - S = \Delta L$ if $\Delta L/c >> t_{coin}$. Then we can confine ourselves to considering

$$\Phi_j(t_1,t_2) = \frac{1}{2}\{\Psi_j(L_1,L_2) + \Psi_j(S_1,S_2)\}. \tag{12}$$

The amplitude in (12) is still a product,

$$\begin{aligned}\Phi_I(t_1,t_2)\Phi_{II}(t_3,t_4) &= \frac{1}{4}(\{\Psi_I(L_I,L_I)\Psi_{II}(L_{II},L_{II}) + \Psi_I(S_I,S_I)\Psi_{II}(S_{II},S_{II})\} + \\ &\quad \{\Psi_I(L_I,L_I)\Psi_{II}(S_{II},S_{II}) + \Psi_I(S_I,S_I)\Psi_{II}(L_{II},L_{II})\}).\end{aligned} \tag{13}$$

This is in a form similar to that of (5). We now want to show how the last term in curly brackets can be made to vanish.

We have shown elsewhere [7] that the two-photon wave function is of the form

$$\Psi(t_1,t_2) = u(t_1 - t_2)v(t_1 + t_2). \tag{14}$$

The function u(t) describes the correlation of the photon pair in space and time. Its width is determined by the single photon bandwidth. This is usually fixed by filters in the output beams of the down-conversion crystal. The function v(t) is of the form

$$v(t) = A(t)e^{-i\frac{\Omega_p t}{2}}, \tag{15}$$

It expresses the fact that the pair is created somewhere within the overlap of the crystal and the pump beam. In general, if the pump beam is nearly monochromatic, A will be a slowly varying function of time.

Using these facts, the second term in curly brackets of (13) can be written as

$$u(t_1 - t_2)u(t_3 - t_4)\{v(t_1 + t_2)v(t_3 + t_4 - \frac{2\Delta L_{II}}{c}) + v(t_1 + t_2 - \frac{2\Delta L_I}{c})v(t_3 + t_4)\}. \tag{16}$$

The *key point* is that we can choose the path lengths so that

$$\Omega_p(\Delta L_I - \Delta L_{II})/c = \pi \tag{17}$$

making the relative phase between the two terms in the bracket of (16) negative. Since A(t) is constant over the measurement time this term will vanish. The first term in curly brackets has two terms with a relative phase of $\Omega_p(\Delta L_I + \Delta L_{II})/c$ which, under the condition stated, becomes $\pi + \Omega_p 2\Delta L_I/c$. Therefore varying the path length while keeping their difference fixed allows one to see interference with oscillations at half the pump wavelength.

The above experiment is for space-time variables. There is a more convenient way to proceed using the polarization of the photons.

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After this paper was completed a related paper on the generation of entangled states from independent sources [8] was called to our attention. We thank Professor Zeilinger for giving us a preprint of this paper.

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References

[1] M. A. Horne, A. Shimony, and A. Zeilenger, Phys. Rev. Lett. **62**, 2209 (1989). M. A. Horne, A. Shimony, and A. Zeilenger, *Quantum Coherence*, edited by J. S. Anandan, (World Scientific Co. Pte. Ltd, Singapore,1990).

[2] B. Yurke and D. Stoler, Phys. Rev. Lett. **68**, 1251 (1992) and Phys. Rev. A **46**, 2229 (1992).

[3] S. M. Tan, D. F. Walls and M. J. Collett, Phys. Rev. Lett. **66**, 252 (1991).

[4] This point of view is the natural one from the point of view of Feynman's approach to quantum mechanics, R. P. Feynman, R. B. Leighton, and M Sands, *The Feynman Lectures on Physics, Quantum Mechanics*, (Addison-Wesley, Reading,MA, 1965).

[5] D. M. Greenberger, M. Horne and A. Zeilenger, in *Bells Theorem, Quantum Theory and Conceptions of the Universe*, edited by M. Kaftos (Kluwer Academis, Dordrect, 1989).

[6] J. D. Franson, Phys. Rev. Lett.**62**, 2205 (1989).

[7] Y. H. Shih and Morton H. Rubin, to be published

[8] M. Zukowski, A. Zeilinger, M. A. Horne and A. K. Ekert, private communication.

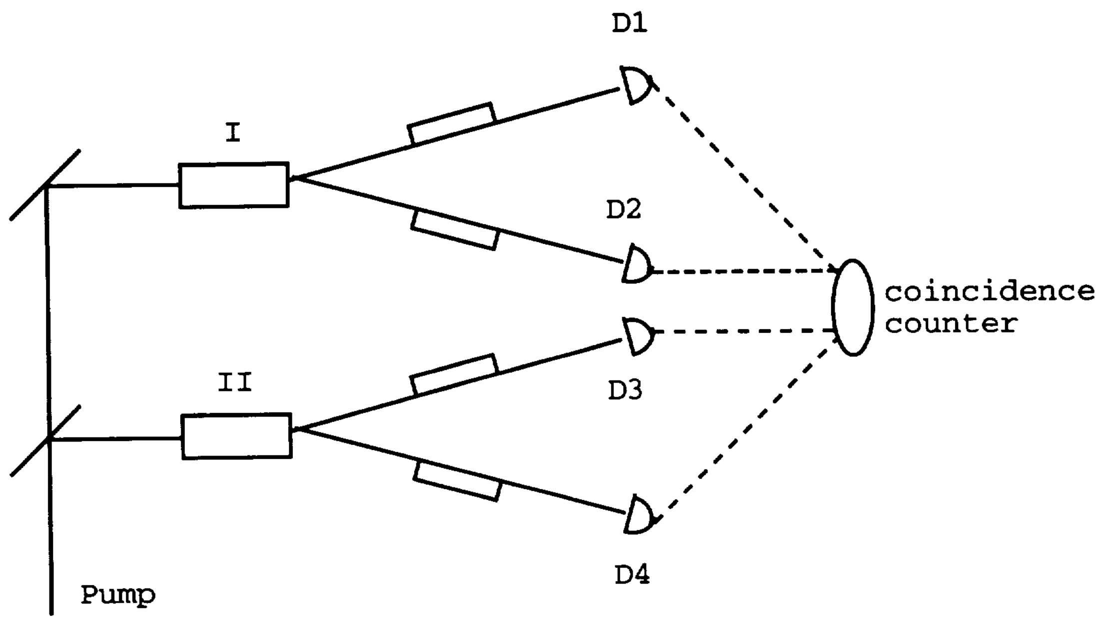

Figure 1: A pump beam enters two crystals which each produce a pair of photons by optical parametric downconversion. Each photon passes through an interferometer and is detected by one of the four detectors (Dk). A coincidence counter (C) gives a count only when all four photons arrive within a fixed time window.