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STUDY OF MULTIPLE CRACKS IN AIRPLANE FUSELAGE BY MICROMECHANICS AND COMPLEX VARIABLES

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SUMMARY

Innovative numerical techniques for two dimensional elastic and elastic-plastic multiple crack problems are presented using micromechanics concepts and complex variables. The simplicity and the accuracy of the proposed method will enable us to carry out the multiple-site fatigue crack propagation analyses for airplane fuselage by incorporating such features as the curvilinear crack path, plastic deformation, coalescence of cracks, etc.

INTRODUCTION

Numerical techniques for plane elastic and elastic-plastic multiple crack problems are presented with the help of micromechanics concepts and complex variables. An amalgamation of the method of singular integral equations for cracks, the boundary element method, and the plastic source method for plastic deformation is achieved in a natural manner with the help of micromechanics tools such as dislocations, point forces, and their dipoles. The formulation is carried out in terms of complex variables to facilitate closed form integration of the boundary and the crack face (singular) integrals and the planar distribution of the plastic sources. For elastic problems, the crack opening displacements are modeled by the continuous distribution of dislocation dipoles such that, with the help of Chebyshev polynomials, the crack-face opening displacements and the crack-tip stress singularity contributions are automatically built in. There is no need to extrapolate the results to get the stress intensity factor. Further, by using complex variables, the crack-face singular integrals are evaluated in closed form. The approach is extended to elastic-plastic multiple crack problems such that the elastic singularity is canceled by the plastic deformation at the crack-tip. The techniques presented in this paper will serve as the key components in the planned FAA computer software for the residual life analysis of aging aircraft under widespread fatigue damage that takes into account such features as the curvilinear crack path, plastic deformation, coalescence of cracks, etc.

MICROMECHANICS TOOLS IN COMPLEX VARIABLES

Muskhelishvili's (ref. 1) complex variable formalism for plane isotropic elasticity uses two analytic functions or complex potential functions, $\phi(z)$ and $\psi(z)$, of a complex variable

$z = x + iy$ to express the solution. The displacement, stress, and strain components are given by

$$2\mu u(z) = \kappa\phi(z) - z\overline{\phi(z)'} - \overline{\psi(z)}, \tag{1}$$

and

$$\begin{aligned} \frac{\sigma_{xx} + \sigma_{yy}}{2} &= \phi(z)' + \overline{\phi(z)'}, \\ \frac{\sigma_{yy} - \sigma_{xx}}{2} + i\sigma_{xy} &= \bar{z}\phi(z)'' + \psi(z)', \end{aligned} \tag{2}$$

where μ is the shear modulus, κ is given by $\kappa = 3 - 4\nu$ in plane strain and $\kappa = (3-\nu)/(1+\nu)$ in plane stress in terms of Poisson's ratio ν. A prime attached to the analytic functions of z indicates differentiation with respect to z and a bar indicates the complex conjugate.

Fundamental Solutions

Consider a point force with the magnitude $f = f_x + if_y$ (per unit thickness) and an edge dislocation with the Burgers vector $b = b_x + ib_y$, independently located at ξ in the infinite isotropic medium. Their solutions are given in the same form (ref. 2)

$$\begin{aligned} \phi^{(s)}(z;\xi) &= -\gamma \log(z - \xi)\ , \\ \psi^{(s)}(z;\xi) &= -k\bar{\gamma}\log(z-\xi) + \gamma\frac{\bar{\xi}}{z-\xi}, \end{aligned} \tag{3}$$

where $k = -\kappa$, $\gamma = f/2\pi(\kappa+1)$ for the point force and $k = 1$, $\gamma = i\mu b/\pi(\kappa+1)$ for the dislocation. The corresponding dipole solutions (i.e., the force and the dislocation dipoles) are given by

$$\begin{aligned} \phi^{(d)}(z;\xi) &= -\gamma\, d\left\{\log(z-\xi)\right\}, \\ \psi^{(d)}(z;\xi) &= -k\bar{\gamma}\, d\left\{\log(z-\xi)\right\} + \gamma d\left\{\frac{\bar{\xi}}{z-\xi}\right\}, \end{aligned} \tag{4}$$

where $d(\cdots) = \frac{\partial}{\partial\xi}(\cdots)d\xi + \frac{\partial}{\partial\bar{\xi}}(\cdots)d\bar{\xi}$ is the total differentiation operator.

Continuous Distributions of the Singularities

The continuous distribution of point forces over an arc L (with arc parameter s) is given, from equation (3), by

$$\begin{aligned} \phi^{(s)}(z) &= -\int_L \Gamma(s)\log(z-\xi)ds, \\ \psi^{(s)}(z) &= \kappa\int_L \overline{\Gamma(s)}\log(z-\xi)ds + \int_L \Gamma(s)\frac{\bar{\xi}}{z-\xi}ds, \end{aligned} \tag{5}$$

with $\Gamma(s) = t/2\pi(\kappa+1)$, where $t = t_x + it_y$ is the traction. The continuous distribution of dislocation dipoles is given, from Equation (4), by

$$\begin{aligned} \phi^{(d)}(z) &= -\int_L \gamma(s)\, d\{\log(z-\xi)\}, \\ \psi^{(d)}(z) &= -\int_L \overline{\gamma(s)}\, d\{\log(z-\xi)\} + \int_L \gamma(s)\, d\left\{\frac{\bar{\xi}}{z-\xi}\right\}, \end{aligned} \tag{6}$$

with $\gamma(s) = i\mu b/\pi(\kappa+1)$, where $b = b_x + ib_y$ is the dislocation.

BOUNDARY ELEMENT METHOD IN COMPLEX VARIABLES

Consider a body R with its boundary ∂R subject to the traction $T = T_x + iT_y$ and the displacement $U = U_x + iU_y$. The displacement field in this body is obtained by assuming that the region R is embedded in an infinite medium and ∂R, which is simply a line marked out in the infinite domain, is covered by a continuous distribution of point forces with density T and by a continuous distribution of dislocation dipoles with the Burgers vector U. This is the physical interpretation of Somigliana's identity (ref. 3). We discretize and approximate the original boundary by a piecewise straight line, $L = \sum_{j=1}^{M} L_j$. The j-th boundary element $L_j = \xi_j\xi_{j+1}$ $(j = 1, 2, \ldots, M)$ has the slope ϕ_j. The boundary traction and the displacement are approximated by constant interpolation functions over an element. Let T_j and U_j be the traction and the displacement of the element, then the potential functions in (5) and (6) are integrated analytically with the result

$$\begin{aligned} \phi_j^{(s)}(z) &= \Gamma_j^{(T)} f_j(z), \\ \psi_j^{(s)}(z) &= -\Gamma_j^{(T)}\left\{g_j(z) + e^{-2i\phi_j} f_j(z)\right\} - \overline{\Gamma_j^{(T)}}\kappa e^{-2i\phi_j} f_j(z), \\ \phi_j^{(d)}(z) &= -\gamma_j^{(U)} s_j(z), \quad \psi_j^{(d)}(z) = \gamma_j^{(U)} m_j(z) - \overline{\gamma_j^{(U)}} s_j(z), \end{aligned}$$

where

$$f_j(z) = \left\{(z-\xi)\log_j(z-\xi) + \xi\right\}\Big|_{\xi_j}^{\xi_{j+1}}, \quad g_j(z) = \bar{\xi}\log_j(z-\xi)\Big|_{\xi_j}^{\xi_{j+1}},$$

$$s_j(z) = \log_j(z-\xi)\Big|_{\xi_j}^{\xi_{j+1}}, \quad m_j(z) = \left.\frac{\bar{\xi}}{(z-\xi)}\right|_{\xi_j}^{\xi_{j+1}},$$

and $\Gamma_j^{(T)} = T_j e^{-i\phi_j}/2\pi(\kappa+1)$, $\gamma_j^{(U)} = i\mu U_j/\pi(\kappa+1)$. For ξ on L_j, the branch line of the logarithm $\log_j(z-\xi)$ is given by a straight line connecting ξ, ξ_j, and ∞; thus the branch cut of the logarithm differs from element to element. The displacement and the traction contributions of the two layers are obtained by substituting these potential functions into (1) and (2). However, it is necessary to separate the real and imaginary parts of the displacement and the traction before establishing the boundary equations. The displacement boundary equations and the traction boundary equations are derived following the standard procedure of the BEM.

Define the Cauchy-type integrals

$$\begin{aligned} T^{(m)}(z) &= -\frac{1}{\pi}\int_{-1}^{1}\frac{\sqrt{1-x^2}U_{m-1}(x)dx}{x-z} \quad (m \geq 0), \\ U^{(m-1)}(z) &= \frac{1}{\pi}\int_{-1}^{1}\frac{T_m(x)dx}{\sqrt{1-x^2}(x-z)} \quad (m \geq 0), \end{aligned} \tag{7}$$

over the interval $-1 \leq x \leq +1$, where $T_m(x)$ and $U_{m-1}(x)$ are the Chebyshev polynomials of the first and second kind and $z = x + iy$ is a complex variable. The integrals in (7) can be evaluated analytically with the result

$$\begin{aligned} T^{(m)}(z) &= \left(z - \sqrt{z^2-1}\right)^m \quad (m \geq 0), \\ U^{(m-1)}(z) &= -\frac{\left(z - \sqrt{z^2-1}\right)^m}{\sqrt{z^2-1}} \quad (m \geq 0). \end{aligned} \tag{8}$$

Single Crack in the Infinite Body

A dislocation dipole over an infinitesimal segment $d\xi$ gives rise to a displacement discontinuity. We identify the displacement discontinuity as the crack opening displacement and call the dislocation dipole as the crack source. Further, the continuous distribution of the crack sources over an arc is called the crack element. Consider a straight center crack of length $2a$ subject to a self-equilibrated traction

$$\pm\tilde{\mathrm{t}} = \pm\left\{ \begin{array}{c} \tilde{t}_x \\ \tilde{t}_y \end{array} \right\}. \tag{9}$$

over its upper $(+)$ and lower $(-)$ surfaces. Select the local coordinate system xy with the coordinate origin at the center of the crack and the x-axis along the crack and introduce the non dimensional coordinates $X = x/a$ and $Z = z/a$, where $z = x + iy$ is a complex variable. The density function of the crack element is given by $\tilde{\gamma}(X) = \gamma(x) = i\mu\delta/\pi(\kappa+1)$ in terms of the crack opening displacement $\delta = \delta_x + i\delta_y$. If we interpolate the density function by

$$\tilde{\gamma}(X) = \frac{i\mu}{\pi(\kappa+1)}\sqrt{1-X^2}\sum_{m=1}^{p}\delta^{(m)}U_{m-1}(X),$$

then the complex potential functions, (4), for the crack element are integrated analytically, with the help of (8), with the result

$$\begin{aligned} \phi^{(d)}(Z) &= \pi\sum_{m=1}^{p} A^{(m)}T^{(m)}(Z), \\ \psi^{(d)}(Z) &= \pi\sum_{m=1}^{p}\left\{\overline{A^{(m)}}T^{(m)}(Z) - mA^{(m)}ZU^{(m-1)}(Z)\right\}. \end{aligned}$$

The displacement and the traction contributions of the crack element can be evaluated by (1) and (2). Of interest is the traction

$$(t_x + it_y)^{\pm}(X) = \pm \frac{2\mu}{(\kappa+1)a} \sum_{m=1}^{p} m\delta^{(m)} U_{m-1}(X) \quad (|X| \leq 1), \tag{10}$$

on the upper and the lower faces of the crack and the stress intensity factor

$$K(\pm 1) = K_I(\pm 1) + iK_{II}(\pm 1) = \frac{2\mu i}{\kappa + 1}\sqrt{\frac{\pi}{a}} \sum_{m=1}^{p} (\pm 1)^{m+1} m\overline{\delta^{(m)}}.$$

The components $\delta^{(m)}$ of the crack opening displacement are determined from Equation (10) and the applied traction (9) by collocation.

Multiple Cracks in the Infinite Body

Consider the problem of N multiple straight cracks, L_j ($j = 1, \ldots, N$), in the infinite body; the individual crack surface is loaded according to (9). First we formulate each crack in the respective local coordinate system and then assemble the contributions from all the cracks in the global coordinate system. The total traction $\mathbf{t}^{(k)+} = \{t_x^{(k)+}, t_y^{(k)+}\}^T$ on the upper surface of the crack L_k is given in the form

$$\mathbf{t}^{(k)+} = \sum_{j=1}^{N} \left\{ \sum_{m=1}^{p_{(j)}} \boldsymbol{\Omega}_{(kj)}^{*(m)} \boldsymbol{\delta}_{(j)}^{(m)} \right\}, \tag{11}$$

where $\boldsymbol{\Omega}_{(kj)}^{*(m)}$ is a coefficient matrix and $\boldsymbol{\delta}_{(j)}^{(m)} = \{\delta_{(j)x}^{(m)}, \delta_{(j)y}^{(m)}\}^T$ ($j = 1, \ldots, N$; $m = 1, \ldots, p_{(j)}$) are the unknown crack opening displacement coefficients, which will be determined, from Equations (11) and (9), by collocation.

Effect of the Finite Boundary

We now consider multiple center cracks in a finite body R whose boundary ∂R is subject to the traction $\mathbf{T} = \{T_x, T_y\}^T$ and displacement $\mathbf{U} = \{U_x, U_y\}^T$ and each crack surface is loaded according to (9). The total traction on the upper surface of crack L_k is given in the form

$$\mathbf{t}^{(k)+} = \sum_{j=1}^{N} \left\{ \sum_{m=1}^{p_{(j)}} \boldsymbol{\Omega}_{(kj)}^{*(m)} \boldsymbol{\delta}_{(j)}^{(m)} \right\} + \sum_{n=1}^{M} \left\{ \mathbf{G}_n^{*(k)} \mathbf{T}_n - \mu \mathbf{H}_n^{*(k)} \mathbf{U}_n \right\}, \tag{12}$$

where the first term in the right hand side comes from (11) and the second and the third terms come from the traction BEM with constant interpolation functions. The quantities $\mathbf{U}_n$ and $\mathbf{T}_n$ are the boundary displacement and the traction vectors and $\mathbf{G}_n^{*(k)}$, $\mathbf{H}_n^{*(k)}$ are coefficient matrices. The total displacement on the non-crack boundary ∂R is given in the form

$$2\mathbf{U}_k = \sum_{j=1}^{N}\left\{\sum_{m=1}^{p(j)} \boldsymbol{\Pi}_{(j)}^{(m)}\delta_{(j)}^{(m)}\right\} + \sum_{n=1}^{M}\left\{\frac{1}{\mu}\mathbf{G}_n\mathbf{T}_n - \mathbf{H}_n\mathbf{U}_n\right\}, \tag{13}$$

where $\boldsymbol{\Pi}_{(j)}^{(m)}$, $\mathbf{G}_n$, and $\mathbf{H}_n$ are coefficient matrices. The solution is obtained by setting up traction equations on the upper surface of each crack from Equations (12) and (9) and displacement boundary equations on ∂R from Equation (13) and the boundary condition on ∂R.

Numerical Results

A single center crack in a plate in uniaxial tension was analyzed by the present method using one Chebyshev polynomial. The same problem, with identical mesh, was analyzed by the crack Green's function BEM, which uses the Green's function that satisfies the traction free crack surface boundary condition automatically. The stress intensity factor results agreed up to seven significant digits. Figures 1, 2, and 3 show two collinear cracks, two parallel cracks, and three parallel cracks, respectively, in the infinite body. The numerical results have been obtained for a large plate, compared to the cracks, using seven Chebyshev polynomials for each crack in each case. Comparison of the numerical results and the results from the stress intensity handbook (ref. 4) is listed in Tables 1, 2, and 3.

PLASTIC SOURCE METHIOD

In the presence of the plastic strain ϵ_{ij}^p the stress-strain relations for isotropic materials in two-dimensions are given by (ref. 2)

$$\sigma_{\alpha\beta} = 2\mu\left(\epsilon_{\alpha\beta} - \epsilon_{\alpha\beta}^p\right) + \lambda^*\left(\epsilon_{\gamma\gamma} - \epsilon_{\gamma\gamma}^p\right)\delta_{\alpha\beta},$$

and

$$\epsilon_{\alpha\beta} = \frac{1}{2\mu}\left(\sigma_{\alpha\beta} - \nu^*\sigma_{\gamma\gamma}\delta_{\alpha\beta}\right) + \epsilon_{\alpha\beta}^p,$$

where $\epsilon_{\alpha\beta}$ is the total strain, $\epsilon_{\alpha\beta}^*$ is the fictitious in-plane plastic strain, called the plane plastic strain, defined by

$$\epsilon_{\alpha\beta}^* = \begin{cases} \epsilon_{\alpha\beta}^p + \nu\epsilon_{33}^p\delta_{\alpha\beta} & \text{(plane strain)} \\ \epsilon_{\alpha\beta}^p & \text{(plane stress)} \end{cases}$$

The elastic moduli λ^* and ν^* are defined by

$$\lambda^* = \begin{cases} \lambda & \text{(plane strain)} \\ \frac{2\lambda\mu}{\lambda+2\mu} & \text{(plane stress)} \end{cases}, \quad \nu^* = \begin{cases} \nu & \text{(plane strain)} \\ \frac{\nu}{1+\nu} & \text{(plane stress)} \end{cases},$$

in terms of Lamé constants, λ and μ, and Poisson's ratio ν. The non-zero out of plane components are given by

$$\begin{aligned}
\sigma_{33} &= \nu\sigma_{\gamma\gamma} - E\epsilon^p_{33} \qquad \text{(plane strain)},\\
\epsilon_{33} &= -\frac{1}{2\mu}\frac{\nu}{1+\nu}\sigma_{\gamma\gamma} + \epsilon^p_{33} \qquad \text{(plane stress)},
\end{aligned}$$

where E is the Young's modulus.

Plastic Element

Consider a region D of plastic deformation where the plastic strain components ϵ^p_{11}, ϵ^p_{22}, ϵ^p_{12}, and $\epsilon^p_{33} = -\epsilon^p_{\gamma\gamma}$ are prescribed. In the numerical implementation the plastic region is discretized into a collection of plastic elements in each of which the plastic strain distribution is approximated by an interpolation function. We use the constant interpolation in this paper so that the problem can be treated as Eshelby's (ref. 5) stress-free transformation problem with constant transformation strain given by the plane plastic strain $\epsilon^*_{\alpha\beta}$. Then the displacement in the entire region is given by a continuous distribution of point forces over the boundary ∂D of the plastic region; the magnitude of the force over a segment $d\zeta$ of the boundary is given by

$$f = \frac{1}{2i}\left(\sigma^*_{11} + \sigma^*_{22}\right)d\zeta - \frac{1}{2i}\left(\sigma^*_{11} - \sigma^*_{22} + 2i\sigma^*_{12}\right)d\overline{\zeta}, \tag{14}$$

where

$$\sigma^*_{\alpha\beta} = 2\mu\epsilon^*_{\alpha\beta} + \lambda^*\epsilon^*_{\gamma\gamma}\delta_{\alpha\beta}$$

The solution is obtained by integrating the fundamental solution of the point force, (3) with (14), over the boundary ∂D with the result

$$\begin{aligned}
\phi^*(z) &= -\int_{\partial D}\gamma^*\log(z-\zeta)d\zeta,\\
\psi^*(z) &= \kappa\int_{\partial D}\overline{\gamma^*}\log(z-\zeta)d\zeta + \int_{\partial D}\gamma^*\frac{\overline{\zeta}}{z-\zeta}d\zeta,
\end{aligned} \tag{15}$$

where

$$\begin{aligned}
\gamma^* &= \frac{1}{2i}\left(\sigma^* - \tau^* e^{-2i\psi}\right),\\
\sigma^* &= \frac{1}{2\pi(1+\kappa)}\left(\sigma^*_{11} + \sigma^*_{22}\right) = \frac{\mu+\lambda^*}{\pi(1+\kappa)}\left(\epsilon^*_{11} + \epsilon^*_{22}\right),\\
\tau^* &= \frac{1}{2\pi(1+\kappa)}\left(\sigma^*_{11} - \sigma^*_{22} + 2i\sigma^*_{12}\right) = \frac{\mu}{\pi(1+\kappa)}\left(\epsilon^*_{11} - \epsilon^*_{22} + 2i\epsilon^*_{12}\right),
\end{aligned}$$

and ψ is the slope of the boundary ∂D.

Consider the polygonal plastic element of a constant plastic strain distribution bounded by N-lines $\Gamma = \sum_{j=1}^{N}\Gamma_j$, where $\Gamma_j = \zeta_j\zeta_{j+1}$ $(j = 1, 2, \ldots, N)$ is the j-th edge extending from corners ζ_j to ζ_{j+1} with the slope ψ_j. The integrals in (15) are evaluated analytically with the result

$$\begin{aligned} \phi^*(z) &= -\sum_{j=1}^{N} \gamma_j^* f_j(z), \\ \psi^*(z) &= \sum_{j=1}^{N} \left\{ -(\kappa \overline{\gamma_j^*} + \gamma_j^*) e^{-2i\psi_j} f_j(z) - \gamma_j^* g_j(z) \right\}, \end{aligned} \tag{16}$$

where

$$f_j(z) = \Big[(z-\zeta)\log_j(z-\zeta) + \zeta\Big]_{\zeta_j}^{\zeta_{j+1}}, \quad g_j(z) = \Big[\overline{\zeta}\log_j(z-\zeta)\Big]_{\zeta_j}^{\zeta_{j+1}},$$

and $\gamma_j^* = \frac{1}{2i}\left(\sigma^* - \tau^* e^{-2i\psi_j}\right)$. The branch cut for the logarithm $\log_j(z-\zeta)$ for ζ located on the line Γ_j is specified as explained earlier for the BEM. The displacement and the stress are obtained by substituting the potential functions in (16) into (1) and (2) following the Eshelby's procedure of stress free transformation. This results in an additional term for the stress inside the plastic element (ref. 2).

Numerical Solution Procedure

The solution procedure whereby the unknown plastic strain distribution is determined was given by Denda and Lua (ref. 6) for standard elastoplastic problems. In order to use the crack source method it is convenient to break the problem into the elastic and the plastic solutions. The former is the elastic solution under the applied load and the latter the solution of the plastic elements. The crack source method is used for the determination of each solution, once for the elastic solution and several times, iteratively, for the plastic solution. Note that each solution gives rise to a $1/\sqrt{r}$ stress singularity. A singularity cancellation scheme, whereby the final solution is obtained by elimination of the total stress intensity factor, is used as a part of the convergence criterion of the procedure. The results will be reported elsewhere.

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Table 1. Two Collinear Cracks ($F_{IA} = K_{IA}/\sigma\sqrt{\pi a}$ and $F_{IB} = K_{IB}/\sigma\sqrt{\pi a}$)

2a/d	F_{IA} (Handbook)	F_{IA} (Numerical)	F_{IB} (Handbook)	F_{IB} (Numerical)
0.05	1.00031	1.0018	1.00032	1.0018
0.1	1.0012	1.0027	1.0013	1.0028
0.2	1.0046	1.0061	1.0057	1.0071
0.3	1.0102	1.0117	1.0138	1.0153
0.4	1.0179	1.0194	1.0272	1.0287
0.5	1.0280	1.0295	1.0480	1.0495
0.6	1.0410	1.0426	1.0804	1.0821
0.7	1.0579	1.0596	1.1333	1.1351
0.8	1.0811	1.0827	1.2289	1.2314
0.9	1.1174	1.1187	1.4539	1.4639

Table 2. Two Parallel Cracks ($F_I = K_I/\sigma\sqrt{\pi a}$)

2a/d	F_I (Handbook)	F_I (Numerical)
0.0	1.0000	1.0011
0.2	0.9855	0.9870
0.4	0.9508	0.9517
0.8	0.8727	0.8732
1.0	0.8319	0.8440
2.0	0.7569	0.7746
5.0	0.6962	0.7129

Table 3. Three Parallel Cracks ($F_{IA} = K_{IA}/\sigma\sqrt{\pi a}$ and $F_{IB} = K_{IB}/\sigma\sqrt{\pi a}$)
(Handbook digital values for F_{IB} are not available.)

2a/d	F_{IA} (Handbook)	F_{IA} (Numerical)	F_{IB} (Handbook)	F_{IB} (Numerical)
0.1	0.99500	0.99687	—	0.99410
0.2	0.98198	0.98379	—	0.97306
0.3	0.96299	0.96430	—	0.94156
0.4	0.94010	0.94100	—	0.90361
0.5	0.91535	0.91650	—	0.86789
0.6	0.89080	0.89254	—	0.82333
0.7	0.86851	0.87041	—	0.78603
0.8	0.85052	0.85062	—	0.75234

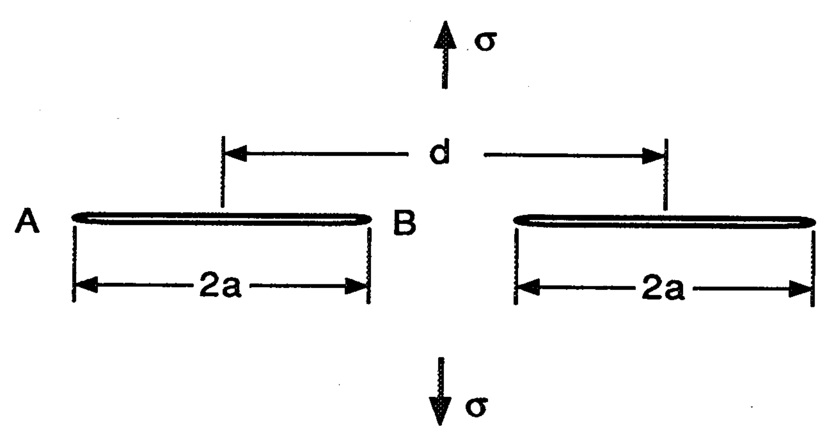

Figure 1: Two collinear cracks in the infinite body under uniaxial tension.

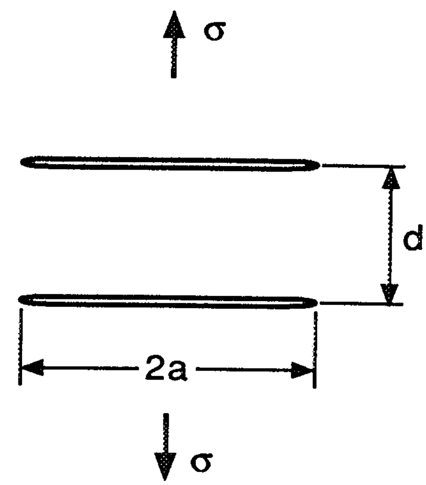

Figure 2: Two parallel cracks in the infinite body under uniaxial tension.

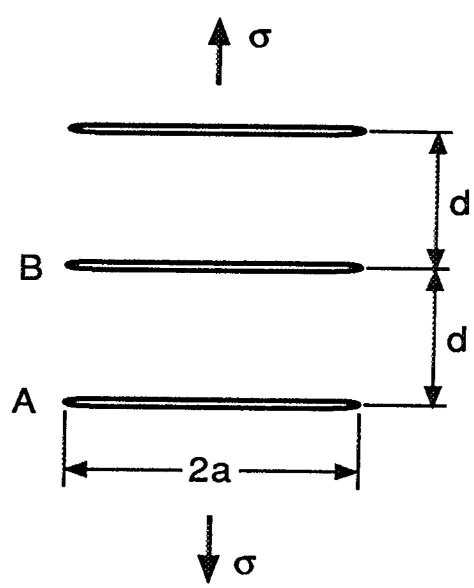

Figure 3: Three parallel cracks in the infinite body under uniaxial tension.