

Mechanical System Reliability for Long Life Space Systems

Prepared by

Michael T. Kowal
Department of Mechanical Engineering
Vanderbilt University
Nashville, Tennessee

Final Report

Grant NAG3-1352

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NASA Lewis Research Center
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1.0 Introduction

1.1 Purpose

The creation of a compendium of mechanical limit states was undertaken in order to provide a reference base for the application of First-Order Reliability Methods to mechanical systems in the context of the development of a system level design methodology. The compendium was conceived as a reference source specific to the problem of developing the noted design methodology, and not an exhaustive or exclusive compilation of mechanical limit states. The compendium is not intended to be a handbook of mechanical limit states for general use.

1.2 Proposed Use of the Compendium

The compendium provides a diverse set of limit-state relationships for use in demonstrating the application of probabilistic reliability methods to mechanical systems. The compendium is to be used in the reliability analysis of moderately complex mechanical system.

1.3 Selection of Limit-States

In determining the limit-states to be included in the compendium, a comprehensive listing of the possible failure modes that could affect mechanical systems was generated. Previous literature on failure modes was studied, and cited failure modes were included. From this, the following classifications for failure modes were derived:

- Wear
- Corrosion
- Fatigue
- Material Degradation

With the definition of the different failure modes, a literature search for each was conducted, with the aim of establishing relationships for each failure mechanism to be used in formulating mechanical limit-states.

The individual failure modes that were determined for each classification are:

Wear

- Adhesive wear
- Abrasive wear
- Lubricated wear
- Fretting wear
- Surface fatigue wear
- Liquid impact erosion

Fatigue

- Low-cycle fatigue
- High-cycle fatigue
- Crack growth

Corrosion

- Erosion-corrosion
- Galvanic corrosion
- Uniform attack
- Pitting corrosion
- Cavitation
- Crevice corrosion
- Stress corrosion cracking
- Selective leaching
- Inter-granular corrosion

Material Degradation

- Thermal degradation
- Radiation damage

Although it was possible to determine the various failure modes that could affect a mechanical system, the identification of relationships for all noted failure modes was not possible. Consequently, the compendium consists of relationships for those failure modes for which reasonably robust relationships exist.

1.3.1 Distribution Properties of Primitive Variables

The compendium does not contain information on the distribution properties of the individual primitive variables. With the exception of fatigue and wear related primitive variables, little research was uncovered that dealt with the distribution properties of other limit-state primitive variables. It is apparent that additional efforts are needed to uncover these distribution properties.

1.3.2 Form of Limit-State Relationships

The result of the investigation into mechanical limit-states indicate that the vast majority of identified relationships are of the form of a power law. All the limit states cited within the compendium, with the exception of those for uniform attack and thermal degradation, take a power law form.

1.4 Space Environment Factors

The environment likely to be encountered by the Space Station is that associated with a low earth orbit (LEO). The difficulty in assessing the impact of individual environment factors arises from the fact that some factors are known with considerable accuracy. Additional problems arise in determining the impact of space environment factors on mechanical systems for an anticipated service life that is longer than previous studies for which data is available. All information relating to space environment factors presented has been synthesized from previous reports¹.

The environment factors likely to affect low earth orbit space-based mechanical systems are:

1. Atomic Oxygen
2. Solar Radiation
3. Plasma
4. Debris and Micrometeoroids
5. Thermal Loading
6. Corrosion

1.4.1 Atomic Oxygen

In low earth orbit atomic oxygen is the predominant species of the atmosphere present. Typically the atomic oxygen is neutral and in a ground state with a low static energy level of 0.1 eV^2 . Due to the orbital velocity (8 km/s) of the station, the atomic oxygen impacts the station with an apparent kinetic velocity of 5.0 eV^3 . Damage occurs as mass loss at surfaces exposed to an incoming flux of high energy atomic oxygen. The deterioration of materials is measured as the volume of material removed normalized to the fluence of atomic oxygen that the surface was exposed to⁴. The determination of the average flux of atomic oxygen is outlined in the referenced document. In addition, consideration should be given to the reflection and scattering of atomic radiation from inert materials that may increase the amount of atomic oxygen impinging on surrounding susceptible surfaces.

¹ Structural & Mechanical Systems Long-Life Assurance Design Guidelines, SwRI Report No. 06-1998-3, Southwest Research Institute, January 6, 1989.

² Structural & Mechanical Systems Long-Life Assurance Design Guidelines, p.77.

³ Structural & Mechanical Systems Long-Life Assurance Design Guidelines, p. 77.

⁴ Structural & Mechanical Systems Long-Life Assurance Design Guidelines, p. 111.

1.4.2 Ultraviolet Radiation

Ultraviolet radiation from the solar flux is an important factor to consider in the design of low earth orbit space-based platforms due to the high energy levels⁵. The average solar flux is 1390 W/m² with a peak intensity of 1425 W/m² resulting from the elliptical earth orbit⁶. Some materials are reflectors of ultraviolet radiation, so consideration of reflected UV radiation should be incorporated in the analysis of all platform components. Consideration should also be made for solar activity, notably solar flares, which can increase the amount of ultraviolet radiation in a low earth orbit.

1.4.3 Ionizing Radiation

"Ionizing radiation consists of protons, electrons, neutrons, and nuclei of elements"⁷. The determination of the exact flux for the various particles is presented in JSC 30425, Space Station Program Natural Environment Definition for Design⁸.

1.4.4 Plasma

The movement of low earth orbit vehicles through the space plasma existing in the low earth orbits results in the generation of an electromagnetic effect⁹. As a result of this electromagnetic effect the platform will become charged due to differences between grounded and non-grounded surfaces. Some researchers have indicated that space platforms in low earth equatorial orbit are not detrimentally affected by charging due to plasma¹⁰. The same authors indicate that polar orbits near the arctic circle experience intense electron fluxes that may affect platform performance.

⁵ Structural & Mechanical System Long-Life Assurance Guidelines, p.83.

⁶ Woodcock, G. R., Space Stations and Platforms, Orbit Book Company, 1986, p. 61.

⁷ Structural & Mechanical System Long-Life Assurance Guidelines, p. 84.

⁸ JSC-30425 Space Station Program Natural Environment Definition for Design, SSCBD BB000258 EFF 01-09-87, January 15, 1987.

⁹ Structural & Mechanical System Long-Life Assurance Guidelines, p. 89.

¹⁰ Martin, A. R., "Polar Platform Plasma Wake and Charging Studies", Proceedings of the ESA/BNSC/CNES Workshop on Solar-Terrestrial Physics on Space

1.4.5 Debris and Micrometeoroids

The low earth orbit is characterized by the presence of space debris and micrometeoroids. Those orbiting particles generated from previous space exploration efforts are defined as space debris. Micrometeoroids are particles of space dust that originated in deep space.

Determining the nature of the space debris to be encountered in low earth orbits involves the tracking and cataloging of a large number of items. Although the determination of the number of particles of a specific size may be known to an accuracy of a factor of 3, generally the estimates for most debris particles is accurate to only an order of magnitude¹¹. To determine the largest size of debris likely to be encountered, an approximate solution is presented in 'Structural & Mechanical Systems Long-Life Assurance Guidelines' (pp. 91-96), and an exact solution is given in JSC-30425 'Space Station Program Natural Environment Definition for Design'.

Micrometeoroids can consist of both lower density particles, which tend to be small and irregularly shaped, and larger, heavier iron and nickel based particles. The worst case scenario is to envision heavy particle fragments (7 grams/cc) at an average velocity of 20 km/s¹². Detailed calculations for determining the meteoroid flux model are given in JSC-30425 'Space Station Program Natural Environment Definition for Design'.

1.4.6 Thermal Loading

The environmental sources of thermal loading on low earth orbit platforms are the sun and the earth. The heating effect seen by the platform is dependent on its altitude, orientation, solar flux, and radiation emitted and reflected by the earth¹³. The movement of the platform into the earth's shadow greatly diminishes the rate of heating. For detailed information on the thermal loading that low earth orbit platforms are likely to experience refer to JSC 30205¹⁴ and JSC 30206¹⁵.

¹¹ Structural & Mechanical System Long-Life Assurance Guidelines, p.91.

¹² Structural & Mechanical Systems Long-Life Assurance Guidelines, p. 97.

¹³ Structural & Mechanical Systems Long-Life Assurance Guidelines, p. 105.

¹⁴ JSC 30205, "External Thermal Environment Data Base Geometric Math Model", Space Station Program Office, June 29, 1985.

¹⁵ JSC 30206, "External Environment Data Base Thermal Math Model", Space Station Program Office, June 29, 1985.

1.4.7 Corrosion

The primary concern in the design of low earth orbit platforms is the deterioration of platform materials due to their interaction with liquids, gases, other reactive materials as well as possible areas for galvanic corrosion¹⁶. The anticipated liquid or gaseous factors include: water, ammonia, gaseous oxygen, hydrogen, nitrogen, and humidity¹⁷. Bacteria and fungus is also anticipated to be a problem. Establishing limits on levels of contamination and degradation of lubricants and gases to be used is critical to the long service life anticipated¹⁸

¹⁶ Structural & Mechanical Systems Long-Life Assurance Guidelines, p. 109.

¹⁷ Structural & Mechanical Systems Long-Life Assurance Guidelines, p. 109.

¹⁸ Structural & Mechanical Systems Long-Life Assurance Guidelines, p. 109.

2.0 Compendium of Mechanical Limit-States

2.1 Wear

Wear is the removal of material from solid surfaces as a result of mechanical action. In most cases the amount of the material removed from the surfaces is small in relation to the overall material mass of the components involved.

Wear processes have been identified to conform to four different forms. The four major forms of wear enumerated by Rabinowicz¹⁹ are:

1. Adhesive wear;
2. Abrasive wear;
3. Corrosive wear;
4. Surface fatigue wear.

In addition to these four major wear processes, there are a number of minor processes that are often categorized as being a wear process.

Models for the following wear processes are presented in sections 1.1 through 1.5:

1. Adhesive wear
2. Abrasive wear
3. Lubricated wear
4. Fretting wear
5. Liquid impact erosion.

¹⁹ Rabinowicz, E., Friction and Wear of Material, J. Wiley, New York, 1965.

2.1.1 Adhesive Wear

Definition

"Adhesive wear occurs when two smooth bodies are slid over each other, and fragments are pulled off one surface to adhere to the other" ²⁰. Once the fragments have been torn from their original surface and attached to the opposing surface, they may reattach to their original surface, or become loose fragments.

Limit-State Formulation

Experimental data indicates that there are three laws of adhesive wear, namely;

1. The amount of wear is generally directly proportional to the load L;
2. The amount of wear is generally proportional to the distance slid, x;
3. The amount of wear is generally inversely proportional to the hardness, p, of the surface being worn away.

Holm ²¹ proposed that the volume worn away could be described by:

$$V = \frac{cLx}{p}$$

where V=volume of material worn away
c=material dependent nondimensional constant
x=sliding distance
L=load
p=hardness of wearing surface (1)

Evidence for this relationship is mixed, with some results being very close to the predicted volumes and other being widely different. Archard ²² presented a model of sliding which allows for the derivation of the above equation, while providing insight into the meaning of the constant, c. From his model we get the following model for the volume of material worn away through adhesive wear;

²⁰ Rabinowicz, E., Friction and Wear of Materials, J. Wiley, New York, 1965.

²¹ Holm, R., Electric Contacts, Almqvist and Wiksells, Stockholm, 1946.

²² Archard, J.F., Contact and Rubbing of Flat Surfaces, Journal of Applied Physics, vol. 24, pp.981-988.

$$V = \frac{kLx}{3p}$$

where k =coefficient of wear
 =probability of any junction
 forming a fragment
 p =flow pressure of softer metal (2)

As can be seen the difference in these equations is that we have replaced c with $k/3$. The only important requirement for the second equation to hold is that the volume of the fragment should be proportional to the cube of the junction diameter.

An alternative form of the second equation is:

$$V = \frac{kA_r x}{3} \quad (3)$$

where A_r =actual area of contact

Although knowledge of the volume worn is important, typically we are more interested in the depth of material worn away. The extension of the above relations to yield the depth of material worn away is given Archard²³;

$$\frac{d}{L} = \frac{K P}{H}$$

where d =depth of wear
 P =nominal pressure
 L =sliding distance
 H =material hardness
 K =coefficient of wear

²³ Archard, J.F., *Wear Theory and Mechanisms*, Wear Control Handbook, eds. M.B. Peterson and W.O. Winer, ASME, New York, 1980.

This form of the wear relationship now permits the estimation of the life of the surface. If we let the sliding distance, L, be expressed as the velocity, v, and the time, t, the relationship becomes:

$$t = \frac{d H}{K P v}$$

The authors go on to demonstrate that the coefficient of wear, K is the proportional of volume worn away to the theoretical worn volume that would have resulted if every asperity contact produced a worn particle.

Model Assumptions

The assumptions of the model are:

1. k=probability of any junction forming a fragment
2. Each junction is in existence throughout the sliding distance, d.

Notes

Some values for the wear coefficients are given on the following page. Additional wear coefficient values can be found in Proceedings of the Conference on Lubrication and Wear (1957 on), and the Transactions of the American Society of Lubrications Engineers (1958-1963).

Additional Sources

Rabinowicz, E., Friction and Wear of Materials, J. Wiley, New York, 1965.

Holm, R., Electric Contacts, Almqvist and Wiksells, Stockholm, 1946.

Archard, J.F., *Contact and Rubbing of Flat Surfaces*, Journal of Applied Physics, vol. 24, pp.981-988.

M.B. Peterson and W.O. Winer, eds., Wear Control Handbook, ASME, New York, 1980.

Selected Wear Coefficients

From Rabinowicz, p. 139²⁴.

Combination	Wear Constant k
Zinc on zinc	160×10^{-3}
Low carbon steel on low carbon steel	45
Copper on copper	32
Stainless steel on stainless steel	21
Copper (on low carbon steel)	1.5
Low carbon steel (on copper)	0.5
Bakelite on bakelite	0.02

From Archard, p. 38²⁵.

MATERIALS		Wear Rate (10^{-6} mm ³ per m sliding)	Hardness (DPN) kg/mm ²	Coefficients of Friction (μ)	Coefficients of Wear (K)
Wearing Surface	Counter Surface				
Mild Steel	Mild Steel	1.57×10^4	186	0.62	7×10^{-3}
60/40 Leaded Brass	Tool Steel	2.4×10^3	95	0.24	6×10^{-4}
PTFE	Tool Steel	2.0×10^2	5	0.18	2.4×10^{-5}
Stellite	Tool Steel	32	690	0.60	5.5×10^{-5}
Ferritic Stainless Steel	Tool Steel	27	250	0.53	1.7×10^{-5}
Polyethylene	Tool Steel	3	17	0.53	1.3×10^{-7}
Tungsten Carbide	Tungsten Carbide	0.2	1300	0.35	1×10^{-6}

From Archard, p. 65²⁶

BEARING MATERIALS		ENVIRONMENT		K	AWN
Wear Surface	Opposing Surface	Atmosphere	Lubricant		
52100 Steel	52100 Steel	Dry Argon	None	1.0×10^{-2}	2.0
Mild Steel	Mild Steel	Air	None	2.3×10^{-3}	2.6
60/40 Leaded Brass	Tool Steel	Air	None	2.0×10^{-3}	2.7
52100 Steel	52100 Steel	Dry Air	None	1.0×10^{-3}	3.0
Stellite	Tool Steel	Air	None	1.8×10^{-5}	4.7
52100 Steel	52100 Steel	Air	Cyclohexane	8.4×10^{-6}	5.1
PTFE	Tool Steel	Air	None	8.3×10^{-6}	5.1
52100 Steel	52100 Steel	Air	Paraffinic Oil	3.2×10^{-7}	6.5
Polyethylene	Tool Steel	Air	None	4.3×10^{-8}	7.3
Aluminium Bronze	Carburised Steel	Air	Gear Lubricant	2.5×10^{-8}	7.6
Carburised Steel	Carburised Steel	Air	Gear Oil	1.6×10^{-9}	7.8
52100 Steel	52100 Steel	Air	Paraffinic Oil/TCP	3.3×10^{-9}	8.5
52100 Steel	52100 Steel	Air	Engine Oil	$< 2.0 \times 10^{-10}$	9.7

²⁴ Rabinowicz, E., Friction and Wear of Materials, J. Wiley, New York, 1965.

²⁵ Archard, J.F., Wear Theory and Mechanisms, Wear Control Handbook, eds., M.B. Peterson and W.O. Winer, ASME, New York, 1980.

²⁶ Archard, J.F., Wear Theory and Mechanisms, Wear Control Handbook, eds. M.B. Peterson and W.O. Winer, ASME, New York, 1980.

2.1.2 Abrasive Wear

Definition

Abrasive wear occurs when a rough hard surface, or a soft surface containing hard particles, slides on a softer surface. As a result of the sliding action, the softer surface has a series of grooves ploughed into its surface. The material removed in the creation of the grooves is typically found to be loose particles.

Limit-State Formulation

Rabinowicz²⁷, assuming that the hard surface was composed of conical asperities, derived the following relationship:

$$\frac{\partial V}{\partial l} = \frac{L \tan \theta}{\Pi \rho} \quad (6)$$

where $\tan \theta$ = weighted average angle of conical asperities

This equation has the same form as the equation derived for adhesive wear. Thus we can use the same relationship as that for adhesive wear with the following value of the coefficient of wear:

$$k_{abr} = 0.96 \tan \theta$$

Some values for the abrasive wear coefficient are given on the following page.

²⁷ Rabinowicz, E., Friction and Wear of Materials, J. Wiley, New York, 1965.

Selected Abrasive Wear Coefficients

Investigator	Wear Type	Fragment Size(μ)	Materials	K(X 10 ⁻³)
Spurr et al. (1957)	2-body	-----	Many	180
Spurr et al. (1957)	2-body	110	Many	150
Avient et al. (1960)	2-body	40-150	Many	120
Lopa (1956)	2-body	260	Steel	80
Kruschov et al. (1958)	2-body	80	Many	24
Samuels (1956)	2-body	70	Brass	16
Toporov (1958)	3-body	150	Steel	6
Rabinowicz et al. (1961a)	3-body	80	Steel	4.5
Rabinowicz et al. (1961b)	3-body	40	Many	2

Reproduced from Rabinowicz, p. 169²⁸.

Source Documents for Table:

Avient, B.W.E., J. Goddard, and M. Wilman, *An Experimental Study of Friction and Wear during Abrasion of Metals*, Proceedings of the Royal Society, Vol A-258, pp. 159-180, 1960.

Kruschov, M.M., and M.A. Babichev, *Resistance to Abrasive Wear of Structurally Inhomogeneous Materials*, Friction and Wear in Machinery, Vol. 12, pp. 5-23, ASME, New York, 1958.

Lopa, M., *A Study of the Influence of Hardness, Rubbing Speed and Load on Abrasive Wear*, B.S. Thesis in Mechanical Engineering, M.I.T. Cambridge, Massachusetts, 1956.

Rabinowicz, E., L.A. Dunn, and P.G. Russell, *The Abrasive Wear Resistance of Some Bearing Steels*, Lubrication Engineering, Vol. 17, pp. 587-593, 1961a.

Rabinowicz, E., L.A. Dunn, and P.G. Russell, *A Study of Abrasive Wear under Three-Body Conditions*, Wear, Vol. 4, pp.345-355, 1961b.

Samuels, L.E., *The Nature of Mechanically Polished Surfaces: The Surface Deformation Produced by the Abrasion and Polishing of 70:30 Brass*, Journal of the Institute for Metallurgy, Vol. 85, pp. 51-62, 1956.

Spurr, R.T., and T.P. Newcomb, *The Friction and Wear of Various Materials Sliding against Unlubricated Surfaces of Different Types and Degrees of Roughness*, Proceedings of the Conference on Lubrication and Wear, Institute of Mechanical Engineers, London, pp. 269-275, 1957.

Toporov, G.V., *The Influence of Structure on the Abrasive Wear of Cast Iron*, Friction and Wear in Machinery, Vol. 12, pp. 39-59, ASME, New York, 1958.

²⁸ Rabinowicz, E., Friction and Wear of Materials, J. Wiley, New York, 1965.

2.1.3 Lubricated Wear

Definition

Lubricated wear occurs when two smooth surfaces are slid over each other in the presence of a lubricating media. The lubricating media serves to partially or completely separate the surface asperities of the opposing surfaces, thereby reducing or eliminating the formation of worn particles.

Failure Mechanism

Lubricated wear occurs when the degree of lubricant separating the two surfaces is insufficient to prevent asperity contact. If asperity contact occurs, then other wear mechanisms can be used to determine the wear.

Even under dry conditions, absorbed gas molecules act as a lubricant, thereby yielding less wear than predicted by theoretical relationships. A more accurate form of the standard wear relationship was offered by Rowe ²⁹:

where k_m = adhesive wear coefficient free of contaminants
 A_m = area of metallic contact
 α = fraction of true area which is metal to metal
 V = volume material worn away
 d = sliding volume
 L = load
 H = material hardness

So α is a measure of the effectiveness of a lubricant for cases where a thick elastohydrodynamic film separates the surfaces, or there is a surface film.

²⁹ Rowe, C.N., *Lubricated Wear*, Wear Control Handbook, eds., M.B. Peterson and W.O. Winer, ASME, New York, 1980.

Rowe then demonstrated:

$$\alpha = \frac{X}{U t_0} e^{-\frac{E}{RT_s}}$$

where X =diameter of area of adsorbed lubricant molecule
 t_0 =fundamental time of vibration of molecule in adsorbed state
 U =sliding velocity
 E =energy of adsorption
 T_s =surface temperature

In order to evaluate the above relationship for α , it is necessary to estimate the surface temperature, or the change in temperature that results from the relative slip of opposing surfaces.

The temperature rise in the contact surface can be approximated by:

$$\Delta T = \frac{gfWU}{8JK_{tc}r}$$

where g =gravitational constant
 J =mechanical equivalent of heat
 K_{tc} =thermal conductivity
 f =coefficient of friction
 r =radius of contact area

When wear particles begin to interact they breakdown the lubricant film and increase lubricant temperature. The result is collapse of lubricant film and catastrophic wear. This transition is marked by the end of mild wear processes, and the move to insufficient lubrication and catastrophic wear.

Once we have determined that the film has not broken down, then we can determine the lubricated wear.

Note that the adhesive wear model was noted earlier to be:

$$\frac{V}{d} = \frac{K L}{H}$$

where V =volume of material worn away
 d =sliding distance
 K =wear coefficient
 L =load
 H =material hardness

So,

$$\frac{K L}{H} = K_m \alpha \frac{L}{H}$$

$$K = K_m \alpha$$

With the relationship for α given previously,

$$K = \frac{X K_m}{t_o} e^{-\frac{E}{RT}}$$

Thus, we can determine values for K_m . Once we determine values for K_m and α , we can determine the wear using the standard adhesive wear equation modified for the lubricated wear case.

2.1.4 Fretting Wear

Definition

Fretting wear is the removal of material at a component interface that is the result of relative movement of the components, usually of such small magnitude that the movement is not detected by visual inspection.

Failure Mechanism

Fretting wear involves 3 possible basic processes³⁰:

1. Mechanical action disrupts oxide films on the surface exposing clean and possibly strained metal, which would be reactive and in atmosphere would oxidize rapidly during the cycle after disruption.
2. Removal of metal particles from the surface in a finely divided form by a mechanical grinding action or by formation of welds at points of contact which are broken at a surface other than the original interface by shearing or fatigue.
3. Oxide debris resulting from either process 1, or 2 is an abrasive powder that damages the surfaces.

In observation of fretting behavior, research has found that³¹:

1. Fretting damage is reduced in a vacuum or inert atmosphere.
2. Debris formed by fretting of iron is largely composed of Fe_2O_3 .
3. Greater damage occurs at low frequencies for a given number of cycles compared with high frequencies.
4. Metal loss increases with load and relative slip.
5. Greater damage occurs below room temperature compared with above room temperature.
6. Damage is greater in dry atmospheric conditions than humid atmospheric conditions.³²

³⁰ Waterhouse, R.B., Fretting Corrosion, Pergamon Press, London, 1972.

³¹ Waterhouse, R.B., Fretting Corrosion

³² Waterhouse, R.B., Fretting Corrosion

Uhlig proposed a model of a regular array of asperities removed by successive cycles³³. It is assumed that the asperities plough in the metal surfaces. From this a model for the weight loss per cycle due to scraping of the oxide film layer was determined as:

$$W_{CORR} = 2nlck \ln\left(\frac{s}{2lf\tau} + 1\right)$$

where: n = number of circular asperities/unit area
 l = distance moved by an asperity in 1/2 cycle
 = amplitude of slip
 c = diameter of asperity
 s = spacing of asperity
 f = frequency
 τ, k = constants

The weight loss per cycle due to the ploughing action is:

$$W_{MECH} = \frac{2k'lp}{p_0} = k_2lP$$

where: P = normal load
 p_0 = yield pressure
 k', k_2 = constants

Uhlig assumes a linear oxidation rate, so:

$$W_{CORR} = \frac{ncks}{f\tau}$$

Note that:

asperity spacing = $n^{0.5}$
 total area of contact = $n\pi (c/2)^2$

³³ Uhlig, H.H., *Mechanism of Fretting Corrosion*, Journal of Applied Mechanics, ASME Transactions, Vol. 76, 1954, pp.401-407.

Thus,

$$W_{CORR} = k_o \frac{P^{0.5}}{f} - k_1 \frac{P}{f}$$

$$\text{where: } k_o = \frac{2k}{\tau \sqrt{p_o \pi}}$$

$$k_1 = \frac{4k}{\tau p_o \pi}$$

Combining these results yields,

$$W_{TOTAL} = \frac{(k_o P^{0.5} - k_1 P) N}{f} + k_2 IPN$$

where: N = total number of cycles

The previous relationships apply to the initial fretting. This initial fretting can result in failure if the component surface is degraded to the point at which satisfactory operation of the element is not possible. Alternatively, this initial fretting can cause a fatigue crack and ultimately cause a fretting fatigue failure. In determining the conditions experienced in a fretting fatigue failure, the following notation will be used:

- S_{MAX} = maximum shear stress on asperities of bridge
- p = normal pressure on asperities
- μ = coefficient of friction
- p_o = yield pressure of weaker material
- H = hardness of bridge material
- S_{alt} = alternating shear stress in material due to alternating bending stress and frictional shear stress
- σ_{alt} = alternating bending stress in material
- σ_y = yield strength of weaker material

Assuming that:

- i. $p_o = 3\sigma_y$
- ii. $\sigma_y = k H$

Assuming that the asperity contact is subjected to a shear stress, μp , then the maximum pressure that the asperity on the bridge can sustain is:

$$p = \frac{k_1 H^2}{k_2 + 4\mu^2}$$

Small elements of the material experience alternating shear stress as a result of the alternating motion, the maximum value of which is :

$$S_{alt} = 0.5(4\mu^2 p^2 + \sigma_{alt}^2)^{0.5}$$

When the alternating shear stress, S_{alt} , reaches the fatigue strength of the material in shear, a fatigue crack will be initiated. The bending stress that will yield this critical value of the alternating shear stress is:

$$\sigma_{alt} = \left(4S_{alt}^2 - \frac{4k_1 H^2}{k_2 + 4\mu^2} \right)^{0.5}$$

Sources

Waterhouse, R.B., Fretting Corrosion, Pergammon Press, London, 1972.

Rabinowicz, E., Friction and Wear of Materials, J. Wiley, New York, 1965.

Perterson, M.B., and W.O. Winer, eds., Wear Control Handbook, ASME, New York, 1980.

2.1.5 Liquid Impact Erosion

Definition

Liquid impact erosion occurs when the action of a liquid striking a solid object results in the removal of the surface material.

Failure Mechanism

There are several models for liquid impact erosion, namely:

- Thiruvengadam's Theory
- Springer's Theory
- Hertzian Theory
- Evan's Elastic Plastic Theory

Each of these theories is discussed below.

Thiruvengadam's Theory of Liquid Impact Erosion

Thiruvengadam developed the notion of erosion strength, S_e .³⁴

S_e =energy absorbing capacity of material per unit volume under erosive forces.

The erosion process is controlled by 2 opposing phenomena:

1. time-dependent efficiency of absorption of impact energy by the material;
2. attenuation of impact pressure due to changing surface topography as the surface material is eroded.

The intensity of a single drop impact is defined as,

$$I_c = \frac{P_w^2}{\rho_w C_w}$$

where I_c =intensity of impact

P_w =pressure imparted to surface by liquid impact

ρ_w =density of liquid

C_w =compressional wave velocity for liquid

³⁴

Thiruvengadam, A., *The Concept of Erosion Strength, Erosion by Cavitation or Impingement*, ASTM-STP-408, American Society for Testing and Materials, 1967, pp. 22-35, quoted by G.F. Schmitt, Jr., *Liquid and Solid Particle Impact Erosion, Wear Control Handbook*, eds., M.B. Peterson and W.O. Winer, ASME, New York, 1980.

The attenuation of the intensity of impact, I_i , is assumed to be:

$$I_i = \left(\frac{A}{R + R_f} \right)^m I_e$$

where: I_i = attenuated intensity

A = proportionality constant

R = mean depth of erosion from original surface

R_f = thickness of liquid layer on surface

The intensity of erosion, defined as the power absorbed by a unit eroded area of material, is designated, I_e :

$$I_e = S_e \frac{dR}{dT}$$

and

$$I_e = n I_i$$

where $n+n(t)$ is time dependent property governing energy absorption efficiency.

This can be constructed into a normalized differential equation, the solution to which is;

$$\bar{\tau} = \frac{n}{\left(1 + \bar{k} \frac{n+1}{n} \int_1^{\bar{\tau}} \bar{n} d\tau \right)^{\frac{n}{n+1}}}$$

This approach has 2 weaknesses:

1. The dependence on the presence of a liquid layer on the surface to attenuate the loading pulses.
2. The parameter, n , is related to the theory, but has no physical meaning in most liquid drop situations, since the liquid layer is either thin or nonexistent.

Springer's Theory of Liquid Impact Erosion

Springer's theory is based on concepts involving metal fatigue³⁵. The model assumes that the incubation period, the acceleration period and the maximum rate period of the characteristic erosion curve can be represented by:

$$M^* = \alpha^*(N^* - N_i^*)$$

where: M^* = erosion mass loss

α^* = rate of mass loss

N^* = number of impacts per site

N_i^* = number of impacts corresponding to the incubation period

Based on Miner's rule in torsion and bending fatigue, Springer derives the expression for impacts in the incubation model:

$$N_i^* = a_1 \left(\frac{S}{P} \right)^{a_2}$$

where: a_1, a_2 = constants

P = avg. interfacial pressure due to water drop impact

and

$$S = \frac{4\sigma_u(b-1)}{(1-2\nu) \left[1 - \left(\frac{\sigma_l}{\sigma_u} \right)^{b-1} \right]}$$

where: ν = Poisson's ratio

σ_u = ultimate tensile strength

σ_l = endurance limit

$$b = \frac{b_2}{\log_{10} \left(\frac{\sigma_u}{\sigma_l} \right)} = \text{derived from S-N curve}$$

From a least square fits of the data, $a_1 = 7 \times 10^{-6}$, and $a_2 = 5.7$. The weakness of this approach is the arbitrary selection of a constant, b , which is applied to all materials.

³⁵ Springer, G.S., Erosion by Liquid Impact, Scripta Publishing Co, J. Wiley & Sons, Washington, D.C., 1976, quoted by G.F. Schmitt, Jr., Liquid and Solid Particle Impact Erosion, Wear Control Handbook, eds., M.B. Peterson and W.O. Winer, ASME, New York, 1980.

Hertzian Impact Theory³⁶

In a collision of a deformable sphere and target, the time dependent radius of contact area is:

$$a(t) = a_1 \sin^2 \left(\frac{\pi t}{T} \right)$$

where: $a_1 = K^{0.2} r v_0^{0.4}$ = maximum contact radius

$T = 2.943 K^{0.4} r v_0$ = duration of bodies in contact

$K = 1.25 \pi \rho_1 \left[\frac{1}{\rho_1 C_1^2} + \frac{1}{\rho_2 C_2^2} \right]$ = elastic properties of impacting bodies

ρ_1, ρ_2 = density of sphere and target

C_1, C_2 = elastic wave velocity of sphere and target

$$C_i^2 = \frac{1}{\rho_i} \frac{E}{(1-\nu^2)}$$

If the liquid is assumed to be a deformable sphere impacting a rigid body, then when the relative velocity between the 2 bodies is 0,

$$a(t) = a_1 = \frac{5}{4} \pi^{0.2} r \left(\frac{V_0}{C_1} \right)^{0.4}$$

Zero relative velocity between the 2 bodies occurs at $t = T/2$.

The magnitude of the liquid impact pressure differs from those of solids, so the following results from the case of water hammer are used:

$$P_w = \rho_w C_w V_0$$

where: ρ_w = density of liquid

C_w = acoustic wave velocity in liquid

V_0 = liquid impact velocity

³⁶ Hertz, H., Miscellaneous Papers, Macmillan and Company, London, 1886, quoted by G.F. Schmitt, Jr., Liquid and Solid Particle Impact Erosion, Wear Control Handbook, eds., M.B. Peterson and W.O. Winer, ASME, New York, 1980.

Accounting for the compressibility of the liquid,

$$P_w = \frac{\rho_w C_w V_0}{1 + \frac{\rho_w C_w}{\rho_t C_t}}$$

where the subscript *t*, denotes the properties of the target.

The impact of a water drop on a rigid surface was found by Engels to be:

$$P_w = \frac{\alpha}{2} \rho_w C_w v_0$$

Evans' Elastic-Plastic Theory

Evans³⁷ proposes that the predicted erosion rate is:

$$V = v_0^{\frac{19}{6}} r^{\frac{11}{3}} \rho^{\frac{19}{12}} Ke^{\frac{-4}{3}} H^{\frac{-1}{4}}$$

where: *V* = volume lost/impact
*v*₀ = impact velocity
r = particle radius
ρ = particle density
Ke = stress intensity factor
H = dynamic hardness

³⁷ Evan, A.G., et al., Impact Damage in Brittle Materials in the Plastic Response Regime, Contract No. 00014-75-C-0069, Report No. SC5023.9TR, Rockwell International Science Center, Thousand Oaks, California, 1976, quoted by G.F. Schmitt, Jr., Liquid and Solid Particle Impact Erosion, Wear Control Handbook, eds., M.B. Peterson and W.O. Winer, ASME, New York, 1980.

Empirical Models

Due to the complex nature of the processes involved in impact erosion, the traditional approach has been the utilization of models fitted to empirical data. Schmitt^{38 39} has proposed that impact erosion be modelled using equations of the form:

$$MDPR = KV^\alpha \sin^\beta \theta$$

where: MDPR = mean depth of penetration rate

K = constant

V = velocity

θ = impact angle

α, β = empirically determined exponents

Schmitt⁴⁰ notes that for uncoated two-dimensionally reinforced composite materials, beta of 2 is a best fit for velocities between 450 to 1700 m/sec.

³⁸ Schmitt, G.F., *Influence of Materials Construction Variables on the Rain Erosion Performance of Carbon-Carbon Composites*, Erosion: Prevention and Useful Applications, ASTM-STP-664, W.F. Adler, ed., American Society for Testing and Materials, 1979, pp.373-405.

³⁹ Schmitt, G.F., *Erosion Rate-Velocity Dependence for Materials at Supersonic Speeds*, Characterization and Determination of Erosion Resistance, ASTM-STP-474, American Society for Testing and Materials, 1970, pp. 323-352.

⁴⁰ Schmitt, G.F., *Liquid and Solid Particle Impact Erosion*, Wear Control Handbook, eds., M.B. Peterson and W.O. Winer, ASME, New York, 1980.

2.2 Corrosion

Corrosion is the destructive attack of a metal by chemical or electrochemical reaction with its environment.⁴¹ The destruction of a metal surface by physical causes is not classified as corrosion.

The major classifications of corrosion given by Fontana⁴² are:

1. Erosion-corrosion
2. Galvanic corrosion
3. Uniform attack
4. Pitting corrosion
5. Cavitation
6. Crevice corrosion
7. Stress corrosion cracking
8. Selective leaching
9. Intergranular corrosion

Models for the first six forms of corrosion were found, and are presented in sections 2.1 through 2.6 that follow. At this time no deterministic relationships were found that accurately predict stress corrosion cracking, selective leaching, or intergranular corrosion.

⁴¹ Uhlig, H., Corrosion and Corrosion Control, 2nd Edition, p.1, J. Wiley & Sons Inc., 1971.

⁴² Fontana, M., Corrosion Engineering, Third Edition, McGraw-Hill Book Co., San Francisco, 1986.

2.2.1 Erosion-Corrosion

Definition

Erosion-corrosion is defined as the accelerated corrosion of a metal as a result of a flowing fluid disrupting or thinning a protective film of corrosion products.

Failure Mechanism

The transition from laminar to turbulent flow occurs over a velocity range and is dependent on the geometry, surface roughness, and liquid viscosity. The conditions of fluid flow and the transition to turbulent flow is predicted using the Reynolds number:

$$Re = \frac{V d}{\gamma}$$

where: V = velocity ($m s^{-1}$)
 d = characteristic specimen length (m)
 γ = kinematic viscosity ($m^2 s^{-1}$)

When a liquid exhibits predominantly turbulent flow, a thin laminar sublayer, of thickness δh , exists near the metal surface as a result of viscous drag. If material is being removed from the metal surface there will be a diffusional boundary layer of thickness δd .⁴³ The Schmidt number describes the relationship between these two boundary layers:

$$Sc = \frac{\gamma}{D}$$

where: D = diffusivity of the relevant species (m^2/s)

As the value of the Schmidt number increases the diffusion layer will become thinner, and its formation occur more rapidly.⁴⁴

⁴³ Poulson, B., *Electrochemical Measurements in Flowing Solutions*, Corrosion Science, vol. 23, no. 4, pp.391-430, 1983.

⁴⁴ Poulson, B., *Electrochemical Measurements in Flowing Solutions*

In most cases the resistance of the surface to mass transfer, both laminar and turbulent regions of flow, is accomplished through the definition of the mass transfer coefficient, K , as:

$$K = \frac{\text{rate of reaction}}{\text{concentration driving force}}$$

The mass transfer rate is more conveniently described by a dimensionless number, the Sherwood number,

$$Sh = \frac{K d}{D}$$

For reactions that are governed by mass transfer, the relationship between the Sherwood, Reynolds, and the Schmidt numbers is found by empirical results to be of the form,

$$Sh = \text{constant} \cdot Re^x \cdot Sc^y$$

Typical experimental results indicate that x is usually 0.3-1, and y is 0.33. These values are applicable in cases where the surface is initially smooth. The case of rough surfaces subjected to mass transfer is outlined below. In addition, the consideration of different surface configurations that may be subject to erosion-corrosion is accommodated by the constant in the above equation, a constant that is geometry dependent. The most common geometries studied, and some results are outlined in the next section.

When a metal is subjected to erosion corrosion it will develop scallop shaped ridges. The roughening due to the creation of these ridges tends to increase the mass transfer rate, and the rate of erosion corrosion. The availability of relationships defining either mass transfer or erosion corrosion from rough surfaces is extremely limited. Recent work has indicated that once a surface is roughened, the rate of mass transfer is governed by roughness, not the geometry of the surface⁴⁵. More importantly, the results indicate that a universal governing relationship for mass transfer for roughened surfaces may exist, namely,

$$Sh = 0.01 Re Sc^{0.33}$$

⁴⁵ Poulson, B., *Mass Transfer From Rough Surfaces*, Corrosion Science, vol. 30, no. 6/7, pp.743-746, 1990

The results indicating such a result are shown in Figure 1, taken from Poulson (Corr. Sc., 1990). Poulson does indicate that the main differences between geometries is their tendency to roughen; he cites that bends in pipes tend to roughen faster, probably as a result of their attaining a critical Reynolds number more easily. Although the smooth surface mass transfer relationships are useful for the prediction of initial rates of erosion corrosion, the roughened surface relationship will govern the erosion corrosion process once the surface is no longer smooth. The precise point at which the roughened surface relationship is applicable has yet to be established.

Symbol	Geometry	Technique	Equation	Ref
n	2.5d 180° bend in 22.6mm tube	copper in .1N HCl + 4g/l Fe ³⁺	$Sh = 0.1 Re^{.33} Sc^{.33}$	16
l	95mm impinging jet at height 95mm		$Sh = 0.65 Re^{1.02} Sc^{.33}$	17
o	rotating cylinder $\frac{d}{\epsilon} = 87$	LCDT	$Sh = 0.16 Re^{.95} Sc^{.33}$	10
•	90mm pipe	plaster dissolution	$Sh = 0.2 Re^{.9} Sc^{.33}$	13

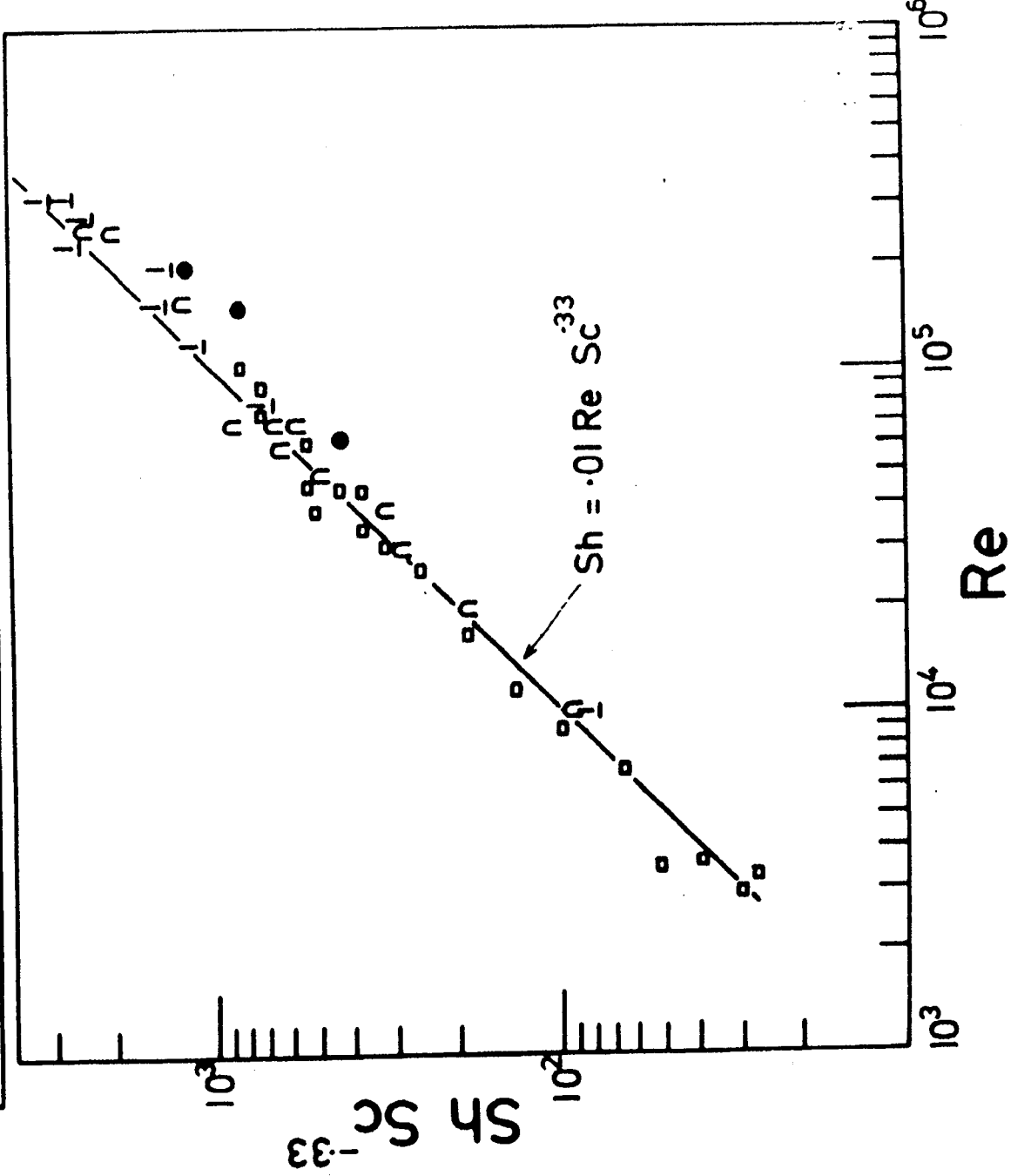

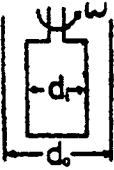
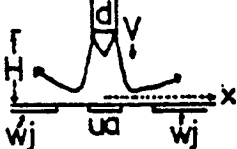
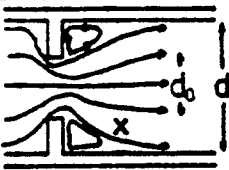
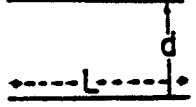


FIG 1 MASS TRANSFER FROM ROUGH SURFACES

Initial Erosion Corrosion Geometry-Dependent Relationships

The following table gives the initial smooth surface erosion corrosion rates.

(From Poulson, B., *Electrochemical Measurements in Flowing Solutions*, *Corrosion Science*, vol. 23, no. 4, pp. 391-430, 1983)

ROTATING DISC		$Re = \frac{\omega r^2}{\nu}$ $Re_c = 17.35 \times 10^5$	$\bar{Sh}_l = 0.6205 Re^{0.5} Sc^{0.33}$ $\bar{Sh}_r = 0.0078 Re^9 Sc^{0.33}$
ROTATING CYLINDER		$Re = dV = \frac{\omega d_i^2}{2\nu}$ $Re_c \sim 200$	$\bar{Sh}_{smooth} = 0.079 Re^7 Sc^{0.56}$ $\bar{Sh}_{rough} = (1.25 + 5.76 \log_{10} d/\lambda)^2 Re Sc^{0.56}$
IMPINGING JET		$Re = dV$ $Re_c \approx 2000$	$\bar{Sh}_{ua} = 1.12 Re^5 Sc^{0.33} (H/d)^{0.51}$ $Sh_{w_j} = 0.65 Re^{0.6} (x/d)^{-1.2}$
NOZZLE OR ORIFICE		$Re_0 = \frac{d_0 V}{\nu}$ $Re_c \sim 600$	$Sh_{max} = 0.276 Re_0^{0.66} Sc^{0.33}$ $\frac{Sh_x}{Sh_{rd}} = 1 + A_x \left[1 + B_x \left(\frac{Re_0^{0.66}}{0.0165 Re^{0.66}} - 21 \right) \right]$
TUBE		$Re = \frac{dV}{\nu}$ $Re_c \approx 2000$	$\bar{Sh}_e = 1614 (Re Sc d/L)^{0.33} \quad \bar{Sh}_r = 0.276 Re^{0.66} Sc^{0.33} (d/L)^{-0.33}$ $\bar{Sh}_{dT} = 0.0165 Re^{0.66} Sc^{0.33}$

Additional Sources

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Heitz, E., *Chemo-Mechanical Effects of Flow on Corrosion*, Corrosion Engineering, vol. 47, no.2, pp. 135-145, 1991.

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Poulson, B., *Mass Transfer From Rough Surfaces*, Corrosion Science, vol. 30, no. 6/7, pp.743-746, 1990.

2.2.2 Galvanic Corrosion

Definition

Galvanic corrosion is the corrosion associated with the current of a galvanic cell made up of dissimilar electrodes.⁴⁶

Failure Mechanism

The fundamental behavior of galvanic corrosion follows the behavior of a dry cell. The relationship between the current flow in a dry cell and the weight of material corroded is given by Faraday's law:

$$W = k I t$$

*where: W=weight of metal reacting
k=electrochemical equivalent
I=current in amperes
t=time in seconds*

Alternatively, the relationship between galvanic corrosion of the anode and the galvanic current is given by the following form of Faraday's Law;

$$W = \frac{t M I}{F n}$$

*where W=weight of metal dissolved to produce galvanic current (grams)
t=time of current flow (sec)
M=atomic weight of anode metal
n=charge of metal ions formed
I=galvanic current in amperes
F=Faraday's constant (96,501 coulombs)*

It is important to distinguish between the open-circuit potential of a system and the corrosion potential of a system. The corrosion potential of the system is not the open-circuit potential of the system since the electrode reactions going on are continuously dissipating energy.

⁴⁶ Uhlig, H., The Corrosion Handbook, J. Wiley & Sons, Inc., New York, 1948.

In determining the galvanic corrosion, the initial step is the establishment of the corrosion potential, ϕ_{corros} . The potential difference of the polarized electrodes, ϕ , is given by⁴⁷:

$$\phi = I_1(R_e + R_m)$$

where: ϕ = potential difference

I_1 = current

R_e = electrolytic resistance

R_m = external metal resistance

On short circuit, the current becomes the maximum current, I_{MAX} , and R_m can be neglected, resulting in the potential difference being⁴⁸:

$$\phi = I_{\text{MAX}}R_e$$

The measured potential of a corroding metal is the corrosion potential, ϕ_{corros} . The value of the maximum current, I_{MAX} , is known as the corrosion current, I_{corros} . The corrosion rate of the anodic areas on a metal surface is related to I_{corros} by Faraday's Law.⁴⁹ By applying Faraday's Law, the corrosion rate per unit area can be expressed as a current density.

The potential of the system changes as the reaction progresses, resulting from net current to or from the electrode, and this process is referred to as polarization. The result is the system potential does not remain constant, resulting in variable current density. There are three causes of polarization⁵⁰:

1. Concentration Polarization

Concentration polarization occurs when, as a result of the current flow, material is deposited on the electrode, decreasing the surface concentration of ions. Infinite concentration polarization is approached when the concentration of ions on the surface approaches zero. The corresponding current density is referred to as the limiting

⁴⁷ Uhlig, H., Corrosion and Corrosion Control, pp. 38-39.

⁴⁸ Uhlig, H., Corrosion and Corrosion Control, p.39.

⁴⁹ Uhlig, H., Corrosion and Corrosion Control, p. 39.

⁵⁰ Uhlig, H., Corrosion and Corrosion Control, pp.42-47.

current density. For conditions where concentration polarization is present, the difference in the material potentials is given by;

$$\phi_2 - \phi_1 = -\frac{RT}{nF} \ln \frac{i_L}{i_L - i}$$

where: $\phi_2 - \phi_1$ = difference in potentials

R = gas constant (8.314 J/deg-mole)

T = absolute temperature

F = Faraday = 96500 C/eq

n = number of electrons involved in the reaction

i_L = limiting current density

i = applied current density

The limiting current density (A/cm²) can be determined from:

$$i_L = \frac{DnF}{\delta t} c \times 10^{-3}$$

where: D = diffusion coefficient for reduced ion

δ = thickness of the stagnant layer of electrolyte on electrode surface

t = transference number of all ions in solution except reduced ion

c = concentration of diffusing ion (moles/liter)

For all ions at 25°C, with the exception of H and OH, D averages about 1×10^{-5} cm²/sec, and the limiting current density can be approximated by:

$$i_L = 0.02 \text{ nc.}$$

2. Activation Polarization

In this case, the polarization is the result of a slow electrode reaction; or equivalently, the electrode requires an activation energy in order to proceed⁵¹. The activation polarization, η , increases with the current density according to:

$$\eta = \beta \log \frac{i}{i_0}$$

Note that both β and η are constant for each metal and environment, and are

⁵¹ Uhlig, H., Corrosion and Corrosion Control, pp.44-46.

temperature dependent. The exchange current, i_o , is the current density equivalent to the equal forward and reverse reactions at the electrode at equilibrium⁵².

3. IR Drop

The polarization includes a drop in the potential for the electrolyte surrounding the electrode.

Calculation of Corrosion Rates

In determining the corrosion rate, the anode-cathode area ratio must first be considered. If A_a is the fraction of the surface that is anode, and A_c is the fraction that is cathode, such that $A_a + A_c = 1$, then:

$$\phi'_c = \phi_c - \beta_c \log \frac{I_c}{A_c j_{oc}}$$

$$\phi'_a = \phi_a + \beta_a \log \frac{I_a}{A_a j_{oc}}$$

where: ϕ'_c = polarized potential-cathode

ϕ'_a = polarized potential-anode

ϕ_c = open-circuit potential-cathode

ϕ_a = open-circuit potential-anode

β_c = Tafel slope-cathode

β_a = Tafel slope-anode

I = current per unit metal surface area

When the system is in steady state, $\phi'_c = \phi'_a = \phi_{\text{corros}}$, and $I_a = I_c = I_{\text{corros}}$, then the maximum corrosion current occurs when:

$$\beta_c(1 - A_c) - \beta_a A_c = 0$$

$$\text{or } A_c = \frac{\beta_c}{(\beta_c + \beta_a)}$$

⁵² Uhlig, H., Corrosion and Corrosion Control, p.45.

If $\beta_c = \beta_a$, then the maximum corrosion rate occurs when $A_c = A_a = 0.5$.

In estimating the corrosion current, if the solution is a deaerated acid, then the following relationship is a useful approximation:

$$\phi_{corros} = - \left[0.059 \text{ pH} + \beta \log \frac{i_{corros}}{i_o} \right]$$

Alternatively, Stern and Geary⁵³, derived the following relationship:

$$I_{corros} = \frac{I_{applied}}{2.3 \Delta \phi} \left(\frac{\beta_c \beta_a}{\beta_c + \beta_a} \right)$$

where: β_a = Tafel constant for anodic reaction

β_c = Tafel constant for cathodic reaction

$\frac{I_{applied}}{\Delta \phi}$ = polarization slope in region near corrosion potential

Once the corrosion current has been determined, the corrosion rate can be found using Faraday's Law.

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Uhlig, H., Corrosion and Corrosion Control, Second Edition, J. Wiley & Sons, New York, 1971.

⁵³ Stern, M., and A. Geary, J. Electrochem. Soc., vol. 104, no. 56, 1958.

2.2.3 Uniform Attack

Definition

Uniform attack is the destruction of a material as a result of an electrochemical reaction with its environment, where the rate of corrosion is constant over the material surface.

Failure Mechanism

No empirical relationship exists to predict the rate of uniform attack of a material. Typically, materials are arranged into 3 categories of susceptibility to uniform attack (low, moderate, high), and little predictive work has been done.

Despite the few models available for uniform attack, the application of an Arrhenius type relationship would permit the prediction of rates of corrosion at differing temperatures and corrosive environments. The use of an Arrhenius relationship assumes that the corrosion rate is constant up to a critical temperature, T^C , and increasing rates of corrosion at temperatures in excess of T^C . In addition, the Arrhenius model assumes that the material is continually exposed to the corrosive environment, with the concentration of corrosives remaining fixed. The model is then^{54 55 56}:

$$\text{For } T < T^C \\ ML = C = \text{Constant}$$

$$\text{For } T \geq T^C \\ ML = Ce^{-\frac{\phi}{k} \left(\frac{1}{T^C} - \frac{1}{T} \right)}$$

where: ML = rate of material loss (ipy)

C = constant corrosion rate specific to material and environment

ϕ = activation energy for corrosion (eV)

k = Boltzmann constant ($0.8617 \times 10^{-4} \text{ eV/K}$)

T = absolute temperature of the environment ($^{\circ}K$)

T^C = absolute temperature-critical ($^{\circ}K$)

The sum of the rates of material loss for each temperature exposure multiplied by the exposure duration should yield the total material loss.

⁵⁴ Uhlig, H., The Corrosion Handbook, J. Wiley & Sons, New York, 1948.

⁵⁵ Cruse et al., Space Transportation System Life Management System Development, Phase 2, SwRI Project No. 06-2924, Report No. 06-2924-1A, Southwest Research Institute, San Antonio, Texas, 1989.

⁵⁶ Cruse et al., STS Life Management System Development, SwRI Project No. 06-2148, Report No. 06-2148-1A, Southwest Research Institute, San Antonio, Texas, 1988.

Additional Sources

Fontana, M., Corrosion Engineering, Third Edition, McGraw-Hill Book Co., San Francisco, 1986.

Uhlig, H., The Corrosion Handbook, J. Wiley & Sons, New York, 1948.

Corrosion, Volume 13, Metals Handbook, 9th ed., ASM International, Metals Park, OH, 1987.

2.2.4 Pitting Corrosion

Definition

"Pitting corrosion is the local dissolution leading to the formation of cavities in passivated metals or alloys that are exposed to aqueous, nearly neutral solution containing aggressive anions."⁵⁷ Pitting corrosion is characterized by the fact that pitting will be not initiated if the anodic potential is below a critical threshold value.

Failure Mechanism

Pitting corrosion is dependent on three factors:

1. The anodic potential exceeding the critical potential for pit nucleation, E_{np} .
2. The time required to form the first pit on a passive metal exposed to a solution containing aggressive anions, referred to as the induction time for pit initiation, τ .
3. The kinetics of pit growth.

The induction time for pit initiation is inversely related to the concentration of chloride ions, Cl^- , or the potential. The induction time for pit initiation is thought to represent the time to penetrate the passive film. Studies by Nishimura and Kudo⁵⁸ demonstrated that τ was proportional to the thickness of the passive film barrier layer.

Pit induction time is greatly affected by the concentration of chloride ions in the solution; the relationship is given by:

$$\frac{1}{\tau} = k ([Cl^-] - [Cl^-]^*)$$

where: $[Cl^-]^*$ = critical Cl^- concentration (pitting if $Cl^- > [Cl^-]^*$)

Hoar and Jacob⁵⁹ determined τ for 18Cr-8Ni stainless steel. The rate of pit initiation, τ , was found to be proportional to the n^{th} power of the Cl^- concentration, for the region where n was between 2.5 to 4.5. Other researchers have determined the values of n for different materials (see attached table from Szklarska-Smialowska).

⁵⁷ Szklarska-Smialowska, Z., Pitting Corrosion of Metals, NACE, Houston, 1986.

⁵⁸ Nishimura, R., and K. Kudo, *Effect of Thickness and Composition of Films on the Breakdown of Passivity of Iron*, Proceedings of the 8th International Congress on Metal Corrosion, Mainz, DECHEMA, Frankfurt am Main, Vol. 1, p. 6 1981, quoted by Z. Szklarska-Smialowska, Pitting Corrosion of Metals, NACE, Houston, 1986.

⁵⁹ Hoar, T.D., and W.R. Jacob, Nature, Vol. 216, p. 1299, 1967, quoted by Z. Sklarska-Smailowska, Pitting Corrosion in Metals, NACE, Houston, 1986.

The determination of pit growth is easy for the potentiostatic case. Szklarska-Smialowska⁶⁰ reported that Engell and Stolica found that the rate of pit growth was:

$$i = i^* + k_1 (t - \tau)^b$$

where: i = total current

i^* = current in passive state

t = total time

τ = induction time

k_1, b are constants

If the passive state current is disregarded and the time, t , is calculated from the beginning of the pit propagation, then the relationship becomes:

$$i = k \cdot t^b$$

In this case, k is dependent on the concentration of the chloride ion, Cl^- . If the number of pits is constant over time, then $b=2$; if the number of pits increases with time, then $b=3$. The assumptions made in this model are:

1. Pits are hemispherical in shape.
2. Current density in the pits is constant; thus the pit radii should increase linearly with time.

Szklarska-Smialowska⁶¹ reported that Forchhammer and Engell studied the growth of the pit radius, r , and the number of pits, N , and found:

$$r = k \cdot t^{\frac{1}{3}}$$

In addition, the research found that the coefficient b , cited above, varied from 0.6 to 1.2 for austenitic stainless steels.

⁶⁰ Engell, H.J., and N.D. Stolica, Z. Phys. Chem., NF, Vol. 20, p. 113, 1959 quoted by Z. Szklarska-Smialowska, Pitting Corrosion of Metals, NACE, Houston, 1986.

⁶¹ Forchhammer, P., and H.J. Engell, Werkst. Korros., Vol. 20, p. 1, quoted by Z. Szklarska-Smialowska, Pitting Corrosion of Metals, NACE, Houston, 1986.

Godard⁶² found that the greatest pit depth, d , in aluminum was proportional to:

$$d = k \cdot t^{\frac{1}{3}}$$

The results of numerous studies found that the relationship proposed for pit growth is not applicable in all situations. This led Szklarska-Smialowska to propose three models:

Case 1: Hemispherical Pit Model ($r=h$)

$$i \sim \frac{r^3}{t} N$$

Case 2: Cap Shaped Pit Model ($r>h$)

$$i = \frac{h^2 R (3 - \alpha)}{t} N$$

Case 3: Cylinder Shaped Pit Model ($r<h$)

$$i \sim \frac{r^2 h}{t} N$$

where: r =pit radius
 R =radius of sphere
 h =cap height
 α = h/r
 N =number of pits

Now to determine the maximum pit depth, one could apply Faraday's law using the current equations outlined above, or the relationship developed by Godard and already discussed.

⁶² Godard, H.P., *Pitting Kinetics*, Canadian Journal of Chemical Engineering, vol. 21, p.167, 1960.

Additional Sources

Fontana, M., Corrosion Engineering, Third Edition, McGraw-Hill Book Co., San Francisco, 1986.

Godard, H.P., *Pitting Kinetics*, Canadian Journal of Chemical Engineering, vol. 38, p.167, 1960.

Szklarska-Smialowska, Z., Pitting Corrosion of Metals, NACE, Houston, 1986.

Uhlig, H., The Corrosion Handbook, J. Wiley & Sons, Inc., New York, 1948.

Order of Reaction Values, n.

(From Szklarska-Smialowska, Z., Pitting Corrosion of Metals, p. 110, 1986)

Metal or Alloy	Aggressive Anion	pH	n	Potential
Al 1199	low Cl ⁻ conc.	0.0	11.1	0.18 V _{SCE}
	higher Cl ⁻ conc.		4.0	
Al 2024 (99.99% Al)	0.004-0.01M Cl ⁻	0.0	3.0	0.18 V _{SCE}
	0.003-0.004M Cl ⁻ (in H ₂ SO ₄)	0.0	8.8	
		3.5	4.8	
Pure, preanodized Al	0.01-1M KCl	6.1	0.1	
	1-3M KCl	5.9	0.9	
Al alloy 7075	Cl ⁻	0.3	8	0.18 V _{SCE}
	Br ⁻	0.3	4	
	I ⁻	0.3	2	
	F ⁻	5.8	3	
	Cl ⁻	5.8	2	
	Br ⁻	5.8	2	
Al 1199	Cl ⁻	3.56	1.5	0.6 V _{Hg/Hg₂SO₄}
	Br ⁻	3.56	2.5	
18Cr-8Ni stainless	Cl ⁻	2.0	2.5-4.5	0.4-0.8 V _{SHE}
Iron	10 ⁻³ M Cl ⁻ in H ₂ SO ₄	0.0	3	0.85-1.75 V _{SHE}
Nickel	low Cl ⁻ conc.	2.0	4	0.5 V _{SCE}
	high Cl ⁻ conc. in H ₂ SO ₄		3	
18Cr-8Ni stainless	Cl ⁻ in H ₂ SO ₄	0.0	2.5	0.2 V _{SCE}
			4.5-5	0.6 V _{SCE}

2.2.5 Cavitation

Definition

Cavitation is a dynamic phenomena that is concerned with the growth and collapse of cavities in a liquid. Cavitation is a liquid phenomena that is the result of pressure reductions in the liquid.

It is important to note that:

1. cavitation can occur in liquids in motion or at rest.
2. cavitation is not restricted to or excluded from occurring at solid boundaries.

Failure Mechanism

At this time no comprehensive model that adequately predicts the behavior of cavitation erosion is available. Existing models of cavitation erosion have focussed on extremely simplified cases involving the collapse of single cavities on a solid surface. The agreement between such models and experimentally observed results varies. This being the case, the best guide to the designer is the avoidance of situations where cavitation can occur.

The cavitation number, σ , is a useful parameter for categorizing cavitating flows and is defined as⁶³:

$$\sigma = \frac{P_{\infty} - P_v}{0.5 \rho V_{\infty}^2}$$

where: P_{∞} = absolute pressure at some reference locality

P_v = vapor pressure (at liquid bulk temperature)

V_{∞} = reference velocity

ρ = liquid density

If the value of σ is large, then no cavitation will occur, while if σ is sufficiently small then cavitation behavior will be well developed. Limited cavitation will occur when the cavitation

⁶³ Knapp, R.T., et al., Cavitation, McGraw-Hill Book Co., New York, 1970.

number is of some intermediate value. This is exhibited by a few bubbles in the flow. The limited cavitation number which describes this state is⁶⁴:

$$\sigma_L \equiv \frac{P_{\infty L} - P_V}{0.5 \rho V_{\infty}^2}$$

Here, $P_{\infty L}$ is the absolute pressure at the reference locality that corresponds to the state of limited cavitation.

The determination of the nature of cavitation has been studied with two different approaches:

1. *Desinent Cavitation*

In this case cavitation is established while holding the velocity constant, and then increasing the pressure until the cavitation disappears at a pressure value, referred to as the desinence pressure, $P_{\infty d}$. The desinent cavitation number is determined from the same relation as that above, with the exception that P_{∞} is replaced by $P_{\infty d}$.

2. *Incipient Cavitation*

In this case velocity is held constant, eliminating all cavitation, and then the pressure is decreased until cavitation appears at the inception pressure, $P_{\infty i}$. The incipient cavitation number is determined by replacing P_{∞} by $P_{\infty i}$ and applying the above relationships.

Additional Sources

Holl, J.W., *Limited Cavitation*, Proceedings of the ASME Fluids Engineering and Applied Mechanics Conference, Northwestern University, Evanston, Illinois, June 16-18, 1969.

Knapp, R.T., J.W. Daily, and F.D. Hammitt, Cavitation, McGraw-Hill Book Co., New York, 1970.

⁶⁴ Knapp, R.T., et al., Cavitation, McGraw-Hill Book Co., New York, 1970.

2.2.6 Crevice Corrosion

Definition

"Crevice corrosion occurs when two or more surfaces in close proximity lead to the creation of a locally occluded region where enhanced dissolution can occur."⁶⁵ The rate of crevice corrosion is usually a function of the depth and width of the crevice. The distinction between crevice corrosion and pitting is that crevice corrosion proceeds at a much faster rate than pitting. This more rapid corrosion rate is a result of the restrictive crevice geometry which permits the crevice electrolytic solution to under change much more rapidly than a pit.

Failure Mechanism

The complex nature of crevice corrosion has made prediction of crevice corrosion rates difficult. At present, no accurate deterministic model of crevice corrosion exists. The current approach favored for the evaluation of crevice corrosion is the application of finite element methods. A number of packaged computer programs are available to predict crevice corrosion based on finite element methods, including:

CHEQMATE-CHEMICAL EQUILIBRIUM WITH MIGRATION AND TRANSPORT EQUATIONS developed by A. Haworth, S.M. Sharland, P.W. Tasker, and C.J. Tweed outlined in Harwell Laboratory Report, NSS-R113, 1988.

HARWELL-presented by C.P. Jackson, *The TSGl Finite-Element Subroutine Library*, AERE Report AERE-R10713, 1982.

⁶⁵ Sharland, S.M., *A Mathematical Model of the Initiation of Crevice Corrosion in Metals*, Corrosion Science, vol. 33, no. 2, p. 183, 1992.

Governing Equations

The transportation of aqueous species i is governed by the mass-balance equation. The mass-balance equation describes the diffusion under concentration gradients, electromigration under potential gradients, and chemical reaction.⁶⁶ The mass-balance equation is:

$$\frac{\partial C_i}{\partial t} = D_i \nabla^2 C_i + z_i U_i F \nabla(C_i \nabla \phi) + R_i$$

where: C_i = concentration of species i

D_i = effective diffusion coefficient

z_i = charge number

R_i = rate of production/depletion of species i by chemical reaction

$$U_i = \text{mobility} = \frac{D_i}{RT}$$

For a crevice geometry that is assumed to be rectangular, the steady-state transport equations for species i is:

$$D_i \left(\frac{\partial^2 C_i}{\partial x^2} + \frac{\partial^2 C_i}{\partial y^2} \right) + \frac{z_i D_i F}{RT} \left[\frac{\partial}{\partial x} \left(C_i \frac{\partial \phi}{\partial x} \right) + \frac{\partial}{\partial y} \left(C_i \frac{\partial \phi}{\partial y} \right) \right] + R_i = 0$$

The boundary conditions are usually then determined.

Sources

Bockris, J., et al. ed., Comprehensive Treatise of Electrochemistry: Volume 4 Electrochemical Materials Science, Plenum Press, New York, 1981.

Sharland, S.M., and P.W. Tasker, *A Mathematical Model of Crevice and Pitting Corrosion-I. The Physical Model*, Corrosion Science, vol. 28, no. 6, pp.603-620, 1988.

Sharland, S.M., *A Mathematical Model of Crevice and Pitting Corrosion-II. The Mathematical Solution*, Corrosion Science, vol. 28, no. 6, pp.621-630, 1988.

⁶⁶ Sharland, S.M., C.P. Jackson, and A.J. Diver, *A Finite-Element Model of the Propagation of Corrosion Cracks and Pits*, Corrosion Science, Vol. 29, no. 9, p. 1153.

Sharland, S.M., C.P. Jackson, and A.J. Diver, *A Finite-Element Model of the Propagation of Corrosion Crevices and Pits*, Corrosion Science, vol. 29, no. 9, pp. 1149-1166, 1989.

Sharland, S.M., *A Mathematical Model of the Initiation of Crevice Corrosion in Metals*, Corrosion Science, vol. 33, no. 2, pp. 183-201, 1992

Walton, J.C., *Mathematical Modeling of Mass Transport and Chemical Reaction in Crevice and Pitting Corrosion*, Corrosion Science, vol. 30, no. 8/9, pp.915-928, 1990.

Watson, M.K., and J. Postlethwaite, *Numerical Simulation of Crevice Corrosion: The Effect of the Crevice Gap Profile*, Corrosion Science, vol. 32, no. 11, pp. 1253-1262, 1991.

Watson, M.K., and J. Postlethwaite, *Numerical Simulation of Crevice Corrosion of Stainless Steel and Nickel Alloys in Chloride Solutions*, Corrosion, vol. 46, no. 7, pp. 522-530, 1990.

2.3 Fatigue

Definition

Fatigue failure is a general term given to the sudden and catastrophic separation of a machine part into two or more pieces as a result of the application of fluctuating loads or deformations over a period of time.

In studying fatigue failures, three areas are of interest, namely;

1. High-cycle fatigue
2. Low-cycle fatigue
3. Crack growth

Physical Process of Fatigue

The fatigue process consists of three stages:

1. Crack initiation phase
2. Subcritical crack propagation
3. Final fracture

Of these stages, most designers are concerned with crack initiation and subcritical crack propagation.

Probability models of fatigue life

Because many of the factors involved are random in nature, the appropriate development of analysis and design methodologies should be probabilistic. Two distributions that have been widely used in fatigue studies are the lognormal and Weibull distributions. Use of the lognormal distribution has been based primarily on arguments of mathematical expediency. However, it has been pointed out by Gumbel that the hazard function for the lognormal model decreases for large values of N^{67} . This does not agree with our physical understanding of progressive deterioration resulting from the fatigue process. Nevertheless, the lognormal distribution often seems to provide a "good fit" of cycles to failure data. The Weibull distribution is based on more physical convincing arguments. Moreover, the Weibull distribution is well-suited for certain procedures of statistical extrapolation to large systems.

⁶⁷ Gumbel, E.J., *Parameters in the Distribution of Fatigue Life*, Journal of Engineering Mechanics Division, ASCE, Vol. 89, No. EM5, October 1963.

Source:

Fatigue Reliability: Introduction, ASCE, Vol.108, NO. ST1, January,1982

J.A.Collins, Fatigue of Materials in Mechanical Design

Freudenthal, A., Reliability Analysis Based on Time to the First Failure, Aircraft Fatigue, Pergamon Press, Inc., London, England, 1972.

Gumbel, E.J., Parameters in the Distribution of Fatigue Life. Journal of the Engineering Mechanics Division, ASCE, Vol. 89, No. EM5, Oct. 1963.

Paul H. Wirsching, Statistical Summaries of Fatigue Data for Design Purposes, STAR, N83-29731.

2.3.1 High-Cycle Fatigue

Definition

High-cycle fatigue is associated with lower stress levels and high numbers of cycles to produce fatigue failure. It is typically associated with cycle lives greater than about 10^4 or 10^5 cycles.

Failure Mechanism

The crack initiation period consumes a substantial percentage of the usable fatigue life in high-cycle fatigue problems where stress fluctuations are low at fatigue-critical locations. The crack propagation period is very short compared with the crack growth period.

The classical model (The Basquin Equation)⁶⁸:

$$NS^m = C$$

or

$$N = CS^{-m}$$

where:

S = stress amplitude, or stress range.

M, C = empirically determined constants which depend on the material and are significantly affected by the environment

N = fatigue life, cycles to failure.

Additional Sources

Fatigue Reliability: Introduction, ASCE, Vol.108, NO. ST1, January, 1982

J.A.Collins, Fatigue of Materials in Mechanical Design

⁶⁸ Basquin, H.O., *The Exponential Law of Endurance Tests*, Proceedings of the ASTM, Vol. 10, Part 2, 1910.

2.3.2 Low-Cycle Fatigue

Definition

Low-cycle fatigue is associated with high stress levels and low number of cycles to produce fatigue failure. It is typically associated with cycle lives from one up to about 10^4 or 10^5 cycles.

Failure Mechanism

When stress fluctuations are high, fatigue cracks initiate quite early and a significant portion of the service life of the component may be spent propagating the crack to critical size. The two phases are of roughly equal importance, in terms of order of magnitude, in low cycle fatigue.

*The general model*⁶⁹

$$\epsilon_a = \frac{\sigma'_f}{E} (2N)^b + \epsilon'_f (2N)^c$$

where:

ϵ_a = strain amplitude.

E = modulus of elasticity.

σ'_f = fatigue strength coefficient.

b = fatigue strength exponent.

ϵ'_f = fatigue ductility coefficient.

c = fatigue ductility exponent.

This model considers elastic strain and plastic strain life separately. The total strain life is the summation of the two.

⁶⁹ Manson, S.S., and M.H. Hirschberg, Fatigue: An Interdisciplinary Approach, Syracuse University Press, Syracuse, New York, 1964.

Langer proposed a simple model for the low-cycle fatigue life⁷⁰. The model is:

$$S = \frac{E}{4 \sqrt{N}} \ln \frac{100}{100-RA} + S_e$$

where:

$$S = 1/2 E \epsilon_f$$

E=modulus of elasticity

N=number of cycles to failure

RA=percent reduction of area in tensile test

S_e=endurance limit

Langer suggests assuming a RA value of 50% which is the minimum specified value. Using this RA value, and rearranging the equation, we can determine the following relationship for the number of cycles to failure:

$$N = \left(\frac{0.1732868 E + S_e}{S} \right)^2$$

Additional Sources

Fatigue Reliability: Introduction, ASCE, Vol.108, NO. ST1, January, 1982

J.A.Collins, Fatigue of Materials in Mechanical Design

Paul H. Wirsching, Statistical Summaries of Fatigue Data for Design Purposes, STAR, N83-29731.

⁷⁰ Langer, B.F., *Design of Pressure Vessels for Low-Cycle Fatigue*, Journal of Basic Engineering, ASME, Vol. 84, No.3, September 1962.

2.3.3 Crack Growth

Failure Mechanism

The crack growth rate can be estimated by Paris' Law, given by:

$$\frac{da}{dN} = C(\Delta K)^n$$

$$K = F(a)S\sqrt{\pi a}$$

where:

a = crack depth for a surface flaw or half-width for a penetration flaw.

$F(a)$ = finite geometry correction factor which may depend on a .

K = stress intensity factor.

S = applied stress.

n, c = experimentally determined constants which depend on the mean cycling stress.

Sources:

Fatigue Reliability: Introduction, ASCE, Vol.108, NO. ST1, January, 1982

J.A.Collins, Fatigue of Materials in Mechanical Design

2.4 Thermal Degradation

Definition

Thermal degradation is the deterioration of the functionality or physical properties of a material due to the effects of temperature. The degradation is due solely to the effects of temperature and does not involve with other materials.

Failure Mechanism

The interest in thermal degradation lies in materials that are non-metallic, since thermal influences on metals can result in changes in the grain structure. The material properties of metals possessing different grain structures is well documented. As a result, this examination of thermal degradation will focus primarily on polymers and insulating materials.

In the case of polymers, the two main types of degradation are:

1. Depolymerisation, which is the breaking of the main polymer chain backbone so that at any intermediate stage the products are similar to the parent material in that the monomer units remain distinguishable. The products of the degradation may be monomer, or they may be volatile chain fragments.⁷¹
2. Substituent reactions result in the attachment of substituents to the backbone of the polymer molecules involved so that the chemical nature of the repeat unit is changed even though the chain structure may remain intact.⁷²

In the case of polystyrene, if the chain scission occurs in a polymer molecule without volatilisation, then:⁷³

$$P_t = \frac{P_o}{(s+1)}$$

where: P_o = initial chain length of the polymer

P_t = chain length of the polymer after time t

s = average number of scissions per molecule

⁷¹ Grassie, N., and G. Scott, Polymer Degradation and Stabilisation, Cambridge University Press, Cambridge U.K., 1985, pp. 23-24.

⁷² Grassie, N., and G. Scott, Polymer Degradation and Stabilisation, p. 24.

⁷³ Grassie, N., and G. Scott, Polymer Degradation and Stabilization, pp.28-29.

From this equation the fraction of broken bonds, α , can be found. If the chain scission is random then $\alpha=kt$ where k =rate constant for chain scission. If the chain contains weak and strong links, then $\alpha=\beta+kt$, where β is the fraction of weak links in the molecule.

In practice the approach is to determine the lifetime of polymers using approaches based on Arrhenius equations. Burnay⁷⁴ reports that a relationship that accounts for both thermal and radiation degradation on polymers is:

$$a(T,D)=\exp\left[-\frac{E}{R}\left(\frac{1}{T_{ref}}-\frac{1}{T}\right)\right]\left[1+k\exp\left[-\frac{Ex}{R}\left(\frac{1}{T_{ref}}-\frac{1}{T}\right)\right]D^x\right]$$

where: E =activation energy

R =gas constant

T =absolute temperature polymer environment

T_{ref} =reference temperature

k =radiation degradation reaction constant

D =dose rate

Note that E , k , and x are empirical constants. The parameter E can be determined from the polymer where radiation is not a problem. Thus, k and x are empirically determined from irradiation data. Burnay further reports that the majority of polymers studied have been found to have $x=1$. For cases where $x=1$, the above relation is simply:

$$a(T,D)=\exp\left[-\frac{E}{R}\left(\frac{1}{T_{ref}}-\frac{1}{T}\right)\right]+kD$$

This relationship is simply the sum of the thermal degradation described by the Arrhenius relationship plus a dose rate component, D , multiplied by a radiation degradation reaction constant, k . In cases where radiation is not significant, the relationship for simple thermal degradation results.

⁷⁴ Burnay, S.G., *A Practical Model for Prediction of the Lifetime of Elastomeric Seals in Nuclear Environments*, Proceedings from 200th National Meeting of the American Chemical Society, Washington, August 26-31, 1990 (Published in Radiation Effects on Polymers, ACS Symposium Series #475)

Sources

A Review of Equipment Aging Theory and Technology, EPRI Report NP-1558, Project 890-1, Final Report, Electric Power Research Institute, Palo Alto, CA, September 1980.

STS Life Management System Development, SwRI Report No. 06-2148-1A, Project No. 06-2148, Southwest Research Institute, San Antonio, TX, November 30, 1988.

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Grassie, N., and G. Scott, Polymer Degradation and Stabilisation, Cambridge University Press, Cambridge, U.k., 1985.

Schnabel, W., Polymer Degradation, Macmillan Publishing Co., Inc, New York, 1981.

Troitskii, B.B., and L.S. Troitskaya, *Mathematical Models of the Initial Stage of the Thermal Degradation of Poly (vinyl Chloride)*, Journal of Polymer Science:Part A:Polymer Chemistry, vol. 28, pp. 2695-2709, 1990.

3.0 Extension of First-Order Reliability Methods to Mechanical System Design

The application of first-order reliability methods (FORM) to mechanical limit states has not been extensively carried out. Two barriers to the widespread use of first-order reliability methods in the design of mechanical systems have been the modeling and validation of first-order techniques to such systems, and the limited availability of software capable of performing such an analysis to mechanical limit-states.

3.1 First-Order Reliability Method Modeling Strategies for Mechanical Limit States

To determine the best modeling approach for first-order reliability methods applied to mechanical limit states, several of the limit states identified in the compendium of limit states were selected and tested. The majority of the limit states that were deemed relevant to mechanical systems were found to be power law relationships. Two limit states were found to follow Arrhenius relationships, namely, uniform corrosion and thermal degradation. The limit states that were modeled included:

- wear
- fretting wear
- pitting
- erosion-corrosion
- uniform corrosion
- galvanic corrosion
- low-cycle fatigue

With the exception of low-cycle fatigue, all the noted limit-states were considered to have a response function of the form:

$$g(\mathbf{R}, \mathbf{S}) = \mathbf{R} - \mathbf{S}$$

In this case R represents either the resistance of the system to the specific degradation effect being modeled, or some maximum allowable degree of system deterioration. Similarly, S represents the actual degree of material stress induced on the system or the actual degree of degradation of the system due to the degradation process under consideration.

Where R represents the maximum allowable degree of system deterioration, it was assumed to be a random variable with a specified distribution and parameters. In all limit states modeled, the actual deterioration experienced by the system, S , was a function of several random variables. The precise nature of the functional representation of the actual deterioration, S , followed that of the corresponding section of the compendium of mechanical limit states outlined in section 2 of this report.

In the case of low-cycle fatigue, the response function was of the form:

$$g(N_{\text{MIN}}, N_{\text{ACTUAL}}) = -N_{\text{MIN}} + N_{\text{ACTUAL}}$$

Here N_{MIN} is the minimum number of cycles the system must operate while in service, and N_{ACTUAL} is the number of cycles to failure. N_{MIN} was assumed to be randomly distributed variable, and N_{ACTUAL} was a function of several random variables following the relationship given in section 2.3.2 of this report.

After extensive examination of the available scientific literature, little information on the distribution of specific limit state random variables was found. In the majority of cases where information on random variables distribution parameters could not be found, the variables were assumed to be either normal or log-normally distributed. The random variables defined in each limit state were assumed to be independent.

The first-order reliability method applied to each response function included both the mean value first order method (MVFO) and the advanced mean value first order method (AMVFO). If the response function (or Z function) is assumed to be smooth, then we can take a Taylor's series expansion of the response function at the mean value⁷⁵. The response function can then be written as:

$$Z(\mathbf{X}) = Z(\boldsymbol{\mu}) + \sum_{i=1}^n \left(\frac{\partial Z}{\partial X_i} \right) * (X_i - \mu_i) + H(\mathbf{X})$$

The first two right hand terms are the first order approximations of the response function, and $H(\mathbf{X})$ represents all higher-order terms. The mean and variance of the response function can be determined if we consider only the first-order terms.⁷⁶ If the distributions of the random variables and their parameters are defined, and if we consider only first order terms in the Taylor series approximation, then the cumulative density function (cdf) of the response function is also defined. The estimation of the response function cdf by linear function is referred to mean value first order (MVFO) approach.

If the response function is non-linear, then its approximation by a first order Taylor series will result in some error. The inclusion of higher order terms in the Taylor series will improve the accuracy of the analysis, although the approach is more difficult. The advanced mean value first order (AMVFO) method includes a simple function, $H(Z)$, to approximate the higher order terms of the Taylor series.

The application of first-order reliability methods to each response functions was conducted using the NESSUS/FPI software developed by Southwest Research Institute under funding the NASA Lewis Research Center. For each limit state modeled, the maximum number of

⁷⁵ Wu, Y.-T., Fast Probability Integration: Theoretical Manual, FPI Version 4.2, Southwest Research Institute, San Antonio, 1989, p. 36.

⁷⁶ Wu, Y.-T., Fast Probability Integration: Theoretical Manual, p.36.

response function levels (Z-levels) of 20 levels was used, and the probability of failure for each level was determined using both MVFO and AMVFO.

The NESSUS/FPI software allows for the response function to be incorporated in several alternative formats. The two approaches that were used are:

1. The user defined subroutine of the form:

$$g(\mathbf{X}) = f(\mathbf{X}_1, \mathbf{X}_2, \dots, \mathbf{X}_n, \mathbf{Z}_0)$$

In this case the response function can have any specified form.

2. A response function of the form:

$$\mathbf{X}_1 = f(\mathbf{X}_2, \mathbf{X}_3, \dots, \mathbf{X}_n, \mathbf{Z}_0)$$

Here one random variable must be separable from the others, and the response function written with the separable variable on the left-hand side. In some cases this is not feasible given the nature of the limit state.

The input data and response function files required to conduct the first order reliability method using FPI for all the limit states modeled are given in Appendix 1.

3.2 Validation of First-Order Reliability Method Modeling Strategies

The accuracy of the first-order reliability method modeling strategies were validated using Monte Carlo simulation. In all cases conventional Monte Carlo analysis was used, and no effort was undertaken to use importance sampling approaches. For each limit state a Monte Carlo simulation using 50,000 samples was employed.

A comparison was made between those results obtained using Monte Carlo simulation and first-order reliability methods. In all cases, the difference between FORM and Monte Carlo are exceptionally small. The cumulative density functions for each response function modeled were obtained and plotted for both the first-order reliability method and Monte Carlo simulation (see Figures 3.2.1 through 3.2.7).

A Kolmogorov-Smirnov test was applied to each limit-state modeled to determine if the cdf found using first-order reliability methods differed from that obtained using Monte Carlo. The Kolmogorov-Smirnov test considers the maximum difference between the two cumulative density functions to determine if they are the same,⁷⁷ or:

$$D_n = \max|F_n(x) - S_n(x)|$$

Here $F(x)$ represents the theoretical or assumed value of the cdf, and $S(x)$ the observed or experimentally determined value of the cdf. For the seven limit states considered, the following are the results of the Kolmogorov-Smirnov tests:

Limit-State	Sample Size (n)	$D_{\text{CRITICAL}}, \alpha=0.05$	D_{MAX}	Significant Difference
Wear	20	0.29	1.22E-2	No
Fretting Wear	17	0.32	1.00E-2	No
Pitting	20	0.29	1.06E-2	No
Erosion-Corrosion	10	0.41	2.07E-2	No
Uniform Corrosion	15	0.34	1.08E-2	No
Galvanic Corrosion	15	0.34	1.62E-2	No
Low-Cycle Fatigue	16	0.33	1.09E-2	No

From these results, we conclude that there is no statistically significant difference between the cumulative density functions obtained using first order and Monte Carlo methods for each limit state.

3.3 Library of Mechanical Limit States

In order to facilitate the implementation of first-order reliability methods for mechanical systems by practitioners a comprehensive library of limits states was programmed into a user definable subroutine. The limit states incorporated into this subroutine were:

- wear
- fretting wear
- pitting
- erosion-corrosion
- galvanic corrosion
- low-cycle fatigue

These limit states were created in a subroutine used within NESSUS/FPI. The entire subroutine is given in Appendix 2.

⁷⁷ Ang, A. H-S., and W.H. Tang, Probability Concepts in Engineering Planning and Design: Volume 1-Basic Principles, J. Wiley, New York, 1975, pp. 277-279.

CDF For Erosion-Corrosion:Monte Carlo, MVFO, AMVFO

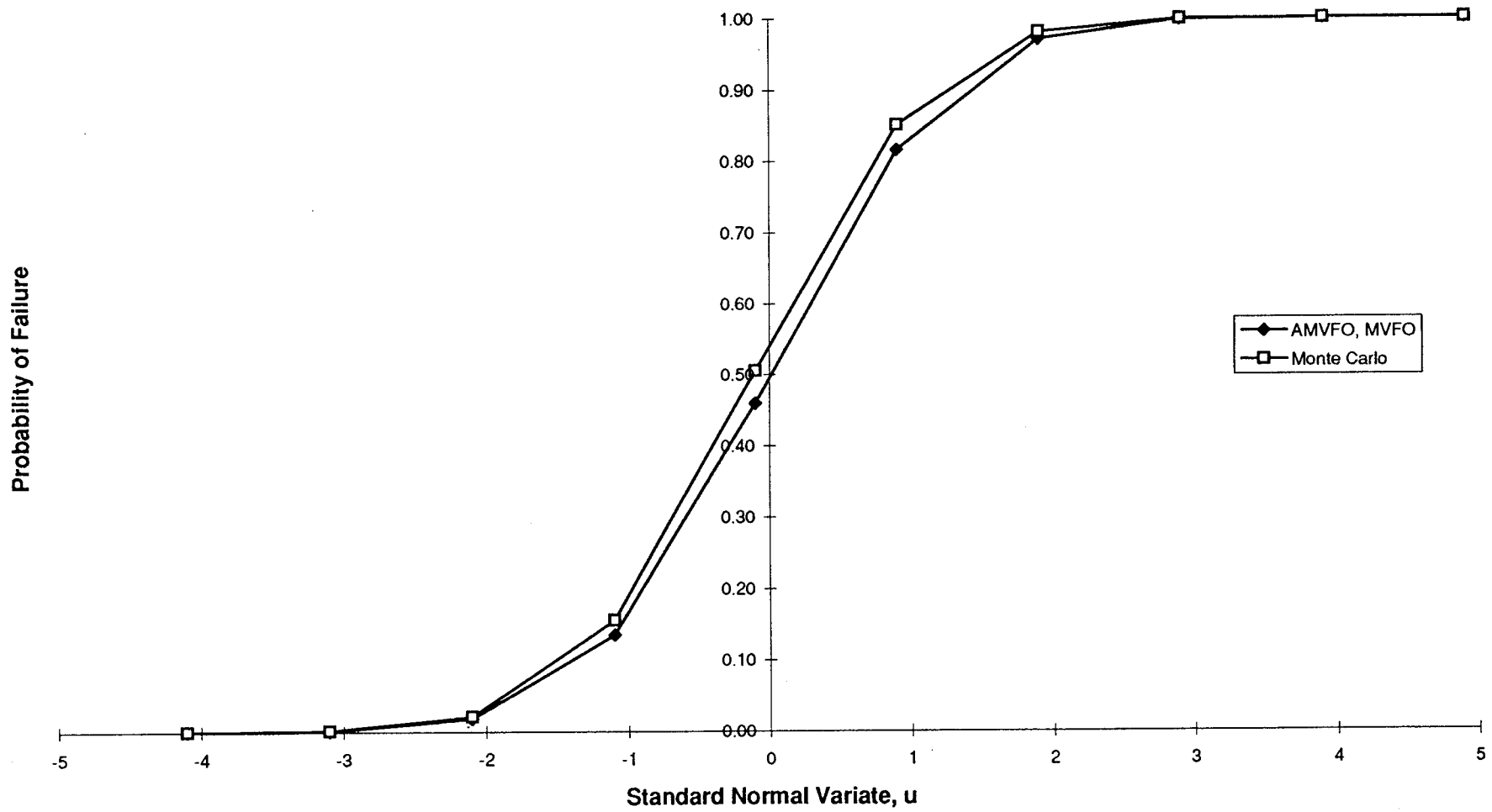


Figure 3.2.1

CDF of Wear

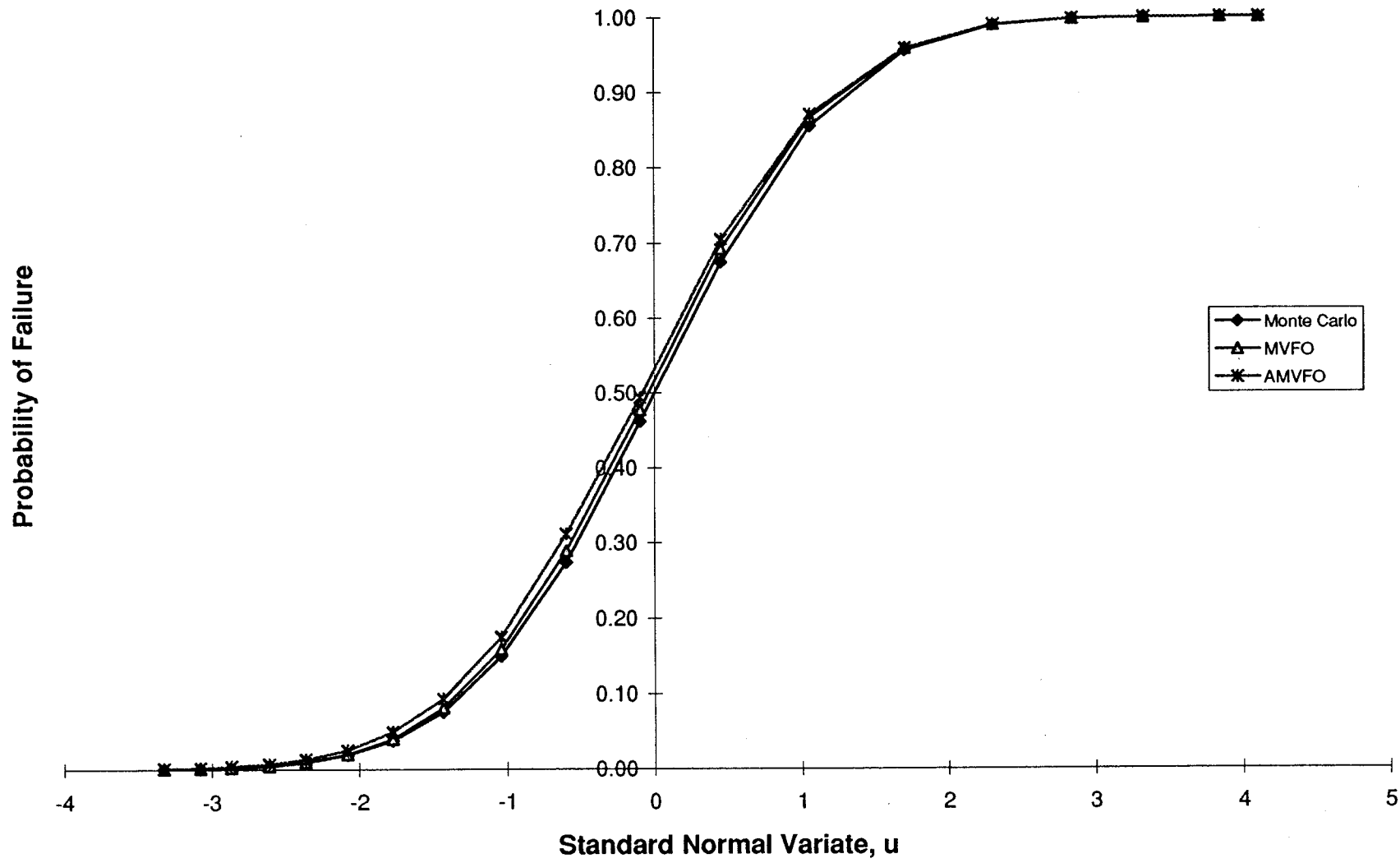


Figure 3.2.2

CDF for Pitting Corrosion

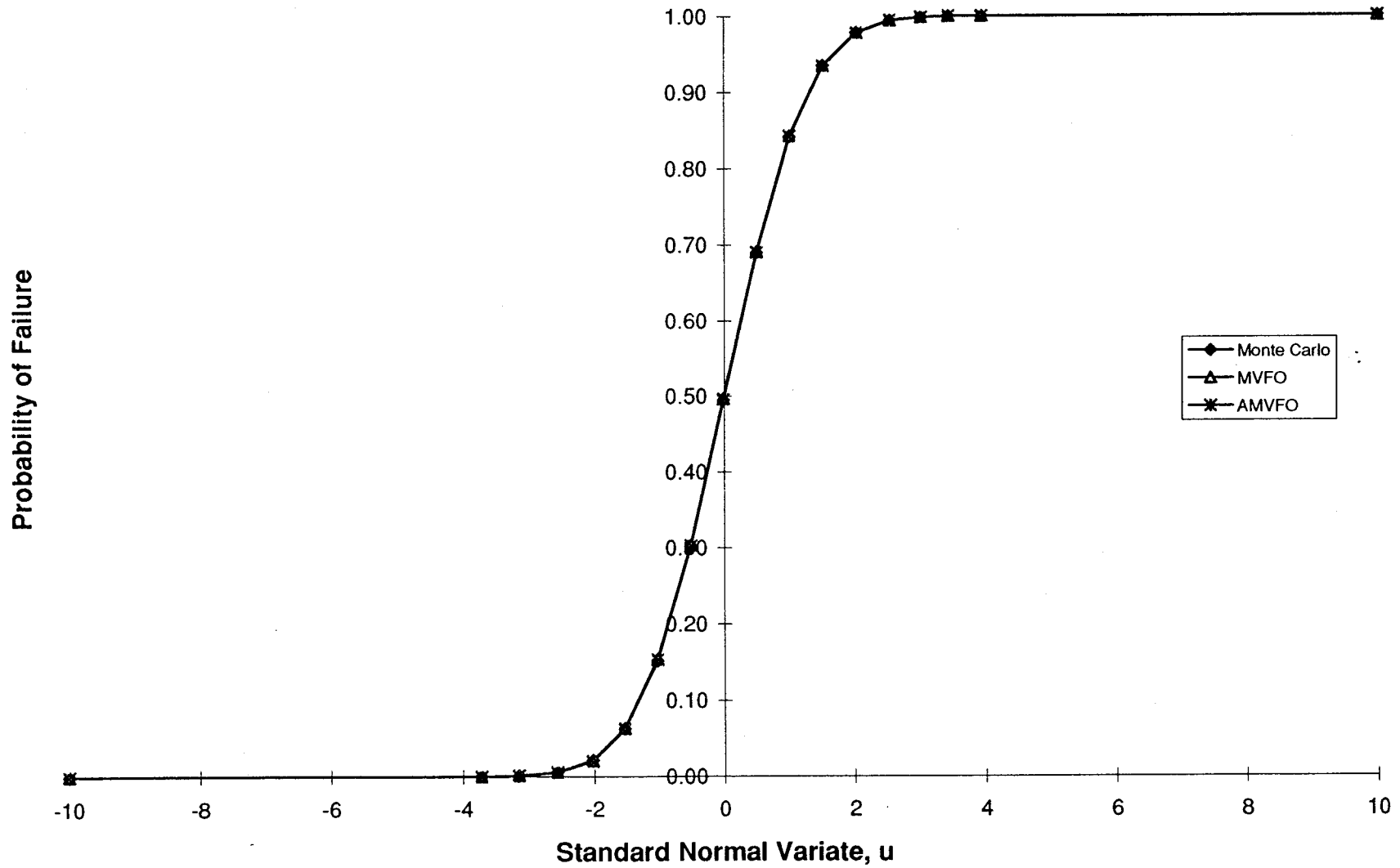


Figure 3.2.3

Probability of Failure

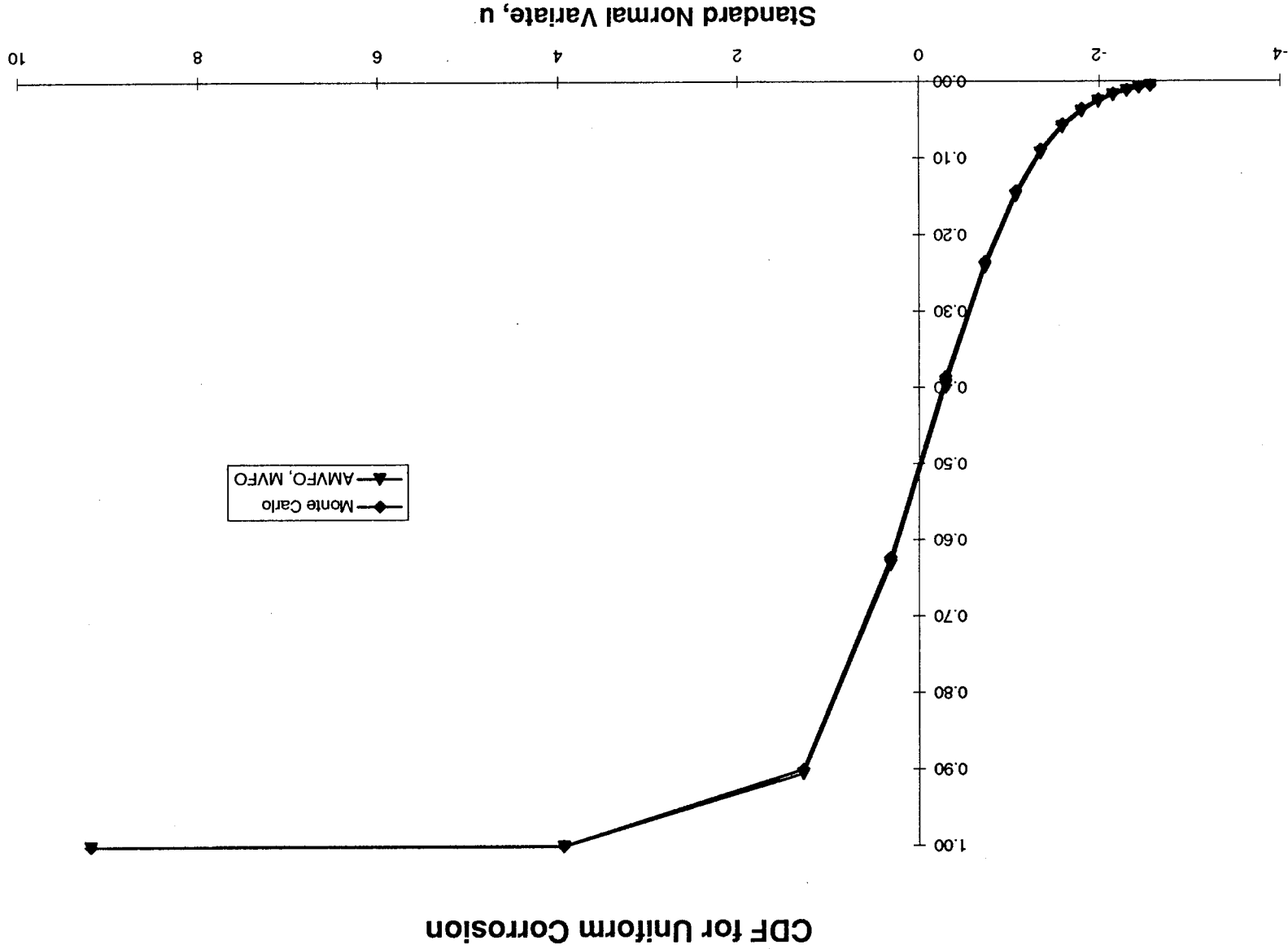


Figure 3.2.4

CDF of Fretting Corrosion

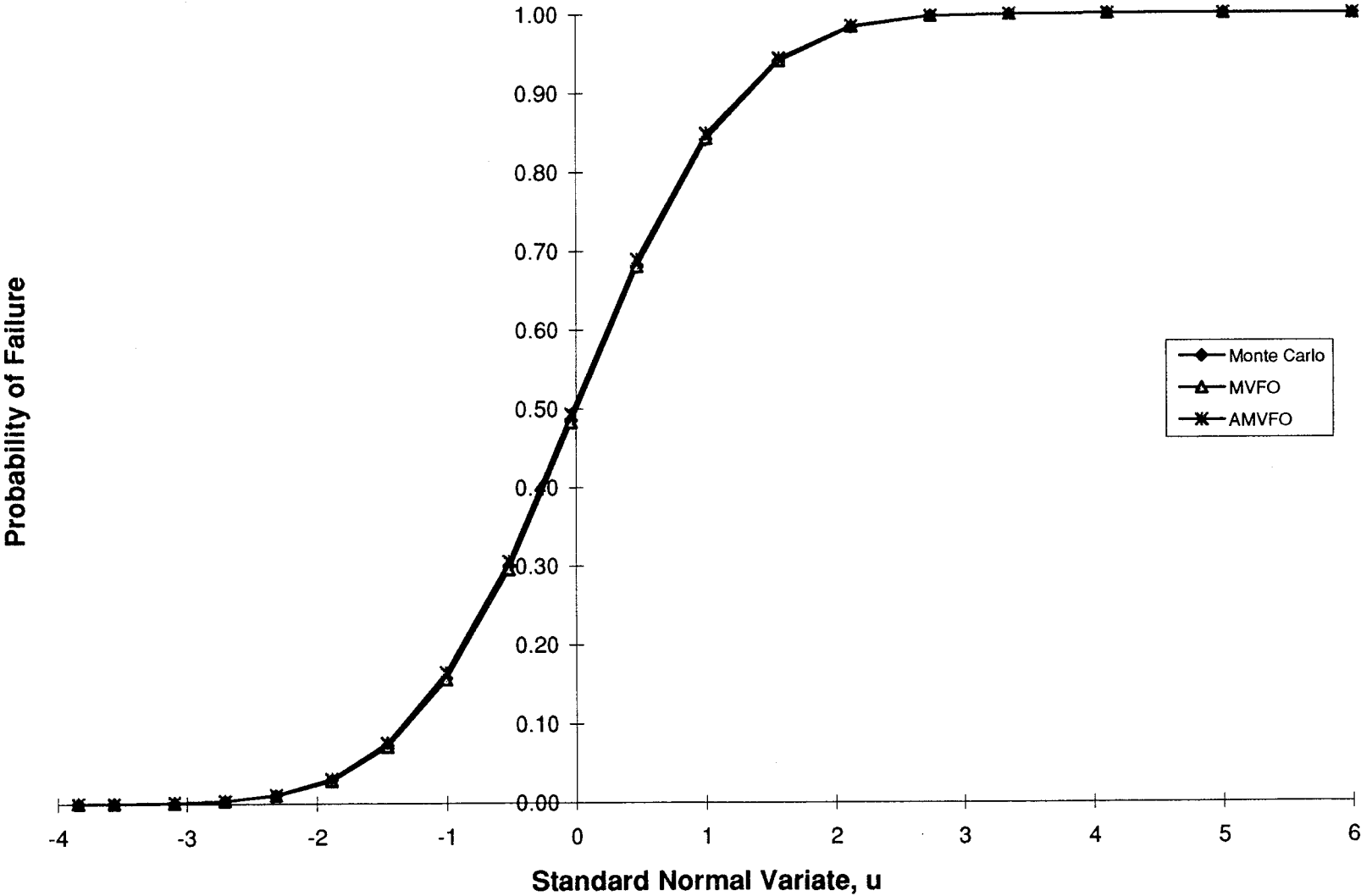


Figure 3.2.5

CDF for Galvanic Corrosion

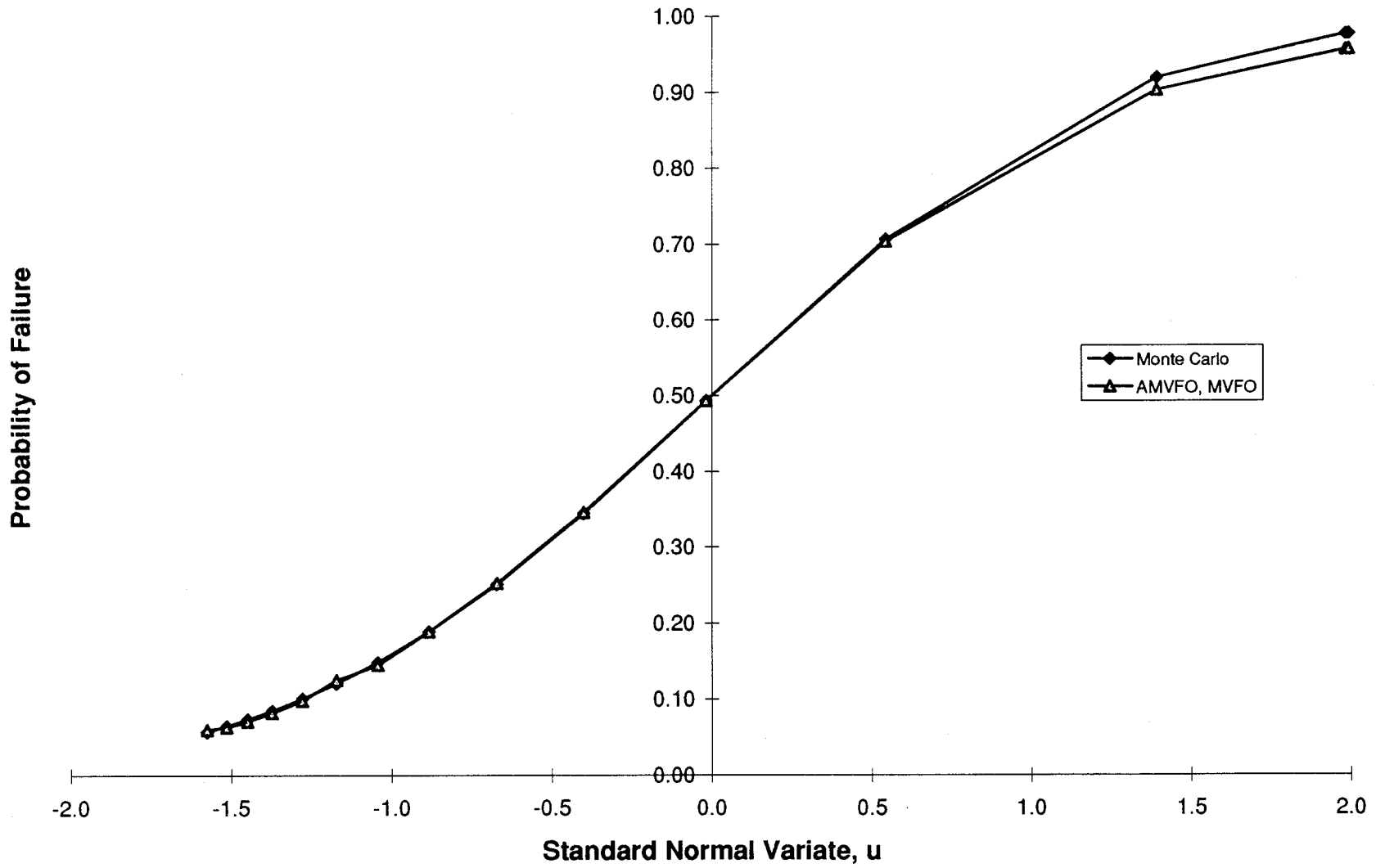


Figure 3.2.6

CDF for Low-Cycle Fatigue

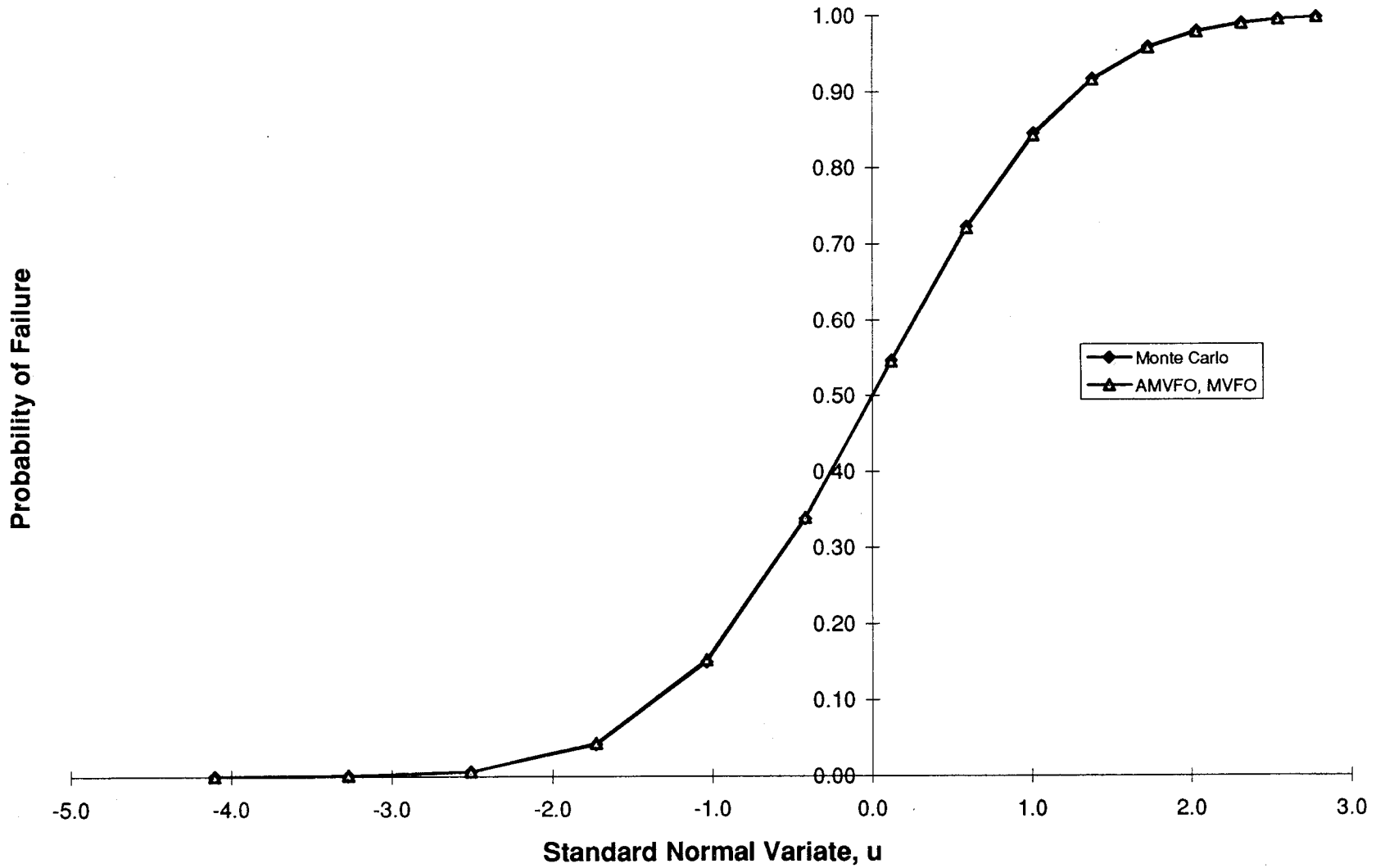
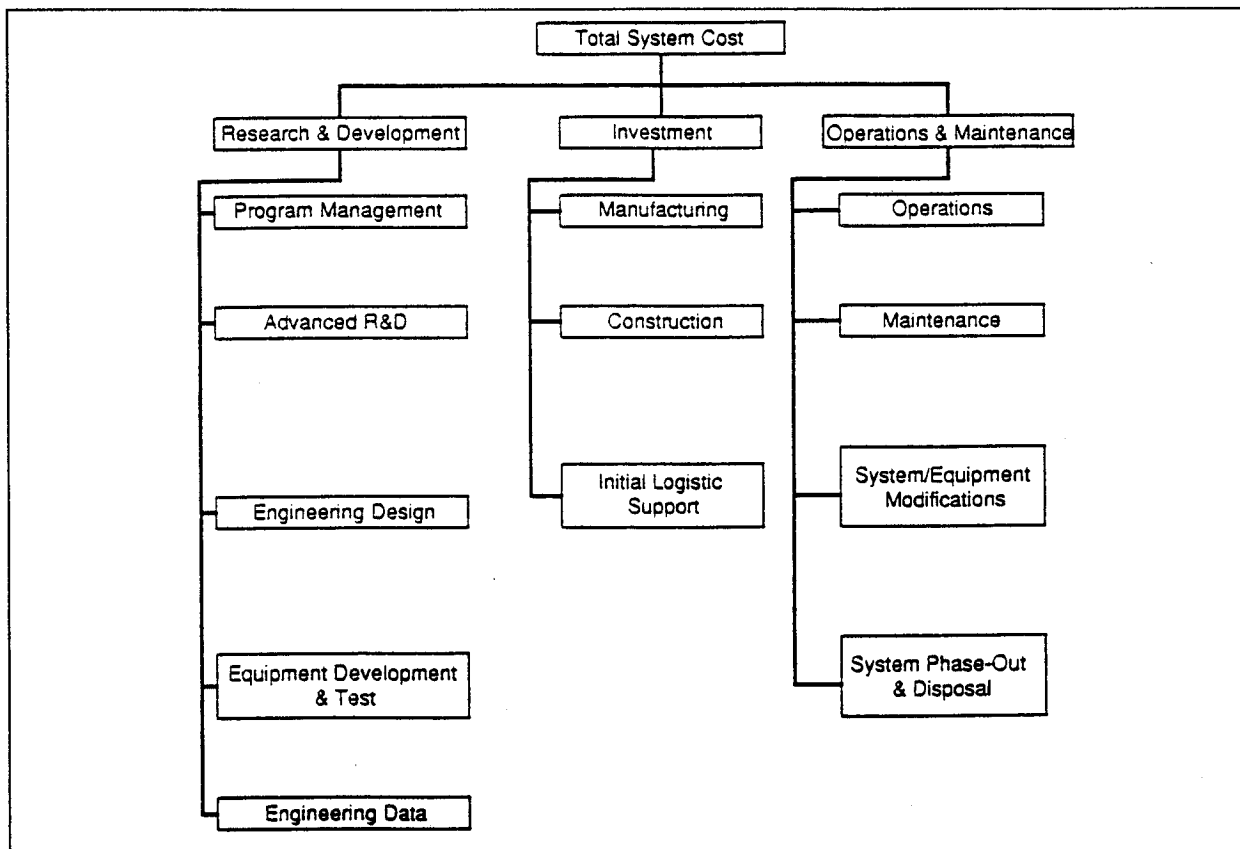


Figure 3.2.7

4.0 Linking Reliability Prediction Methods and Product Life-Cycle Cost

4.1 Motivation for Establishing Reliability-Cost Linkages

The life-cycle cost is the summation of the development, acquisition, use, and disposal costs associated with a product. The life-cycle cost of a product can be further broken down into more discrete costs elements. It is possible to distinguish the life-cycle cost elements on the basis of development cost, investment, and operation and maintenance. A typical breakdown of the life-cycle cost elements is shown below.



The life-cycle cost of a product is directly influenced by its design and configuration. This relationship is clear if we consider the simple case of the impact of design on investment costs. The incorporation of new processes or materials into a product could require the acquisition of new manufacturing equipment.

Linking life-cycle costs to random variables would address several shortcomings with existing costing and design methods. First, current cost sensitivity analysis is conducted by varying one variable at a time. This does not consider the effect of correlation between variables and the effect of variable interactions that may occur. Individually estimating the impact of design

related variables on product cost through this one variable approach can lead to misleading or erroneous conclusions.

Second, present cost methods do not determine cost as a function of randomly distributed variables. No consideration is given to the distributional properties of the random variables. Presently the life-cycle costs are determined by assuming point estimates for each life-cycle cost element. The result is that information on the variability of life-cycle cost elements are not available to the manager and designer. Identification of the major cost elements that display high variability would permit managers and designers to focus attention on reducing cost variability and thereby avoid cost overruns.

Third, existing approaches do not yield a relationship between overall system life-cycle costs and individual cost variables. Determining such a relationship would provide the designer with information on how product costs could be reduced.

Fourth, since present methods do not provide a cost-variable relationship it is not possible to determine a minimum life-cycle cost solution for the product design.

Fifth, the determination of a cost-variable relationship and the use of first-order reliability methods would allow designers to undertake cost/reliability tradeoffs in the initial product design stages of development.

In order to conduct the cost/reliability tradeoff analysis, it would be useful to know the nature of the relationship between current reliability prediction methods and primitive random variables, as well as the relationship between life-cycle cost elements and primitive random variables. The relationships needed include:

- First-order reliability methods and primitive random variables
- Taguchi and Design of Experiment (DOE) methods and primitive random variables

4.2 Linking First-Order Reliability Methods and Primitive Variables

The linkage between the first-order reliability method and primitive variables has already been established. If the response function in standard normal space is considered, then:

$$g(\mathbf{Y}) = f(y_1, y_2, \dots, y_n)$$

In this case, the reliability index, β , is the minimum distance to the linearized limit state surface, called the most probable point, \mathbf{y}^* , or:

$$\beta = \min(\mathbf{y}^{*T} \cdot \mathbf{y}^*)^{1/2}$$

If we consider only a first-order Taylor series approximation to the limit state function at the most probable point, then the direction cosines of the outward normal vector at the most probable point are:⁷⁸

$$\alpha_i = \frac{\frac{\partial g}{\partial y_i}}{\left(\sum_i \left(\frac{\partial g}{\partial y_i} \right)^2 \right)^{1/2}}$$

The direction cosines, α_i , represent the sensitivity of the reliability index, β , to changes in y_i , at the most probable point. This sensitivity measure assumes that the distributional parameters of the variable concerned remains the same. An alternative form for this sensitivity measure can be determined if a first-order approximation to the response function is assumed. In this case the variance of the response function can be shown to be:

$$\sigma_g^2 = \sum_i \left(\frac{\partial g}{\partial y_i} \right)^2 \sigma_{y_i}^2$$

Then the sensitivity coefficients can be determined to be⁷⁹:

$$\beta_{y_i} = \left(\frac{\partial g}{\partial y_i} \right) \frac{\sigma_{y_i}}{\sigma_g}$$

In addition to knowing the sensitivity of the reliability index to changes in individual variable values, its sensitivity to changes in variable distributional parameters is useful. Previously the reliability index was defined as being:

$$\beta(p_i) = |\mathbf{y}^*| = (\mathbf{y}^{*T} \cdot \mathbf{y}^*)^{1/2}$$

⁷⁸

Melchers, R.E., Structural Reliability: Analysis and Prediction, Ellis Horwood Ltd., West Sussex, England, 1987, pp. 108-112.

⁷⁹

Cruse, T.A., Class Instructional notes, 1993.

Now the sensitivity of the reliability index to changes in the variable distribution parameters can be derived as follows:⁸⁰

$$\frac{\partial}{\partial p_i} \beta(p_o) = \frac{\partial}{\partial p_i} \left[(\mathbf{y}^{*T} \cdot \mathbf{y}^*)^{1/2} \right] = \frac{\partial}{\partial p_i} \left[\left((\mathbf{y}_1^*)^2 + (\mathbf{y}_2^*)^2 + \dots + (\mathbf{y}_n^*)^2 \right)^{1/2} \right]$$

This can be rewritten as:

$$\frac{\partial}{\partial p_i} \beta(p_o) = \frac{\mathbf{y}^{*T} \cdot \frac{\partial}{\partial p_i} \mathbf{y}^*}{(\mathbf{y}^{*T} \cdot \mathbf{y}^*)^{1/2}} = \frac{1}{\beta} \mathbf{y}^{*T} \cdot \frac{\partial}{\partial p_i} \mathbf{y}^*$$

From these relationships the sensitivity of reliability index to changes in either the variable distribution parameters or the variable values can be determined.

4.3 Linking Taguchi/Design of Experiment Methods to Cost Primitive Variables

Design of experiments (DOE) and Taguchi methods are statistical procedures used to determine and quantify the relationship between a measured performance criteria of a system and the measurable settings of identified variables. The performance of interest is referred to as the response or response variable, and the variables whose values are being altered are referred to as experimental factors. An experiment is defined as a set of specified factor settings and a measured response.

Both design of experiments and Taguchi methods aim to do three things:

1. Designing a set of different experiments that will yield a set of measurable values of the response desired.
2. Determine and quantify the relationship between the response variable and the experimental factors. This relationship is expressed as a mathematical model.
3. Using the mathematical model for the response, determine the value of all factors that results in the optimal response value.

⁸⁰

Madsen, H.O., S. Krenk, and N.C. Lind, Methods of Structural Safety, Prentice-Hall, Englewood Cliffs, New Jersey, 1986, pp.120-121.

There are three different types of factors:

1. *Signal factors* are those factors which are controlled by the user of the product or system.
2. *Noise factors* are those factors which are either beyond the control of the designer or changes in them are prohibitably expensive to modify.
3. *Control factors* are those factors which can be specified by the designer of the system.

Although design of experiments and Taguchi methods both seek to determine the relationship between the response and factors, they do differ in some regards. The differences between design of experiments and Taguchi methods are:

1. Design of experiments seeks to determine the relationship between the response and factors without consideration of the effect of noise factors. Design of experiments seeks to eliminate noise factors from the relationship through the use of techniques such as blocking and randomization. Taguchi methods are interested in determining the impact of noise factors on the response, in order to reduce the influence of noise factors on the response. In seeking factor levels which minimize the response sensitivity to noise, Taguchi methods attempt to reduce response variance under conditions likely to be experienced in actual use and outside of controlled experimental conditions.
2. Taguchi methods consider interactions between factors as undesirable. By avoiding interaction effects, if the inclusion of additional factors in the mathematical model becomes necessary, far fewer additional experiments will be required.

Despite these differences, both design of experiments and Taguchi methods can be used to determine the relationship between the response and factors. Judicious application of either method will yield similar results.

Taguchi methods can be used to determine product cost in two different ways. First, the response of interest could be a specific life-cycle cost element, and the resulting relationship would link the cost element with factors or random variables. Second, the response of interest could be a measurable quality characteristic of the product, and the relationship would link the quality measure with factors or random variables. In this case, a penalty function is employed to assess a cost to the process for the production of units whose quality deviates from targeted or optimal values.

In order to determine the linkage between product cost and primitive variables, the sensitivity of the response to the three types of factors must be considered. If y denotes the response of interest (either cost or quality), $\mathbf{n}=(n_1, n_2, \dots, n_i)$ denotes a vector of i noise factors, $\mathbf{s}=(s_1, s_2, \dots, s_j)$ denotes a vector of j signal factors, and $\mathbf{c}=(c_1, c_2, \dots, c_k)$ denotes a vector of k control factors, then:

$$y = f(\mathbf{n}, \mathbf{s}, \mathbf{c})$$

The deviation of the response, y , caused by the deviations in the three different factor types is:

$$dy = \left[\frac{\partial f}{\partial n_1} \right] dn_1 + \left[\frac{\partial f}{\partial n_2} \right] dn_2 + \dots + \left[\frac{\partial f}{\partial n_i} \right] dn_i + \left[\frac{\partial f}{\partial s_1} \right] ds_1 + \dots + \left[\frac{\partial f}{\partial s_j} \right] ds_j + \left[\frac{\partial f}{\partial c_1} \right] dc_1 + \dots + \left[\frac{\partial f}{\partial c_k} \right] dc_k$$

If the control and signal factors are held constant, then the deviation in the response caused by deviations in the noise factors is:

$$dy = \left[\frac{\partial f}{\partial n_1} \right] dn_1 + \dots + \left[\frac{\partial f}{\partial n_i} \right] dn_i$$

or, alternatively:

$$\Delta y = \left[\frac{\partial f}{\partial n_1} \right] \Delta n_1 + \dots + \left[\frac{\partial f}{\partial n_i} \right] \Delta n_i$$

Now if the relationship between the response and the factors is linear, then:

$$Y = \sum_1^i a_i n_i$$

and the expected response and variance are given by:

$$E(Y) = \sum_1^i a_i E(n_i)$$

$$\text{Var}(Y) = \sum_1^i a_i^2 \sigma_{n_i}^2 + \sum_{i \neq m}^i \sum_{m=1}^i a_i a_m \rho_{im} \sigma_{n_i} \sigma_{n_m}$$

Recalling that if n_i and n_m are independent, then $\rho_{im}=0$; if the deviations of the noise factors are independent, then the variance of the response is:

$$\sigma_y^2 = \left[\frac{\partial f}{\partial n_1} \right]^2 \sigma_{n_1}^2 + \dots + \left[\frac{\partial f}{\partial n_i} \right]^2 \sigma_{n_i}^2$$

In this case, the $[\partial f / \partial n_i]^2$ terms represent the sensitivity coefficients of the variance of the response, y , to changes in the individual noise factors. Similarly, the deviation in the response due to deviations in the signal and control factors are respectively:

$$dy = \sum_1^j \left[\frac{\partial f}{\partial s_j} \right] ds_j$$

$$dy = \sum_1^k \left[\frac{\partial f}{\partial c_k} \right] dc_k$$

And the variance of the response with respect to the signal and control factors are respectively:

$$\sigma_y^2 = \sum_1^j \left[\frac{\partial f}{\partial s_j} \right]^2 \sigma_{s_j}^2$$

$$\sigma_y^2 = \sum_1^k \left[\frac{\partial f}{\partial c_k} \right]^2 \sigma_{c_k}^2$$

If the response being considered was a life-cycle cost element, then the sensitivity of the cost element to changes in the random variables can be found. These sensitivities can then be used to determine which random variables have the greatest significant impact on the cost element under consideration, and an assessment of how changes in the variables could reduce the cost can be done. Conducting this type of analysis for each life-cycle cost element could then be incorporated into an overall cost analysis of the product. Since the life-cycle cost of a product is the sum of the individual life-cycle cost elements, then if the variance of each cost element is known, the variance of the total life-cycle cost can be found. The variance of the total life-cycle cost would be:

$$\text{Var(LCC)} = \sum_1^p \sigma_{\text{COST}_p}^2 + \sum_{p \neq q}^p \sum_1^p \rho_{pq} \sigma_{\text{COST}_p} \sigma_{\text{COST}_q}$$

Again, if the individual life-cycle cost elements are independent, then the second term of the equation drops out and the variance of the total life-cycle cost is the sum of the individual cost element variances.

If the response being considered is a measurable quality characteristic, then the response is used in a penalty function to determine the cost of product quality deviating from the targeted value. A function that targets a nominal quality measure is given by a quadratic loss function, or:

$$\text{Loss}(y) = c(y - t)^2$$

Here c is the cost associated with missing the quality target, t is the nominal quality target, and y is quality measure. Using the preceding relationships the variance in the response, the quality measure, can be determined. If the variance of c and t are also known, then the expected value and variance of the loss function can be determined, if the assumption that c , t , and y are independent is made. This expected value and variance of the loss function can be determined as follows:

$$\text{Loss}(y) = y^2c - 2ytc + t^2c$$

Using the relationships for the expected value and variance for products of independent variables, then:

$$E(\text{Loss}(y)) = [E(y)]^2 E(c) - 2E(y)E(t)E(c) + [E(t)]^2 E(c)$$

$$\sigma_{\text{Loss}}^2 = [E(y^2)]^2 E(c) - \mu_y^2 \mu_c - 2[E(y^2)E(t^2)E(c^2) - \mu_y \mu_t \mu_c] + [E(t^2)]^2 E(c^2) - \mu_t^2 \mu_c$$

If a different type of penalty function is desired, then a similar approach can be used to determine the expectation and variance of the penalty function.

4.4 Integrated Reliability/Life-Cycle Cost Design Method

The ability to determine the sensitivity of both the reliability and the cost of a proposed design makes it possible for the designer to undertake a tradeoff analysis between product cost and reliability. The major steps in the proposed design method are:

1. *Establish an Initial Design.*

An initial design that encompasses all the major attributes anticipated of the product must be developed. This design will serve as the basis on which improvements will be made to enhance its reliability and cost.

2. *Conduct a Reliability Analysis of the Proposed Design.*

On the basis of the proposed design a complete probabilistic reliability analysis should be performed. The steps to complete this analysis are:

- Identifying the dominant failure modes likely to affect the proposed design. Efforts should be directed to determining those failure modes that will be the likely cause of the majority of product failures, and not defining a comprehensive list of possible failure modes.
- Select appropriate limit-state models for each identified failure mode.
- Determine the sensitivity of the proposed design's reliability to its primitive variables through the application of First-Order Reliability Methods (FORM).

3. *Conduct a Life-Cycle Cost Analysis of the Proposed Design.*

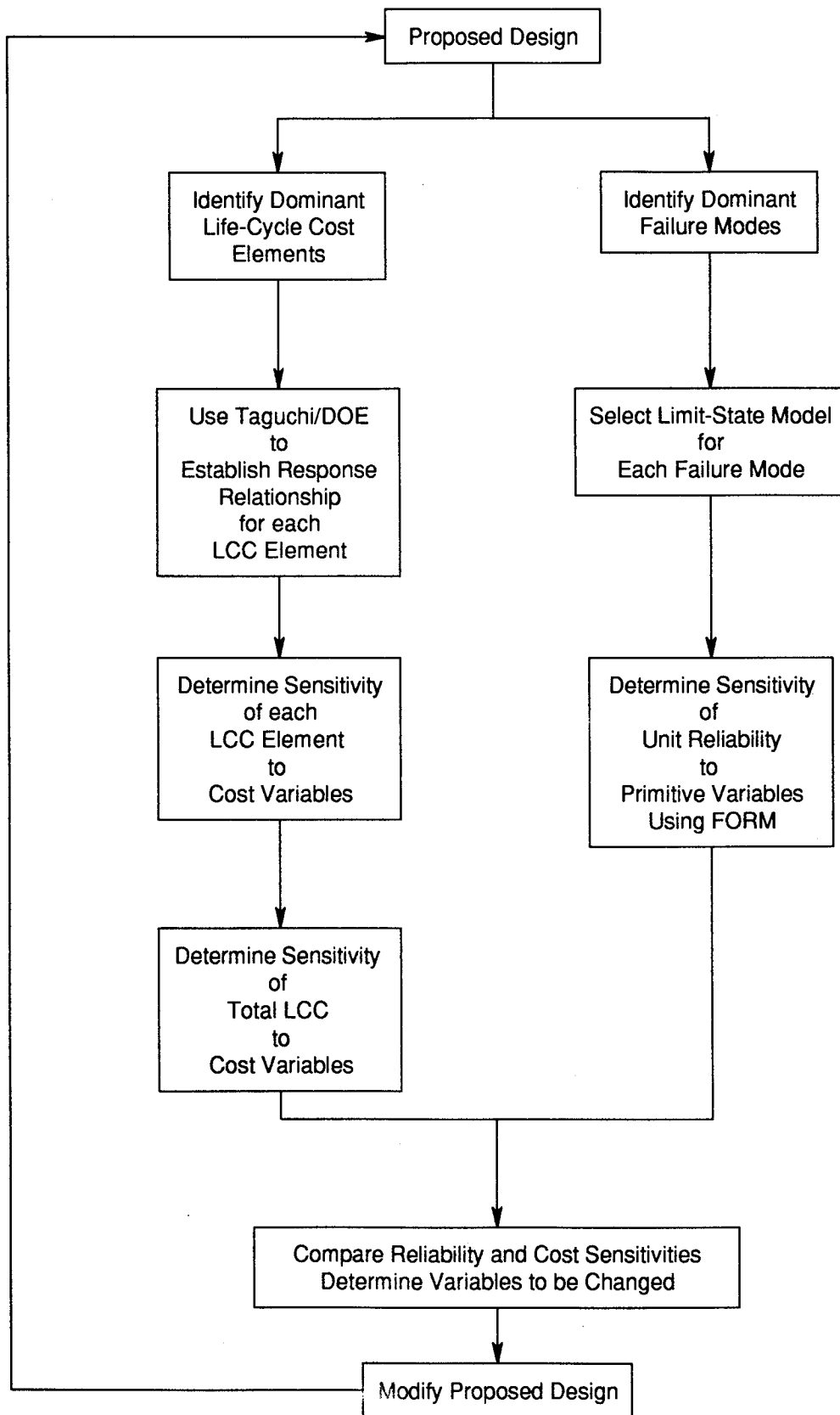
Using the proposed design, an analysis of the anticipated product life-cycle cost should be undertaken using Taguchi/Design of Experiments methods. The steps to complete this analysis are:

- Identify the dominant life-cycle cost elements for the proposed design. The focus should be on identifying those cost elements that constitute the majority of the product life-cycle costs.
- Use Taguchi/Design of Experiments methods to determine the relationship between cost and random variables for each identified life-cycle cost element.
- Determine the sensitivity of each life-cycle cost element to the cost-related random variables.
- Determine the sensitivity of the total life-cycle cost to the cost-related random variables.

4. *Compare Reliability and Cost Sensitivities.*

Having determined the design's reliability and cost sensitivities, an examination of these results should be undertaken to identify those variables, which when changed from initial values, will result in improved reliability and reduced cost. Once the variables values that will be changed have been determined, the design should be modified to reflect the change, and the process repeated.

The overall process for this integrated design method is presented in the accompanying diagram.



Integrated Reliability-Life-Cycle Cost Design Method

5.0 Summary of Research Findings

The findings of the research effort are:

1. The identification of characteristics of space environment that affect space-based mechanical systems.
2. The creation of a compendium of mechanical limit states. The compendium defined the various limit states that affect mechanical system. The physical and environmental variables that affect each mechanical limit state are defined.
3. Accurate first-order reliability method modeling strategies are identified for each type of mechanical limit state identified in the compendium of limit states.
4. The accuracy of the proposed first-order reliability method modeling strategies has been established by comparison of the probabilistic reliability results with those obtained using Monte Carlo simulation.
5. A stand-alone subroutine encompassing a library of mechanical limit state response functions has been developed to facilitate the application of first-order reliability methods to mechanical systems by engineers and designers.
6. Derivation of the sensitivity of product life-cycle cost to design random variables through the application of Taguchi and design of experiment methods.
7. A proposed integrated reliability-life-cycle cost design method that enables engineers and designers to conduct reliability/cost tradeoff analyses for proposed product designs.

In conclusion, the research has demonstrated the validity and accuracy of applying probabilistic reliability methods to mechanical limit states, as well as providing an integrated design approach for the consideration of both reliability and product cost in the product development process.

Appendices

Appendix 1

NESSUS/FPI
Input and Respon.f
Files

94/11/11
14:57:37

adwear1.dat

1

*FPI

FPI 3.0 USER DEFINED RESPONSE FUNCTION; ADHESIVE WEAR

*RVNUM 7
*GFUNCTION 10
*DATASETNM 0
*METHOD 1
*PRINTOPT 1
*ANALTYPE 1
*END
*ZLEVELS 20
-0.12651E-01 -0.11337E-01 -0.10024E-01 -0.87099E-02 -0.73962E-02
-0.60825E-02 -0.47688E-02 -0.34551E-02 -0.21414E-02 -0.82765E-03
0.48606E-03 0.17998E-02 0.31135E-02 0.44272E-02 0.57409E-02
0.70546E-02 0.83683E-02 0.96820E-02 0.10996E-01 0.12309E-01

C*MONTE

C50000 8812 0.0

*EXACTPRM

2, 1, 0

739.2405

*DEFNANVR

W

5.0E-3 1.0E-4 2.0

H

402.0 67.0 2.0

RK

1.7E-5 1.7E-6 2.0

RL

10.0 5.0 2.0

BL

2.5 0.200 2.0

D

1.0 0.100 2.0

R

35.0 5.0 2.0

*END

94/01/27
16:58:25

respon.f.~6~

1

```
FUNCTION RESPON(XSTAR)
  IMPLICIT DOUBLE PRECISION (A-H,O-Z)
  DOUBLE PRECISION H,K,L,BL,D,R,W,PI,HP,WRACT,G
  DIMENSION XSTAR(100)
```

```
C
C---INCLUDE ANY NEEDED COMMON BLOCKS
C
C   USER DEFINE RESPONSE FUNCTION
C       **GFORM = 6 ON INPUT DATA TO USE THIS OPTION
C
C   USER CAN DEFINE ANY RESPONSE FUNCTION DESIRED
C   CLOSED FORM OR NUMERICAL - WHATEVER
C
C   INPUT VARIABLE - XSTAR
C
C   THE NEEDED INPUT VARIABLES MUST BE INPUT AS RANDOM VARIABLES
C   TO FPI.  THE ORDER OF THE VARIABLES IN XSTAR WILL CORRESPOND
C   TO THE ORDER OF INPUT TO FPI.
C
C   IF THE INPUT VARIABLE IS DETERMINISTIC, THEN INPUT THE STANDARD
C   DEVIATION AS 0.0.
C
C   Remember that variable names K thru N are INTEGER ONLY!!!
C
C   ADHESIVE WEAR, VARIABLES DETERMINISTIC, STD DEV=0.
C
H = XSTAR(1)
K = XSTAR(2)
L = XSTAR(3)
BL= XSTAR(4)
D = XSTAR(5)
R = XSTAR(6)
W= XSTAR(7)
PI = 3.141592654
HP = 1422.0*H
WRACT=(K/HP)*(L/BL)*R*1051200.0
G=W-WRACT
RESPON=G
C
RETURN
END
```

94/11/27
16:16:28

fretting1.dat

1

```
*FPI
FPI 3.0 USER DEFINED RESPONSE FUNCTION; Fretting
*RVNUM      8
*GFUNCTION  6
*DASETNM    0
*METHOD    1
*PRINTOPT   1
*ANALTYPE   1
*END
*ZLEVELS    20
-0.69179e1  -0.61559e1  -0.53938e1  -0.46318e1  -0.38698e1
-0.31078e1  -0.23458e1  -0.15837e1  -0.82172    -0.59701e-1
 0.70232    0.14643e1   0.22264e1   0.29884e1   0.37504e1
 0.45124e1  0.52744e1   0.60365e1   0.67985e1   0.75605e1
C*MONTE
C50000      8812      0.0
*DEFNVR
P
5000      500      2.0
N
67800     700      2.0
F
9.0       1.5      2.0
L
3.6E-3    7.0E-4   2.0
K0
5.05E-6   5.0E-7   2.0
K1
1.52E-8   1.5E-9   2.0
K2
4.16E-6   4.2E-7   2.0
WALL
7.2       0.7      2.0
*END
```

94/11/27
16:06:15

respon.f

1

```
FUNCTION RESPON(XSTAR)
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
DOUBLE PRECISION P,N,F,L,K0,K1,K2,WALL,WACT
DIMENSION XSTAR(100)
C
C---INCLUDE ANY NEEDED COMMON BLOCKS
C
C   USER DEFINE RESPONSE FUNCTION
C       **GFORM = 6 ON INPUT DATA TO USE THIS OPTION
C
C   USER CAN DEFINE ANY RESPONSE FUNCTION DESIRED
C   CLOSED FORM OR NUMERICAL - WHATEVER
C
C   INPUT VARIABLE - XSTAR
C
C   THE NEEDED INPUT VARIABLES MUST BE INPUT AS RANDOM VARIABLES
C   TO FPI.  THE ORDER OF THE VARIABLES IN XSTAR WILL CORRESPOND
C   TO THE ORDER OF INPUT TO FPI.
C
C   IF THE INPUT VARIABLE IS DETERMINISTIC, THEN INPUT THE STANDARD
C   DEVIATION AS 0.0.
C
C   Remember that variable names K thru N are INTEGER ONLY!!!
C
C *****FRETTING*****
C
C   Model Form:  Wact=((K0*P^0.5)-K1*P)N/F+K2*L*P*N
C   Limit state: G= Wallow-Wactual
C   Variables passed from FRETTING*.DAT:
C   P=normal load (psi)
C   N=total number of cycles
C   F=cycles per second
C   L=slip amplitude
C   K0=constant
C   K1=constant
C   K2=constant
C   WALL=Allowable wear
C   WACT=Actual wear
C
C   P = XSTAR(1)
C   N = XSTAR(2)
C   F = XSTAR(3)
C   L = XSTAR(4)
C   K0 = XSTAR(5)
C   K1 = XSTAR(6)
C   K2 = XSTAR(7)
C   WALL = XSTAR(8)
C   WACT=((K0*(P**0.5))-(K1*P))*(N/F)+(K2*L*P*N)
C   G=WALL-WACT
C   RESPON=G
C
C   RETURN
C   END
```

94/11/27
14:36:21

pitting1.dat

1

```
*FPI
  FPI 3.0 USER DEFINED RESPONSE FUNCTION; Pitting
*RVNUM          3
*GFUNCTION      6
*DATASETNM      0
*METHOD        1
*PRINTOPT       1
*ANALTYPE       1
*END
*ZLEVELS        20
-0.15426e2  -0.13725e2  -0.12023e2  -0.10321e2  -0.8619e1
-0.69172e1  -0.52153e1  -0.35135e1  -0.18116e1  -0.10979
 0.15921e1  0.32939e1  0.49957e1  0.66976e1  0.83994e1
 0.10101e2  0.11803e2  0.13505e2  0.15207e2  0.16909e2
C*MONTE
C50000      8812      0.0
*DEFNANVR
C
9.778      0.25      2.0
T
160        6          2.0
D
53.00      3.0       2.0
*END
```

94/11/27
14:29:08

respon.f

1

```
FUNCTION RESPON(XSTAR)
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
DOUBLE PRECISION C,T,D
DIMENSION XSTAR(100)

C
C---INCLUDE ANY NEEDED COMMON BLOCKS
C
C   USER DEFINE RESPONSE FUNCTION
C       **GFORM = 6 ON INPUT DATA TO USE THIS OPTION
C
C   USER CAN DEFINE ANY RESPONSE FUNCTION DESIRED
C   CLOSED FORM OR NUMERICAL - WHATEVER
C
C   INPUT VARIABLE - XSTAR
C
C   THE NEEDED INPUT VARIABLES MUST BE INPUT AS RANDOM VARIABLES
C   TO FPI.  THE ORDER OF THE VARIABLES IN XSTAR WILL CORRESPOND
C   TO THE ORDER OF INPUT TO FPI.
C
C   IF THE INPUT VARIABLE IS DETERMINISTIC, THEN INPUT THE STANDARD
C   DEVIATION AS 0.0.
C
C   Remember that variable names K thru N are INTEGER ONLY!!!
C
C *****PITTING*****
C
C   Model Form:  Dact=KT^0.33
C   Limit state: G=Dallow-Dact
C   Variables passed from Pitting*.dat:
C   K=constant
C   T=time in months
C   D=maximum allowable pit depth
C
C
C   C = XSTAR(1)
C   T = XSTAR(2)
C   D = XSTAR(3)
C   G=D-(C*(T**0.33333333))
C   RESPON=G
C
C   RETURN
C   END
```

94/06/22
12:07:51

uniform.dat

1

*FPI

FPI 3.0 USER DEFINED RESPONSE FUNCTION; Uniform Attack Corrosion

*RVNUM 6

*GFUNCTION 6

*DATASETNM 0

*METHOD 1

*PRINTOPT 1

*ANALTYPE 1

*END

*ZLEVELS 15

-0.11717 -0.10712 -0.97071E-01 -0.87022E-01 -0.76973E-01

-0.66923E-01 -0.56874E-01 -0.46825E-01 -0.36776E-01 -0.26727E-01

-0.16678E-01 -0.66285E-02 0.34207E-02 0.13470E-01 0.23519E-01

C*MONTE

C50000 8812 0.0

*DEFNVR

BK

8.62E-05 2E-06 2.0

C

0.001 0.0001 2.0

A

1.1 0.2 2.0

TC

423 0.5 2.0

TA

475 0.5 2.0

UC

0.0157 0.0004 2.0

*END

94/11/29
12:07:38

respon.f

1

```
FUNCTION RESPON(XSTAR)
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
DOUBLE PRECISION BK,C,A,TA,TC,UC,UACT
DIMENSION XSTAR(100)

C
C---INCLUDE ANY NEEDED COMMON BLOCKS
C
C   USER DEFINE RESPONSE FUNCTION
C       **GFORM = 6 ON INPUT DATA TO USE THIS OPTION
C
C   USER CAN DEFINE ANY RESPONSE FUNCTION DESIRED
C   CLOSED FORM OR NUMERICAL - WHATEVER
C
C   INPUT VARIABLE - XSTAR
C
C   THE NEEDED INPUT VARIABLES MUST BE INPUT AS RANDOM VARIABLES
C   TO FPI.  THE ORDER OF THE VARIABLES IN XSTAR WILL CORRESPOND
C   TO THE ORDER OF INPUT TO FPI.
C
C   IF THE INPUT VARIABLE IS DETERMINISTIC, THEN INPUT THE STANDARD
C   DEVIATION AS 0.0.
C
C   Remember that variable names K thru N are INTEGER ONLY!!!
C
C *****UNIFORM CORROSION*****
C
C   Model Form:  $Uact=C*exp(-phi/(bk*(1/TA-1/TC)))$ 
C   Limit state:  $G= UC-Uact$ 
C   Variables passed from UNIFORM*.DAT
C   BK=Boltzmann constant
C   C=constant corrosion rate
C   A=activation energy (eV), phi
C   TC=absolute critical temperature
C   TA=absolute actual temperature
C   UC=allowable uniform corrosion (ipy)
C   Uact=actual uniform corrosion
C
C
C
C
C   BK = XSTAR(1)
C   C = XSTAR(2)
C   A = XSTAR(3)
C   TC = XSTAR(4)
C   TA = XSTAR(5)
C   UC = XSTAR(6)
C   UACT= C* exp(-A/(BK*(1/TA-1/TC)))
C   G=UC-UACT
C   RESPON=G
C
C
C   RETURN
C   END
```

```
*FPI
FPI 3.0 USER DEFINED RESPONSE FUNCTION; erosion corrosion
*RVNUM      12
*GFUNCTION   6
*DATASETNM   0
*METHOD     1
*PRINTOPT    1
*ANALTYPE    1
*END
*ZLEVELS    10
0.04      0.03      0.02      0.01      0.00
-0.01     -0.02     -0.03     -0.04     -0.05
C*MONTE
C50000      8812      0.0
*DEFNVR
FM
0.055847   0.005   2.0
RF
7800       200     2.0
R
1000       50      2.0
C
0.25       0.05   2.0
D
2.0E-9     1E-10  2.0
DO
0.1        0.001  2.0
V
0.1035     0.0025 2.0
FV
0.001      0.00025 2.0
X
0.0165     0.0020  2.0
Y
0.86       0.05   2.0
Z
0.33       0.03   2.0
W
0.55       0.005  2.0
*END
```


94/06/22
12:15:23

respon.f.~8~

1

```
FUNCTION RESPON(XSTAR)
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
DOUBLE PRECISION FM,RF,R,C,D,DO,V,FV,X,Y,Z,W,WACT,G
DIMENSION XSTAR(100)
```

```
C
C---INCLUDE ANY NEEDED COMMON BLOCKS
C
C USER DEFINE RESPONSE FUNCTION
C **GFORM = 6 ON INPUT DATA TO USE THIS OPTION
C
C USER CAN DEFINE ANY RESPONSE FUNCTION DESIRED
C CLOSED FORM OR NUMERICAL - WHATEVER
C
C INPUT VARIABLE - XSTAR
C
C THE NEEDED INPUT VARIABLES MUST BE INPUT AS RANDOM VARIABLES
C TO FPI. THE ORDER OF THE VARIABLES IN XSTAR WILL CORRESPOND
C TO THE ORDER OF INPUT TO FPI.
C
C IF THE INPUT VARIABLE IS DETERMINISTIC, THEN INPUT THE STANDARD
C DEVIATION AS 0.0.
C
C Remember that variable names K thru N are INTEGER ONLY!!!
C
C Erosion Corrosion
C
C
C FM = XSTAR(1)
C RF = XSTAR(2)
C R = XSTAR(3)
C C = XSTAR(4)
C D = XSTAR(5)
C DO = XSTAR(6)
C V = XSTAR(7)
C FV = XSTAR(8)
C X = XSTAR(9)
C Y = XSTAR(10)
C Z = XSTAR(11)
C W = XSTAR(12)
C WACT=(4/3)*(FM/RF)*C*X*(D/DO)*((DO*R*V/FV)**Y)*((FV/(R*D))**Z)
C G=W-WACT
C RESPON=G
C
C RETURN
C END
```

C-2

94/11/27
15:28:46

galvanic1.dat

1

```
*FPI
FPI 3.0 USER DEFINED RESPONSE FUNCTION; Galvanic Corrosion
*RVNUM          5
*GFUNCTION      6
*DATASETNM     0
*METHOD       1
*PRINTOPT      1
*ANALTYPE      1
*END
*ZLEVELS       20
-0.950e1  -0.900e1  -0.850e1  -0.800e1  -0.750e1
-0.700e1  -0.650e1  -0.600e1  -0.550e1  -0.500e1
-0.450e1  -0.400e1  -0.350e1  -0.300e1  -0.250e1
-0.200e1  -0.150e1  -0.100e1  -0.500    0.000
C*MONTE
C50000      8812      0.0
*DEFNANVR
T
31.5E6  5E6      2.0
M
26.0    0.001    2.0
I
5.0E-4  5.0E-5    2.0
N
2.0     1.0      2.0
WALL
7.7E-2  8.0E-3    2.0
*END
```

94/11/27
15:22:59

1

respon.f

```
FUNCTION RESPON(XSTAR)
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
DOUBLE PRECISION T,M,I,N,WALL,F
DIMENSION XSTAR(100)
```

```
C
C---INCLUDE ANY NEEDED COMMON BLOCKS
C
C   USER DEFINE RESPONSE FUNCTION
C       **GFORM = 6 ON INPUT DATA TO USE THIS OPTION
C
C   USER CAN DEFINE ANY RESPONSE FUNCTION DESIRED
C   CLOSED FORM OR NUMERICAL - WHATEVER
C
C   INPUT VARIABLE - XSTAR
C
C   THE NEEDED INPUT VARIABLES MUST BE INPUT AS RANDOM VARIABLES
C   TO FPI.  THE ORDER OF THE VARIABLES IN XSTAR WILL CORRESPOND
C   TO THE ORDER OF INPUT TO FPI.
C
C   IF THE INPUT VARIABLE IS DETERMINISTIC, THEN INPUT THE STANDARD
C   DEVIATION AS 0.0.
C
C   Remember that variable names K thru N are INTEGER ONLY!!!
C
C *****GALVANIC CORROSION*****
C
C   Model Form:  $W_{act} = tMI / (Fn)$ 
C   Limit state:  $G = W_{allow} - W_{actual}$ 
C   Variables passed from GALVANIC*.DAT:
C   T=time in seconds
C   M=atomic weight of anode metal
C   I=galvanic current in amperes
C   F=Faraday's constant (96,501 coulombs)
C   N=charge of metal ions formed
C
C
C   T = XSTAR(1)
C   M = XSTAR(2)
C   I = XSTAR(3)
C   N = XSTAR(4)
C   WALL = XSTAR(5)
C   F = 96501
C   G=WALL-T*M*I/(F*N)
C   RESPON=G
C
C
C   RETURN
C   END
```

94/11/27
15:56:11

lcf1.dat

1

*FPI

FPI 3.0 USER DEFINED RESPONSE FUNCTION; Low-cycle fatigue

*RVNUM 4

*GFUNCTION 6

*DATASETNM 0

*METHOD 1

*PRINTOPT 1

*ANALTYPE 1

*END

*ZLEVELS 20

-0.94952e3 -0.84267e3 -0.73581e3 -0.62896e3 -0.52211e3

-0.41526e3 -0.30841e3 -0.20156e3 -0.94708e2 0.12143e2

0.11899e3 0.22585e3 0.33270e3 0.43955e3 0.54640e3

0.65325e3 0.76010e3 0.86695e3 0.97380e3 0.10807e4

C*MONTE

C50000 8812 0.0

*DEFNVR

E

3.0E7 3.0E6 2.0

SE

37500 4000 2.0

S

2.0E5 2.0E4 2.0

NMIN

700 70 2.0

*END

94/11/27
15:50:32

respon.f

1

```
FUNCTION RESPON(XSTAR)
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
DOUBLE PRECISION E,SE,S,NMIN,NACT
DIMENSION XSTAR(100)

C
C---INCLUDE ANY NEEDED COMMON BLOCKS
C
C   USER DEFINE RESPONSE FUNCTION
C       **GFORM = 6 ON INPUT DATA TO USE THIS OPTION
C
C   USER CAN DEFINE ANY RESPONSE FUNCTION DESIRED
C   CLOSED FORM OR NUMERICAL - WHATEVER
C
C   INPUT VARIABLE - XSTAR
C
C   THE NEEDED INPUT VARIABLES MUST BE INPUT AS RANDOM VARIABLES
C   TO FPI.  THE ORDER OF THE VARIABLES IN XSTAR WILL CORRESPOND
C   TO THE ORDER OF INPUT TO FPI.
C
C   IF THE INPUT VARIABLE IS DETERMINISTIC, THEN INPUT THE STANDARD
C   DEVIATION AS 0.0.
C
C   Remember that variable names K thru N are INTEGER ONLY!!!
C
C *****Low-Cycle Fatigue*****
C
C   Model Form:  NACT=((0.1732868*E+SE)/S)^2
C   Limit state: G= -(NMIN-NACT)
C   Variables passed from LCF*.DAT:
C   E=modulus of elasticity of metal
C   SE=endurance limit
C   S=1/2Eet
C   NACT=predicted number of cycles to failure
C   NMIN=minimum number of cycles for service
C
C
C   E = XSTAR(1)
C   SE = XSTAR(2)
C   S = XSTAR(3)
C   NMIN = XSTAR(4)
C   NACT = ((0.1732868*E+SE)/S)**2
C   G=-(NMIN-NACT)
C   RESPON=G
C
C
C   RETURN
C   END
```

Appendix 2

FPI Subroutine Library
of
Selected Mechanical Limit States

94/11/24
10:59:48

user.f

1

```
SUBROUTINE USER (N,X,Z0)
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
DIMENSION X(100)
COMMON /MENU/ IEQ,NCOEF,NPOW,COEF(100),POW(100)

C
C
C Define the reponse function Z=Z(X1, X2,...,Xn)
C Define the limit state function as G=Z-Z0 where Z0=a value of Z
C The problem of failure is PF=PROB.(G<0)=PROB.(Z<Z0)
C The limit state FORTRAN statement must be provided in this form:
C   X(1)=Function (Z0, X(2), X(3),...,X(N))
C
C
C   IF(IEQ.NE.0)GOTO 100

C For GFORM=0.0
C
C   X(1)=5.0*100000*x(2)**3/6
C
C RETURN

C
C 100 GOTO (1,2,3,4,5,6), IEQ

C   WRITE(6,*) '**ERROR* EQUATION NUMBER > SPECIFIED IN SUBROUTINE'
C   WRITE(*,*) '**ERROR* EQUATION NUMBER > SPECIFIED IN SUBROUTINE'
C   STOP

C
C *****EROSION CORROSION*****
C Limit state for erosion corrosion g=Wear allowed -Wear actual
C Wear Actual=(4/3) (FM/RF)C*X*(D/DO)*((DO*R*V/FV)**Y)((FV/(R*D))**Z)
C X(1)=allowable wear
C X(2)=molar mass of metal subject to erosion,FM, (kg/mol)
C X(3)=density of eroded metal,RF, (kg/m^3)
C X(4)=bulk oxygen concentration,C (mol/m^3)
C X(5)=constant,X
C X(6)=diffusion coefficient of transferred species, D (m^2/s)
C X(7)=pipe diameter, DO (m)
C X(8)=fluid density, R (kg/m^3)
C X(9)=mean fluid velocity, V, (m/s)
C X(10)=fluid viscosity, FV
C POW(1)=constant, Y
C POW(2)=constant, Z
C COEF(1)=4/3

C
1   A=(X(2)/X(3))*X(4)*X(5)*(X(6)/X(7))
   B=((X(7)*X(8)*X(9)/X(10))**POW(1))*((X(10)/(X(8)*X(6)))**POW(2))
   X(1)=Z0+COEF(1)*A*B
   GOTO 300

C
C *****WEAR*****
C Limit state for wear problems. g=Wear allowed-Wear actual
C Wear actual=kpd/h
C X(1)=allowable wear
C X(2)=Brinell hardness of material
C X(3)=wear coefficient
C X(4)=applied load
C X(5)=apparent contact area
C X(6)=rotating shaft diameter, 1 if a non-shaft problem
C X(7)=RPM for rotating shaft case, 1 if non-shaft case
C COEF(1)=total operating lifetime(minutes)/1422 for shaft case
C           =total operating lifetime(minutes)*velocity/1422 for non-shaft case
```

```
C *****
C
2   X(1)=Z0+COEF(1)*X(3)*X(4)*X(7)/(X(2)*X(5))
    GOTO 300
C
C *****PITTING*****
C Limit state for pitting: g=Depth allowed-Depth actual
C Depth actual=KT^0.33
C X(1)=maximum allowable pit depth
C X(2)=constant
C X(3)=time in months
C
3   X(1)=Z0+X(2)*(X(3)**0.33)
    GOTO 300
C
C *****FRETTING*****
C Limit state for fretting: g=Depth allowed-Depth actual
C Depth actual=((K0*(P**0.5))-(K1*P))*(N/F)+(K2*L*P*N)
C X(1)=maximum allowable fretting depth
C X(2)=normal load, P (psi)
C X(3)=total number of cycles, N
C X(4)=cycles per second, F
C X(5)=slip amplitude, L (inches)
C X(6)=constant, K0
C X(7)=constant, K1
C X(8)=constant, K2
C
4   A=((X(6)*(X(2)**0.5))-(X(7)*X(2)))*(X(3)/X(4))+(X(8)*X(5)*X(2)*X(3))
    X(1)=Z0+A
    GOTO 300
C
C *****GALVANIC CORROSION*****
C Limit state for galvanic corrosion: g=Max. mass loss-Actual mass loss
C Actual mass loss=t*M*I/(F*N)
C X(1)=maximum allowable mass loss
C X(2)=time in seconds, T
C X(3)=atomic weight of anode metal, M
C X(4)=galvanic current in amperes, I
C X(5)=charge of metal ions formed, N
C F=Faraday's constant (96,501 coulombs)
C
5   F=96501
    X(1)=Z0+X(2)*X(3)*X(4)/(F*X(5))
    GOTO 300
C
C *****LOW-CYCLE FATIGUE*****
C Limit state for low-cycle fatigue: g= Actual cycles-Min. req'd cycles
C Actual cycles=((0.1732868*E+SE)/S)^2
C X(1)=Minimum number of cycles to failure, NMIN
C X(2)=modulus of elasticity of metal, E
C X(3)=endurance limit, SE
C X(4)=S=1/2Eet
C
6   X(1)=(((0.1732868*E+SE)/S)**2)-Z0
    GOTO 300
C
C
300 CONTINUE
    RETURN
    END
```


Appendix 3

Reliability and Maintainability Research Review

Reliability and Maintainability Research Review

The study of reliability and maintainability of systems has tended to focus on deterministic modelling of failure modes, with little consideration for the development of integrated system models to accurately predict system performance. Furthermore, little research has been focussed on determining the nature of the relationships between product and process primitive variables and their impact on R&M.

Considerable research has been done in the development of reliability and maintainability measures which use a more traditional approach. Amstader (1971) provides a comprehensive coverage of all reliability and maintainability issues from the traditional statistical perspective, with considerable detail concerning reliability apportionment, and system reliability prediction. Kapur and Lamberson (1977) examine the standard reliability techniques, but then extend their work to include reliability design. Their work focusses on the linkage between reliability measurement and engineering design and how design factors impact overall system reliability and maintainability. Smith (1976) also studies the linkage between reliability, maintainability and engineering design, but devotes less attention to the different measurements of R&M performance.

The derivation of first-order reliability method relationships is demonstrated in both Melchers (1987), and Ang and Tang (1984). These authors derive the relationships for the reliability index as well as the most probable point.

The solution of reliability problem using first-order methods for non-linear limit states is presented by Rackwitz and Fiessler (1978). The authors present an algorithm to update estimates of the most probable point using a gradient search approach.

The determination of system reliability estimates for systems with 2 limit states is presented by Ditlevsen (1979). The case of numerous limit states is solved by using upper and lower bounds by Cruse et al.(1992).

The management of the product development process to insure the incorporation of reliability and maintainability characteristics is essential, and has been the focus of considerable research. A comprehensive examination of the managerial issues surrounding R&M design considerations is given by Dhillon and Reiche (1985). The authors consider the organization of engineering effort to facilitate R&M measures being met, the proper allocation of engineering resources, appropriate documentation of the development effort, and procedures for design evaluation and review. Particular attention is given to accurate determination of system life cycle costs and anticipating system warranty costs. The employment of a systems approach to product development has been suggested by several authors. Churchman (1971) provided an explanation of the essential elements of a systems approach, while Blandford et al. (1985) proposed alternative systems approaches. The system approach is favored because it offers a structured development process and facilitated monitoring of development progress.

The determination of linkages between activities in the design process was studied by Steward (1981). A methodology was proposed for the identification of informational and procedural dependencies amongst the product development tasks. The dependencies were then characterized through the use of a Design Structure Matrix.

In managing the product development process attention has focussed on the need to more closely integrate all engineering functions early in the development as a means of ensuring that essential product performance characteristics are anticipated in the design process. The employment of concurrent engineering as a means of achieving this early design integration was studied by Nevins et al. (1989). The authors examined the benefits and pitfalls of concurrent engineering and identified the informational linkages between the engineering function and the manufacturing function.

Contrasting sharply with the view of design as an integration activity, a mechanistic approach is offered by Bodensteiner and Priest (1988) who regard product development and manufacture as a technical process focussed on design, test, and production of a product. They advocate the use of a DOD Template approach to reduce the risks associated with getting new products through the research stage and into manufacturing.

Although product development and design have the greatest direct impact on system R&M performance, most research has focused on predictive methodologies for R&M based on component or system testing. Crow (1984) proposed a method to determine the potential of a system for reliability growth. He determined a relationship for the failure rate of a series system as a function of time, then extended to failure rate model to consider the effect of immediate repair of failures and a time lag prior to repair of failures. In addition, analytical results indicating the maximum possible growth potential of a system and confidence interval estimates of the growth potential were found.

The problems of long development times and testing periods for system reliability were discussed by Kasouf and Weiss (1984). The problems of some systems is that system storage times greatly exceed operational times and operational testing is exceptionally expensive. In addition, the delivery of sufficient systems for reliability testing may be so long as to result in R&M testing being done as initial units become operational. The authors noted factors which affect the typical bathtub curve, and the need for TAAF [test, analyze and fix] programs at the subsystem level.

Bazovsky and Benz (1984) developed a model of the reliability of mechanical systems based on the age of the system. The authors initially develop a model of the reliability of mechanical components using renewal theory, and then assuming a series system determine the results for the overall mechanical system. Simulation of the system was used to study the effect of five different part replacement policies on the system reliability.

The determination of interval estimates of system reliability was examined by Winterbottom (1984). The interval estimates were derived from component test data for both serial and parallel systems. Several different cases were studied including that of binomial test data, Poisson approximations to binomial results, systems with exponential times to failure, and

Bayesian methods.

The use of growth tests as opposed to reliability demonstration tests is contrasted in Smith (1984). The author considers the difficulties in using demonstration reliability tests under severe time constraints, and the advantage that reliability growth tests have in these situations. Parameters of the growth tests such as growth rates, initial MTBF, establishing planned growth rates, time to isolate and verify failures, and the impact of corrective action policies are enumerated as well as their impact on the testing regime.

The incorporation of reliability growth testing into the routine engineering development program allows the estimation of the reliability growth without dedicated testing. The accumulation of reliability testing data from planned engineering tests is proposed by Bentz and Hutchinson (1984). The authors point out the limitations of such a process, namely that all failure modes may not be apparent when conducting tests at such an early stage in the development plan.

The impact on system reliability resulting from errors in component reliability were examined by Fishman (1990). For series systems it was found that sampling variation in component reliability measurements resulting in overstating the system reliability as the number of components increased. For parallel systems, the system reliability was understated as the number of components increased. The author demonstrates what limit conditions must exist in order for the sample system reliability to converge to the true system reliability.

A different approach has been suggested by Siegrist (1988), who studied the situation where system control is transferred between system modules according to a Markov chain. The system was considered to possess a final state defining mission success. The author determined analytical relationships for the system reliability, system sensitivity to changes in individual component reliability, the mean number of system cycles to failure, and the individual component reliabilities required for a specified system reliability.

Comparison of four well known theoretical bound approximations was conducted by Romeu (1989). The comparison of the bounds was using Monte Carlo simulation of a system comprised of 10 subsystems in series. The results indicated that the Mann-Grubbs bound was best for coverage, and that the Mann-Grubbs was very close to the El Mawazini-Buehler bound. Both bounds were close to the true system reliability.

The deterministic and analytical modelling of system R&M performance depends on considerable testing of components and subassemblies. However, there are many situations where the employment of such tests are impractical, and for these cases the application of Bayes theorem to reliability has been studied. Chay (1984) showed how the prior distribution can be obtained, and how the Bayes theorem can be applied in order to determine estimates of the reliability parameters and the uncertainty bounds of a system. In addition, the contrast between using conjugate prior, discretization as an approximation to continuous prior distribution, and numerical integration techniques was demonstrated.

An interrelationship between product reliability and computational modelling is found in

the analysis of process assembly variation. The prediction and optimization of assembly variation can be done either through methods such as limit stacking, root-mean-square, or variation simulation analysis. Craig (1991) points out that the major limitation of limit stacking and root-mean-square techniques is the difficulty in considering 2 and 3 dimensional variation effects. Thus, the variation simulation analysis uses Monte Carlo simulation and 3-D geometry to predict assembly variation.

The study of uncertainty and sensitivity analysis for computer models of complex processes is critical to all simulation modelling of processes. Iman and Helton (1988) demonstrated the superior performance of Latin hypercube sampling and regression analysis over both response surface methodology and differential analysis.

Aggarwal (1989) discusses system which have a non-zero probability of success even when a subcomponent fails. This property, called criticality, is defined and a methodology for its apportionment and use in defining system reliability are presented.

The case of systems composed of components which can take on a number of operating states (called multistates) is addressed by Shao and Kapur (1989). The authors consider a multilevel system composed of multistate components and present a modular decomposition method. In addition, the authors present a convenient means of building structure functions for decompositions as well as the efficiency index for any level modular decomposition.

The linkages between reliability and maintainability of systems and program productivity are presented by Fedor (1989). The author points out that improvement of system R&M will have dramatic effect on the productivity of the development program.

The application of simulation in deriving reliability estimates for systems with extremely low failure rates is discussed by Geist and Smotherman (1989). Importance sampling is used to reduce the number of simulation runs required to estimate the reliability of ultrahigh reliable systems. The accuracy and methodology associated with importance sampling is contrasted with that required for HARP (Hybrid Automated Reliability Predictor) and SURE (Semi-Markov Unreliability Range Estimator).

A Bayesian method for the estimation of system reliability during product development is proposed by Kaplan et al. (1990). The authors present a stepwise process for assessing the failure data from early testing. The process allows for the determination of system reliability at any stage in the development process, and to track the reliability growth as the development process advances.

An estimation of the bounds on state probabilities of system components based on Markov chains is discussed by Laemmel and Shooman (1990). The bounds are combined with merging and decomposition methods.

In developing and managing production facilities, simulation has been widely adopted. Rajamani and Singh (1991) used simulation to assist in the design of a consumer electronics assembly line. The simulation results were used to validate a proposed design based on

cost/return criteria for the process. Singh (1991) highlights that inappropriate application of simulation can result in misinterpretation of results or erroneous conclusions being made. A methodical approach to simulation model development is advocated which stresses frequent comparison between the model and the system being simulated. Simulation methods have been used for the modelling and design of warehouses (Senko and Suskind, 1990), automated storeroom systems (Gogg and Sands, 1990), automated guided vehicle systems (Lee et al., 1990), dual-kanban production systems (Potts et al., 1989), and new facilities (Marmon, 1991).

Simulation techniques have been extended to include models of flexible manufacturing systems (FMS) and to evaluate the performance of the process under differing conditions. Hatono et al. (1991) considered FMS models based on stochastic petri nets in a hierarchical structure, and used simulation to evaluate the performance of the process under differing conditions. Simulation was used by Chakraborty and Ankiah (1989) to determine the main factors which affect the manufacturing system reliability and have an impact on the overall system configuration. The authors were able to demonstrate the applicability of simulation in determining the product and process factors which affect manufacturing system reliability as well as the sensitivity of the production system to changes in the factor values. Taha and de la Parra (1989) propose a methodology for the simulation of manufacturing systems for the determination of system reliability. Calabria et al. (1988) examined the reliability of a series production system using simulation methods and determined relationships between system production efficiency, target dependability, no failure probability, and the MTBF of production equipment.

The key difficulty with the use of simulation methods is that a large number of different operating conditions of the manufacturing system must typically be modelled in order to determine the production system characteristics. A solution to this problem was presented by Mishra and Pandey (1989) who demonstrated that for FMS facilities that the use of design of experiments to determine simulated operating characteristics of the production system did not differ significantly from results obtained by traditional methods.

The nature of the linkages between product and process design have considerable impact on product reliability and maintainability. A methodology linking product and process design is given by De Toni and Zipponi (1991) who identify the nature of the product-process linkages that exist in the 3 major level of product composition: component, subassembly, and finished product. The authors define the predominant product and process attributes at each level of the product composition.

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