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Influence of Flow on Interface Shape Stability in Low Gravity

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Science objectives

- 1) Understand the influence in low gravity of flow on interface shape. For example, document and control the influence of axial flow on the Plateau-Rayleigh instability of a liquid bridge.
- 2) Extend the ground-based density-matching technique of low gravity simulation to situations with flow; that is, develop Plateau chamber experiments for which flow can be controlled.

Relevance of science and potential applications

Containerless containment of liquids by surface tension has broad importance in low gravity. For space vehicles, the behavior of liquid/gas interfaces is crucial to successful liquid management systems. In microgravity science, free interfaces are exploited in various applications. Examples include float-zone crystal growth, phase separation near the critical point of liquid mixtures (spinoidal decomposition) and quenching of miscibility gap molten metal alloys. In some cases, it is desired to stabilize the capillary instability while in others it is desired to induce capillary breakup. In all cases, understanding the stability of interface shape in the presence of liquid motion is central.

Research approach

Both analytical/numerical and experimental approaches are employed.

Stability analyses include linear and nonlinear techniques. The linear stability approach has been used to analyze the shape stability of a cylindrical interface containing axial shear flows, both isothermally- and thermocapillary-driven[1, 2]. Computational feasibility currently limits this approach to base states that are separable flows, effectively, the axial-infinite interfaces. It is now well-known that infinite cylindrical interfaces can be stabilized[3, 4, 5]. For finite interfaces an alternative approach is needed. In the limit of no motion, minima of the free-energy functional are obtained using the calculus of variations supplemented by numerical branch-tracing[6]. For weak motion (creeping flow), we extended this approach below using a modified functional. Near the singularity represented by the Plateau-Rayleigh limit, bifurcation theory using Liapunov-Schmidt reduction is a natural tool for the solution of the appropriate nonlinear Euler-Lagrange equation. All these analytical/numerical tools lend themselves to understanding the physics of stability in terms of simple competition mechanisms.

As for the experimental approach, a dynamic Plateau chamber has been built and is used to study liquid bridges held captive by rod-ends and embedded in a controlled surrounding flow. Theory has guided the experiments to a particular window in parameter space. Such guidance is crucial since interesting stabilization effects occur over narrow parameter ranges for this problem.

Science results

The stability of liquid bridges of finite extent near the Plateau-Rayleigh length is the main subject of this report. The bridge is sheared axially by embedding it in a pipe flow. Finite amplitude deviations from cylindrical shape are accounted for by a nonlinear theory. The predictions so far are limited to small deviations from no flow, no gravity, and the Plateau-Rayleigh limit. They explain the experimental observation of a slight stabilization to lengths *beyond* the Plateau-Rayleigh limit.

The schematic of the experimental configuration is shown in figure 1. The bridge liquid attaches to each rod end with a circular contact-line of radius r_o . A smaller diameter rod (radius r_i) connects the two end rods. This annular geometry influences the pressure at the interface when the bridge liquid is in motion. In the absence of motion, the interface shape and its stability are controlled by dimensionless bridge length L , volume \bar{V} , and the gravity to surface tension strength B (Bond number). The strength of gravity is controlled by the density imbalance $\Delta\rho$ (bridge - surrounding liquid). In the absence of motion the connecting rod plays no role. By controlling the flow rate of the external liquid, between the outer tube wall (radius r_t) and the interface, traction at the interface transmits motion to bridge liquid. In summary, with motion, the shape and stability of the interface depend on a dimensionless flow rate C (capillary number), two radius ratios and the viscosity ratio in addition to the three parameters that determine shapes and stability under static conditions.

Upward flow is driven by a pressure gradient that opposes the hydrostatic gradient of a heavy bridge. This is the interesting case. In one protocol, B and C are fixed and the bridge length is quasistatically increased maintaining a cylindrical volume ($\bar{V} = 1$). Breakup is recorded at length L_c . The experiment is repeated for a different C . The symbols in figure 2 show the results. The opposing effects of gravity and flow are evident: a maximum length occurs near $B = C$. Flow effectively cancels density imbalance. The solid curve is the destabilization by gravity (theory) from the Plateau-Rayleigh limit ($L_c = 2\pi$) in the absence of motion ($C = 0$). Unexplained is the apparent stabilization to length *greater* than 2π .

A different experimental protocol fixes the bridge length for given B . A volume is chosen and the flow rate C is increased/decreased until breakup occurs. Figure 3 show these results. The data are connected by a solid line representing interpolation (dotted indicates extrapolation). Note that as the bridge length approaches 2π the effects of flow on breakup become nonlinear and, indeed for $L = 6.099$, there is apparently an 'island' of stability for cylindrical volumes: bridges exist in the presence of flow that would otherwise breakup. Details of these experiments are found in [7, 8, 9].

The nonlinear analysis is based on a model that accounts for the external flow by imposing a shear stress τ on the interface of surface tension σ . The dissipation of this single phase configuration with deformable interface is minimized. The normal stress balance is a modified Young-Laplace equation and also corresponds to the nonlinear Euler-Lagrange equation of a modified functional. Solutions of the nonlinear equation with their stabilities are obtained by Liapunov-Schmidt reduction using bifurcation theory. Perturbation parameters measure deviations from no flow, no gravity and the Plateau-Rayleigh limit. The following dimensionless groups arise (cylindrical volume is assumed):

$$\bar{C} = \tau r_o / \sigma \quad (1)$$

$$B = r_o^2 \Delta\rho g / \sigma \quad (2)$$

$$A = (r_o - r_i) / r_o \quad (3)$$

$$\lambda = L / (2\pi r_o) - 1. \quad (4)$$

λ is treated as the primary bifurcation parameter. $\gamma = (\bar{C}, B)$ are the perturbation unfolding parameters. In the equilibrium state ($\gamma = 0$), the Young-Laplace equation is recovered which has a linear operator with null space ϕ spanned by $\sin(k\pi x)$. The bifurcation diagram shows a subcritical pitchfork structure. The nontrivial branch near the singular point takes the form:

$$\epsilon\phi + w(x). \quad (5)$$

$w(x)$ is in the complement of the null space and is $o(\epsilon)$. The bifurcation equation is $g = 0$ and the universal unfolding is

$$g = -\frac{\lambda\epsilon}{2} - \frac{3\epsilon^3}{32} + (\bar{C}p - B) + \bar{C}q\epsilon^2, \quad (6)$$

$$C = \bar{C}p(A) \quad (7)$$

Details of the analysis and the form of functions $p(A)$ and $q(A)$ are given elsewhere[10]. The classical subcritical pitchfork is recovered for no flow and no gravity ($C = B = 0$). For gravity ($B \neq 0$) and no flow ($C = 0$) the perturbation results of[11] are recovered. The presence of creeping flow without gravity is always destabilizing ($B = 0, C \neq 0$). Thus, the physics at work here is different from that in the infinite cylinder where inertial effects are responsible for stabilizing pressures. There, finite Reynolds number flows can stabilize without gravity[1]. Our main result is the perturbed bifurcation structure with flow and gravity for $B = C$. This case shows somewhat surprisingly that even though each of two effects is separately destabilizing, together they can stabilize. The maximum stability limit is plotted in figure 4 ($A = 0.5$) as a function of imposed shear and gravity imbalance. We see that the destabilizing effect of gravity can be overcome by the pressure field induced by the shear flow. In the operating regime where the stability limit is extended beyond 2π , the liquid column does not have a perfect cylindrical shape. The predictions of figure 4 are to be compared to the data of figure 2.

Theory (additional results from the past year – peripheral to the above discussion)

- 1) Stability (instability) of a static bridge equilibrium ($B = 0$) is immediate once the family of equilibria to which it belongs is identified; direct calculation of the second variation is circumvented[6].
- 2) All known families of static bridge equilibria ($B = 0$) are ultimately connected and thereby inherit their states of instability (number of unstable modes) ultimately from the stability of the sphere[6].

Experiment (additional results from the past year – peripheral to the above discussion)

- 3) A comparison of pairs of relatively immiscible liquids suitable for use in a dynamic Plateau chamber with density balance within $10^{-4} g/cm^3$ has been presented. Pure water and the isomeric system of 2-, 3- and 4-fluorotoluene is one preferred pair. Pure water and the homologous system of chlorocyclohexane and chlorocyclopentene is another[12].
- 4) Further observation and analysis of the collapse of the soap-film bridge have been performed. The soap-film collapse is viewed as a prototype collapse.

Research plan

Theory

The nonlinear analysis presented above is limited by the single phase assumption of the model. Moreover, even with the model, there is limited validity in amplitude ϵ , length deviation λ and C and B . We shall attempt to remove these limitations. Another goal is to make precise the relationship between our approach and that of energy stability theory (with linking parameters).

Experiment

Further tests of stabilization guided by the new theoretical results are planned. Exploratory experiments will probe the influence of a time-dependent driving flow on the stability limit of bridges, and the coalescence of droplets in the presence of flow.

Conclusion

We have shown in theory and in experiment that shear can stabilize a finite liquid column beyond the Plateau-Rayleigh limit, among other results. Surprisingly, a slight density imbalance is required. Two imperfections (density and flow) conspire to stabilize even though each on its own destabilizes. Apparently, only lengths slightly longer than the Plateau-Rayleigh length are achievable.

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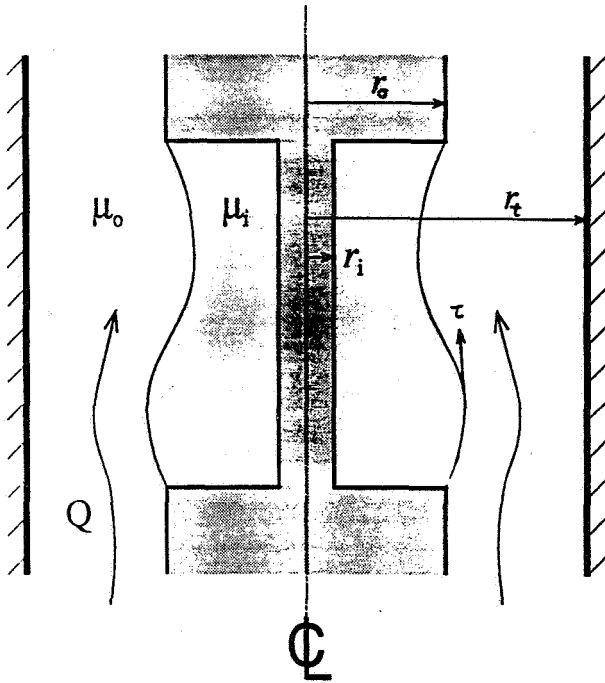


Figure 1 Definition sketch of the experiment.

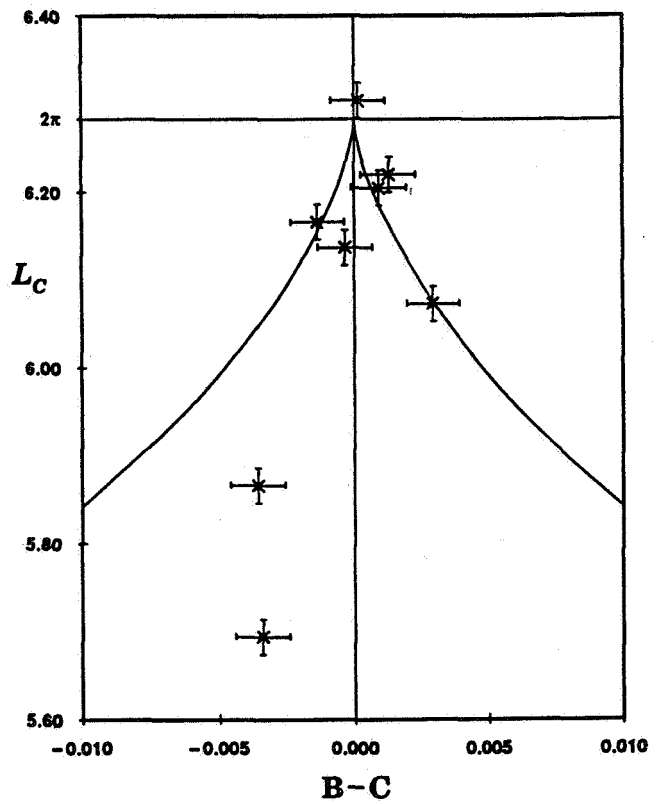


Figure 2 - detail of $L_C(B-C)$ peak for $V = 1.00 \pm 0.02$ and $B = 0.0029$ with comparison to static $L_C(B)$ curve (80% 4-FT, 20% 2-FT, $r_t = 2.130r_o$)

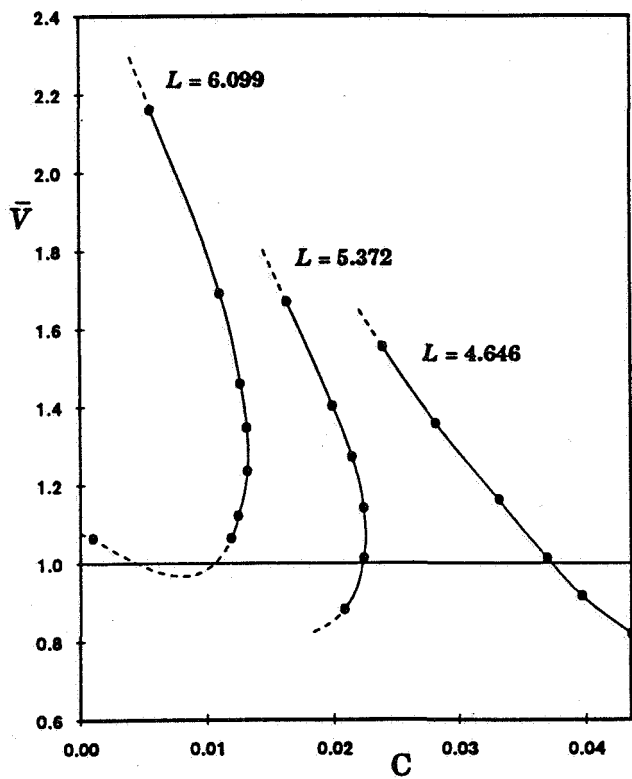


Figure 3 - liquid bridge stability limits in terms of volume and capillary number with $B = 0.0080 \pm 0.0005$ (80% 4-FT, 20% 2-FT, $T = 19.95^\circ\text{C}$, $r_i = 0.532r_o$, $r_e = 2.130r_o$)

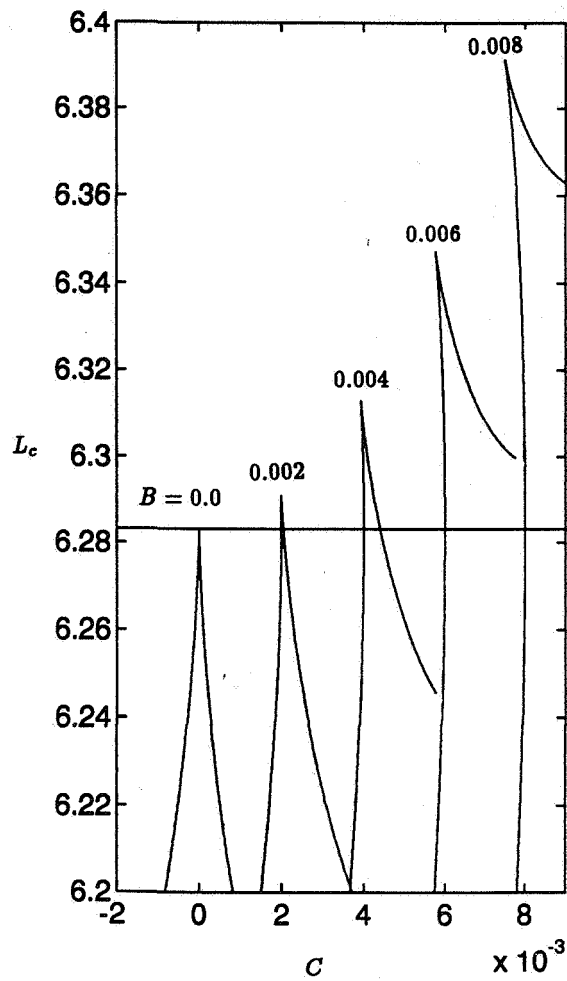


Figure 4 Critical length L_c of bridge versus C
 $A = 0.5$