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# Multi-Objective Decision-Making Under Uncertainty: Fuzzy Logic Methods

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Prepared for the  
Computing in Aerospace 10 Meeting  
sponsored by the American Institute of Aeronautics and Astronautics  
San Antonio, Texas, March 28-30, 1995



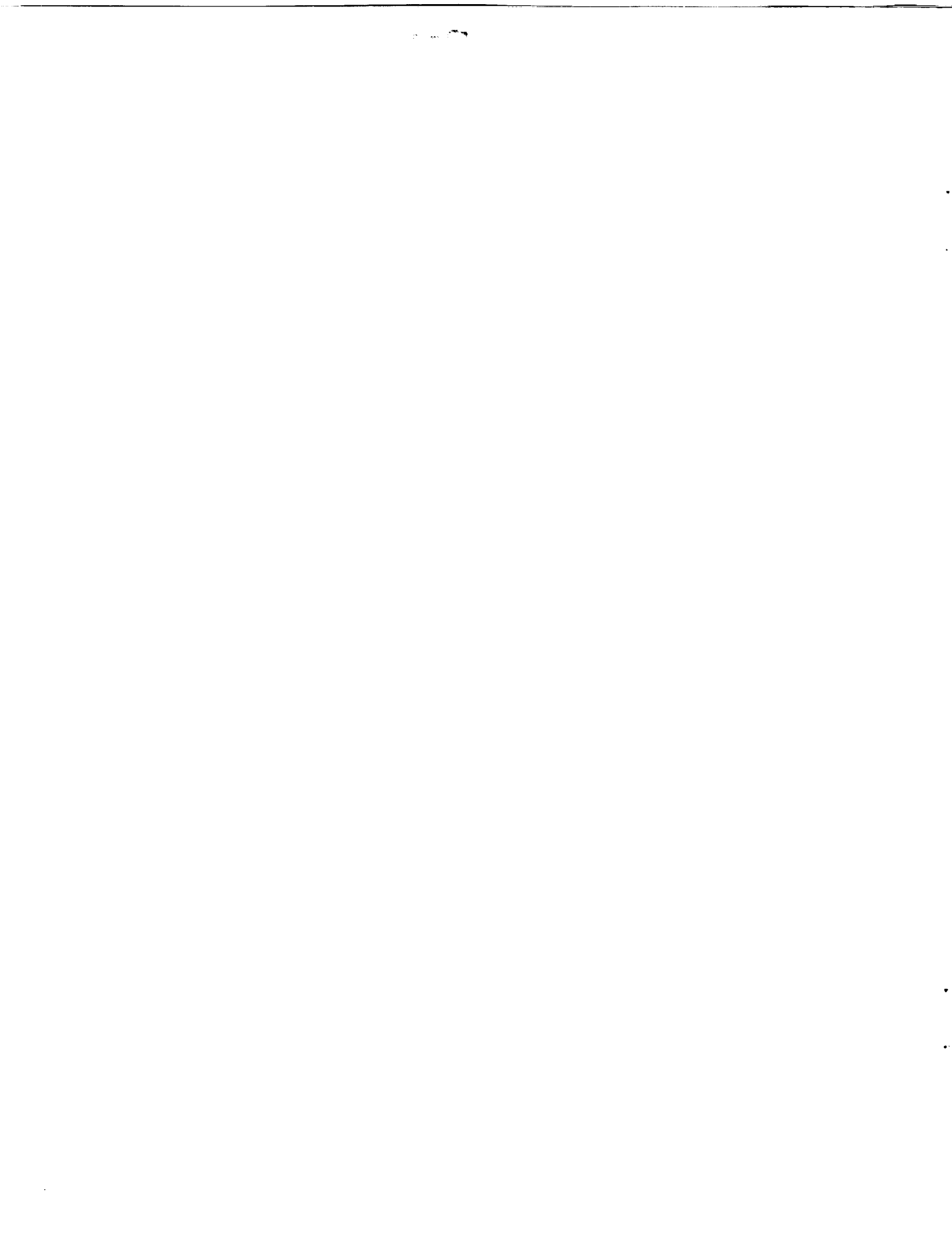
National Aeronautics and  
Space Administration

(NASA-TM-106796) MULTI-OBJECTIVE  
DECISION-MAKING UNDER UNCERTAINTY:  
FUZZY LOGIC METHODS (NASA, Lewis  
Research Center) 16 p

N95-17269

Unclass

G3/20 0033884



# MULTI-OBJECTIVE DECISION-MAKING UNDER UNCERTAINTY: FUZZY LOGIC METHODS

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## Abstract

Selecting the best option among alternatives is often a difficult process. This process becomes even more difficult when the evaluation criteria are vague or qualitative, and when the objectives vary in importance and scope. Fuzzy logic allows for quantitative representation of vague or fuzzy objectives, and therefore is well-suited for multi-objective decision-making. This paper presents methods employing fuzzy logic concepts to assist in the decision-making process. In addition, this paper describes software developed at NASA Lewis Research Center for assisting in the decision-making process. Two diverse examples are used to illustrate the use of fuzzy logic in choosing an alternative among many options and objectives. One example is the selection of a lunar lander ascent propulsion system, and the other example is the selection of an aeration system for improving the water quality of the Cuyahoga River in Cleveland, Ohio. The fuzzy logic techniques provided here are powerful tools which complement existing approaches, and therefore should be considered in future decision-making activities.

## Nomenclature

AHP	Analytic Hierarchy Process
BV	Best value of a criterion
CH <sub>4</sub>	Methane
ClF <sub>5</sub>	Chlorine Tetrafluoride
DO	Dissolved oxygen
IME	Integrated Modular Engine
JSC	NASA Johnson Space Center
LeRC	NASA Lewis Research Center
L <sub>h</sub>	Final composite index value
LH <sub>2</sub>	Liquid hydrogen
L <sub>max</sub>	Maximum of final composite index values
L <sub>min</sub>	Minimum of final composite index values

LO <sub>2</sub>	Liquid oxygen
MMH	Monomethyl hydrazine
N <sub>2</sub> O <sub>4</sub>	Nitrogen tetroxide
S	First-level index value
U	Total utility value
U <sub>L</sub>	Left utility value
U <sub>R</sub>	Right utility value
w	Weighting factor
WV	Worst value of a criterion
Z	Evaluation criterion actual value
μ <sub>G</sub>	Minimizing set
μ <sub>M</sub>	Maximizing set

## Introduction

Engineers and managers are often required to make decisions on the basis of objectives or criteria which vary widely in scope and complexity (for example, performance, cost and schedule). Adding to the difficulty of the process is that many of the criteria are by their very nature vague and difficult to quantify. The decision-maker must combine the vague criteria with criteria which are known quantitatively to obtain the best possible alternative. Without systematic approaches to the process one cannot be sure that the proper decision has been made.

Recently efforts have used fuzzy logic to assist in the decision-making process.<sup>1-5</sup> Fuzzy logic is a super-set of conventional logic which allows for degrees of truth - truth values between true and false. The concepts of fuzzy logic become especially useful when the values of the decision criteria are not only vague but uncertain. An example of such a case might be the selection of a disposal site for hazardous waste material.<sup>5</sup> In this example the criteria may include transportation of the material, surface water quality, ground water quality, and aesthetics, as well as other factors. Transportation may be quantified in

terms of number of miles, which may be known with a high degree of certainty. Surface and ground water quality may refer to the amount of waste which could run off the site or leach into the ground, respectively. These factors may also be quantified, but the uncertainty is large because of the lack of data on existing systems. Finally, the aesthetics of the hazardous waste operation is generally considered qualitative; therefore, the decision-maker must convert vague linguistic descriptions such as *good* or *poor* to quantitative rankings. Fuzzy logic methods can be used for combining criteria which are vague and uncertain with those which are well-known to assist in the selection of an alternative.

This paper will present the concepts of fuzzy set theory, the basis for fuzzy logic, including a description of the differences between classical and fuzzy set theory. In addition, the report will describe methods from the literature and those developed at NASA Lewis Research Center which use fuzzy logic to assist in the multi-objective decision-making process. As part of the discussion on methodology, DECISION MANAGER, software developed at NASA LeRC to automate the decision-making process, will be presented. The fuzzy logic methods shown here were originally applied to aerospace applications; namely, the methods have been used to evaluate rocket engine and space launch vehicle concepts.<sup>1</sup> However, the methods have wide applicability, especially in civil engineering disciplines. This paper will provide two diverse examples of the use of the fuzzy logic methods described here. One example will be the selection of a space chemical rocket engine for lunar lander applications, and the other will be the selection of an aeration system for improving the water quality of the Cuyahoga River in Cleveland, Ohio.

### Fuzzy Set Theory

Fuzzy logic is based on fuzzy set theory, which was developed in 1965 by Lotfi Zadeh.<sup>6</sup> In classical set theory, the basis for most decision-making processes, objects are defined either as being a member of a set or not a member. Therefore, mathematically, there are two values for the degree of membership: 1 (member) and 0 (nonmember). Conventional sets are also known as bivalent sets because two values are possible. Fuzzy set theory, on the other hand, declares that everything is a matter of degree, and sets can have imprecise boundaries. Therefore, in fuzzy set theory, membership can gradually transition between membership and nonmembership. Fuzzy sets are also known as multivalent sets.

The concept of a fuzzy set is best illustrated by an example. Consider the set of *deep* lakes, where *deep* is a vague or *fuzzy* term. According to bivalent set theory, a

discrete dividing point is necessary for defining membership. In this case lakes over 40 m in depth might be called *deep*, and would have a membership value of 1. All lakes less than 40 m in depth would therefore be *shallow* and hence have a membership value of 0. In a fuzzy set, however, a lake that is 25 m deep might be called *somewhat deep*, with a value of 0.4 for the degree of membership. A lake 80 m deep would be *very deep* and therefore have a membership value of 1. The comparison between these sets is shown in Fig. 1.

As illustrated in the above example, the concept of a fuzzy set makes sense when real world situations are examined. Fuzzy sets become especially useful when applied to multi-objective decision-making processes. In most decision-making situations the input data are vague and contain a high degree of uncertainty. Uncertainty can be treated with probabilistic methods.<sup>7</sup> However, probabilistic methods require that the data have a statistical basis, and that the imprecision is a result of randomness in the system. Because many of the input variables in a decision-making process do not have a statistical basis, other methods are required. In these cases ranges of values are used to describe the uncertainty. A trapezoidal fuzzy set can be used to mathematically represent this range. An example of a trapezoidal fuzzy set is shown in Fig. 2, which describes the range of values for the purchase price of an automobile. When buying an automobile we usually do not have data for the mean price or the statistical variance. However, we do have a range of values in mind from previous experience. In this case we would define prices within the *most likely* interval (\$9,000 to \$11,000) as having a membership value of 1, and prices outside of the *largest likely* interval (\$8,000 to \$12,000) as having a membership value of 0. The membership value is assumed to be linear between the *most likely* and *largest likely* values, thus providing the trapezoidal shape. Fuzzy set theory then allows for manipulation of these fuzzy sets to obtain the best possible alternative.

### Methodology

The process for selecting an alternative includes several steps: defining the alternatives and criteria, determining the importance factors for the criteria, specifying the raw scores for each alternative with respect to the criteria, and calculating the final scores to rank the alternatives. Two scoring methods employing fuzzy logic concepts are presented in this paper: arithmetic averaging and fuzzy set methods. The details of these methods follow.

The first step in the decision-making process is to define the alternatives and the criteria for evaluating the alternatives. Because the criteria vary in importance

(for example, cost may be more important than reliability to the decision-maker), weighting factors must be used to indicate relative importance. The weighting factors may simply be selected, or techniques such as the Analytic Hierarchy Process<sup>3,8</sup> may be used. In the Analytic Hierarchy Process the criteria are compared against each other systematically. The result is a matrix of paired comparisons. By solving the matrix for the eigenvector of the maximum eigenvalue, the weighting factors are calculated. Examples of this technique are provided in references 1, 3, 5, and 8.

Once the alternatives and criteria have been determined, the trapezoidal fuzzy sets are specified which give values of the criteria for each alternative. As discussed previously, these fuzzy sets characterize the uncertainty in the criteria values. Note that if the trapezoid is reduced to a single vertical line, no uncertainty is present and a *crisp* number results. Because the units of the criteria values, or "raw scores," are different, the raw scores must be transformed into an index value to allow for direct comparison, as described in references 1-4. This transformation normalizes the fuzzy sets in relation to the best and worst values for each criterion. For each criterion value, Z, the first-level index value, S, is obtained as follows:

If Best Value (BV) > Worst Value (WV):

$$\begin{aligned} S &= 1 && (Z > BV) \\ S &= (Z - WV)/(BV - WV) && (WV < Z < BV) \\ S &= 0 && (Z < WV) \end{aligned}$$

If Best Value (BV) < Worst Value (WV):

$$\begin{aligned} S &= 1 && (Z < BV) \\ S &= (Z - WV)/(BV - WV) && (BV < Z < WV) \\ S &= 0 && (Z > WV) \end{aligned}$$

For example, if reliability is the criterion, higher values are better than lower values and the first set of equations is used. If, on the other hand, cost is the criterion, then lower values are better and the second set of equations is used. Because there are four values which define the trapezoidal fuzzy set (corresponding to the most likely and largest likely intervals), there will be four index values for each criterion. Figure 3 illustrates this transformation for the purchase price of an automobile. For example, the first-level index values of 0.3, 0.4, 0.6, and 0.7 correspond to prices of \$12000, \$11000, \$9000, and \$8000, respectively. The best and worst values can be defined using the largest and smallest values of all the

alternatives (\$12000 and \$8000, for example), or these values can be chosen to fall outside of these bounds (for instance, a best value for price could be \$5000).

Once the first-level index values have been obtained, the final composite index values must be determined for each alternative. Two methods can be used to obtain these final index values: arithmetic averaging and fuzzy set theory. In arithmetic averaging<sup>1-4</sup> the first-level index values for each criterion are multiplied by the corresponding weighting factors and then added together. Mathematically this is given as follows:

$$L_h = \sum_i w_i S_i$$

Four values will result for the final composite index value for each alternative, corresponding to a fuzzy set.

Because the arithmetic averaging method may provide results which are dominated by a few high scores in the selection criteria,<sup>1</sup> another method was developed at NASA LeRC to determine the final composite index values. In this method, based on fuzzy set theory,<sup>5,9</sup> the first-level index values are raised to the power of the weighting factors to give weighted rankings. The weighted scores give the degree to which the alternative meets the criterion. Then, according to fuzzy set theory, the minimum of the weighted scores is used for the final composite value. The minimum represents the intersection of the sets, because all the criteria are necessary to the final decision. Mathematically, the composite index value is represented as follows for the fuzzy set method.

$$L_h = \min \left( S_i^{w_i} \right)$$

As in the arithmetic averaging method, four values will result representing the four corners of the trapezoidal fuzzy set.

Because the fuzzy sets which result from the final composite index values will overlap, a method is required to obtain a discrete score to rank the alternatives. One method uses the maximizing and minimizing set concepts of fuzzy logic as shown in references 1-4. This method is illustrated in Fig. 4. The maximizing set is defined as follows:

$$\begin{aligned} \mu_M &= (L_h - L_{\min}) / (L_{\max} - L_{\min}) && L_{\min} < L_h < L_{\max} \\ \mu_M &= 0 && \text{otherwise} \end{aligned}$$

The maximizing set intersects the trapezoidal fuzzy set for each alternative in two places, as shown in Fig. 4. The right utility value,  $U_R$ , is the largest of these two intersec-

tion values. In a similar manner the minimizing fuzzy set is calculated as follows:

$$\mu_G = (L_h - L_{\max}) / (L_{\min} - L_{\max}) \quad L_{\min} < L_h < L_{\max}$$

$$\mu_G = 0 \quad \text{otherwise}$$

The value for the left utility value,  $U_L$ , is then determined from the maximum of the two intersection points between the minimizing set and the fuzzy set for the alternative. The ranking value, or total utility value, for each alternative is then calculated as follows:

$$U = \frac{U_R + 1 - U_L}{2}$$

Because the decision-making calculations may become tedious, especially if the analysis includes many alternatives and criteria, software was developed at NASA LeRC to automate the process. This software, called DECISION MANAGER, was developed for a Microsoft Windows operating environment and runs on a system with a 80386 or higher microprocessor. The software currently has the capability for 16 alternatives and 28 criteria. The initial screen layout for DECISION MANAGER is shown in Fig. 5.

To begin an analysis in DECISION MANAGER the user selects the *Define Alternatives* option on the screen by clicking on the button with a mouse. A new screen will then be displayed which allows the user to enter a 10-character name for each alternative followed by a detailed description of that alternative. When the user has finished defining all the alternatives the criteria must then be specified. This is done by clicking on the *Define Criteria* button. Again, a new screen will be displayed where the user enters the name of each criterion, the criterion description, best score, and worst score. In addition, the user can choose to enter a weight for each criterion or use the Analytic Hierarchy Process to determine the weights. Once the alternatives and criteria are defined, the raw scores are entered by the clicking on *Input Raw Scores for Alternatives* after choosing the *Raw Score Type* on the initial screen. DECISION MANAGER allows for single values, ranges of values, or normal probabilistic distributions for the raw scores. For the *Range of Values* option most likely ranges and largest likely ranges are input, whereas for the *Normal Distribution* option the mean and standard deviation are entered.

Finally, after the alternative, criteria, and raw scores have been input, the user can determine the preferred alternative by using the arithmetic averaging or the fuzzy set scoring method. By clicking on either of the buttons a new screen showing the preferred order of alter-

natives will be displayed. DECISION MANAGER also allows the user to see the effect of changing the weighting factors on the criteria by displaying the preferred order of alternatives when all the criteria have the same importance. In addition, when the fuzzy set method is chosen DECISION MANAGER displays the limiting criterion for each alternative. Once the analysis is complete the user may choose to save the data by selecting the *File* menu on the initial screen and choosing the *Save As* option.

#### Example Application: Lunar Lander Propulsion System

Concepts are currently being considered for the ascent propulsion system of a lunar lander vehicle. In a trade study performed by NASA Johnson Space Center in 1993, fourteen options were examined using thirty-one different criteria.<sup>10</sup> The JSC study employed the Analytic Hierarchy Process to select the best propulsion system. In the AHP method used in the JSC study, paired comparisons were made not only to determine the weighting values of the criteria, but also for comparing options against each other. Details of this technique are provided in reference 8. In the present study fuzzy logic methods were employed to determine the best alternative among the fourteen options, and these results were compared against those obtained from the original JSC study. In the present study only the top 15 criteria were used in the evaluation to simplify the example. In addition, the present study accounted for uncertainty in the criteria values; uncertainty was not considered in the original analysis.

The lunar lander ascent propulsion options are shown in Table 1 and the selection criteria are provided in Table 2. The propulsion options were based on propellant type and configuration (such as pump-fed versus pressure-fed). The relative weights for the selection criteria were obtained from reference 10. The raw scores for the alternatives are provided in Table 3. For quantitative criteria such as numbers of components, number of operations, number of instrumentation locations, abort response time, and number of subsystems (criteria A-D, G, H, I, O), ranges of values were used on the basis of data provided in reference 10 and judgements made by this author. The values for other criteria were obtained through qualitative assessments in the original study. For instance, hardware readiness was rated on a scale of 1 to 9, with 9 meaning *excellent*. In the current study no uncertainty was considered for these qualitative criteria, and only the discrete values from the JSC study were used. Therefore, the present study combined uncertain quantitative data with discrete qualitative data.

The results from the analysis are provided in Table 4. The results of the arithmetic averaging method

showed some differences in comparison to the original JSC study. The "ClF<sub>5</sub>/N<sub>2</sub>H<sub>4</sub> Both Stages" system was preferred in the arithmetic averaging method, whereas "N<sub>2</sub>O<sub>4</sub>/MMH" system was preferred for the Analytic Hierarchy Process. In addition, the arithmetic averaging method showed the "LO<sub>2</sub>/LH<sub>2</sub> Pressure" option to be much higher in the order of preferences. Although the order was somewhat different, both methods resulted in the same top three alternatives. Differences in order were primarily the result of the method used to convert the raw scores to index values. The AHP uses paired comparisons between criteria values to obtain these index values, whereas the arithmetic weighting method normalizes the raw scores on the basis of best and worst values. The inclusion of uncertainty in the raw scores also affected the final score; the AHP is limited to discrete values.

When the fuzzy set scoring method was used, the "LO<sub>2</sub>/LH<sub>2</sub> Pressure" option gave the highest final score. Examination of the data showed that the number of flight operations was the limiting factor (the factor which gave the minimum score) for most of the options in determining the final scores by using the fuzzy set scoring method. Because this criterion had a high weight relative to many of the other factors, the fuzzy set scoring method emphasized this criterion. Although the "ClF<sub>5</sub>/N<sub>2</sub>H<sub>4</sub> Both Stages" system had fewer flight operations than the "LO<sub>2</sub>/LH<sub>2</sub> Pressure" system, the ascent engine hardware readiness criterion was the limiting factor for the "ClF<sub>5</sub>/N<sub>2</sub>H<sub>4</sub> Both Stages" system.

Examining the results in Table 4 clearly shows that different methods can provide different results. In these cases the decision-maker must examine the information provided by each method. For instance, the fuzzy set scoring method uses the minimum values of the weighted score, thus emphasizing the attributes which can hinder future development. This method, however, does not include all attributes as does the arithmetic averaging method. As shown in reference 1, the arithmetic averaging method emphasizes the attributes which are good, but this method can also ignore attributes which can negatively affect design. Therefore, in making a decision under uncertainty, both arithmetic averaging and fuzzy set scoring methods should be used.

#### Example Application: Cuyahoga River Aeration Options

Recent studies have been conducted in the Cuyahoga River in Cleveland, Ohio to examine the impairments to the use of the river.<sup>11</sup> One key finding of these studies was that low levels of dissolved oxygen (DO) exist in the Cuyahoga River. This problem is most severe in the navigation channel, the last 5.6 miles of the river before it empties into Lake Erie. Low levels of dis-

solved oxygen can lead to reduced quantity and variety of aquatic life. Several factors enter into the low DO levels, including periodic dredging of the river to allow ship navigation. Because of the increased depth of the river following dredging, the Cuyahoga River flows at a rate slower than what would occur naturally, thus reducing the natural aeration in the river. Therefore, dredging is seen as a significant factor in the low oxygen levels in the Cuyahoga River. Options are currently being examined to improve the dissolved oxygen levels in the navigation channel.

One potential option for increasing the oxygen levels is artificial aeration of the navigation channel. On the basis of this previous work and the physical characteristics of the Cuyahoga River, submerged aeration appears to be the most feasible near-term option for the reaeration of the river.<sup>12,13</sup> In this study five submerged aeration systems were examined by using fuzzy logic techniques to determine the optimum system for the Cuyahoga River. Diagrams of these systems are provided in Fig. 6. Both fuzzy set and arithmetic averaging scoring methods were used to assess these systems.

The criteria for evaluating the aeration systems are provided in Table 5. The weighting factors for the criteria were obtained from a previous study which examined aeration options without consideration of uncertainty.<sup>14</sup> The raw scores for each of the options are shown in Table 6. The transfer efficiency, initial cost, and operating cost were considered to be quantitative but uncertain. Estimates of these parameters were obtained from the literature.<sup>12,13,15</sup> The other criteria were qualitative in nature. For instance, the coarse and fine diffusers have low potential for interfering with navigation, and therefore receive a score of 0.8, whereas the sparge turbine aerator will have extreme interference with navigation and therefore receive a score of 0. (It should be noted that in any evaluation where values are assigned to linguistic descriptions, the score of zero should be reserved for extreme cases, such as in the case of the sparge turbine aerator. A score of zero implies that the alternative cannot meet the minimum requirements to be considered in the evaluation.)

The results of the analysis are provided in Table 6 for both the arithmetic averaging and the fuzzy set scoring methods. As shown in the table, the preferred option appeared to be the coarse diffusion aeration system. The results did not depend on the scoring method, giving high confidence in the preferred order of alternatives calculated. In the fuzzy set analysis the limiting criteria was the transfer efficiency for the coarse bubble diffuser, the operating cost for the static mixer and fine bubble diffuser, and navigation interference for the jet aerator and

sparge turbine aerator. Note that although the analysis showed that the scoring method did not affect the results, the criteria weights will impact the preferred order of selection. If all the criteria are assumed to have equal weights, the fine bubble aerator and jet aerator will provide the highest score, followed by the coarse bubble diffuser, static mixer, and sparge turbine aerator. Therefore, the selection of the aeration system is highly sensitive to the relative importance of the criteria in this case.

### Concluding Remarks

A study was performed to demonstrate the use of fuzzy logic techniques to assist in decision-making under uncertainty. Such situations occur often in engineering applications, especially in cases where a statistical database does not exist for the criteria or where the criteria themselves are qualitative in nature. This paper described methods from the literature and those developed at NASA Lewis Research Center to assist in selecting the best alternative when the inputs are vague or qualitative. Examples were provided to illustrate the use of the methods not only for aerospace disciplines but also for civil engineering applications. In addition, the DECISION MANAGER software developed at NASA Lewis Research Center was described. DECISION MANAGER automates the decision-making process, allowing for rapid comparison of alternatives.

The fuzzy logic techniques described by this report provide powerful tools for the evaluation of various alternatives under uncertainty. Fuzzy logic allows for quantifying vague or qualitative criteria, a common occurrence in engineering evaluations. It is important, however, in any evaluation that multiple techniques be used in arriving at a final decision to assure that the best decision has been made. Therefore, two scoring methods were described in this report: arithmetic averaging and fuzzy set methods. As demonstrated in the examples, differences in the order of alternatives may occur when the results of the scoring methods are compared, providing the decision-maker with further insight into the selection process. The techniques illustrated by this report do not replace engineering judgement. However, the fuzzy logic methods described here provide a systematic approach to the often difficult process of decision-making in an uncertain environment.

### Acknowledgment

The author wishes to thank Peter Rutledge of NASA Headquarters for his support of this effort.

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**TABLE 1.- LUNAR LANDER ASCENT PROPULSION OPTIONS.**

Number	Description
1	N <sub>2</sub> O <sub>4</sub> /MMH Baseline
2	LO <sub>2</sub> /N <sub>2</sub> H <sub>4</sub>
3	ClF <sub>3</sub> /N <sub>2</sub> H <sub>4</sub>
4	N <sub>2</sub> O <sub>4</sub> /MMH Optimized
5	LO <sub>2</sub> /CH <sub>4</sub> Pressure
6	N <sub>2</sub> O <sub>4</sub> /MMH Pump
7	LO <sub>2</sub> /CH <sub>4</sub> Pump
8	LO <sub>2</sub> /LH <sub>2</sub> Pump
9	LO <sub>2</sub> /LH <sub>2</sub> Single Stage
10	LO <sub>2</sub> /LH <sub>2</sub> 1½ Stage
11	ClF <sub>3</sub> /N <sub>2</sub> H <sub>4</sub> Both Stages
12	LO <sub>2</sub> /LH <sub>2</sub> IME 2 Stage
13	LO <sub>2</sub> /LH <sub>2</sub> Pressure
14	LO <sub>2</sub> /LH <sub>2</sub> IME 1½ Stage

**TABLE 2.- LUNAR LANDER ASCENT PROPULSION SELECTION CRITERIA.**

Designation	Criterion Description	Weight
A	Total number of components	.1148
B	Number of abort operations	.0987
C	Number of flight operations	.0987
D	Number of instrumentation locations	.0963
E	Ascent engine hardware readiness	.0894
F	Descent engine hardware readiness	.0894
G	Number of return engine components	.0608
H	Number of unique components	.0608
I	Abort response time	.0573
J	Descent stage launch operations index	.0470
K	Ascent stage launch operations index	.0470
L	Redundancy	.0390
M	Readiness of ascent pressurization/tank/feed	.0344
N	Readiness of descent pressurization/tank/feed	.0344
O	Number of subsystems	.0321

TABLE 3.- RAW SCORES FOR LUNAR LANDER PROPULSION OPTIONS.

Criterion	Option													
	1		2		3		4		5		6		7	
	Most Likely Interval	Largest Likely Interval	Most Likely Interval	Largest Likely Interval	Most Likely Interval	Largest Likely Interval	Most Likely Interval	Largest Likely Interval	Most Likely Interval	Largest Likely Interval	Most Likely Interval	Largest Likely Interval	Most Likely Interval	Largest Likely Interval
A	490-542	464-568	447-494	423-517	437-483	414-506	437-483	414-506	456-504	432-528	658-728	624-762	666-736	631-771
B	3-5	3-5	4-6	4-6	3-5	3-5	3-5	3-5	4-6	4-6	7-9	7-9	7-9	7-9
C	62-66	60-68	67-71	65-73	62-66	60-68	62-66	60-68	69-73	67-75	83-87	81-89	83-87	81-89
D	211-233	200-244	180-200	171-209	175-193	166-202	175-193	166-202	191-211	181-221	263-291	249-305	278-308	264-322
E	9	9	3.75	3.75	3.25	3.25	4	4	3.75	3.75	3.5	3.5	3.6	3.6
F	7	7	7	7	7	7	7	7	7	7	7	7	7	7
G	133-147	126-154	95-105	90-100	86-95	81-99	86-95	81-99	105-116	99-121	307-339	291-355	314-348	298-364
H	104-114	98-120	115-127	109-133	104-114	98-120	104-114	98-120	111-123	105-129	124-137	117-143	122-134	115-141
I	0-.5	0-.5	0-.5	0-.5	0-.5	0-.5	0-.5	0-.5	0-.5	0-.5	1.3-1.7	1.3-1.7	1-1.5	1-1.5
J	.44	.44	.44	.44	.44	.44	.44	.44	.44	.44	.44	.44	.44	.44
K	.66	.66	.59	.59	.65	.65	.66	.66	.62	.62	.61	.61	.51	.51
L	.644	.644	.644	.644	.644	.644	.644	.644	.644	.644	.644	.644	.644	.644
M	7	7	7	7	3.25	3.25	7	7	6.3	6.3	7	7	6.3	6.3
N	7	7	7	7	7	7	7	7	7	7	7	7	7	7
O	10-12	10-12	11-13	11-13	10-12	10-12	10-12	10-12	11-13	11-13	11-13	11-13	12-14	12-14

Criterion	Option													
	8		9		10		11		12		13		14	
	Most Likely Interval	Largest Likely Interval	Most Likely Interval	Largest Likely Interval	Most Likely Interval	Largest Likely Interval	Most Likely Interval	Largest Likely Interval	Most Likely Interval	Largest Likely Interval	Most Likely Interval	Largest Likely Interval	Most Likely Interval	Largest Likely Interval
A	714-790	677-828	410-454	389-475	418-462	396-484	216-238	204-250	368-406	348-426	442-489	419-513	262-290	248-304
B	11-13	11-13	7-9	7-9	6-8	6-8	3-5	3-5	6-8	6-8	5-7	5-7	7-9	7-9
C	88-92	86-94	87-91	85-93	88-92	86-94	24-28	22-30	85-89	83-91	56-60	54-62	84-88	82-90
D	291-321	275-337	198-218	187-229	198-218	187-229	90-100	85-105	280-310	266-325	211-233	200-244	205-227	194-238
E	7	7	4.8	4.8	4.8	4.8	3.25	3.25	2.1	2.1	4	4	2.1	2.1
F	7	7	9	9	9	9	3.25	3.25	2.1	2.1	7	7	9	9
G	363-401	344-420	346-382	328-400	357-395	338-414	86-95	81-99	172-190	163-199	71-79	68-83	146-162	139-169
H	107-119	102-124	76-84	72-88	82-90	77-95	61-67	57-70	91-101	86-106	103-113	97-119	64-70	60-74
I	.5-1.5	.5-1.5	0-.5	0-.5	1-1.5	1-1.5	0-.5	0-.5	1-1.5	1-1.5	0-.5	0-.5	1-1.5	1-1.5
J	.44	.44	.42	.42	.41	.41	.63	.63	.58	.58	.44	.44	.59	.59
K	.48	.48	.71	.71	.75	.75	.65	.65	.6	.6	.59	.59	.78	.78
L	.644	.644	.085	.085	.085	.085	.644	.644	.644	.644	.644	.644	.085	.085
M	7	7	7	7	7	7	3.25	3.25	3	3	7	7	3	3
N	7	7	9	9	7	7	3.25	3.25	3	3	7	7	6	6
O	14-16	14-16	6-8	6-8	6-8	6-8	7-9	7-9	11-13	11-13	10-12	10-12	5-7	5-7

TABLE 4.- PREFERRED ORDER OF LUNAR LANDER PROPULSION ALTERNATIVES.

Analytic Hierarchy Process (Original JSC Study)		Fuzzy logic/ Arithmetic averaging scoring		Fuzzy logic/ Fuzzy set scoring	
Option	Score	Option	Score	Option	Score
N <sub>2</sub> O <sub>4</sub> /MMH Optimized	.756	ClF <sub>5</sub> /N <sub>2</sub> H <sub>4</sub> Both Stages	.904	LO <sub>2</sub> /LH <sub>2</sub> Pressure	.963
N <sub>2</sub> O <sub>4</sub> /MMH Baseline	.739	N <sub>2</sub> O <sub>4</sub> /MMH Baseline	.826	ClF <sub>5</sub> /N <sub>2</sub> H <sub>4</sub> Both Stages	.897
ClF <sub>5</sub> /N <sub>2</sub> H <sub>4</sub> Both Stages	.733	N <sub>2</sub> O <sub>4</sub> /MMH Optimized	.755	N <sub>2</sub> O <sub>4</sub> /MMH Baseline	.881
ClF <sub>5</sub> /N <sub>2</sub> H <sub>4</sub>	.693	LO <sub>2</sub> /LH <sub>2</sub> Pressure	.698	N <sub>2</sub> O <sub>4</sub> /MMH Optimized	.881
LO <sub>2</sub> /N <sub>2</sub> H <sub>4</sub>	.653	ClF <sub>5</sub> /N <sub>2</sub> H <sub>4</sub>	.692	ClF <sub>5</sub> /N <sub>2</sub> H <sub>4</sub>	.866
IME LO <sub>2</sub> /LH <sub>2</sub> 1½ Stage	.595	LO <sub>2</sub> /N <sub>2</sub> H <sub>4</sub>	.674	LO <sub>2</sub> /N <sub>2</sub> H <sub>4</sub>	.801
LO <sub>2</sub> /CH <sub>4</sub> Pressure	.580	LO <sub>2</sub> /CH <sub>4</sub> Pressure	.652	LO <sub>2</sub> /CH <sub>4</sub> Pressure	.766
LO <sub>2</sub> /LH <sub>2</sub> Pressure	.552	LO <sub>2</sub> /LH <sub>2</sub> Single Stage	.568	N <sub>2</sub> O <sub>4</sub> /MMH Pump	.433
LO <sub>2</sub> /LH <sub>2</sub> Single Stage	.515	IME LO <sub>2</sub> /LH <sub>2</sub> 1½ Stage	.520	LO <sub>2</sub> /CH <sub>4</sub> Pump	.433
LO <sub>2</sub> /LH <sub>2</sub> 1½ Stage	.481	LO <sub>2</sub> /LH <sub>2</sub> 1½ Stage	.466	IME LO <sub>2</sub> /LH <sub>2</sub> 1½ Stage	.400
N <sub>2</sub> O <sub>4</sub> /MMH Pump	.436	N <sub>2</sub> O <sub>4</sub> /MMH Pump	.213	IME LO <sub>2</sub> /LH <sub>2</sub> 2 Stage	.366
IME LO <sub>2</sub> /LH <sub>2</sub> 2 Stage	.420	LO <sub>2</sub> /CH <sub>4</sub> Pump	.196	LO <sub>2</sub> /LH <sub>2</sub> Single Stage	.289
LO <sub>2</sub> /LH <sub>2</sub> Pump	.407	IME LO <sub>2</sub> /LH <sub>2</sub> 2 Stage	.187	LO <sub>2</sub> /LH <sub>2</sub> Pump	.246
LO <sub>2</sub> /CH <sub>4</sub> Pump	.350	LO <sub>2</sub> /LH <sub>2</sub> Pump	.176	LO <sub>2</sub> /LH <sub>2</sub> 1½ Stage	.246

TABLE 5.- AERATION SYSTEM SELECTION CRITERIA.

Criterion	Weighting Factor
Transfer Efficiency	.091
Initial Cost	.212
Operating Cost	.212
Clogging Potential	.152
Navigation Interference	.273
Mixing Capability	.030
Icing Potential	.030

TABLE 6.- RAW SCORES FOR AERATION SYSTEM OPTIONS.

Criterion	Option									
	Coarse		Fine		Static		Sparge		Jet	
	Most Likely Interval	Largest Likely Interval	Most Likely Interval	Largest Likely Interval	Most Likely Interval	Largest Likely Interval	Most Likely Interval	Largest Likely Interval	Most Likely Interval	Largest Likely Interval
Transfer eff. (lb/hp-hr)	.9-1.1	.7-1.2	1.6-2.0	1.4-2.5	.9-1.1	.7-1.2	1.2-1.5	1-1.7	1.6-2.2	1.4-3
Initial Cost (\$/1000)	200-300	150-500	200-300	150-500	200-300	150-500	200-300	150-500	550-700	400-800
Operating Cost (\$/1000)	120-180	100-220	500-600	400-700	500-600	400-700	500-600	400-700	150-300	100-400
Clogging Potential	.5	.5	.2	.2	1	1	.5	.5	.8	.8
Navigation Interference	.8	.8	.8	.8	.5	.5	0	0	.2	.2
Mixing Capability	.5	.5	.5	.5	.5	.5	.8	.8	.8	.8
Icing Potential	1	1	1	1	.8	.8	.5	.5	.8	.8

TABLE 7.- PREFERRED ORDER OF AERATION SYSTEM OPTIONS.

Fuzzy logic/ Arithmetic averaging scoring		Fuzzy logic/ Fuzzy set scoring	
Option	Score	Option	Score
Coarse	.883	Coarse	.973
Static	.662	Static	.915
Fine	.623	Fine	.887
Jet	.468	Jet	.734
Sparge	.213	Sparge	.000

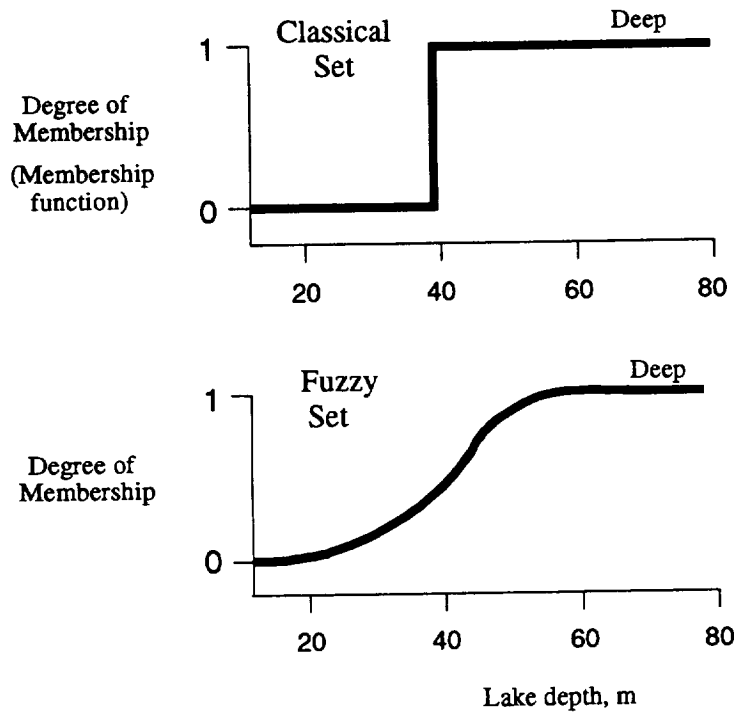


Figure 1.- Comparison of a classical set with a fuzzy set for the set of deep lakes.

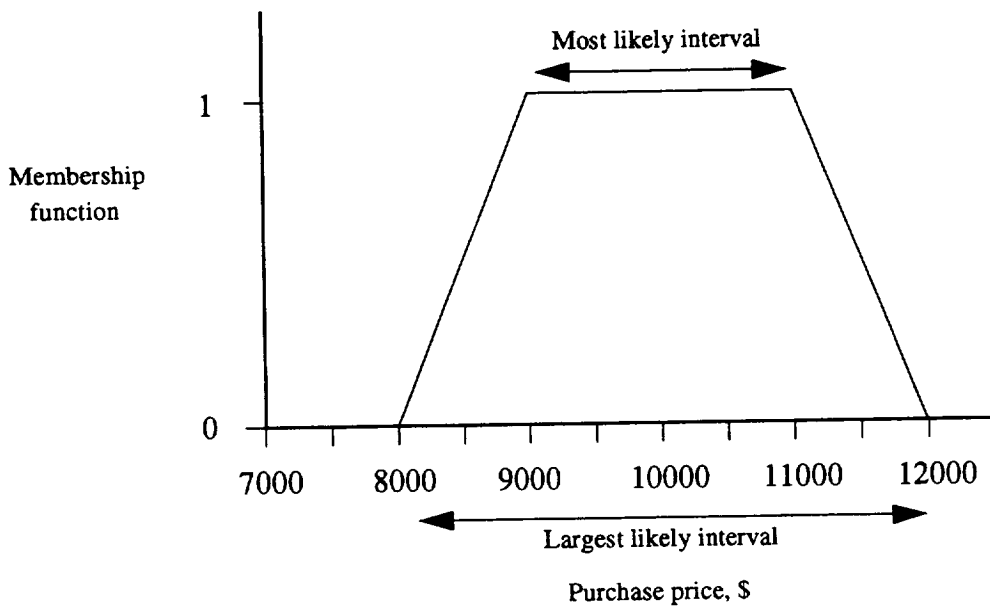


Figure 2.- Trapezoidal fuzzy set for automobile purchase price.

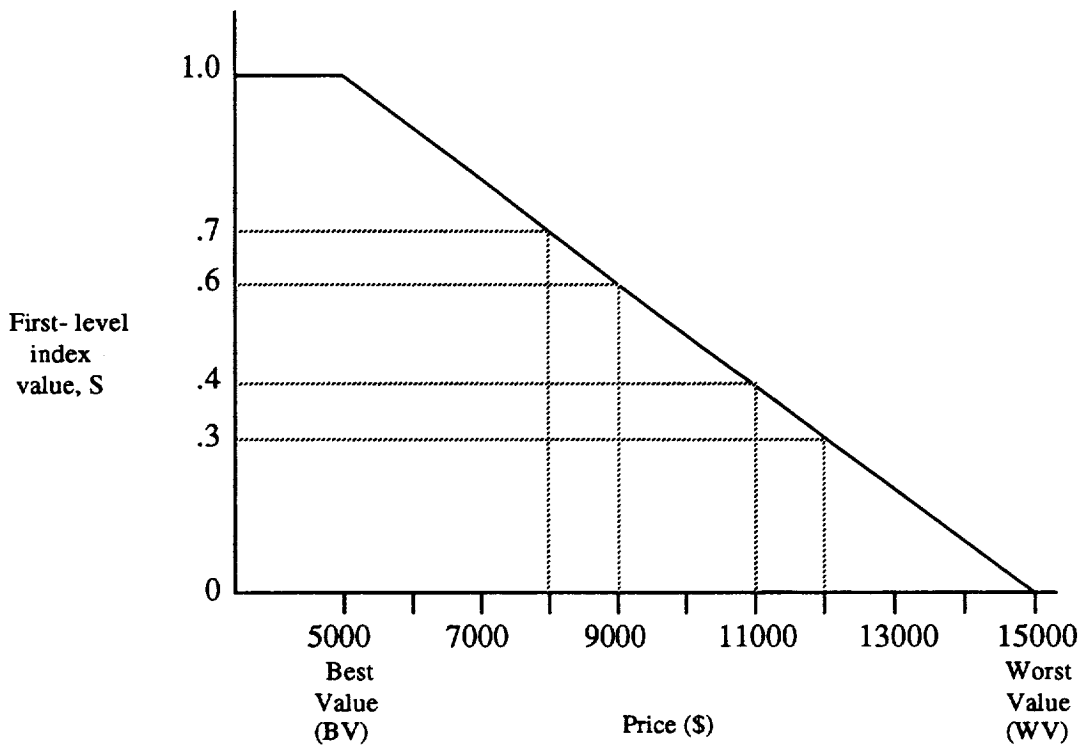


Figure 3.- Example of transforming raw score to first-level index value.

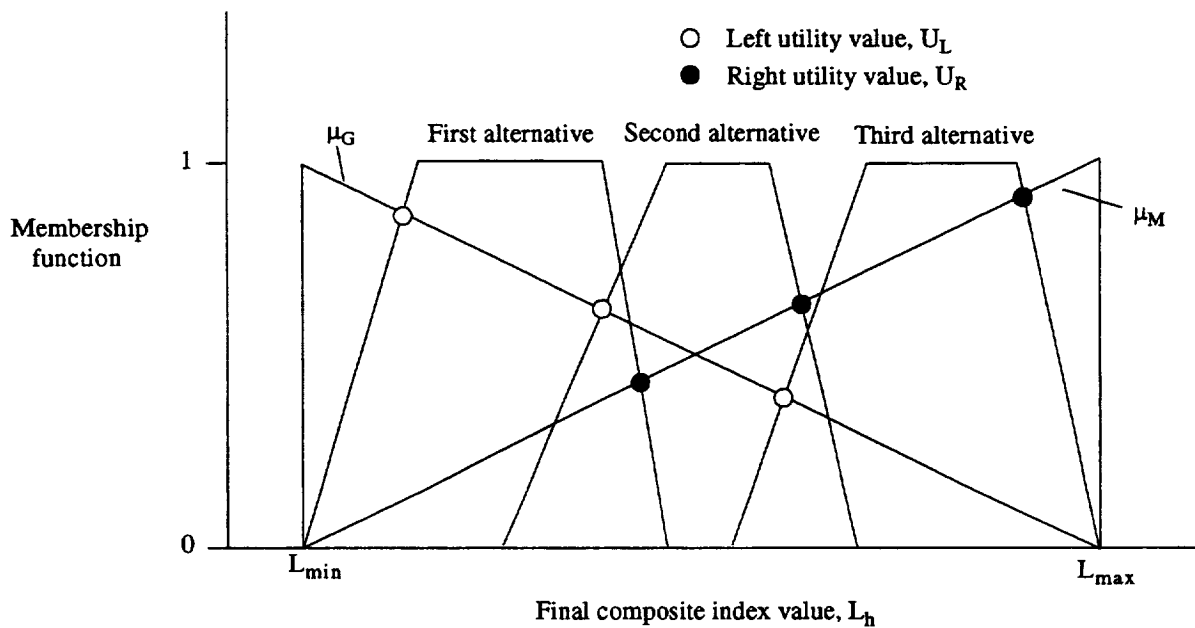


Figure 4.- Ranking method of fuzzy sets.

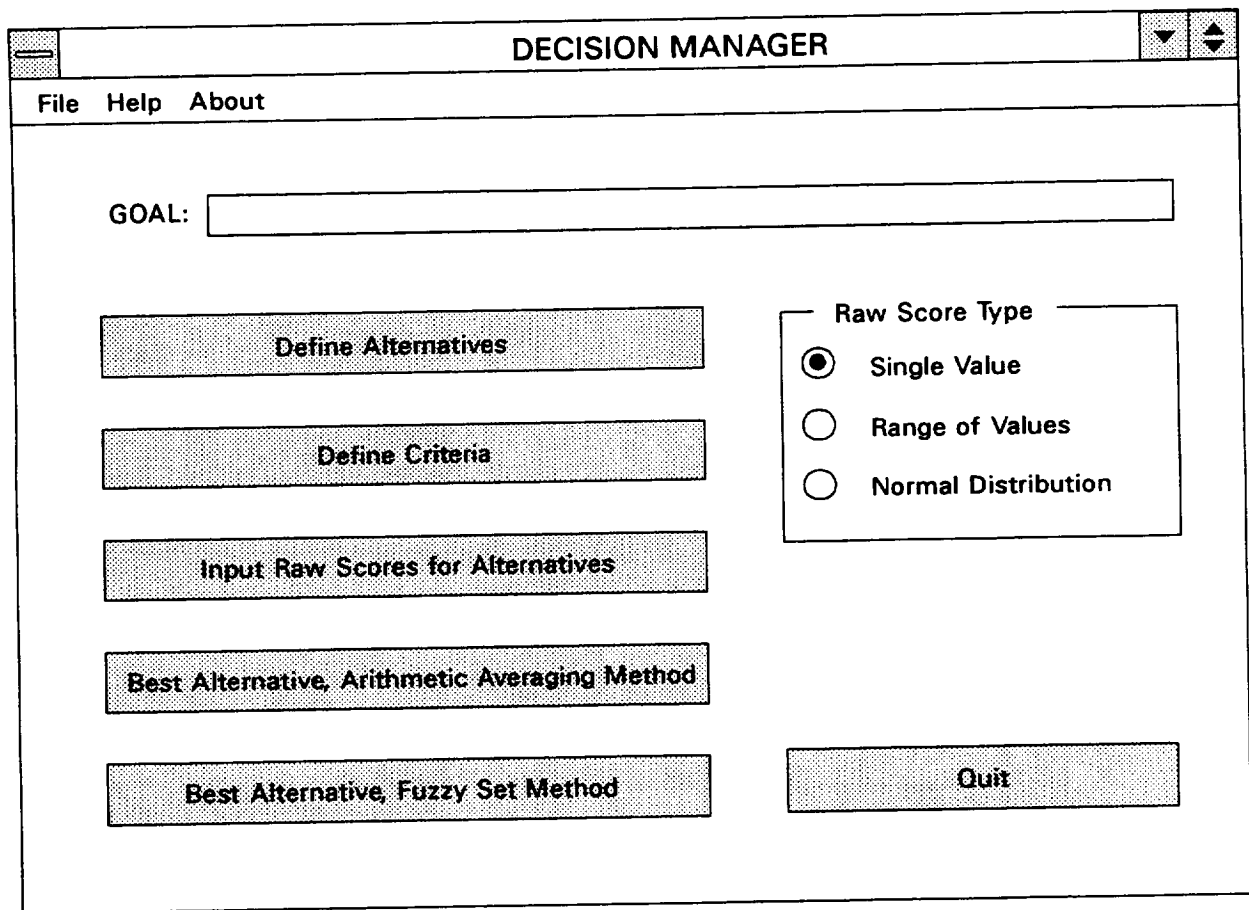


Figure 5.- DECISION MANAGER software, initial screen layout.

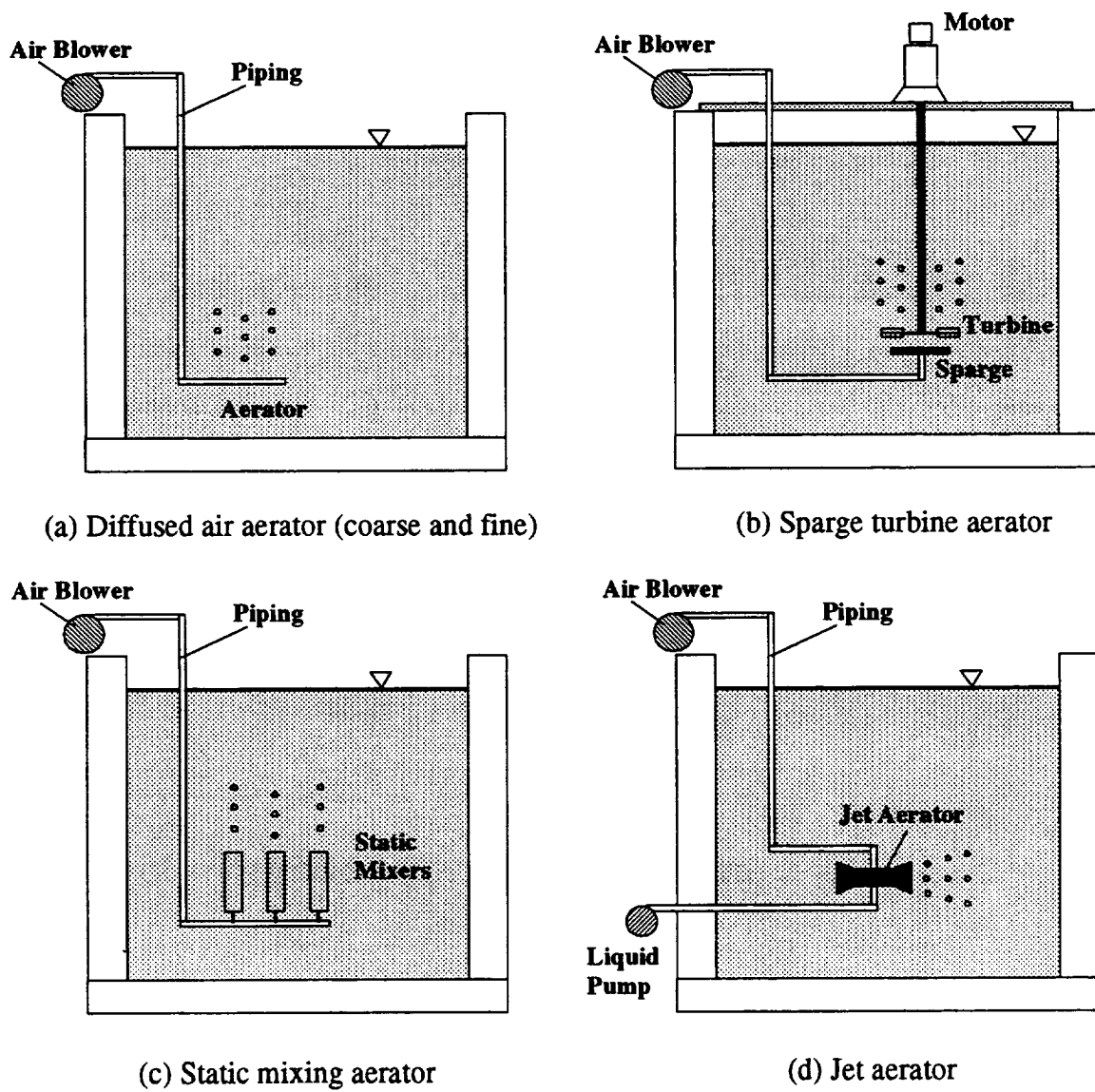


Figure 6.- Aeration system options.





# REPORT DOCUMENTATION PAGE

*Form Approved*  
OMB No. 0704-0188

Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.

<b>1. AGENCY USE ONLY</b> <i>(Leave blank)</i>	<b>2. REPORT DATE</b> December 1994	<b>3. REPORT TYPE AND DATES COVERED</b> Technical Memorandum	
<b>4. TITLE AND SUBTITLE</b> Multi-Objective Decision-Making Under Uncertainty: Fuzzy Logic Methods		<b>5. FUNDING NUMBERS</b>  WU-323-41-22	
<b>6. AUTHOR(S)</b> Terry L. Hardy			
<b>7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)</b>  National Aeronautics and Space Administration Lewis Research Center Cleveland, Ohio 44135-3191		<b>8. PERFORMING ORGANIZATION REPORT NUMBER</b>  E-9263	
<b>9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)</b>  National Aeronautics and Space Administration Washington, D.C. 20546-0001		<b>10. SPONSORING/MONITORING AGENCY REPORT NUMBER</b>  NASA TM-106796	
<b>11. SUPPLEMENTARY NOTES</b> Prepared for the Computing in Aerospace 10 Meeting sponsored by the American Institute of Aeronautics and Astronautics, San Antonio, Texas, March 28-30, 1995. Responsible person, Terry L. Hardy, organization code 5320, (216) 433-7517.			
<b>12a. DISTRIBUTION/AVAILABILITY STATEMENT</b>  Unclassified - Unlimited Subject Category 20  This publication is available from the NASA Center for Aerospace Information, (301) 621-0390.		<b>12b. DISTRIBUTION CODE</b>	
<b>13. ABSTRACT</b> <i>(Maximum 200 words)</i>  Selecting the best option among alternatives is often a difficult process. This process becomes even more difficult when the evaluation criteria are vague or qualitative, and when the objectives vary in importance and scope. Fuzzy logic allows for quantitative representation of vague or fuzzy objectives, and therefore is well-suited for multi-objective decision-making. This paper presents methods employing fuzzy logic concepts to assist in the decision-making process. In addition, this paper describes software developed at NASA Lewis Research Center for assisting in the decision-making process. Two diverse examples are used to illustrate the use of fuzzy logic in choosing an alternative among many options and objectives. One example is the selection of a lunar lander ascent propulsion system, and the other example is the selection of an aeration system for improving the water quality of the Cuyahoga River in Cleveland, Ohio. The fuzzy logic techniques provided here are powerful tools which complement existing approaches, and therefore should be considered in future decision-making activities.			
<b>14. SUBJECT TERMS</b> Fuzzy logic; Uncertainty; Decision making		<b>15. NUMBER OF PAGES</b> 16	
		<b>16. PRICE CODE</b> A03	
<b>17. SECURITY CLASSIFICATION OF REPORT</b> Unclassified	<b>18. SECURITY CLASSIFICATION OF THIS PAGE</b> Unclassified	<b>19. SECURITY CLASSIFICATION OF ABSTRACT</b> Unclassified	<b>20. LIMITATION OF ABSTRACT</b>

National Aeronautics and  
Space Administration

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