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ATMOSPHERIC PROPAGATION AT LARGER LATERAL DISTANCES FROM THE FLIGHT TRACK

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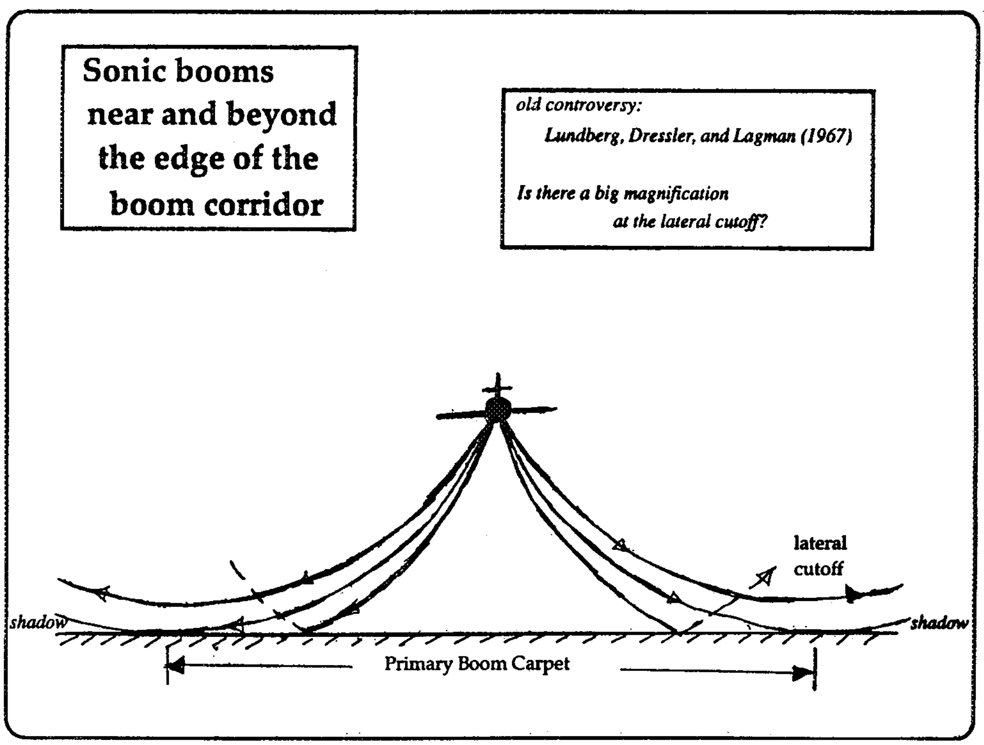
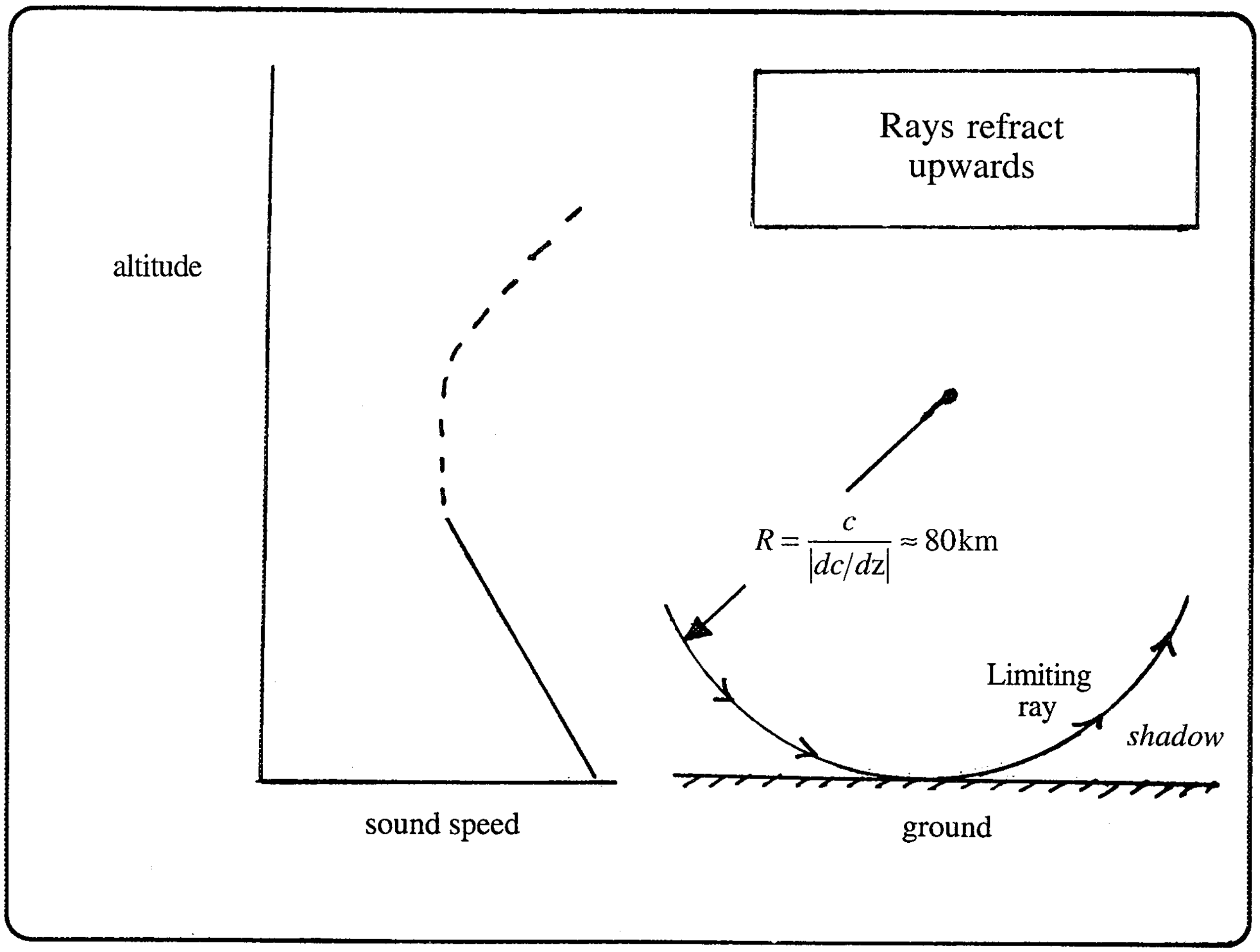
INTRODUCTION

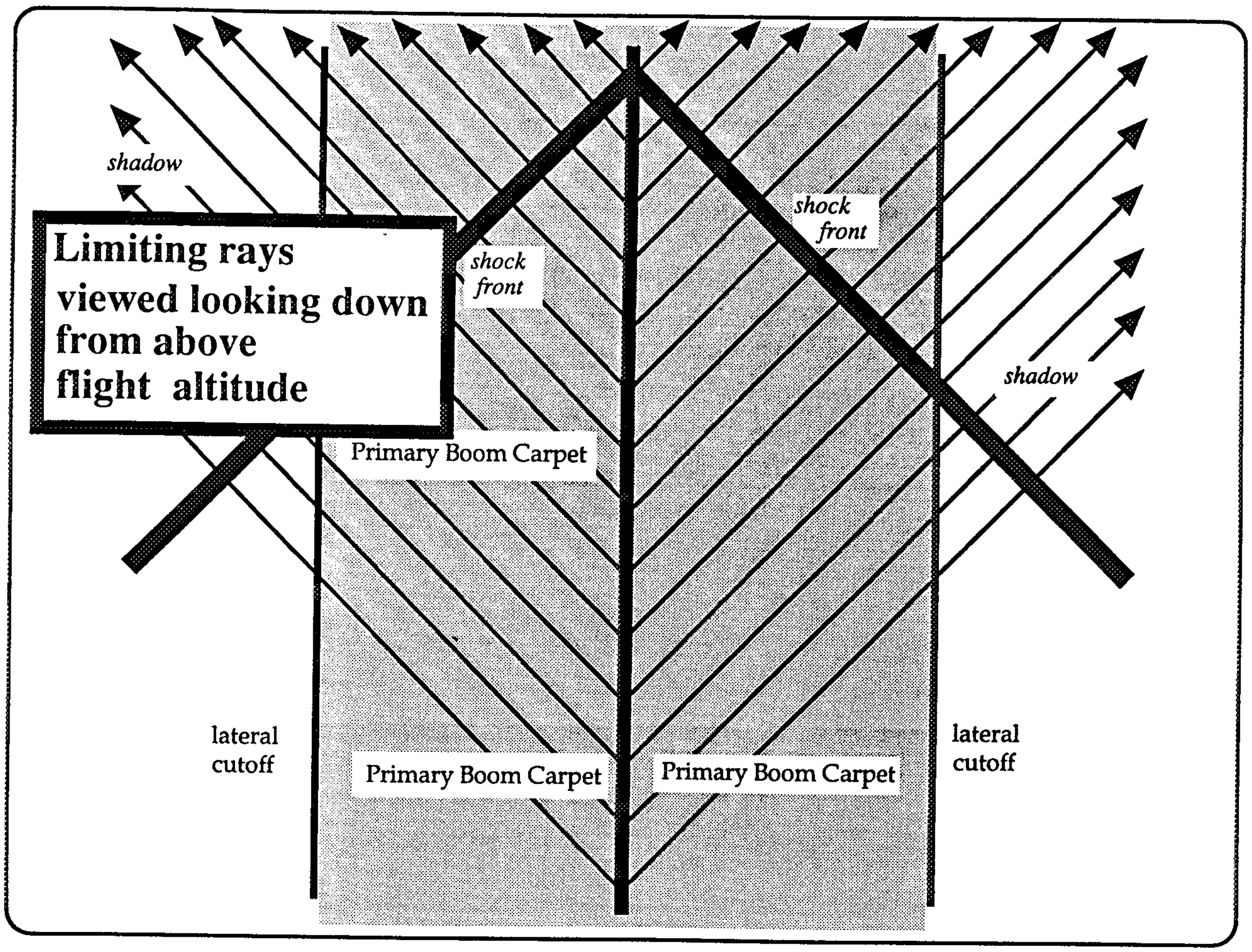
Sonic booms received on the ground tend to be restricted to a region of finite lateral extent below the flight track. This occurs because of refraction and because the effective speed of sound, even with winds taken into account, decreases with altitude in the lower atmosphere. Not all rays proceeding initially downwards from the flight track within an allowable range of initial directions will reach the ground. The restricted region which can be reached by rays impacting the ground is known as the primary carpet. However, weak rumbles are heard in the nominal shadow zone beyond the edge of this carpet. A full wave theory is necessary for explaining waveforms in that region, and the present paper gives a matched asymptotic expansion technique for a suitable approximate full wave theory that involves a relatively small number of parameters. The outer solution is derived from the structure of the system of rays that impact near the corridor edge; the inner solution involves a solution of the parabolic equation and results in the special functions encountered in the diffraction of sound over the tops of hills. (Work supported by Armstrong Aerospace Medical Research Laboratory, Human Systems Division (AFSC), United States Air Force, Wright-Patterson AFB, Ohio 45433-6573.)

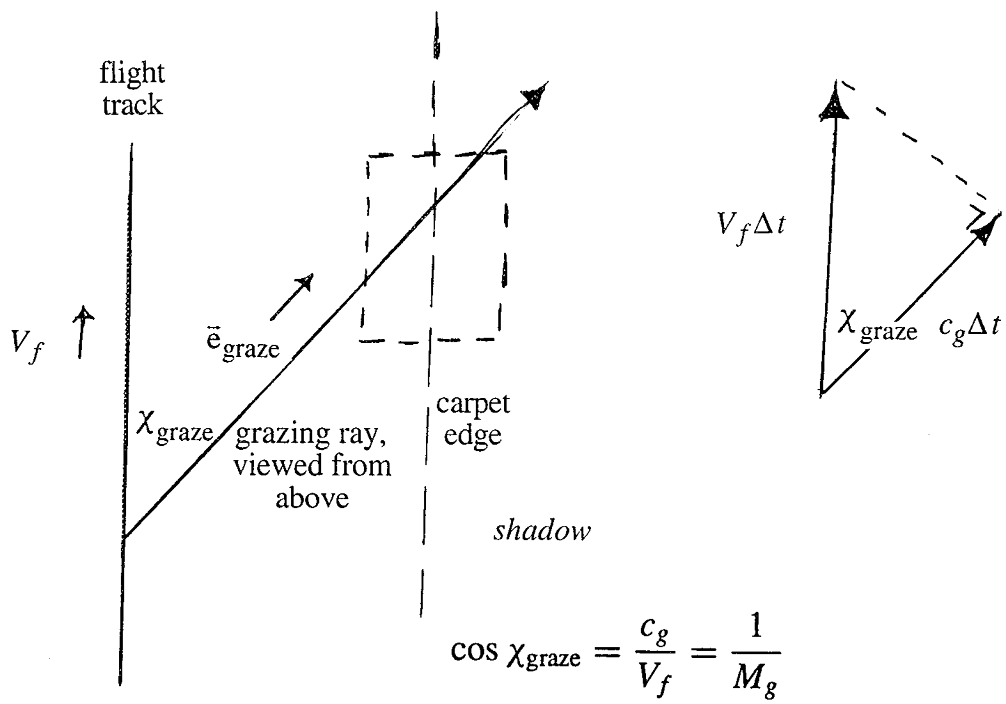


RAYS UNDER INFLUENCE OF LINEAR SOUND SPEED GRADIENT



LIMITING RAYS FROM ABOVE

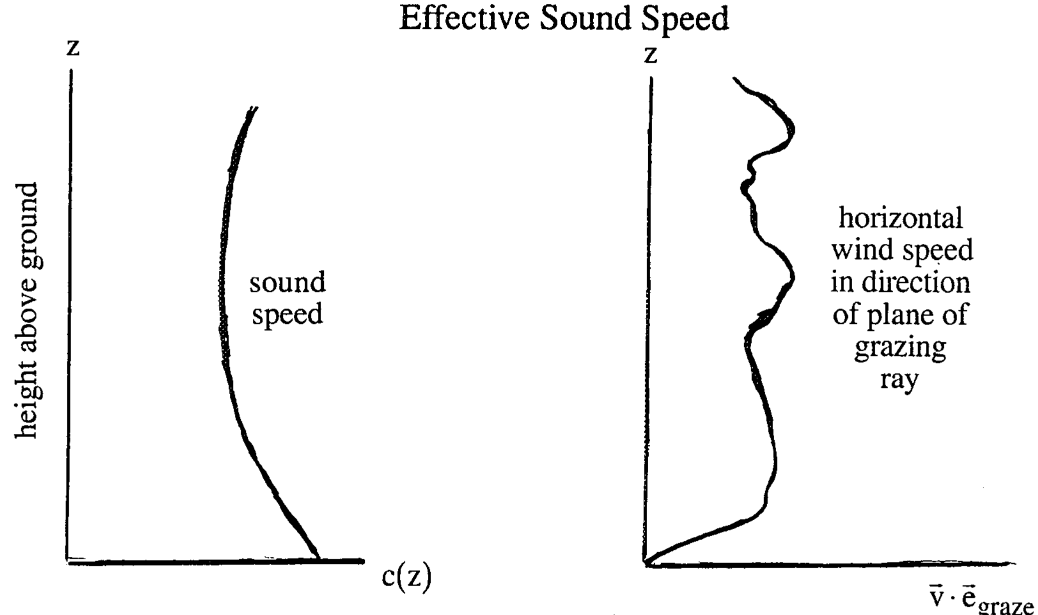




$$\cos \chi_{graze} = \frac{c_g}{V_f} = \frac{1}{M_g}$$

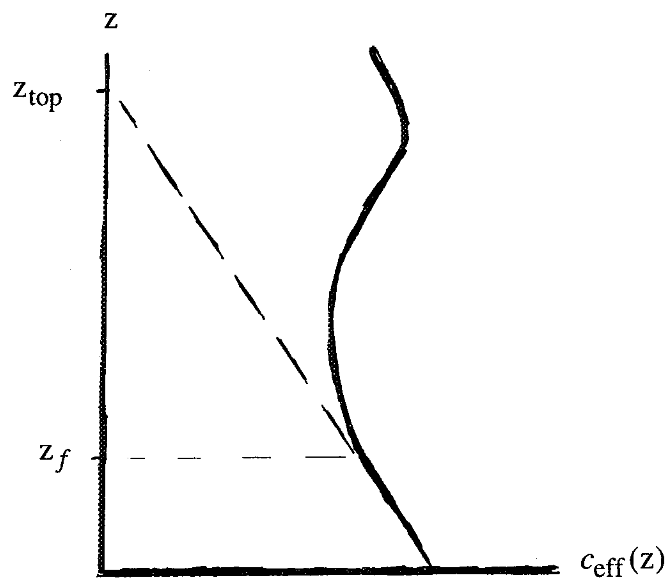
M_g = flight Mach number relative to sound speed at ground

Effective Sound Speed



$$c_{\mathrm{eff}}(z) = c(z) + \vec{\mathbf{v}}(z) \cdot \vec{\mathbf{e}}_{\mathrm{graze}}$$

$\vec{\mathbf{v}}(z) \cdot \vec{\mathbf{e}}_{\mathrm{graze}}$ = horizontal wind component in direction of grazing ray

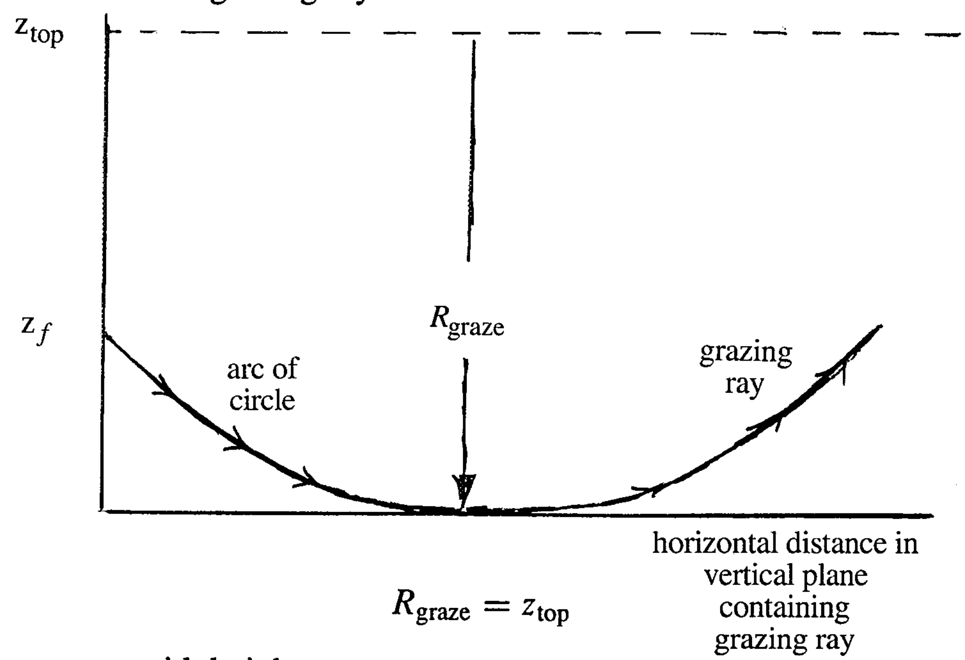


z_{top} = height of apparent top of atmosphere, based on linear extrapolation of sound speed beyond flight altitude

For US Standard Atmosphere without winds:

$$z_{top} = 88.4\,\text{km}$$

Radius of curvature of the grazing ray

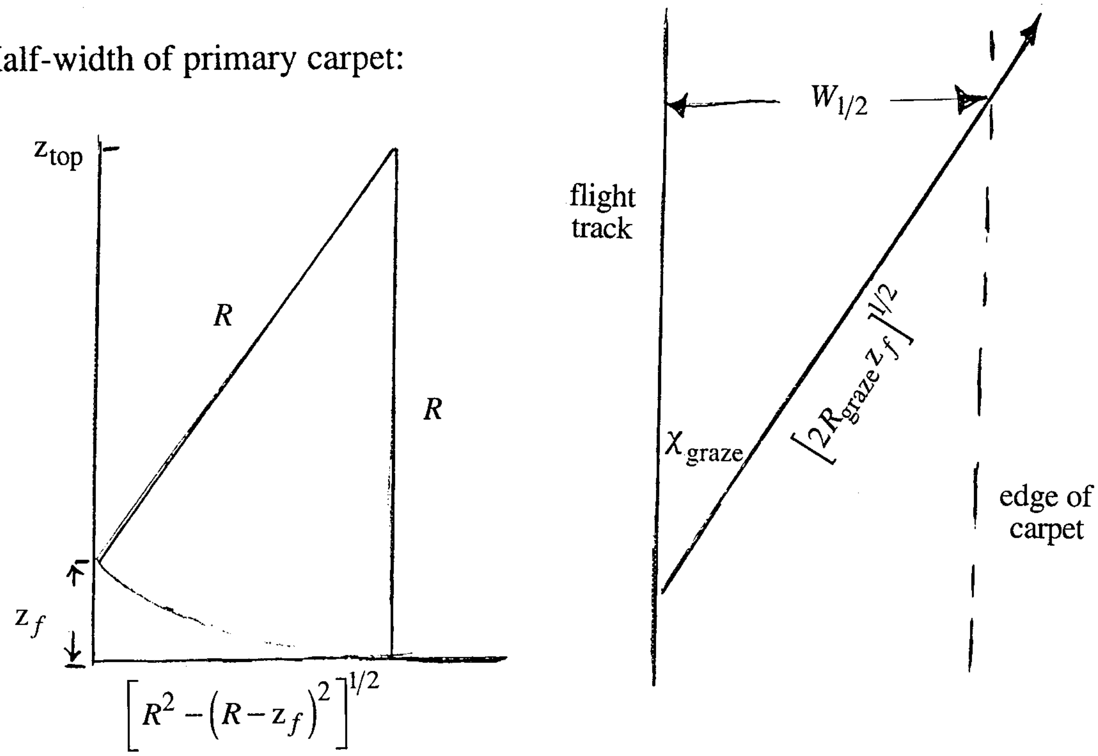


$$R_{graze} = z_{top}$$

If dc/dz not constant with height, use

$$R_{graze} = \frac{4c_g}{z_f}\left[\int_0^{z_f} \frac{dz}{(c_g - c)^{1/2}}\right]^2$$

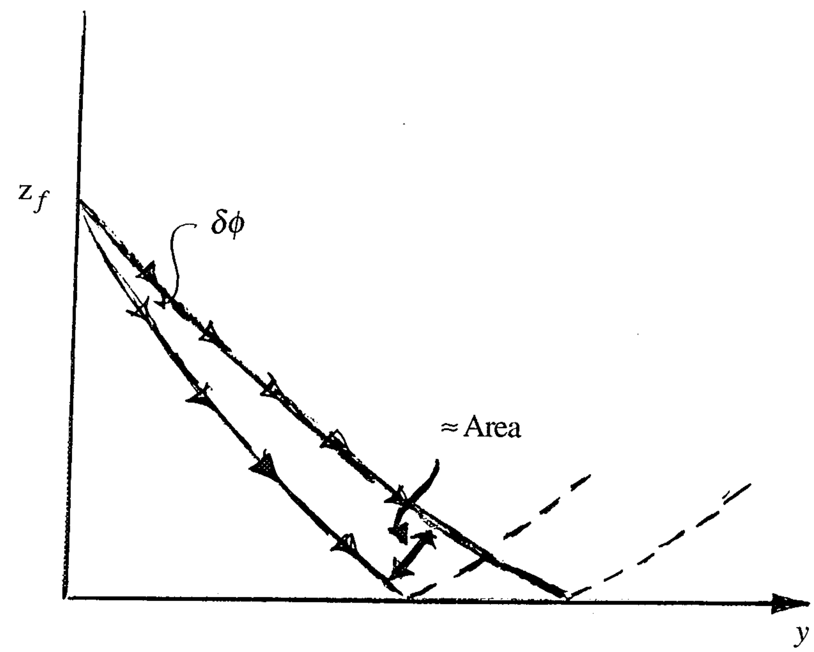
Half-width of primary carpet:



$$\text{half-width} = [2R_{graze} z_f]^{1/2} \sin \chi_{graze}$$

$$\sin \chi_{graze} = \left[1 - (1/M_g)^2\right]^{1/2}$$

Ray-tube area at ground:

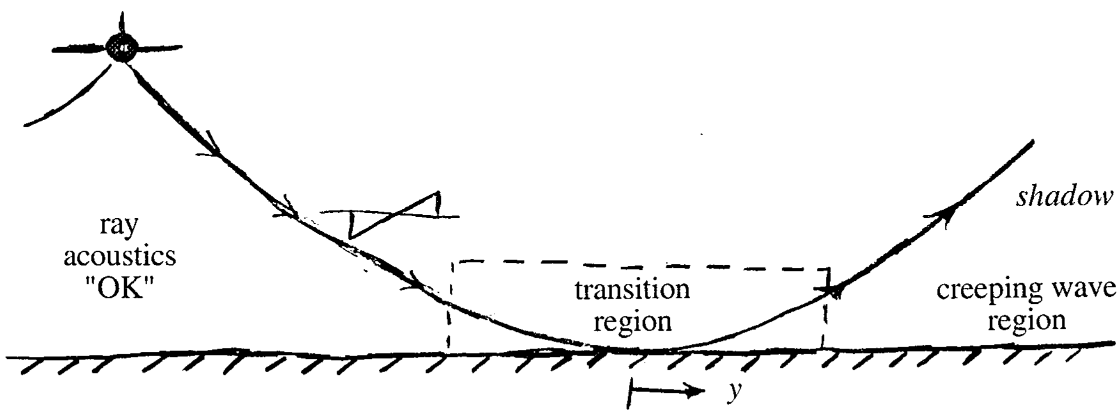


$$\text{Area} = (\text{constant})[y^2 + z_f^2]^{1/2}\delta\phi$$

just as if rays were spreading cylindrically

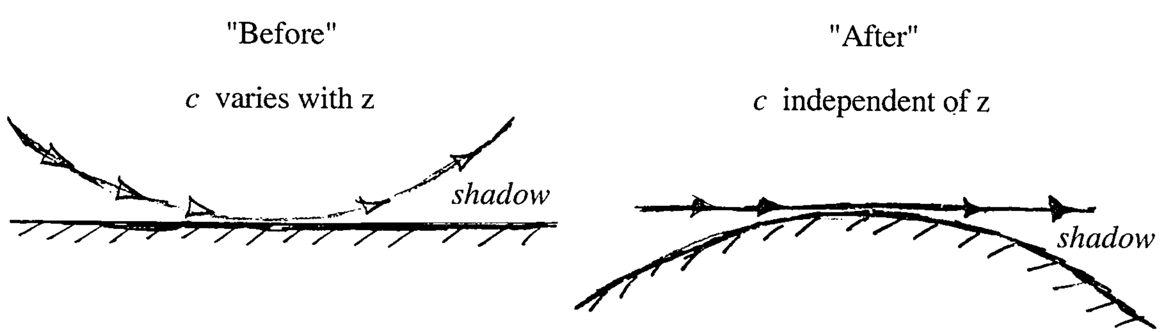
Result is for limiting case of high Mach number; no significant difference is expected at moderate Mach number.

Concept of a transition region:

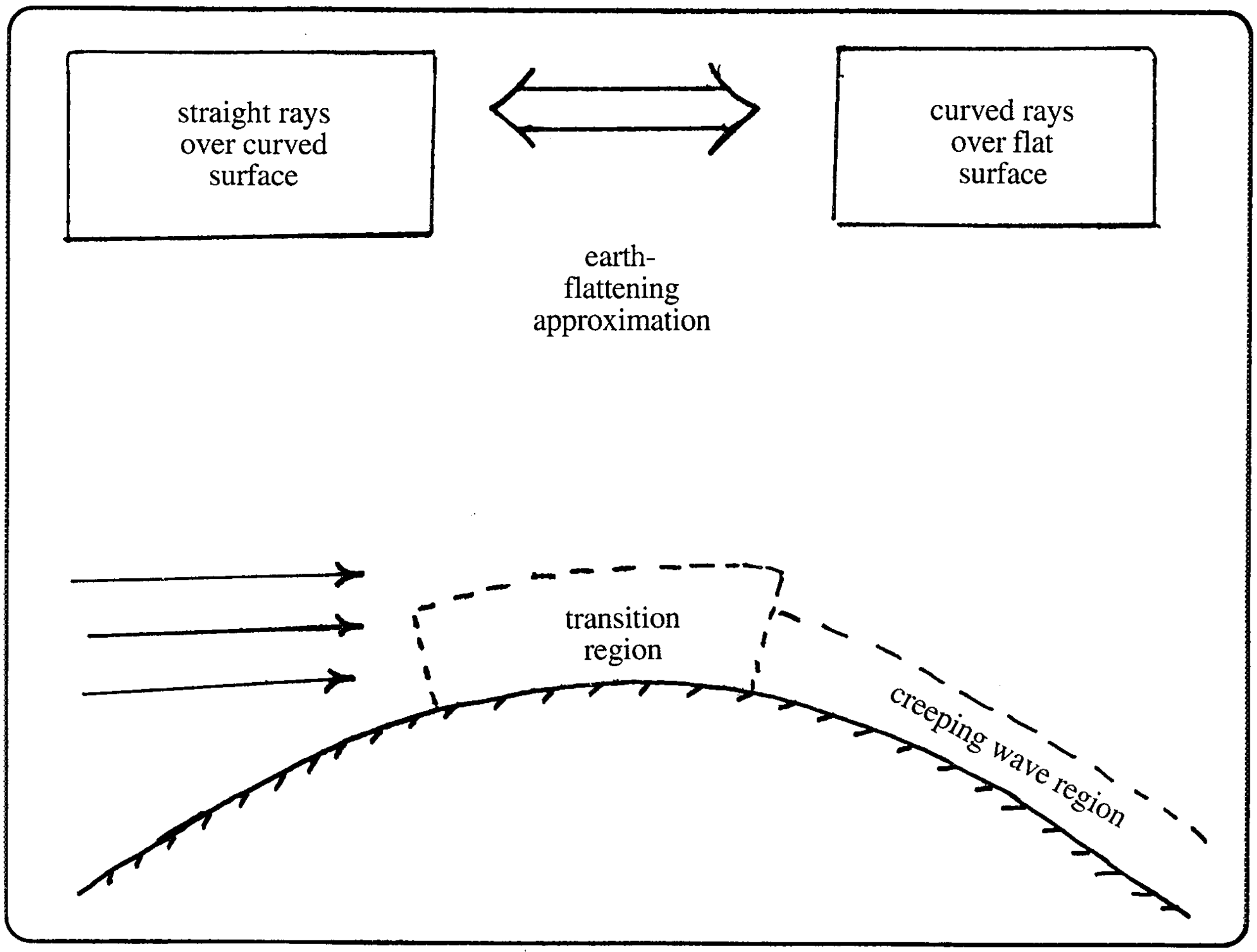


Region near the ground and centered at edge of primary carpet, within which ray acoustics is not valid

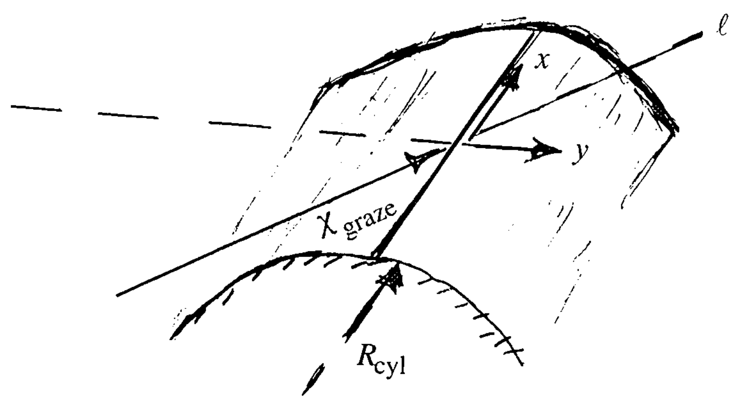
Converse of Earth-flattening approximation:



- Use a set of curvilinear coordinates where grazing ray appears straight, but ground appears curved
  - Edge of primary carpet becomes line on top of cylinder
  - Grazing ray becomes horizontal straight line
  - Crosses the top of cylinder at oblique incidence with angle χ_{graze}


straight rays
over curved
surface
curved rays
over flat
surface
earth-
flattening
approximation
transition
region
creeping wave region

Cylinder radius of curvature:



- Grazing-ray's path:

$$x = \ell \cos \chi_{\text{graze}}, \quad y = \ell \sin \chi_{\text{graze}}$$

- Distance of ray above ground:

$$\Delta z = \frac{y^2}{2R_{\text{cyl}}} = \frac{\ell^2}{2R_{\text{graze}}}$$

- Conclude that

$$R_{\text{cyl}} = R_{\text{graze}} \sin^2 \chi_{\text{graze}}$$

Plane wave obliquely incident on a cylinder:

- Trace velocity matching principle:

$$p = p(t - [x/V_f], y, z)$$

$$\frac{\partial}{\partial x} \rightarrow \frac{1}{V_f}\frac{\partial}{\partial t}$$

- Wave equation:

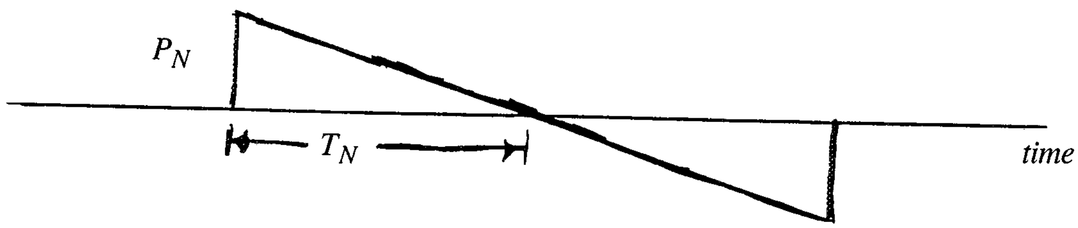
$$\frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} - \left(\frac{1}{c^2} - \frac{1}{V_f^2}\right)\frac{\partial^2 p}{\partial t^2} = 0$$

Results for normal incidence apply with transformation

$$\frac{1}{c^2} \rightarrow \frac{1}{c^2} - \frac{1}{V_f^2} = \frac{1}{c^2}\sin^2 \chi_{\text{graze}}$$

Waveform going into transition region:

- Use N-wave of form



P_N = peak amplitude of N-wave
T_N = positive phase duration of N-wave

- Determine P_N and T_N by extrapolation of waveform received on the ground within the primary carpet, taking into account
    - Pressure doubling on reflection at ground
    - Geometrical spreading along ray tubes
    - Lengthening with propagation distance of T_N due to non-linear effect
    - Extra decrease of P_N with propagation distance due to non-linear effect

Fourier transform solution:

- Let incident plane wave be of the form:

$$p_{\text{inc}} = P_N f_N(\tau/T_N)$$

$$\tau = t - [y/c]\sin\chi_{\text{graze}} - [x/c]\cos\chi_{\text{graze}}$$

$$f_N(\tau/T_N) = \int_{-\infty}^{\infty} e^{-i\omega\tau}\hat{f}_N(\omega T_N)T_N d\omega$$

- Then the total solution is of the form

$$p = P_N T_N \int_{-\infty}^{\infty} e^{-i\omega(t-[x/V_f])}\hat{f}_N(\omega T_N)\mathrm{TF}(y, z, \omega)d\omega$$

- where the transfer function TF satisfies

$$\frac{\partial^2\mathrm{TF}}{\partial y^2} + \frac{\partial^2\mathrm{TF}}{\partial z^2} + \frac{\omega^2}{c^2}\sin^2\chi_{\text{graze}}\mathrm{TF} = 0$$

For points on the ground:

- The transfer function is given by

$$\mathrm{TF}(y, z, \omega) = e^{i[\omega/c]y \sin \chi_{\mathrm{graze}}} G(\xi, 0, q)$$

$$G(\xi, 0, q) = \pi^{-1/2} \int_{-\infty}^{\infty} \frac{e^{i\alpha\xi}}{w_1'(\alpha) - q w_1(\alpha)} d\alpha$$

$$w_1(\alpha) = 2\pi^{1/2} e^{i\pi/6} \mathrm{Ai}(\alpha e^{i2\pi/3})$$

$$\xi = y/[2^{1/3} L_\omega]; \quad q = i(\omega c^{-1} R_{\mathrm{cyl}} \sin \chi_{\mathrm{graze}})^{1/3} \rho c / Z_S$$

$$L_\omega = \frac{R_{\mathrm{cyl}}^{2/3}}{(\omega c^{-1} \sin \chi_{\mathrm{graze}})^{1/3}} = \frac{R_{\mathrm{graze}}^{2/3} \sin \chi_{\mathrm{graze}}}{(\omega c^{-1})^{1/3}}$$

- Recall that

$$R_{\mathrm{cyl}} = R_{\mathrm{graze}} \sin^2 \chi_{\mathrm{graze}}$$

Hard ground limit:

- Transmitted pressure reduces to

$$p = P_N T_N \int_{-\infty}^{\infty} e^{-i\omega T_N\{\tau/T_N\}} \hat{f}_N(\omega T_N) G(\xi, 0, 0) d\omega$$

$$\tau = t - [y/c] \sin \chi_{\text{graze}} - [x/c] \cos \chi_{\text{graze}}$$

$$G(\xi, 0, 0) = \pi^{-1/2} \int_{-\infty}^{\infty} \frac{e^{i\alpha\xi}}{w_1'(\alpha)} d\alpha$$

$$\xi = (y/L_{\text{tran}})(\omega T_N)^{1/3}$$

- Characteristic length for N-wave deterioration through and beyond the transition region

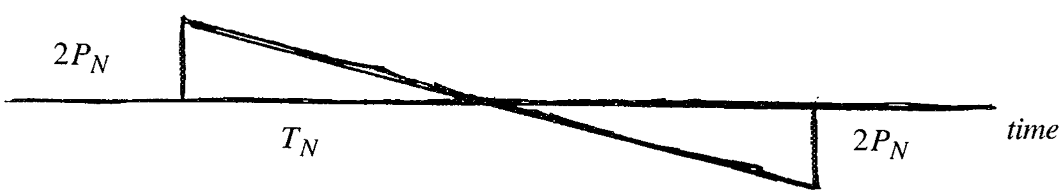
$$L_{\text{tran}} = 2^{1/3} R_{\text{graze}}^{2/3} \sin \chi_{\text{graze}} (c T_N)^{1/3}$$

Before entering the transition region:

$$G(\xi, 0, 0) \rightarrow 2e^{-i\xi^3/3} \quad \text{as } \xi \rightarrow -\infty$$

and one recovers the N-wave

$$p \rightarrow 2P_N f_N(\tau_{\text{before}}/T_N)$$



$$\tau_{\text{before}} = t - [y/c]\sin\chi_{\text{graze}} - [x/c]\cos\chi_{\text{graze}} + [y^3/(6R^2_{\text{graze}}c)]\frac{1}{\sin^3\chi_{\text{graze}}}$$

Apparent phase velocity in lateral direction is

$$v_{ph} \approx \frac{c}{\sin\chi_{\text{graze}}}\left[1 + \frac{y^2}{2R^2_{\text{graze}}\sin^4\chi_{\text{graze}}}\right]$$

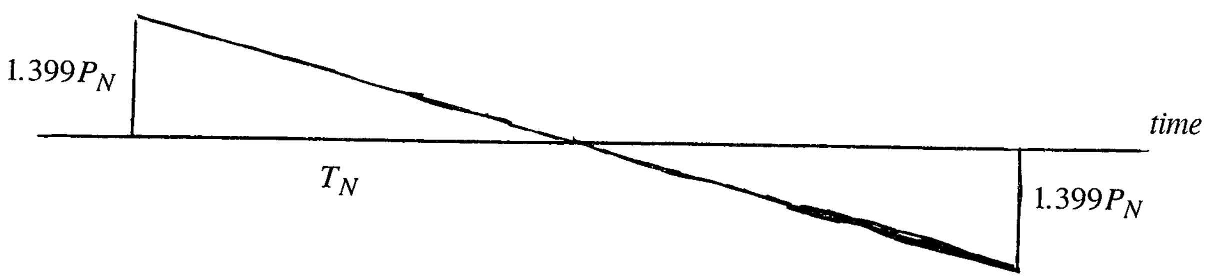
At precisely the edge of the carpet boundary:

$y = 0$, so $\xi = 0$ for all ω

$$\left\{G(\xi, 0, 0)\right\}_{\xi=0} = 1.399$$

Thus one has an N-wave, but of reduced amplitude

$$p = 1.399 P_N f_N(\tau/T_N)$$



$$\tau = t - [x/c] \cos \chi_{\text{graze}}$$

Representative value for:

$$L_{\text{tran}} = 2^{1/3} R_{\text{graze}}^{2/3} \sin \chi_{\text{graze}} (cT_N)^{1/3}$$

Let $M_g = 2$, so $\sin \chi_{\text{graze}} = \sqrt{3}/2$
Let $c = 0.34\,\text{km/s}$
Let $R_{\text{graze}} = 88\,\text{km}$
Then

$$\left(\frac{cT_N}{R_{\text{graze}}}\right)^{1/3} = 0.0834$$

and one deduces

$$L_{\text{tran}} = 0.09 R_{\text{graze}} = 8\,\text{km} = 5\,\text{miles}$$

Compare with representative value of carpet half-width

$$W_{1/2} = [2 \cdot 88 \cdot 12]^{1/2} \sqrt{3}/2 = 40\,\text{km} = 25\,\text{miles}$$

To study deterioration of front of shock,
sufficient to take incident wave as a step function:

$$p = \frac{P_N}{\pi} \int_0^\infty \frac{1}{\omega} \mathrm{Im}\left[e^{-i\omega\tau} G(\xi, 0, 0)\right] d\omega - \frac{1}{2} G(0, 0, 0) P_N$$

- To examine the contribution of the higher-frequency components, it suffices to use the creeping wave approximation:

$$G \to 1.8325 e^{e^{-i\pi/6} K \xi} \quad \text{as} \quad \xi \to +\infty$$

where $K = 1.0188$. Result is (for $y > 0$)

$$p = P_N \Phi_{\mathrm{creep}}(\tau/\tau_{\mathrm{char}}) - \frac{1}{2} G(0, 0, 0) P_N$$

$$\Phi_{\mathrm{creep}}(\eta) = \frac{1.8325}{\pi} \int_0^\infty \frac{1}{\Omega} e^{-[\sqrt{3}/2]\Omega^{1/3}} \sin(\Omega\eta - [1/2]\Omega^{1/3}) d\Omega$$

$$\tau_{\mathrm{char}} = \frac{(1.088)^3 y^3}{2cR_{\mathrm{graze}}^2 \sin^3 \chi_{\mathrm{graze}}}$$

For analysis of field data:

- Need at outset to identify
    - nominal location of carpet edge, origin of y
    - Mach number relative to sound speed at ground, to get χ_{graze}
    - Grazing ray radius of curvature R_{graze}, from meteorological profiles
    - N-wave amplitude P_N before reflection at ground
    - Positive phase duration T_N at the carpet edge
- Theory predicts that peak waveform versus extra lateral distance should fall on a "universal curve" if one plots p_{peak}/P_N versus y/L_{tran}
- Theory predicts that waveform shape is similar for all waveforms recorded at same value of y/L_{tran}

Rough characterization of wave evolution beyond the transition region,

- Begin with an N-wave of amplitude of $1.399 P_N$ at the carpet edge ($y = 0$).
- $|G(\xi, 0, 0)|$ drops to 1/2-th of its zero-frequency value at $\xi = 1.05$.
- Frequencies which propagate with negligible attenuation to lateral distance y from carpet edge are those for which

$$\omega < \frac{2cR_{\text{graze}}^2 \sin^3 \chi_{\text{graze}}}{y^3}$$

- Propagation to lateral distance y from carpet edge is roughly equivalent to passing an N-wave of peak overpressure $1.399 P_N$ through a low-pass filter with cut-off frequency

$$\omega_{\text{cut-off}} = \frac{2cR_{\text{graze}}^2 \sin^3 \chi_{\text{graze}}}{y^3}$$