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Solution of Matrix Equations Using Sparse Techniques

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The solution of large systems of matrix equations is key to the solution a large number of scientific and engineering problems.

Tradition has it that iterative methods persist for CFD and direct methods for Structures spplications. With the increase in computational power (over 3 orders of magnitude this decade) problem sizes with full detail that could not have even been considered tractable are now solved routinely. The equation solvers used for structures applications have advanced from the use of full matrix (LINPACK, LAPACK BLAS-3) to band solvers to variable band and skyline solvers to sparse matrix solvers with corresponding increases in performance. It appears that for large-scale structural analysis applications sparse matrix methods have a significant performance advantage over other methods This talk will describe the latest sparse matrix solver developed at Langley which if not the fastest in the world is among the best. It can routinely solve in excess of 263,000 equations in 40 seconds on one Cray C-90 processor.

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Dr. Majdi Baddourah received the Ph D. in the Department of Civil Engineering at Old Dominion University in 1991. He has been employed by Lockheed Engineering and Sciences Company since then in support of the Computational Structures Branch at NASA Langley Research Center. Dr. Baddourah is widely recognized for contributing to the development of software to exploit scalable high-performance computers for structural analysis applications including the solution of large systems of equations (approaching 1 million) by both direct and iterative methods.

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Lockheed Engineering and Sciences Co.

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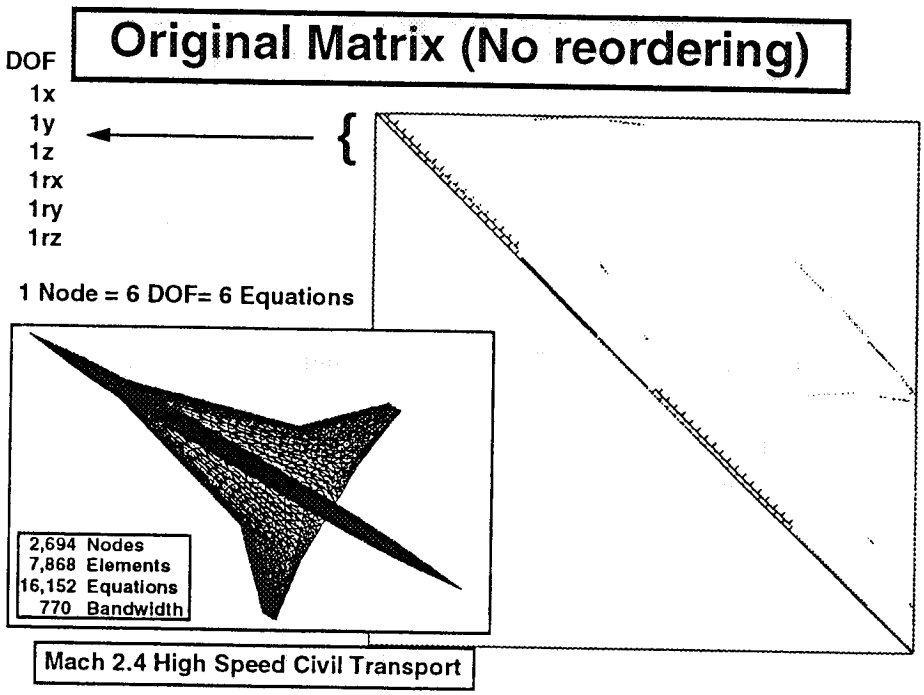
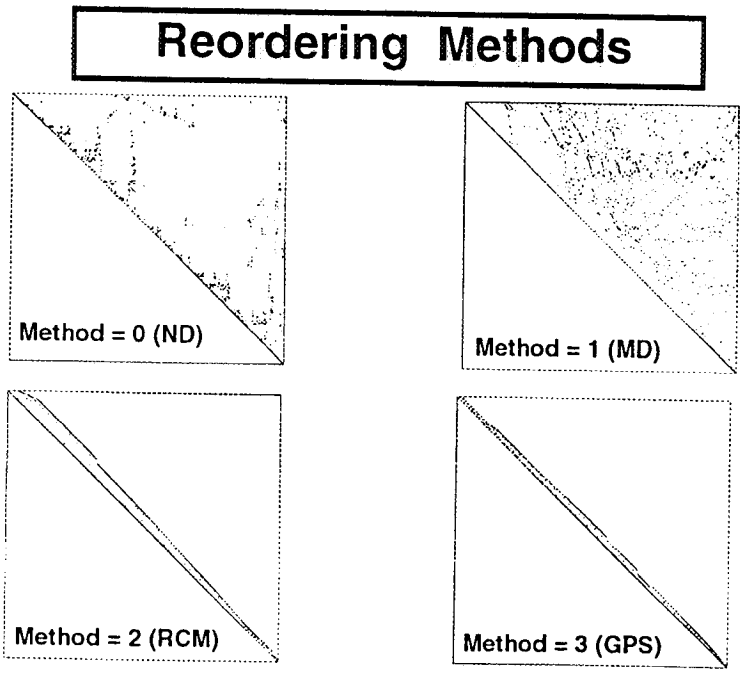
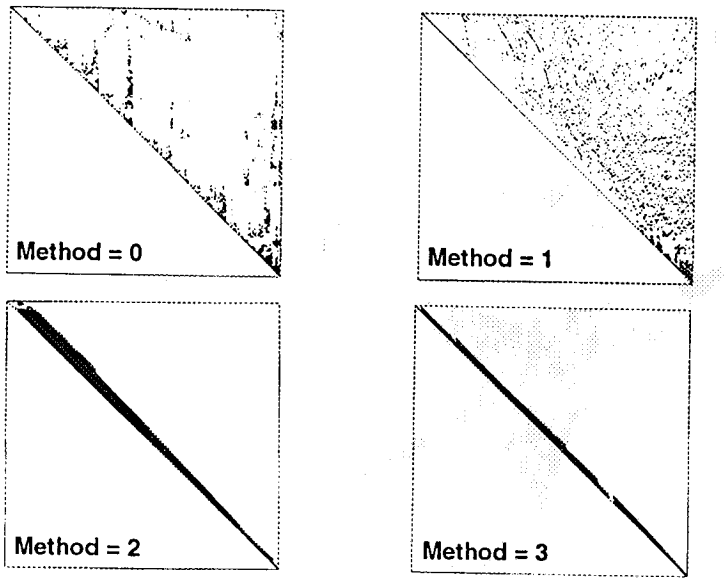
Outline

- Matrix Storage
- Reordering
- Factoring
- Results (Computational Structures and Fluids)
- Conclusion

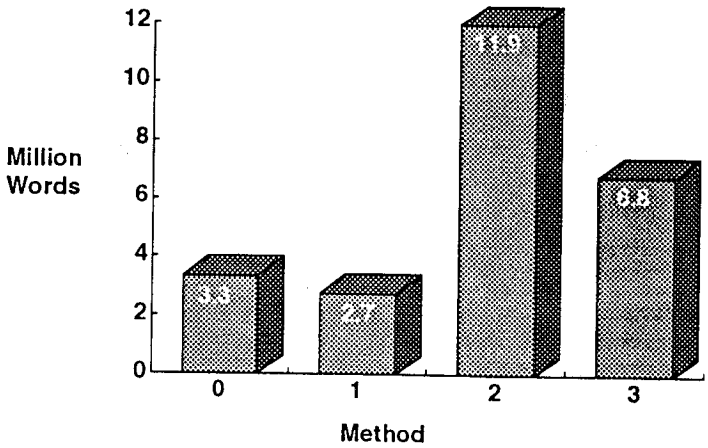

Original Matrix (No reordering)
DOF
1x
1y
1z
1rx
1ry
1rz
1 Node = 6 DOF= 6 Equations
2,694 Nodes
7,868 Elements
16,152 Equations
770 Bandwidth
Mach 2.4 High Speed Civil Transport


Reordering Methods
Method = 0 (ND)
Method = 1 (MD)
Method = 2 (RCM)
Method = 3 (GPS)

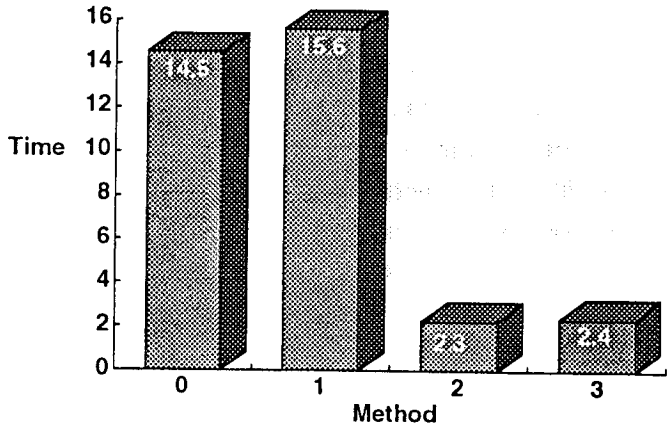
Matrix After Factoring



Matrix Storage Memory Requirement

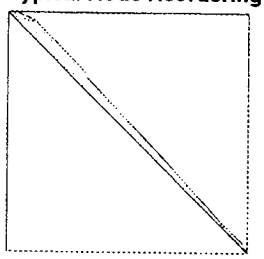


Reordering Time

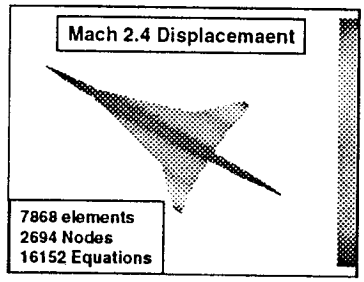


Equation Reordering Reduces Solution Time

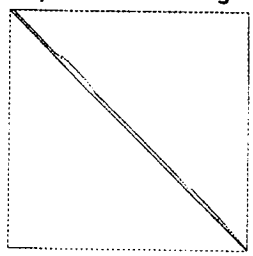
Typical Node Reordering



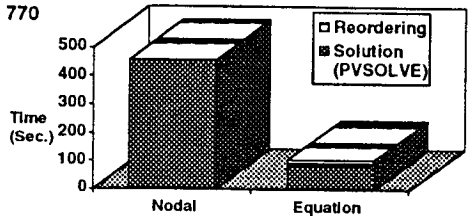
Maximum Band = 1266
Average Band = 770



Equation Reordering



Maximum Band = 609
Average Band = 347



Reordering Method

Factoring Matrix

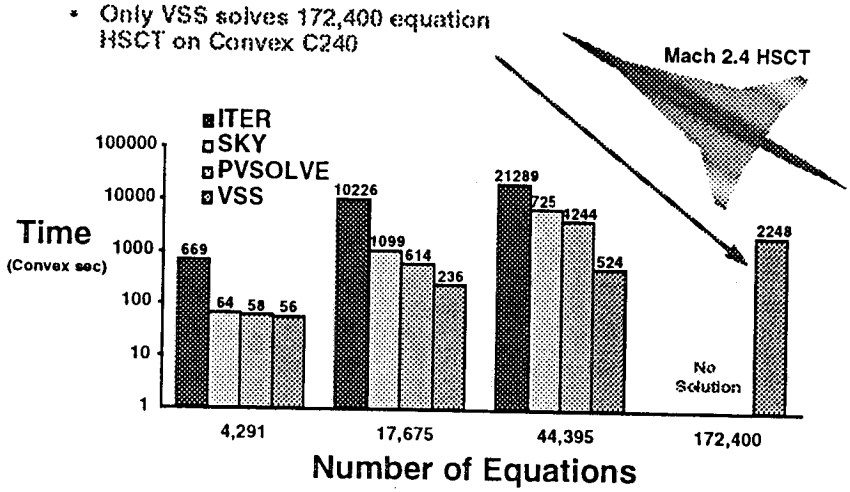
- Banded or full:
  - easy to vectorize.
  - problem size limit.
- General sparse:
  - difficult to vectorize.
  - fewer operations.
  - indirect addressing.

Results

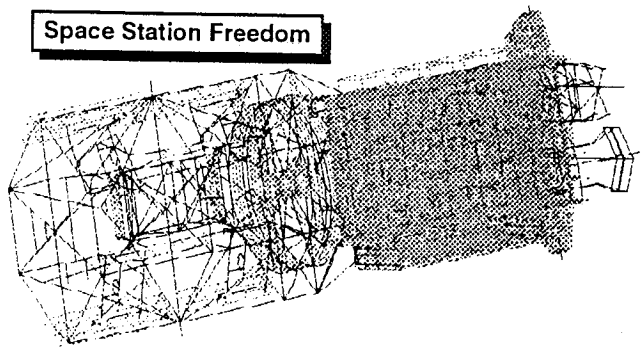
- High Speed Civil Transport
- Space Station
- CFD Application
- Automotive Application

Mach 2.4 HSCT Results

* Only VSS solves 172,400 equation HSCT on Convex C240



Space Station Application



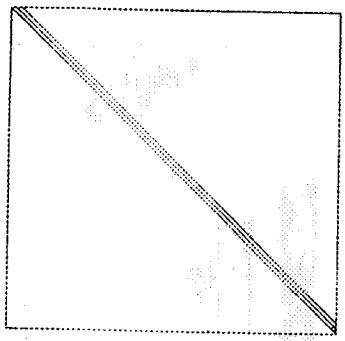
111893 Equations
1664984 Non-zero terms
97 solution secs*

* Using 1 Cray Y-MP processor and Solid State Disk at NAS

Beam Elements
Triangular Elements
Quadrilateral Elements

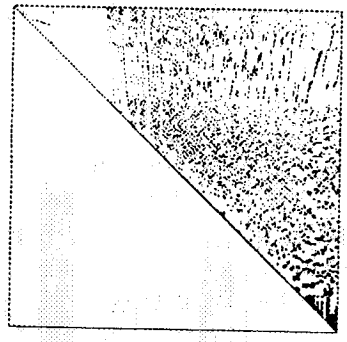
CFD Application

Before Reordering



Number of Equations = 15360
Number of Cofficients = 257797

After Reordering with fill



Number of Cofficients = 3081995

1 Cray C-90 Solution Time = 6.7 Seconds

Automotive Application

44,188 Nodes
48,894 Elements
263,574 Equations
NASA solution took 78 sec for full static analysis
(on 1 Cray C-90 processor)
- fastest solver known to date -
(32 sec reordering, 45 sec factor and 1 sec F/B)

CRAY Sparse solver took 102 sec for full static analysis
Banded Solver took 2500 sec for full static analysis

Conclusion

- Sparse solvers are preferred for large-scale structures.
- Sparse Solver outperforms iterative solver which can have convegence problems.
- Sparse Solver can be used for CFD applications
- Sparse solvers uses minimum memory.