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A Trajectory Preprocessor for Antenna Pointing

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A trajectory-preprocessing algorithm has been devised which matches antenna angular position, velocity, and acceleration to those of a target. This eliminates vibrations of the antenna structure caused by discontinuities in velocity and acceleration commands, and improves antenna-pointing performance by constraining antenna motion to a linear regime. The algorithm permits faster acquisition times and preserves antenna-tracking capability in situations where there would otherwise be an unacceptably sudden change in antenna velocity or acceleration. A simulation of DSS 13 shows that this preprocessor would reduce servo error to 1 mdeg during acquisition of a low-Earth-orbiting satellite.

I. Introduction

When a large antenna is moved in a sudden or jerky manner, the ensuing vibration of the structure can adversely affect pointing accuracy. A fast-moving antenna which stops in a precipitous manner upon acquiring a target may vibrate enough to lose lock. Even in situations for which the vibration amplitude does not immediately produce an unacceptable pointing error, the servo control may enter a nonlinear, and possibly unstable, region. This problem can arise when the servo controller attempts to apply an excessive velocity or acceleration in an attempt to track a sudden command change [1].

In the past, Deep Space Network pointing requirements have been for slow antenna motion. While DSN antennas are required to track at up to 0.4 deg per sec, they are not required to meet pointing requirements for other than sidereal targets.[1] The prospect of very accurate acquisition of low-Earth-orbiting satellites (with angular velocities in excess of even 0.4 deg per sec) is a relatively new idea. The anticipation of such requirements has led to the present work.

Trajectory preprocessing is one method used to preserve antenna-pointing integrity. The principal idea is to make antenna motion smoother during target acquisition, although the algorithm can be applied to all antenna motion commands. The present antenna control system is nonlinear. Its behavior is governed by the nonlinearity (acceleration and velocity limits), its inputs, the initial conditions, and the linear subsystem frequency response (the controller bandwidth). This has resulted in a system with complicated switching rules for changing the controller bandwidth as a function of inputs. A change in a velocity or acceleration limit affects the switching rules. The system can still work poorly for some sets of inputs, initial conditions, and parameters.

[1] W. Scherr, *Deep Space Communications Complex Subsystem Functional Requirements, Antenna Mechanical Subsystem (1991 through 1997)*, JPL D-1179, Rev. C (internal document), Jet Propulsion Laboratory, Pasadena, California, September 1, 1992.

Since commands which are in violation of these limits cause the system to behave nonlinearly, a simple method for allowing the system to operate in a linear regime is to force command angles to conform to the limits. Trajectory preprocessing performs this task.

Even if an antenna were perfectly rigid and could be moved at will at accelerations of up to 1 deg/sec^2, it could still overshoot a target during a high-speed acquisition. When the difference in speeds between average acquisition velocity and tracking velocity can easily exceed a deg/sec, some algorithm is needed to ensure that the acquisition time remains reasonable for a target which may not be visible for more than a minute or two in the first place. This provides a further motivation for considering trajectory preprocessing.

The problem to be solved by this preprocessor is to find an optimal, or at least an adequate, path from an initial antenna position, velocity, and acceleration to some target trajectory. The first idea that was considered for finding this path was to use the calculus of variations. This idea was abandoned for three reasons. First, the method is overly complex. Second, the calculation time can be large. The method is iterative, and the calculation time is indeterminate, making it unsuitable for a real-time system. Finally, although the calculus of variations can give a least-time solution, it is not easy to include constraints that will prevent sudden changes in the slopes of the antenna velocity and acceleration profiles.

II. The Three-Region Method

The method described in this article involves three regions of antenna motion and was inspired by the following scenario. A target is far away and one wishes to move the antenna towards the target trajectory as quickly as possible. So one begins by accelerating the antenna to its maximum speed; that is region 1. Then, in region 2 one moves the antenna at maximum speed until one is near the target. Finally, in region 3, the antenna is decelerated until it matches the apparent target angular position and angular velocity.

As applied to a low-Earth-orbiting satellite, the initial scenario envisioned the following preprocessor steps:

(1) Input a set of target positions and velocities (θ and v) for both local azimuth and elevation as a function of time using known orbital parameters (updated by optical detection).

(2) Choose an intercept (acquisition) $\theta_{el} = \theta_{0_{el}}$. From this obtain the time interval, T, from the start of region 1 to the intercept time, as well as θ_{az} and both v_{az} and v_{el}. Input the initial antenna pointing (θ_0 and v_0) and the maximum angular velocity for the antenna (v_{max}). Input or calculate maximum angular accelerations for the antenna ($a_{max} > 0$).

(3) If necessary, try a different maximum acceleration (remaining less than or equal to the specified limit) or a later acquisition time. Set a flag if acquisition is not possible before $\theta_{el} > \theta_{max}$.

The same algorithm can be used for reacquisition during tracking (without the flag). One does not need to start by choosing the acquisition elevation. A total acquisition time can be input instead. This time can be chosen by noting the distance in position and the change in velocity that must occur. Using one's experience from previous acquisitions, it will often be possible to choose an acquisition time within a second of the minimum. Should one's choice be too small, the algorithm can be rerun with steadily increasing total times input. In practice, the maximum acceleration may simply be set to the specified acceleration limit.

For acquisition, in both azimuth and elevation, it is best to chase the target rather than approach it head-on; this lessens the required change in velocity. That means that θ_0 should be a little less than θ. Thus, while waiting for the target to appear, the antenna will generally be pointing at a place in the trajectory close to that anticipated by an optical acquisition aid. The antenna will typically have an initial velocity of approximately zero in elevation (or it might badly overshoot the target). When the target shows up "late," the Earth's rotation will cause it to move in azimuth as a function of lateness, so the antenna may initially be moving slowly in azimuth. The formula for the minimum time to acquire is then $T = v_f/A$, where A is the average acceleration.

When $A = a_{max}$, we get the fastest acquisition. An a_{max} in elevation of 0.5 deg/sec gives $T = 2v_f$, where v is in deg/sec and T is in sec.

In our acquisition schemes, the average A will often be $a_{max}/2$ or less, giving a minimum acquisition time of $4v_f$ sec. When v_f is about 0.5 deg/sec, this results in a minimum acquisition time on the order of 2 sec.

This minimum acquisition time applies only when

$$\Delta\theta_{el} = \frac{v_f^2}{2A}$$

If, for example, we initially aim at the acquisition point instead of a point $v_f^2/2A$ below it, the minimum time increases from T to $T/\sqrt{2} - 1$.

Figure 1 shows the matching in elevation of antenna pointing with that of a target. Figure 2 shows the antenna angular velocity for this example. In these figures, the antenna is aimed a little above the acquisition point. Figure 3 shows how the algorithm matches angular position when the antenna is initially aimed a little below this point.

The acquisition time will often be determined by the change in v_{el}. The final azimuthal velocity is usually much smaller, and there may even be an initial azimuthal velocity. However, if the antenna is simply waiting several degrees from the acquisition, θ_{az}, the azimuthal acquisition time can easily become the total acquisition time.

III. Acquisition Schemes

Four acquisition schemes are considered in this article:

Scheme 1: Match initial and final angular position and velocities. Use the maximum allowable acceleration throughout each acceleration region.

Scheme 2: The same as scheme 1, but use a sinusoidal acceleration pattern to avoid large discontinuities in acceleration.

Scheme 3: The same as scheme 2, but match the final acceleration as well.

Scheme 4: The same as scheme 2, but match both initial and final accelerations.

Each of these ideas has some merit. The calculations needed to implement the trajectory-preprocessing algorithm are similar for any of the first three schemes.

A. The First Acquisition Scheme

Scheme 1 has the advantage of speed. By using the maximum acceleration, the target can be reached more quickly. Even when the increased initial overshoot and ensuing oscillations are taken into account, the total acquisition time may be minimized in some situations. On the other hand, when the angular distance to the target is large and the maximum allowable pointing error is small, this scheme is inappropriate.

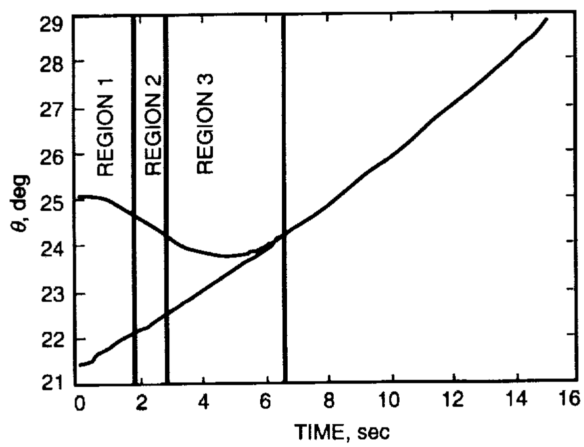

Fig. 1. Matching of antenna and target positions for constant accelerations (elevation only).

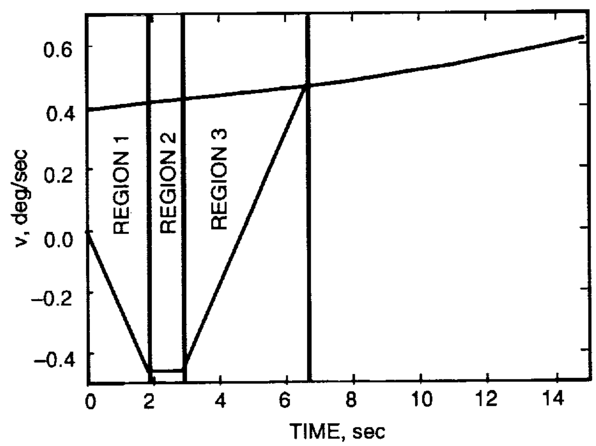

Fig. 2. Matching of antenna and target velocities for constant accelerations (elevation only).

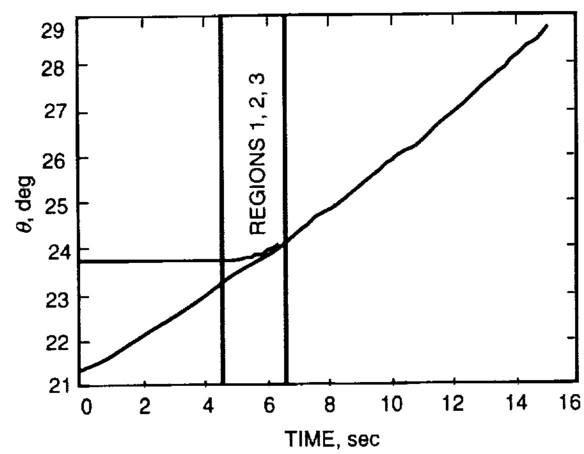

Fig. 3. Matching of antenna and target positions for elevation only (chasing target).

This scheme uses the maximum acceleration in each acceleration region. Although this introduces discontinuities in acceleration, it permits a straightforward calculation of the acquisition parameters. Then $a = \pm a_m = \pm a_{max}$ in regions 1 and 3. By integrating twice, we get the velocities and positions as a function of time in all three regions. The following calculations must be performed both for elevation and for azimuth:

For region 1,

$$v = v_0 + a_1 t \tag{1}$$

$$\theta = \theta_0 + v_0 t + \frac{a_1 t^2}{2} \tag{2}$$

For region 2,

$$v = v_2 = v_0 + a_1 t_1 = v_f - a_3 t_3 \tag{3}$$

$$\theta = \theta_0 + v_0 t_1 + v_2 (t - t_1) + \frac{a_1 t_1^2}{2} \tag{4}$$

where $T = t_1 + t_2 + t_3$.

For region 3,

$$v = v_f - a_3(T - t) \tag{5}$$

$$\theta = \theta_f - v_f(T - t) + a_3\frac{(T - t)^2}{2} \tag{6}$$

1. Calculation of Acquisition Parameters. In the above equations, we do not yet know the signs of a_1 or a_3. Nor do we know the durations, t_1 or t_3, or the constant velocity, v_2. These are calculated as follows. Input the initial and final antenna positions (azimuth and elevation), θ_0 and θf,

$$\Delta\theta = \theta_f - \theta_0$$

as well as the initial and final antenna velocities (azimuth and elevation), v_0 and v_f,

$$\Delta v = v_f - v_0$$

and the total time,

$$T = t_1 + t_2 + t_3$$

as well as the maximum acceleration, a_m, and velocity, v_m (in both azimuth and elevation), which must be less than or equal to the requirements, a_{max} and v_{max} (in both azimuth and elevation).

From the above inputs, output t_1, t_2, t_3, and v_2 and also determine a_1 and a_3. To obtain these outputs, the following formulas are used. First, a normalized angular position, x, and a normalized angular velocity, y, are calculated:

$$x = \frac{\Delta\theta}{a_m T^2} - \frac{v_0}{a_m T} \tag{7}$$

$$\frac{y = \Delta v}{a_m T} \tag{8}$$

Next, ε_0 and ε_f, intermediate variables that are used to find the signs of the accelerations in regions 1 and 3, are determined:

$$\varepsilon_0 = \varepsilon_f = -1 \text{ when } y \le 0 \text{ and } y + \frac{y^2}{2} \le x \le -\frac{y^2}{2} \tag{9}$$

$$\varepsilon_0 = \varepsilon_f = 1 \text{ when } y < 0 \text{ and } \frac{y^2}{2} \le x \le -y - \frac{y^2}{2} \tag{10}$$

$$\varepsilon_0 = 1 \text{ and } \varepsilon_f = -1 \text{ when } y > 0 \text{ and } x > y - \frac{y^2}{2} \text{ or } y \le 0 \text{ and } x > -\frac{y^2}{2} \tag{11}$$

$$\varepsilon_0 = -1 \text{ and } \varepsilon_f = 1 \text{ when } y > 0 \text{ and } x < \frac{y^2}{2} \text{ or } y \le 0 \text{ and } x < y + \frac{y^2}{2} \tag{12}$$

These give us a_1 and a_3:

$$a_1 = \varepsilon_0 a_m \tag{13}$$

$$a_3 = \varepsilon_f a_m \tag{14}$$

Now the velocity in region 2 can be found:

$$\text{for } \varepsilon_0 = \varepsilon_f, \qquad y_2 = \frac{y^2\varepsilon_f - 2x}{2(y\varepsilon_f - 1)} \tag{15}$$

$$\text{for } \varepsilon_0 \neq \varepsilon_f, \qquad y_2 = \frac{y\varepsilon_f - 1 + \sqrt{y^2\varepsilon_0\varepsilon_f - 2y\varepsilon_f + 2x(\varepsilon_f - \varepsilon_0) + 1}}{\varepsilon_f - \varepsilon_0} \tag{16}$$

$$v_2 = a_m T y_2 + v_0 \tag{17}$$

Finally, the time intervals can be deduced:

$$t_1 = \frac{v_2 - v_0}{a_1} \tag{18}$$

$$t_3 = \frac{v_f - v_2}{a_3} \tag{19}$$

$$t_2 = T - t_1 - t_3 \tag{20}$$

Once the above parameters have been obtained, position and velocity commands are determined from Eqs. (1) through (6).

The main problem with this acquisition scheme is the discontinuity in antenna acceleration at the borders of regions 1 and 3. The problem is most serious at the border of region 3, where transient phenomena may significantly affect antenna-pointing accuracy immediately after acquisition. To see this, consider the example of Figs. 1 and 2. In this, Example (1),

$$\begin{aligned}
\theta_0 &= 25.104 \text{ deg} \\
\theta_f &= 24.253 \text{ deg} \\
v_0 &= -0.001 \text{ deg/sec} \\
v_f &= 0.479 \text{ deg/sec} \\
T &= 6.6 \text{ sec} \\
|a| &= 0.25 \text{ deg/sec}^2
\end{aligned}$$

which has the solution

$$v_2 = -0.46 \text{ deg/sec}$$
$$t_1 = 1.85 \text{ sec}$$
$$t_2 = 1 \text{ sec}$$
$$t_3 = 3.75 \text{ sec}$$

Figure 4 shows anticipated servo error during and immediately following acquisition for this example using a simulation in which a PI controller is applied to a model of a 34-m Deep Space Network antenna, namely DSS 13 in Goldstone, California. This servo controller, which uses both proportional (P) and integral (I) feedback terms, is in use at DSS 13. The controller and the model of the antenna are described in [2].

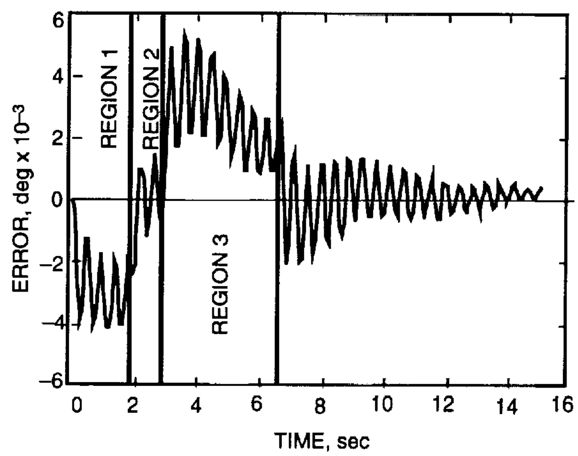

Fig. 4. Anticipated servo error for Example (1) with constant acceleration in regions 1 and 3 (elevation only).

2. Derivation of Results. This section gives a derivation of the results of Eqs. (7) through (20). Begin by conserving the total angular distance:

$$v_1 t_1 + v_2 t_2 + v_3 t_3 = \Delta\theta \tag{21}$$

The values t_1 and t_3 are obtained from the change in velocity divided by the acceleration. This immediately gives Eqs. (18) and (19).

The values v_1 and v_3 are the average angular velocities in regions 1 and 3. Since the acceleration is constant in each of these regions,

$$v_1 = \frac{v_2 + v_0}{2} \text{ and } v_3 = \frac{v_2 + v_f}{2} \tag{22}$$

When Eqs. (18), (19), and (22) are substituted in Eq. (21), the result is

$$\frac{v_2^2 - v_0^2}{2a_1} + v_2 t_2 + \frac{v_f^2 - v_2^2}{2a_3} = \Delta\theta \tag{23}$$

By substituting $T - t_1 - t_3$ for t_2 in Eq. (23), we get

$$-\frac{(v_2 - v_0)^2}{a_1} + 2v_2 T + \frac{(v_f - v_2)^2}{a_3} = 2\Delta\theta \tag{24}$$

Further substitution of the definitions of x, y, y_2, ε_0, and ε_f from Eqs. (7), (8), (17), (13), and (14) gives

$$y_2^2(\varepsilon_f - \varepsilon_0) + 2y_2(1 - y\varepsilon_f) + y^2\varepsilon_f - 2x = 0 \tag{25}$$

When $\varepsilon_0 = \varepsilon_f$, which is often the case, Eq. (15) follows directly from Eq. (25). When $\varepsilon_0 \neq \varepsilon_f$, Eq. (16) immediately results from applying the quadratic formula to Eq. (25). However, the sign in front of the square root in Eq. (16) is yet to be determined. This is done as follows. Note that

$$t_2 = T - \frac{v_f}{a_3} + \frac{v_0}{a_1} - \frac{v_2}{a_1} + \frac{v_2}{a_3} + \frac{v_0}{a_3} - \frac{v_0}{a_3}$$

which gives

$$\frac{t_2}{T} = 1 - y\varepsilon_f + y_2(\varepsilon_f - \varepsilon_0)$$

The square root term in Eq. (16) is also equal to $1 - y\varepsilon_f + y_2(\varepsilon_f - \varepsilon_0)$. Since t_2/T must be positive, the plus sign must be chosen in front of the square root.

The next step is to find out where the solutions for y_2 described in Eqs. (15) and (16) are valid. The values of x and y for which a solution can be found will be referred to as an "area of validity," which can be plotted on a graph of y versus x. This area is in "phase space," and as it increases, a larger number of combinations of target positions and velocities can be matched. Starting from Eq. (16), the minimum and maximum values of x as a function of y_2 are obtained by setting $dx/dy_2 = 0$. This gives $y_2 = (y\varepsilon_f - 1)/(\varepsilon_f - \varepsilon_0)$ and $t_2 = 0$. This means that the discriminant of the square root in Eq. (16), t_2^2, is 0 at the borders of the area of validity of solutions to Eq. (25). Setting the discriminant to zero gives

$$x = \frac{1 - 2y\varepsilon_f + y^2\varepsilon_0\varepsilon_f}{2(\varepsilon_0 - \varepsilon_f)} \tag{26}$$

At the external boundaries of the area of validity, $\varepsilon_0 \neq \varepsilon_f$. Thus, the area of valid solutions is bounded by the curves

$$x = \frac{y^2 + 2y - 1}{4} \qquad (-1 \leq y \leq 1)$$

and

$$x = \frac{1 + 2y - y^2}{4} \qquad (-1 \leq y \leq 1)$$

This area of validity is shown in Fig. 5. Notice that solutions for which x and y have the same sign are favored (have more phase space) than those which do not.

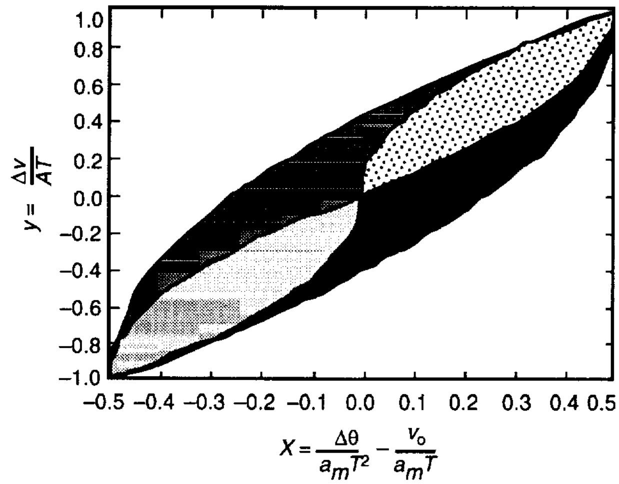

Fig. 5. Area of valid solutions ($a_m = a_{max/2}$ and $a_o = a_f = 0$).

In these calculations, we have ignored the situation in which v_{max} corresponds to $|y_{max}| < 1$, which would cut off the region of valid solutions at $y < 1$ or $y > -1$. This can be dealt with as follows: Note that for the antenna velocity to exceed a velocity limit, either v_o, v_f, or v_2 must exceed that limit. If v_0 exceeds the limit, we already have a problem, but it is one that has little to do with the trajectory preprocessor. If v_f exceeds the limit, we cannot acquire at that point, but we can try other values of T for which v_f may be smaller. If v_2 is the only culprit, we increase T to a point at which v_2 is acceptable.

Figure 5 also shows internal borders in the area of validity. These borders define the regions where a_1 and a_3 are both negative, both positive, of opposite sign with a_1 positive, and of opposite sign with a_1 negative, respectively. At these borders, $t_1 = 0$ (which gives $y_2 = 0$) or $t_3 = 0$ (which gives $y_2 = y$). Substituting in Eq. (25),

$$y_2 = 0 \Rightarrow x = \frac{y^2 \varepsilon_f}{2} \tag{27}$$

while

$$y_2 = y \Rightarrow x = y - \frac{y^2 \varepsilon_0}{2} \tag{28}$$

By defining the borders of each of the four internal regions, Eqs. (27) and (28) permit us to write down Eqs. (9) through (12), completing our derivation of the formulas used in the first acquisition scheme.

B. The Second Acquisition Scheme

To avoid the discontinuities in acceleration, one can choose a sinusoidal acceleration pattern. This trades acquisition rate for pointing accuracy; the price that is paid for improved accuracy is a factor of two in average antenna angular acceleration. The preprocessor matches position and velocity as before. The antenna angular acceleration is still not matched to that of the target, but it is zero rather than the maximum allowable acceleration at acquisition. The idea is to let $a = \pm a_m(1 - \cos 2\pi\omega)$ for some ω and for $a_m < a_{max}/2$. In particular, for region 1,

$$a = a_1 \left(1 - \cos \frac{2\pi t}{t_1}\right)$$

Integrating with respect to t,

$$v = v_0 + a_1 \left(t - \frac{t_1}{2\pi} \sin \frac{2\pi t}{t_1} \right) \tag{29}$$

Integrating again,

$$\theta = \theta_0 + v_0 t + a_1 \left(\frac{t^2}{2} - \frac{t_1^2}{4\pi^2} + \frac{t_1^2}{4\pi^2} \cos \frac{2\pi t}{t_1} \right) \tag{30}$$

For region 2,

$$a = 0$$

$$v = v_2 = v_0 + a_1 t_1 = v_f - a_3 T_3 \tag{31}$$

$$\theta = \theta_0 + v_0 t_1 + v_2 (t - t_1) + \frac{a_1 t_1^2}{2} \tag{32}$$

For region 3,

$$a = a_3 \left(1 - \cos \frac{2\pi (T - t)}{t_3} \right)$$

Integrating with respect to $(T - t)$,

$$v = v_f - a_3 \left(T - t - \frac{t_3}{2\pi} \sin \frac{2\pi (T - t)}{t_3} \right) \tag{33}$$

$$\theta = \theta_f - v_f (T - t) + a_3 \left(\frac{(T - t)^2}{2} - \frac{t_3^2}{4\pi^2} + \frac{t_3^2}{4\pi^2} \cos \frac{2\pi (T - t)}{t_3} \right) \tag{34}$$

Note that da/dt is not merely finite everywhere; it goes to 0 at the borders of each region.

At first, these equations seem more complicated than those for constant accelerations, but the solutions for their parameters are identical. Equations (7) through (20) still hold. The acceleration is not constant, but Eq. (22) is still valid. The maximum value of a_m is reduced by a factor of 2, but Eqs. (13) and (14) are unchanged. The same four solutions exist with the same boundaries in x and y, as represented by Fig. 5. The only difference is that Eqs. (1) through (6) have been replaced by Eqs. (29) through (34).

Example (1) still has the same solution for t_1, t_2, t_3, and V_2. However, the anticipated servo error has been reduced because the acceleration changes more smoothly.

Figure 6 shows the matching of antenna and target angular positions in Example (1) for acquisition scheme 2. Figure 7 shows the corresponding matching of velocities. Note that the velocity slope is zero at acquisition for the antenna but not for the target; there is still a slight discontinuity in acceleration at acquisition. Figure 8 gives the anticipated servo error for Example (1) using the second acquisition scheme.

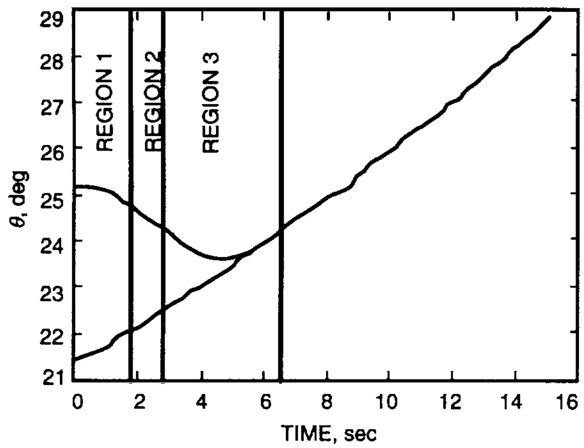

Fig. 6. Matching the target position for raised cosine acceleration (elevation only).

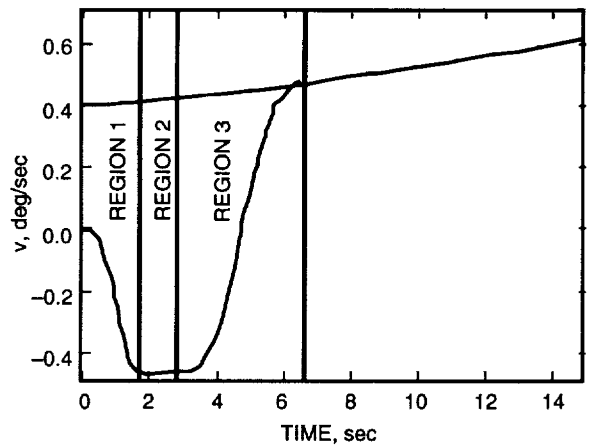

Fig. 7. Matching the target velocity for raised cosine acceleration (elevation only).

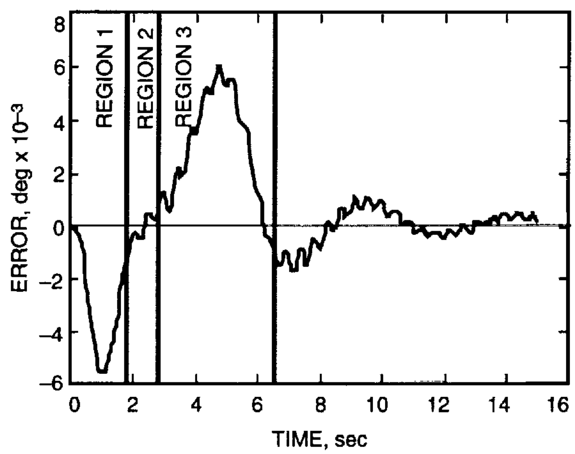

Fig. 8. Anticipated servo error (for Example (1) with raised cosine acceleration in regions 1 and 2 (elevation only).

Since T has already been chosen prior to the calculation of t_1 and t_3, it may seem unnecessary to set a_m to $a_{max}/2$ when a smaller a_m would suffice. Also, potentially huge values of da/dt (jerk) can arise in this trajectory preprocessing scheme. These high values of jerk are produced when one is near the internal borders within the region of valid solutions in Fig. 5, since t_1 or t_3 approaches 0. Since $|a|$ goes from 0 to $2a_m$ and back in each region, the average $|da/dt|$ is $4a_m/t_3$ in region 3. The maximum $|da/dt|$ is $8a_m/t_3$ in region 3, and as t_3 approaches 0, this number becomes larger until the increase is filtered out by the servo mechanism. Actually, this gives us a very small servo error because for the PI controller, the maximum error caused by a sudden pulse does not exceed the total displacement caused by the pulse. Thus, the PI controller does not react much to a t_3 of 0.1 sec because $1/2a_{av}t^2$ for an average a of 0.125 deg/sec^2 is only 0.6 mdeg, which is an upper bound on the servo transient error. This is illustrated by Example (2), for which

$$\theta_0 = 23.7618 \text{ deg}$$

$$\theta_f = 24.253 \text{ deg}$$

$$v_f = 0.479 \text{ deg/sec}$$

$$v_0 = -0.001 \text{ deg/sec}$$

$$|a| = 0.25 \text{ deg/sec}^2$$
$$T = 2 \text{ sec}$$
$$t_1 = 1.9 \text{ sec}$$
$$t_2 = 0.0 \text{ sec}$$
$$t_3 = 0.1 \text{ sec}$$
$$v_2 = 0.4665 \text{ deg/sec}$$

Figure 9 shows the anticipated servo error for Example (2) with acquisition scheme 2.

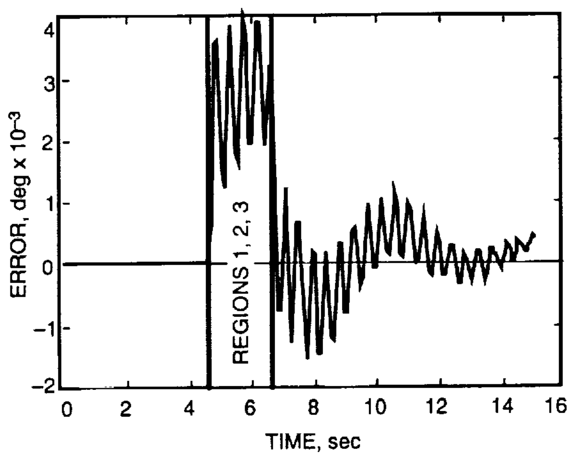

Fig. 9. Anticipated servo error during acquisition for Example (2), (t_3 = 0.1 sec, t_2 = 0) (elevation only).

If it is thought that the servo controller would react adversely to such a sharp acceleration pulse, one can modify a_m so that neither t_1 nor t_3 is small, or at least choose a reasonable a_m to begin with and modify either a_m or t if necessary. For example, one can choose an a_m that gives $t_2 = 0$ and if t_1 or t_3 is small anyway, one simply increases T. From Eq. (26), $t_2 = 0$ gives

$$y^2 \varepsilon_0 \varepsilon_f - 2y\varepsilon_f + 1 = 2(\varepsilon_0 - \varepsilon_f)x$$

Picking $\varepsilon_0 \varepsilon_f = -1$,

$$y^2 + 2y\varepsilon_f \left(1 - \frac{2x}{y}\right) - 1 = 0$$

Let

$$\Omega = 1 - \frac{2x}{y} = \frac{\Delta v T - 2\Delta\theta + 2v_0 T}{\Delta v T}$$

Then

$$\frac{\Delta v}{a_m T} = y = -\varepsilon_f \Omega \pm \sqrt{\Omega^2 + 1} \tag{35}$$

The sign in front of the square root in Eq. (35) and the value of ε_f are readily determined by observing that y has the same sign as $\Delta\nu$ and that $|y| \leq 1$. This determines y and gives

$$a_m = \frac{\Delta v}{yT}$$

The $a = 0$ region can also be omitted from the algorithm entirely. The resulting scheme would have no intermediate region of maximum velocity for large changes in antenna position and would increase acquisition time as a result. If by some chance,

$$\frac{\Delta v}{T} = \frac{v_f^2 - v_0^2}{2\Delta\theta}$$

(and even with some latitude in choosing t, it is unlikely that this will occur), then a one-region solution is possible where

$$a = \frac{T}{\Delta v}\left(1 - \cos\frac{2\pi t}{T}\right)$$

If not, then $\varepsilon_0\varepsilon_f = -1$, and y is determined from Eq. (35), which also gives a_m, while

$$y_2 = \frac{y \pm 1}{2}$$

By leaving out the $a = 0$ region, overall accelerations are reduced. However, there is an increased flexibility in maintaining a three-region algorithm, and picking a_m prior to solving for y_2 may be impractical in some situations.

C. The Third Acquisition Scheme

In our examples, the target acceleration and servo controller are such that the sudden discontinuity in acceleration at acquisition does not significantly increase the pointing error. However, there is a straightforward way to avoid this discontinuity by modifying the previous acquisition scheme so that it matches the target acceleration at acquisition instead of acquiring with zero acceleration. One simply switches to a frame of reference which has an acceleration equal to that of the target. Figure 10 shows the anticipated servo error for Example (1) in this case.

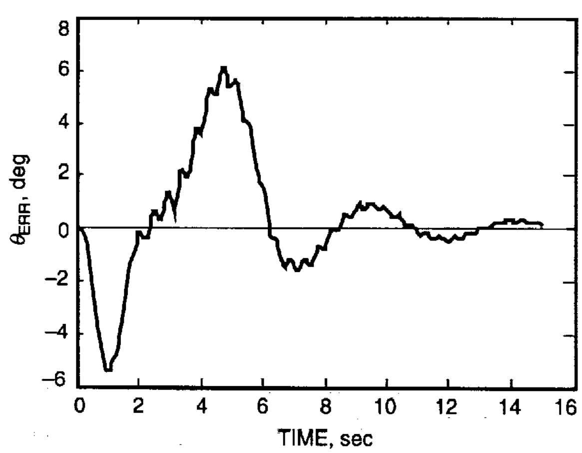

Fig. 10. Servo error for acquisition scheme 3 and constant velocity track. Acquisition is at t = 6.6 sec.

Let a_f be the acceleration of the target at $t = T$. Then match a position of $\theta_f - 1/2aT^2$ and a velocity of $v_f - aT$.

This means that $\Delta\theta$ now equals $\theta_f - \theta_0 - ((1/2)aT^2)$ and $\Delta v = v_f - v_0 - aT$. Once again, Eqs. (7) through (20) remain valid. However, care must be taken to ensure that the actual antenna accelerations and velocities never exceed the maxima. The acceleration can be kept within bounds by choosing $a_m = a_{max} - |a_f|/2$. But the velocity is trickier, as the actual velocity in region 2 is no longer constant. For a situation in which the antenna must traverse a large angular distance to acquire a target, it might at first appear that the maximum velocity will be reached at the end of region 2. This is not the case. At the end of region 2, the antenna has an acceleration of a_f, so the velocity is still increasing. The maximum velocity is reached when the acceleration first reaches zero in region 3.

The difference between this scheme and the first scheme is that Eqs. (1) through (6) are now replaced with the following equations:

For region 1,

$$a = a_1 \left(1 - \cos \frac{2\pi t}{t_1}\right) + a_f$$

$$v = v_0 + a_1(t - \frac{t_1}{2\pi} \sin \frac{2\pi t}{t_1} + a_f t \tag{36}$$

$$\theta = \theta_0 + v_0 t + a_1 \left(\frac{t^2}{2} - \frac{t_1^2}{4\pi^2} + \frac{t_1^2}{4\pi^2} \cos \frac{2\pi t}{t_1}\right) + \frac{1}{2} a_f t^2 \tag{37}$$

For region 2,

$$a = a_f$$

$$v = v_2 + a_f t \tag{38}$$

$$\theta = \theta_0 + v_0 t_1 + v_2(t - t_1) + \frac{a_1 t_1^2}{2} + \frac{a_f t^2}{2} \tag{39}$$

For region 3,

$$a = a_3 \left(1 - \cos \frac{2\pi(T-t)}{t_3}\right) + a_f$$

$$v = v_f - a_3 \left(T - t - \frac{t_3}{2\pi} \sin \frac{2\pi(T-t)}{t_3}\right) + a_f t \tag{40}$$

$$\theta = \theta_f - v_f(T-t) + a_3 \left(\frac{(T-t)^2}{2} - \frac{t_3^2}{4\pi^2} + \frac{t_3^2}{4\pi^2} \cos \frac{2\pi(T-t)}{t_3}\right) + \frac{a_f t^2}{2} \tag{41}$$

D. The Fourth Acquisition Scheme

Our final scheme applies to situations where the antenna has an initial acceleration a_0 and a final acceleration a_f. In this scheme, both the initial and final accelerations are matched by the preprocessor. We will assume that an initial value of a_m is chosen either by setting it to $a_{max}/2$, by looking at Eq. (35) (possibly modified by adding or subtracting a_0 or a_f), or by some as yet undetermined method. Inputs are then θ_0, θ_f, v_a, v_f, A_0, A_f, A_m, T, v_{max}, and a_{max} (for both azimuth and elevation). Equations (1) through (6) are now replaced by the following:

For region 1,

$$a = a_1 \left(1 - \cos \frac{2\pi t}{t_1}\right) + \frac{a_0}{2}\left(1 + \cos \frac{\pi t}{t_1}\right)$$

Note that $da/dt = 0$ both at $t = 0$ and $t = t_1$. By integrating,

$$v = v_0 + \left(a_1 + \frac{a_0}{2}\right) t + \frac{t_1}{2\pi}\left(a_0 \sin \frac{\pi t}{t_1} - a_1 \sin \frac{2\pi t}{t_1}\right) \tag{42}$$

$$\theta = \theta_0 + v_0 t + \left(a_1 + \frac{a_0}{2}\right) \frac{t^2}{2} + \frac{t_1^2}{4\pi^2}\left[2a_0\left(1 - \cos \frac{\pi t}{t_1}\right) - a_1\left(1 - \cos \frac{2\pi t}{t_1}\right)\right] \tag{43}$$

For region 2,

$$a = 0$$

$$v = v_2 = v_0 + \left(a_1 + \frac{a_0}{2}\right) t_1 = v_f - \left(a_3 + \frac{a_f}{2}\right) t_3 \tag{44}$$

$$\theta = \theta_0 + v_0 t_1 + v_2(t - t_1) + \left(a_1 + \frac{a_0}{2}\right) \frac{t_1^2}{2} + \frac{a_0 t_1^2}{\pi^2} \tag{45}$$

For region 3,

$$a = a_3\left(1 - \cos \frac{2\pi(T-t)}{t_3}\right) + \frac{a_f}{2}\left(1 + \cos \frac{\pi(T-t)}{t_3}\right)$$

$$v = v_f - \left(a_3 + \frac{a_f}{2}\right)(T - t) - \frac{t_3}{2\pi}\left(a_f \sin \frac{\pi(T-t)}{t_3} - a_3 \sin \frac{2\pi(T-t)}{t_3}\right) \tag{46}$$

$$\theta = \theta_f - v_f(T-t) - \left(a_3 + \frac{a_f}{2}\right)\frac{(T-t)^2}{2} - \frac{t_3^2}{4\pi^2}\left[2a_f\left(1 - \cos \frac{\pi(T-t)}{t_3}\right) - a_3\left(1 - \cos \frac{2\pi(T-t)}{t_3}\right)\right] \tag{47}$$

When sign (a_0) = sign (a_1),

$$|a_1| = \frac{1}{2}(2a_m - |a_0|)$$

Otherwise $|a_1| = a_m$. When sign (a_f) = sign (a_3),

$$|a_3| = \frac{1}{2}(2a_m - |a_f|)$$

Otherwise $|a_3| = a_m$.

The solution from the previous acquisition schemes no longer applies. Although y_2 has a similar form, ϵ_0 and ϵ_f are changed. The parameters are derived as follows. Once again, $v_1 t_1 + v_2 t_2 + v_3 t_3 = \Delta\theta$. Let

$$\beta_0 = \frac{1}{a_1 + (a_0/2)}$$

$$\beta_f = \frac{1}{a_3 + (a_f/2)}$$

Then

$$t_1 = \beta_0(v_2 - v_0)$$

$$t_3 = \beta_f(v_f - v_2)$$

From Eq. (45) we can calculate the change in position from $t = 0$ to $t = t_1$. Dividing by t_1, we get v_1:

$$v_1 = \frac{v_0 + v_2}{2} + \frac{t_1 a_0}{\pi^2}$$

Equation (22) is no longer valid, so the calculation becomes more complex:

$$v_3 = \frac{v_f + v_2}{2} + \frac{t_3 a_f}{\pi^2}$$

Plugging in,

$$(v_f - v_2)^2 \left[\beta_f + \frac{2\beta_f^2 a_f}{\pi^2}\right] - (v_0 - v_2)^2 \left[\beta_0 - \frac{2\beta_0^2 a_0}{\pi^2}\right] = 2\Delta\theta - 2v_2 T$$

Let

$$\epsilon_f = a_m \left(\beta_f + \frac{2\beta_f^2 a_f}{\pi^2}\right) \text{ and } \epsilon_0 = a_m \left(\beta_0 - \frac{2\beta_0^2 a_0}{\pi^2}\right)$$

Then

$$(v_f - v_2)^2 \epsilon_f - (v_0 - v_2)^2 \epsilon_0 = 2a_m(\Delta\theta - v_2 T)$$

Changing variables,

$$y_2^2(\epsilon_f - \epsilon_0) - 2y_2(y\epsilon_f - 1) + y^2\epsilon_f - 2x = 0$$

When $\epsilon_f = \epsilon_0$, we get Eq. (15). Otherwise,

$$y_2(\epsilon_f - \epsilon_0) = y\epsilon_f - 1 \pm \sqrt{y^2\epsilon_0\epsilon_f - 2y\epsilon_f + 2x(\epsilon_f - \epsilon_0) + 1} \tag{48}$$

which is the same as Eq. (16) except that the sign in front of the square root can be negative.

Since ϵ_0 can have at most 2 values, ϵ_f can have at most 2 values, and the sign in front of the square root can have at most 2 values, at worst one needs to solve Eq. (48) eight times to see if a valid value of y_2 can be obtained. In practice one does not need to try all eight possibilities. If $y > 0, \epsilon_0$ and ϵ_f cannot both be negative. If $y < 0, \epsilon_0$ and ϵ_f cannot both be positive. So only six cases remain.

It is tempting to simply write down the equations for the borders of each region and pick which solution is valid. That would determine ϵ_0 and ϵ_f. However, this method is not practical in general, as will be illustrated by trying it for $a_0 = -a_m$ and $a_f = a_m$. When a_1 and a_3 are both negative,

$$\epsilon_0 = -1 + \frac{2}{\pi^2} \text{ and } \epsilon_f = -2 + \frac{8}{\pi^2}$$

The minimum x is found by setting y_2 to y. From Eq. (48),

$$x = y - \frac{y^2\epsilon_0}{2}$$

Here, $-1 \leq y \leq 0$. When $-1 + a_f/2a_m = -1/2 \leq y \leq 0$, the maximum x is determined by setting $y_2 = 0$. Now Eq. (48) gives

$$x = \frac{y^2\epsilon_f}{2}$$

For $-1 \leq y < -1/2 = -1 + a_f/2a_m$, the maximum x is found by setting $y_2 = 1 + 2y$. Here Eq. (48) gives

$$x = y^2 + \frac{y}{2}(\epsilon_f + 6) + \frac{1}{2}(3\epsilon_0 + 4)$$

When a_1 and a_3 are both positive,

$$\epsilon_0 = 2 + \frac{8}{\pi^2} \text{ and } \epsilon_f = 1 + \frac{2}{\pi^2}$$

Now it takes three curves to describe the minimum x. For

$$0 \leq y \leq \frac{2}{\epsilon_0 + \epsilon_f}, \text{ then} \quad y_2 = 0 \Rightarrow x = \frac{y^2}{2}\epsilon_f$$

$$\frac{2}{\epsilon_0 + \epsilon_f} \leq y \leq +1 + \frac{a_0}{2a_m}, \text{ then} \quad y_2 = y \Rightarrow x = y - \frac{y^2}{2}\epsilon_0$$

$$1 + \frac{a_0}{2a_m} \leq y \leq 1, \text{ then} \quad y_2 = 1 - y \Rightarrow x = y^2(2\epsilon_f - 3)y + \frac{1}{2}(4 - 3\epsilon_f)$$

The maximum x also requires three curves. When

$$0 \le y \le \frac{1}{\epsilon_0}, \text{ then} \qquad y_2 = y \Rightarrow x = y - \frac{y^2\epsilon_0}{2}$$

$$\frac{1}{\epsilon_0} \le y \le \frac{1}{\epsilon_f}, \text{ then} \qquad y_2 = \frac{y\epsilon_f - 1}{\epsilon_f - \epsilon_0} \Rightarrow x = \frac{y^2\epsilon_0\epsilon_f - 2y\epsilon_f + 1}{2(\epsilon_0 - \epsilon_f)}$$

$$\frac{1}{\epsilon_f} \le y \le 1, \text{ then} \qquad y_2 = 0 \Rightarrow x = \frac{y^2\epsilon_f}{2}$$

When a_1 is positive but a_3 is negative, the minimum and maximum x require three curves each.

$$\epsilon_o = 2 + \frac{8}{\pi^2} \text{ and } \epsilon_f = -2 + \frac{8}{\pi^2}$$

The minimum x is as follows. For

$$\frac{1}{2(\epsilon_0 - 1)} < y < 1 + \frac{a_0}{2a_m}, \text{ then} \qquad y_2 = \frac{2y+1}{4} \Rightarrow x = \frac{1 - 2y\epsilon_f - 4y^2}{8}$$

$$0 < y < \frac{1}{2(\epsilon_0 - 1)}, \text{ then} \qquad y_2 = y \Rightarrow x = y - \frac{y^2\epsilon_0}{2}$$

$$-1 + \frac{a_f}{2a_m} < y < 0, \text{ then} \qquad y_2 = 0 \Rightarrow x = \frac{y^2\epsilon_f}{2}$$

The maximum x is as follows. For

$$\frac{1}{\epsilon_0} < y < 1 + \frac{a_0}{2a_m}, \text{ then} \qquad y_2 = y \Rightarrow x = y - \frac{y^2\epsilon_0}{2}$$

$$0 < y < \frac{1}{\epsilon_0}, \text{ then} \qquad y_2 = \frac{1 - y\epsilon_f}{4} \Rightarrow x = \frac{1 - 2y\epsilon_f + \epsilon_f\epsilon_0 y^2}{8}$$

$$-1 + \frac{a_f}{2a_m} < y < 0, \text{ then} \qquad y_2 = \frac{2y+1}{4} \Rightarrow = \frac{1 - 2y\epsilon_f - 4y^2}{8}$$

When a_1 is negative but a_3 is positive, we obtain our final six curves.

$$\epsilon_0 = -1 + \frac{2}{\pi^2} \text{ and } \epsilon_f = 1 + \frac{2}{\pi^2}$$

The minimum x is as follows. For

$$\frac{1}{\epsilon_f} < y < 1, \text{ then} \qquad y_2 = 0 \Rightarrow x = \frac{y^2 \epsilon_f}{2}$$

$$0 < y < \frac{1}{\epsilon_f}, \text{ then} \qquad y_2 = \frac{y\epsilon_f - 1}{2} \Rightarrow x = \frac{-y^2 \epsilon_f \epsilon_0 + 2y\epsilon_f - 1}{4}$$

$$-1 < y < 0, \text{ then} \qquad y_2 = \frac{y-1}{2} \Rightarrow x = \frac{y^2 + 2y\epsilon_f - 1}{4}$$

The maximum x is as follows. For

$$\frac{1}{2\epsilon_f - 1} < y < 1, \text{ then} \qquad y_2 = \frac{y-1}{2} \Rightarrow x = \frac{y^2 + 2y\epsilon_f - 1}{4}$$

$$0 < y < \frac{1}{2\epsilon_f - 1}, \text{ then} \qquad y_2 = 0 \Rightarrow x = \frac{y^2 \epsilon_f}{2}$$

$$-1 < y < 0, \text{ then} \qquad y_2 = y \Rightarrow x = y - \frac{y^2 \epsilon_0}{2}$$

For other values of a_0 and a_f, different curves determine the boundaries of the regions. The difficulties in finding the internal borders of the area of validity are sufficiently great that were this algorithm to be implemented, one would simply try all six candidate solutions and pick the first one that worked.

Figure 11 shows the area of valid solutions for $a_0 = -a_m$ and $a_f = a_m$ superimposed on the one for $a_0 = a_f = 0$ shown in Fig. 5. Note the loss in phase space for valid solutions in the $a_f = -a_0 = a_m$ case.

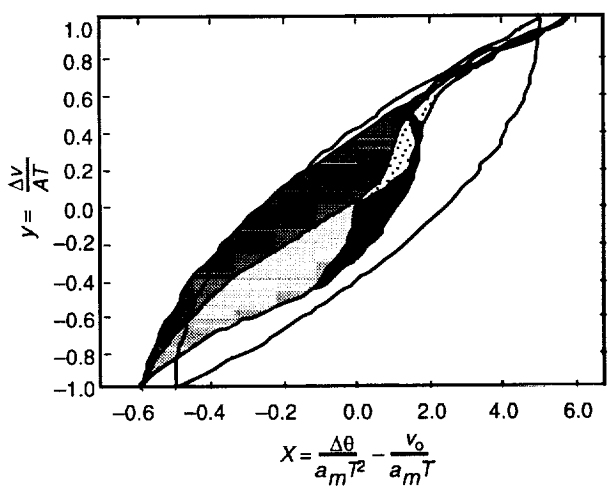

Fig. 11. Shrinkage of region of valid solutions for $a_f = -a_0 = a_m$.

IV. Discussion and Conclusions

Acquisition scheme 4 is overly complex even if one does not calculate the internal borders of the area of validity. Such an algorithm would be very difficult to implement and maintain. Scheme 3 should suffice for high-speed acquisitions and is recommended as an option on Deep Space Network antennas. The other schemes produce acceleration discontinuities which may cause undesirable excitations of the antenna structure.

The algorithm can be run with the entire set of commands output at once. However, in practice, it is not necessary to calculate all commands before implementing the first one. It is sufficient to find an acceptable acquisition time, T. The position and velocity commands can be calculated in real time as the antenna moves. If it is inconvenient to calculate the commands in real time, there may be enough processor time to calculate them in advance and store them.

Since the trajectory preprocessor cannot be used to supply commands until a satisfactory acquisition time is calculated, the question arises of how to find the acquisition time quickly. The problem is not trivial, as the target may have a trajectory that is very difficult to match. There are a number of possible strategies for picking a candidate acquisition time. A trade-off is involved. If the processor is so slow that it may take several tenths of a second to discover if a candidate acquisition time will work, one must be conservative in one's choice of candidate acquisition times. Only a few candidates can be tried. The candidate which is finally selected may not be optimal, but the time lost in finding a better solution may more than make up for the time saved by the improved answer. On the other hand, if thousands of candidate solutions can be tried in a second, one should expect to find an acquisition time that is within a fraction of a second of optimal. Mere processor speed does not guarantee success, as the processor may need to be shared with other tasks. Nor should it be forgotten that each target position may need to be translated in a relatively time-consuming manner to an equivalent antenna command. There may be coordinate conversions to apply, refraction must be taken into account, and subreflector squint must be corrected for, along with a host of other tabled or modeled systematic pointing errors. A trajectory preprocessor algorithm should not be developed for an antenna system unless one has a reasonable knowledge of the required and available processor time, both before and during the time period in which preprocessor commands are to be output.

When the trajectory preprocessor is used to match the trajectory of a sidereal object, the acquisition time is easy to estimate. There is usually no hurry to acquire the object, but even if there were, the preprocessor could handle the situation rather easily. A problem arises when the algorithm is used to match the trajectory of a fast-moving object that may be moving at 10 to 50 percent of the maximum antenna angular rate at acquisition.

It is understandable that one might wish to construct a simple, general algorithm to generate candidate acquisition times. One can guess a time of 1 sec, and should that be insufficient, continue with guesses of 2, 4, 8, 16, 32, 64, and 128 sec until a solution is found. If powers of two seem inappropriate, one can try powers of 3, 1.5, 1.2, or whatever. One can be satisfied with the first acceptable solution, or one can backtrack, looking for an even better one. Another idea is to start with the maximum $\Delta\theta/v_{max}$ and $2\Delta v/a_{max}$. Any of these ideas may be acceptable, but it seems far better to produce a carefully constructed table of candidate acquisition times for each given mode, especially for low-Earth-orbiting satellites. A simple default mode can be included as well.

This trajectory preprocessor algorithm depends greatly on the ability to predict exactly where the antenna will be when preprocessor commands are to begin. Any confusion resulting from misapplication of pointing corrections or differences between hoped for and actual position will result in a discontinuity in command position, which is precisely the problem that trajectory preprocessing is supposed to avoid. Choosing anticipated commands for the initial antenna position and velocity does no good if the antenna is pointed elsewhere. Using the actual antenna position does little good if the antenna is moving quickly and the preprocessor commands are due to start only a second or two later. Other initialization errors are possible that could render the preprocessor ineffective. For example, if incorrect or inappropriate velocity or acceleration limits are used, so that the antenna cannot respond properly to the preprocessor commands, antenna control will be back into the nonlinear region that the preprocessor was designed to rescue it from. As long as care is taken to avoid mistakes of this sort, the preprocessor should serve a useful function.

There is no reason to demand that an antenna controller always be in a mode for which trajectory preprocessing is in use. The preprocessor can be an option used especially in cases for which very accurate tracking is required or for which control problems are anticipated. For this reason, an acquisition scheme that matches target acceleration may be favored over one that does not; the preprocessor may be used primarily when one wishes to avoid what sometimes seem like small acceleration discontinuities. However, use of a preprocessor as an option does not mean that it should be considered as an ad hoc feature rather than an integral part of a control system design. The issues of how and where to fit preprocessing into a system should be addressed even if it is not yet decided whether or not such an algorithm will be implemented. This would avoid problems that may arise when one attempts to add it after the rest of the system is complete.

As pointing requirements become more strict and tracking speeds increase, trajectory preprocessing will become a more and more valuable option to improve antenna control. The algorithm described in this article could be put to good use in the Deep Space Network.

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