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TDRS ORBIT DETERMINATION BY RADIO INTERFEROMETRY

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ABSTRACT

In support of a NASA study on the application of radio interferometry to satellite orbit determination, MITRE developed a simulation tool for assessing interferometric tracking accuracy. The Orbit Determination Accuracy Estimator (ODAE) models the general batch maximum likelihood orbit determination algorithms of the Goddard Trajectory Determination System (GTDS) with the group and phase delay measurements from radio interferometry. ODAE models the statistical properties of tracking error sources, including inherent observable imprecision, atmospheric delays, clock offsets, station location uncertainty, and measurement biases, and through Monte Carlo simulation, ODAE calculates the statistical properties of errors in the predicted satellite state vector. This paper presents results from ODAE application to orbit determination of the Tracking and Data Relay Satellite (TDRS) by radio interferometry. Conclusions about optimal ground station locations for interferometric tracking of TDRS are presented, along with a discussion of operational advantages of radio interferometry.

INTRODUCTION

As part of its effort to assess cost and performance benefits of various emerging technologies, NASA sponsored a series of studies on the application of radio interferometry to satellite tracking. Though astronomers had applied radio interferometry to astrometry for decades prior, it was not until the late 1960s that interferometry was proposed for use in satellite orbit determination. In an experiment devised by Irwin Shapiro, Alan Whitney, and others, very long baseline interferometric (VLBI) measurements were made on the TACSAT I communications satellite in geosynchronous orbit (GEO), and the semi-major axis of the orbit was measured with accuracy on the order of several hundred meters [1]. Subsequent experiments performed in the 1980s by Jim Ray, Curt Knight, and others to determine the position of the Tracking and Data Relay Satellite (TDRS) yielded accuracy on the order of 75 meters [2]. Such orbit determination accuracy, which derives from the extremely high precision of the group delay and phase delay observables, makes radio interferometry an attractive option for satellite tracking.

Operational considerations are also a benefit of radio interferometry in satellite orbit determination, because the group and phase delay measurements are made completely passively. Whereas the existing Bilateral Ranging Transponder System (BRTS) is taxing on TDRS communications resources, radio interferometry can derive its measurements from any signal, including the signal intended for the TDRS user community. Therefore, an interferometric orbit determination system for TDRS would eliminate traffic for tracking on the TDRS transponder. Because an interferometric tracking system would be passive, it would place no design constraints on the space segment, and it would therefore provide backward compatibility with all generations of TDRS. These potential operational and accuracy benefits led NASA to investigate radio interferometry for future TDRS tracking applications.

NASA sponsored a series of studies to investigate whether an operational radio interferometry system could provide TDRS orbit determination services (1) at lower cost, (2) at greater accuracy, and (3) across considerably smaller baselines than BRTS. Contributors to these studies included Interferometrics, Inc., where a Small Business Innovative Research (SBIR) contract was executed to demonstrate hardware and software that would provide group delay measurements on TDRS with VLBI. CSC performed an assessment for the Goddard Space Flight Center (GSFC) on a variety of TDRS tracking alternatives, including VLBI and Connected Element Interferometry (CEI) systems. The Jet Propulsion Laboratory (JPL) sponsored a series of experiments to determine CEI accuracy from its Goldstone facility. For its part of the effort, MITRE assessed optimal site locations and programmatic considerations of an operational interferometric TDRS orbit determination system.

For accuracy assessment purposes, MITRE developed a Monte Carlo simulation tool, the Orbit Determination Accuracy Estimator (ODAE), that models error sources in orbit determination with VLBI and CEI systems. In ODAE, the user can specify a satellite orbit, any set of ground stations between which group or phase delay measurements are to be made, and the statistical properties of the errors in those measurements. Upon each iteration of the Monte Carlo simulation, the orbit of the satellite is determined based on measurements with errors added, and the errors in the resulting satellite ephemerides are recorded. Thus, the user may study the statistical properties of the error in the batch orbit determination process resulting from the use of group or phase delay measurements.

We applied ODAE to study the effects of varying satellite and measuring station geometries on orbit determination accuracy. This paper presents an assessment of optimal siting for TDRS tracking by radio interferometry. A discussion of the operational and programmatic considerations of an interferometric tracking system are also presented.

THE ODAE MODEL

ODAE, which was implemented in Mathematica to allow maximum flexibility, models the batch maximum likelihood orbit determination process applied in the Goddard Trajectory Determination System (GTDS) [3]. The user specifies a reference true satellite orbit, a set of observing stations (earth-based or space-based), the observation types, and the times at which measurements are to be made. Given a set of observations on the satellite (e.g., radar measurements, group or phase delay measurements, or pseudorange measurements), ODAE determines the set of parameters (e.g., state vector, clock offsets, or atmospheric parameters) that best fit the observations. Upon each iteration of its Monte Carlo simulation, ODAE injects errors of user-specified statistical properties into various parts of the orbit determination process. ODAE computes the error of the measured parameters at each iteration, and at the end of the simulation, ODAE computes the statistical characteristics of the error.

Error sources that can be modeled by ODAE include inherent measurement imprecision, station location uncertainty, atmospheric delays, and clock offsets. The user must specify the statistical properties of the error sources. Trajectory propagation schemes available in ODAE for dynamic orbit determination range from the two-body approximation to numerical integration of the fully disturbed equations of motion. A detailed mathematical specification of the coordinate frame, force models, and numerical integration techniques used in ODAE are given in Reference 4. The only significant deviation from the GTDS approach to orbit determination is the use of Bulirsch-Stoer rational function extrapolation for numerical integration [5, 6]. For the numerical integration of the equations of satellite motion, the Bulirsch-Stoer technique has been shown to provide the same precision as more traditional techniques, such as predictor-corrector integration or Runge-Kutta integration, but at reduced computational cost [4, 7].

For short-term dynamic orbit determination accuracy studies, it is often sufficient to apply simplified trajectory propagation schemes for the sake of reducing computation time. Absolute trajectory propagation accuracy is not of concern for the assessment of the relative effects of changes in geometry or measurement errors. For the study on TDRS tracking by radio interferometry, we were concerned only with the effect of ground station geometry on initial orbit determination accuracy, and so dynamics came to play only over the time of signal propagation from the satellite to the tracking stations. Therefore, we applied the two-body approximation for trajectory propagation and state transition matrix computation.

Since its initial application to the problem of optimal ground station siting for interferometric tracking of TDRS, MITRE has applied ODAE to a variety of problems, including an assessment of Space Surveillance Network Improvement Program (SSNIP) tracking accuracy on various classes of orbits, and an assessment of the accuracy of GPS for satellite telemetry, tracking, and command (TT&C).

INTERFEROMETRY OVERVIEW

Consider an interferometric orbit determination scenario in which O is the origin of an earth-centered inertial (ECI) coordinate system, $\mathbf{r}$ is the position vector of a satellite with respect to O, $\mathbf{b}_1$ and $\mathbf{b}_2$ are the position vectors of two ground stations from which measurements are to be made, and $\mathbf{d}_1$ and $\mathbf{d}_2$ are the position vectors of the satellite with respect to those ground stations, as pictured in Figure 1. The position vectors $\mathbf{r}$, $\mathbf{b}_1$, $\mathbf{b}_2$, $\mathbf{d}_1$, and $\mathbf{d}_2$ are all functions of time. The sum of a station position vector, $\mathbf{b}_k$, and the satellite position vector measured from that station, $\mathbf{d}_k$, is simply the satellite position vector $\mathbf{r}$; therefore, $\mathbf{d}_k = \mathbf{r} - \mathbf{b}_k$. If the propagation rate, c, of the signal through the atmosphere is known, then the transit time, T_k, of the signal from the satellite at point P to ground station number k at point B_k will be given by

$$T_k = \frac{1}{c}\left|\mathbf{d}_k\right| = \frac{1}{c}\sqrt{(\mathbf{r}-\mathbf{b}_k)\cdot(\mathbf{r}-\mathbf{b}_k)} \quad (1)$$

Note that in equation (1), the vectors $\mathbf{r}$ and $\mathbf{b}_k$ are measured at slightly different times. Now, the true group delay, τ, between stations i and j is the differential transit time of the signal between these two sites:

$$\tau = T_j - T_i \quad (2)$$

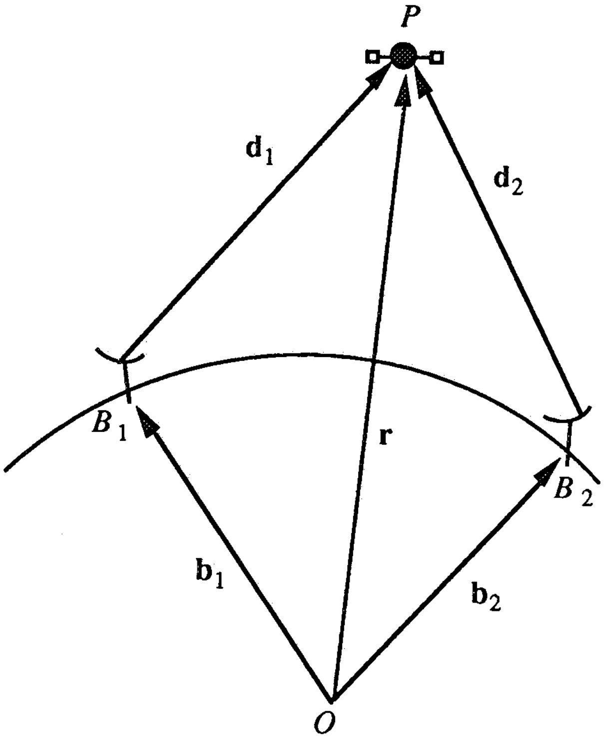

Figure 1. Illustration of the Interferometric Measurement Scenario

During the Monte Carlo simulation, ODAE computes measured group delay by adding measurement or atmospheric fluctuation errors to the true group delay as computed from equations (1) and (2). The solution of the orbit determination problem on each iteration of the simulation, as described in Reference 7, follows the GTDS maximum likelihood estimation approach, one step of which is the computation of the Jacobian, or matrix of partial derivatives of equation (2) with respect to the state vector parameters at epoch.

For phase delay measurements, ODAE converts phase delay into equivalent group delay, as described in Reference 7. This computation can be accomplished so long as the cycle ambiguity can be determined from *a priori* information about the satellite's position vector. ODAE can model both the case where cycle ambiguity is unknown and the case where it is known. We assumed the latter in this study.

ODAE APPLICATION TO TDRS

In this section, we assess the level of orbit determination accuracy that can be attained for a geosynchronous satellite with radio interferometry, and we draw conclusions about optimal station-satellite geometry. The results are applied to recommend optimal ground station siting for orbit determination of TDRS by radio interferometry.

Radio interferometry with baselines the size of BRTS's, which are intercontinental, would translate the high level of observable group delay accuracy into greatly improved TDRS tracking accuracy. However, it was NASA's desire instead to accept only a modest improvement in accuracy while reducing system cost and ameliorating other operational considerations by greatly shortening the baselines. This led naturally to the study of a CEI-based system, where baselines are very short. Because of the requirement for a CEI system to have interferometer sites connected by fiberoptic cable in a temperature-controlled environment, the cost of lengthening baselines is very high. We constrained our baselines to 20 km maximum length for the purposes of this study.

We used ODAE to assess position determination accuracy on a GEO satellite for a sample interferometer siting scenario, and we determined the effects of varying the relative satellite to ground station geometry. Because the effect only of relative geometry was to be studied initially, it was not necessary to select true TDRS ephemerides or true potential ground station locations. The reference orbit chosen was geosynchronous with a 4° inclination and a subsatellite longitude of 18°W. To provide three independent baselines across which phase delay could be measured, we constrained four CEI sites to lie on the vertices of a square with a 20 km baseline, as shown in Figure 2. The site latitudes, longitudes, and altitudes for this reference scenario are given in Table 1. ODAE modeled simultaneous phase delay measurements across the baselines from station 2 to station 1, station 3 to station 1, and station 4 to station 1 (denoted 2-1, 3-1, and 4-1, respectively). These baselines are illustrated in bold in Figure 2.

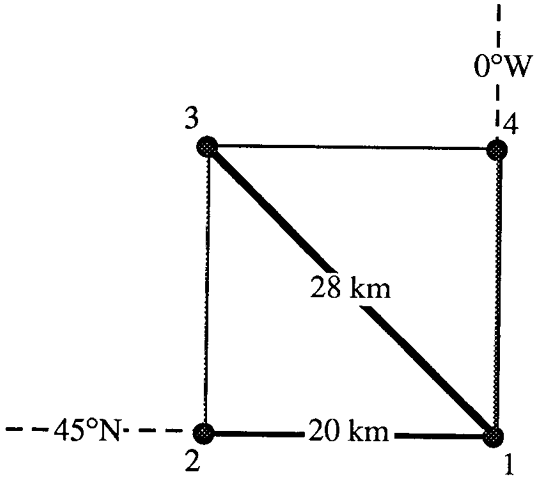

Figure 2. CEI Station Locations

An extension of Alan Whitney's work [8] shows that the theoretically achievable precision of the phase delay observable, σ_ϕ, is given by

$$\sigma_\phi = \frac{1}{2\pi(SNR)\nu} \qquad (3)$$

where ν is the center frequency, in Hz, sampled by the interferometer, and *SNR* is the signal-to-noise ratio. Since the TDRS downlink to White Sands is centered at 14 GHz and $SNR \approx 50$, the theoretically achievable precision of the phase delay observable is 0.23 picosec. While no TDRS tracking experiments were performed with JPL's CEI equipment at Goldstone, observations were made on natural radio sources at 8.4 GHz to assess the precision of the phase delay observable [9, 10]. JPL demonstrated the standard deviation in the phase delay observable to be approximately 1 picosec, which is 70% larger than the theoretically achievable value given by equation (3). Extrapolating this result to the theoretically achievable phase delay precision for TDRS, we estimated the practically achievable precision to be $0.23 \times 1.7 \approx 0.4$ picosec. We took this measurement error to be independently normally distributed across each baseline.

Table 1. CEI Station Locations for Reference Scenario

Station Number	Geodetic Lat. (°N)	Longitude (°E)	Altitude (km)
1	45.00000	0.0000	0.1
2	45.00000	-0.2545	0.1
3	45.17997	0.0000	0.1
4	45.17997	-0.2545	0.1

For the initial study, it was assumed that there were no equipment biases, that there were no atmospheric delay errors, that all station were connected by fiberoptic cable to one clock and frequency standard, that there were no local oscillator offsets between the four stations, and that station positions were known with perfect accuracy. Thus, the pure effect of measurement geometry and observable precision on orbit determination could be assessed.

ODAE Monte Carlo simulation of the orbit determination scenario described above with 200 iterations showed a 1σ root-mean-squared (RMS) position vector accuracy of 3.2 km. We also assessed the accuracy that can be attained with the use of other combinations of baselines. It is practical to have one site in common for all three measurements so that the common site can act as the correlation center at which the phase delay observables are generated. For the particular satellite and ground station locations in this scenario, selection of three measurements where one station is common to each pair (i.e., 2-1, 3-1, 4-1; or 1-2, 3-2, 4-2; or 1-3, 2-3, 4-3; or 1-4, 2-4, 3-4) results in a 1σ RMS position vector accuracy of 3.2 km. Thus, there is no geometrically-preferred common site for the measurements.

The orbit determination scenario described above was the starting point for the assessment of the effects of varying interferometric measurement geometry on orbit determination accuracy. Since only relative geometry matters, and since it would have been more cumbersome to vary the positions of four ground stations, we instead varied the satellite's initial position vector.

First, we studied the effect of relative interferometer baseline size on orbit determination accuracy. Satellite range from station 1 was varied while keeping the elevation angle and azimuth angle from that station constant. Because the baseline sizes are small relative to the range to GEO, the range, elevation angle, and azimuth angle from each of the other three stations are close to those of the first. For the sample orbit determination scenario described above, range from each site to the satellite is approximately 37,850 km, the elevation angle is approximately 39°, and the azimuth angle is approximately 155°. As shown in Figure 3, the smaller the range to the satellite for a constant baseline length (or, equivalently, the longer the baselines across which phase delay is measured relative to the range to the satellite), the greater the position vector accuracy.

Next, we assessed the effect of satellite azimuth angle on orbit determination accuracy. The azimuth angle of the satellite at station 1 in the original scenario was varied while keeping the range and elevation angle from that station constant. The results indicate that for a configuration of four interferometric ground stations at the vertices of a square, position error is maximized when the satellite's azimuth angle is an integer multiple of 90°, and position error is minimized when the satellite's azimuth angle is an odd integer multiple of 45°.

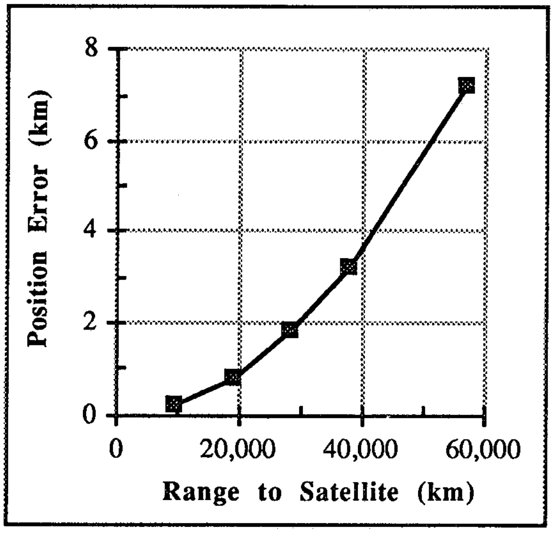

Figure 3. Position Error vs. Range to Satellite

Finally, we assessed the effect of satellite elevation angle on orbit determination accuracy in this scenario. The elevation angle of the satellite at station 1 was varied while keeping the range and azimuth angle from that station constant. As can be seen in Figure 4, for this particular orbit determination scenario, position error increases monotonically with elevation angle. Thus, based on the criterion of minimizing ephemeris error due only to error in the phase delay measurement, optimal viewing geometry is at the lowest possible elevation angle, and the scenario becomes degenerate when the satellite is at zenith.

A tradeoff is suggested by the geometrical result that greater orbit determination accuracy is attained at lower elevation angles. The tradeoff arises because statistical models of the variation in signal propagation rate through the troposphere show that, because a signal must pass through more of the troposphere as the elevation angle of the satellite decreases, errors in predicting signal propagation rate increase as elevation angle decreases [11]. Moreover, errors in predicting propagation rate due to tropospheric fluctuations tend to be the dominant error source in overall accuracy for CEI systems [12]. Thus, we sought to determine the optimal elevation angle for CEI measurements with consideration of both measurement error and tropospheric delay error.

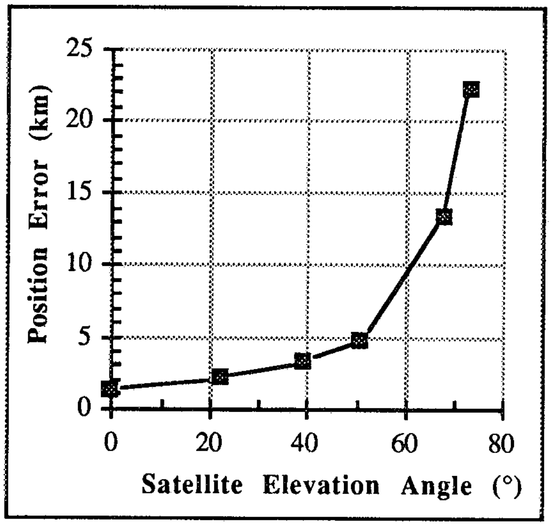

Figure 4. Position Error vs. Satellite Elevation Angle

We modeled tropospheric fluctuations between each interferometer site and the satellite as being independent and normally distributed. The assumption of independence is based on the fact that water vapor cells can be of several kilometers in diameter, and so tropospheric delay errors from each site can in fact be independent. From Reference 11, we computed the elevation angle dependence of the standard deviation in tropospheric delay error for 100 second measurement arcs of phase delay. The results are shown in Table 2.

Table 2. Tropospheric delay error as a function of elevation angle

Elevation Angle (°)	Tropospheric Delay Error (picosec)
10	7.5
20	5.7
30	4.6
40	3.9
50	3.3
60	3.0

For varying satellite elevation angles, we used ODAE to model error due to tropospheric fluctuations as well as inherent phase delay imprecision. The resulting 1σ position errors are shown in Figure 5. As can be seen, the optimal satellite elevation angle is approximately 30°. In the conclusions section of this paper, we show how these results can be applied to optimally siting a CEI system for TDRS orbit determination.

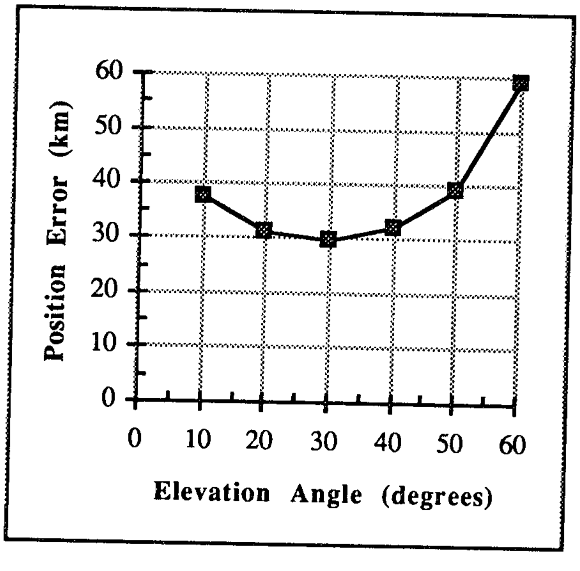

Figure 5. Position Error vs. Elevation Angle with Tropospheric Effects Included

CONCLUSIONS

We have derived conclusions about optimal geometry for orbit determination of a GEO satellite by radio interferometry. These results can be applied to the problem of optimally siting a CEI system to track TDRS. For a particular TDRS satellite, and for a configuration of four interferometer sites located at the vertices of a square, a geographical position should be chosen so that the satellite's elevation angle is as close to 30° as possible, and the square should be oriented so that the satellite's azimuth angle is an odd integer multiple of 45°. For TDRS-W at 171°W, the maximum elevation angle visible within the -20 dB contour of the White Sands downlink is in southern California at approximately 20° elevation. For TDRS-E at 41°W, an elevation angle near 30° can be attained within the -20 dB contour of the White Sands downlink by siting a CEI system in eastern Louisiana or western Mississippi.

DISCUSSION

Having determined optimal siting for a CEI TDRS tracking system, we return to a brief discussion of operational considerations. As stated previously, benefits include freedom from requirements placed on the space segment, the potential for excellent orbit determination accuracy, and the ability to locate the system entirely within the United States. It is expected that these benefits would ameliorate cost and operational constraints. Estimates have placed required staffing levels for an interferometric TDRS tracking system in the range from 10 to 20 full-time equivalent staff [13]. With respect to initial costs, Interferometrics demonstrated prototype hardware and correlation software for less than one million dollars [14]. Expected development and production costs for an operational system are expected to be an order of magnitude larger [13]. Finally, we note that interferometry offers low technological risk because it has been successfully applied in a number of related fields for several decades.

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