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NASA Technical Memorandum 106801

Thermal Analysis of a Functionally Graded Material Subject to a Thermal Gradient Using the Boundary Element Method

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Prepared for the
International Conference on Composites Engineering
sponsored by the International Community on Composites Engineering
New Orleans, Louisiana, August 28–31, 1994

National Aeronautics and
Space Administration

THERMAL ANALYSIS OF A FUNCTIONALLY GRADED MATERIALSUBJECT TO A THERMAL GRADIENT USING THE BOUNDARY ELEMENT METHOD

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SUMMARY

The boundary element method is utilized in this study to conduct thermal analyses of functionally graded composites, materials in which the internal microstructure or properties are explicitly tailored in order to obtain an optimal response, on the micromechanical (constituent) scale. A unique feature of the boundary element formulations used here is the use of circular shape functions to convert the two-dimensional integrations of the composite fibers to one-dimensional integrations. Using the computer code BEST-CMS, the through the thickness temperature profiles are computed for a representative material with varying numbers of fibers and fiber spacing in the thickness direction. The computed temperature profiles are compared to those obtained using an alternate analytical theory which explicitly couples the heterogeneous microstructure to the global analysis. The boundary element results compared favorably to the analytical calculations, with discrepancies that are explainable based on the boundary element formulation. The results serve both to demonstrate the ability of the boundary element method to analyze these types of materials, and to verify the accuracy of the analytical theory.

INTRODUCTION

In the analysis of composite materials at the micromechanical (constituent) level, traditional analytical methods concentrate on calculating the effective properties of a representative volume element based on the constituents' properties and geometry (ref. 1). The representative volume element (RVE) is assumed to be very small when compared to the overall dimensions of the composite. On the macroscopic scale, the composite is then assumed to be composed of a homogeneous material with properties that do not vary as a function of location. The composite effective properties are determined by evaluating the behavior of an RVE at a point within the composite. The overall philosophy of this methodology is that the local and global analyses can be decoupled.

A recent advance in composites is the development of materials in which the internal microstructure of the composite is tailored in order to obtain a desired response. In these materials, which have been named functionally graded materials, the microstructure can be tailored using several techniques. For example, an explicitly graded interfacial layer can be placed between two homogeneous materials, the fiber spacing within a composite can be explicitly set in order to obtain an optimal response, or the fibers in a composite can be given varying properties, sizes or shapes. By intentionally tailoring the geometry or properties of a material, undesired material response such as warping due to a through the thickness temperature gradient can be eliminated.

In developing a procedure to be utilized in analyzing functionally graded materials, Aboudi, Pindera and Arnold (ref. 2) analyzed a composite plate subject to a thermal gradient through the thickness in which the fiber spacing in the thickness direction was explicitly set. The thermal gradient was imposed by setting the temperature at the bottom of the composite plate to 500 °C and the temperature at the top of the composite plate to 0 °C. The thickness dimension of the plate was finite, but the other two dimensions were assumed to be infinite. Figure 1 gives a schematic of the composite plate which was analyzed. Uniform, linear, quadratic and cubic fiber spacings were considered, and composite plates with 1-3, 5, 8, 12, 16, and 20 fibers through the thickness were examined. In this manner, the response of a composite plate with a small number of fibers through the thickness could be examined.

To calculate the thermomechanical response of a composite plate with a small number of fibers through the thickness subject to a temperature gradient, Aboudi, et. al. developed a higher order theory for functionally graded materials (HOTFGM) in which the heterogeneous microstructure of the composite is explicitly coupled to the global analysis. Through this coupling, the representative volume element is considered to be the entire column of fibers and their surrounding matrix through the entire composite thickness. This coupling allows the temperatures and stresses in each of the phases (fiber and matrix) to be unique, and to be different from the temperature and stress that would be present in a homogenized continuum.

Aboudi, et al. calculated the local temperature and stress profiles in the thickness direction which resulted from the globally applied temperature gradient. The analysis results obtained using HOTFGM indicated piecewise linear temperature and stress profiles through the thickness of the material. Continuum theory, on the other hand, predicted purely linear temperature and stress profiles. The linear profiles predicted by continuum theory were a result of an assumption central to this theory; on the global level the material is assumed to be homogeneous with "smeared" properties computed from the constituent properties. Based on the results of this analysis, the conclusion was made that traditional continuum micromechanics theory might not yield accurate local temperature and stress results under the following conditions: when the applied temperature gradient is large with respect to the fiber diameter, the fiber diameter is large with respect to the overall composite thickness, and the number of fibers through the thickness is small.

To verify the accuracy of an analytical theory, alternative analyses using discrete modeling methods are often used, of which the finite element method is the most common approach. To verify HOTFGM, the boundary element method (BEM) was utilized. A joint program between the State University of New York at Buffalo and NASA Lewis Research Center has been underway for several years to examine the possible application of the boundary element method to composite micromechanical analyses. The motivation behind using the boundary element method was the ability of BEM to model a three-dimensional structure with surface discretization only, which could be an advantage when developing complex composite micromechanical models. Several previous studies have been conducted (refs. 3 to 7), in which the applicability of the boundary element methodology to composite micromechanical modeling has been examined.

For this particular study, the through the thickness temperature profiles for a representative material are computed using the boundary element method for a unidirectional composite with varying number of fibers and fiber spacing in the thickness direction. The boundary element results are compared to results obtained using HOTFGM. The objectives of this study are to validate the methodology incorporated into HOTFGM, and to demonstrate the ability of the boundary element method to analyze the thermal behavior of functionally graded materials.

ANALYSIS METHODOLOGY

The BEST-CMS (Boundary Element Solution Technology - Composite Modeling System) computer code was utilized to conduct the boundary element analyses presented in this paper (ref. 8). BEST-CMS includes the capability to conduct elasto-static, heat conduction and thermoelastic analyses. The heat conduction capability is what is specifically examined in this study. The major difference between the boundary element method and the more commonly utilized finite element method is that while in the finite element method the entire volume of a domain is discretized, in the boundary element method only the outer surface of the domain is discretized.

To eliminate the detailed discretization that would be required in order to explicitly model the fibers of the composite, specially formulated "Fiber" elements have been developed in which only the centerline of the fiber is defined using line elements and the fiber radius is specified in the input file. The "Fiber" elements are then placed within the outer surface of the composite matrix. The fiber surfaces and the variation of the field variables in the plane of the fiber cross section are represented through the use of trigonometric circular shape functions and closed form analytical expressions within the boundary element formulation. To calculate the variation of the field variables along the length of the fiber, numerical integration is performed.

As mentioned in the introduction, the motivation behind using the boundary element method to verify the HOTFGM approach is the success that has been achieved in using the finite element method for verifying closed form analytical methods for composite micromechanical analyses (ref. 9). For unidirectional laminates for which a single unit cell model is appropriate, a finite element mesh is not overly difficult to construct. However, for more complex architectures such as a functionally graded material in which several layers of fibers (often with varied spacing between the layers) are required, the construction of an appropriate finite element mesh becomes much more complex and time consuming.

The boundary element methods used in BEST-CMS provide a method to reduce the mesh complexity of a three-dimensional composite structure in which the fibers and matrix are explicitly modeled, particularly for complex architectures. The composite matrix is modeled by discretizing only the outer surface, which eliminates the need for an interior volumetric mesh. Additionally, by modeling the fibers using one-dimensional line elements, the modeling of a complex fiber architecture can be simplified. Again, while for simple composite architectures these modeling simplifications may be trivial if at all applicable, for complex architectures the time and effort saved in the generation of a model of a composite material on the micromechanical level has the potential to be significant.

There are several limitations to the current formulation and implementation of the boundary element methods in BEST-CMS and the BEST-CMS code itself which should be discussed. Fiber ends as free surfaces cannot be represented due to the insert element formulation, which results in the fibers lying entirely within the outer surface of the matrix. The primary effect of this assumption is that heat flow must be transferred from the fiber through a small portion of matrix before it reaches the model outer surface. Additionally, while theoretically detailed interior temperature distributions can be computed for any location within the fiber or matrix, these routines in BEST-CMS are still being refined in order to give accurate results. As a result of these limitations, the temperature values obtained in the results are taken on the outer boundary of the model, for which the fiber end assumption discussed above results in some discrepancies in the temperature results, both qualitatively and quantitatively. These discrepancies will be examined further when the results are specifically discussed. Additionally, since neither multi-point constraints or accurate interior stress calculations are currently available within BEST-CMS, only the temperature response due to heat conduction is considered in this study, instead of the full thermomechanical response including both temperature and stress profiles.

ANALYSIS SETUP

The analyses presented in this study involve the computation of the local temperature profile in the thickness direction of a composite plate subjected to a thermal gradient imposed by specifying temperatures on the top and bottom faces of the plate, as is shown in figure 1. The composite system that is examined for this study is composed of SiC (SCS-6) fibers with a fiber diameter of 142 μm embedded within a titanium alloy (Ti-15V-3Cr-3Sn-3Al or Ti-15-3 for short) matrix, with a fiber volume fraction of 0.40. A unidirectional [0] fiber orientation was utilized. The material properties of the fiber and matrix, obtained from (ref. 2), are listed in table I. The fibers are assumed to be perfectly bonded to the matrix. If the fibers were not perfectly bonded, a thermal interface with an appropriate thermal resistance value (see ref. 8 for further details) would have to be imposed to account for the fact that with a weak interface the heat flow between the fiber and matrix is restricted. The material properties are also assumed not to vary with temperature. This assumption is necessary because of the nature of the boundary element formulation, in which the entire region enclosed by a group of boundary elements must have constant material properties throughout the entire enclosed volume. Allowing the material properties to vary with temperature would result in the properties varying throughout the volume due to the differences in local temperatures.

For this study, plates with one, three and five fibers through the thickness were analyzed. In order to maintain a constant volume fraction, as the number of fibers with constant diameter through the thickness was increased, the model thickness was appropriately increased. For the plates with three and five fibers through the thickness, both constant and linear fiber spacings were examined. A sample boundary element model for the plate with three fibers through the thickness is shown in figure 2. As can be seen in figure 2 and its accompanying schematic, the model includes two fibers in the width direction, and the model depth is set equal to the width, in order to avoid extraneous edge effects as much as possible. To check the possible effect of model width on the results, models with an increased number of fibers in the width direction were examined, and the results were found not to vary from those obtained by using two fibers in the width direction. Eight noded quadrilateral elements were used to model the composite matrix, and three noded line elements were used to model the fibers.

A temperature of 500 °C was specified on the bottom face of the model, and a temperature of 0 °C was specified on the top face of the model. The temperature values were taken on the front face of the model, along the path indicated in the schematic shown in figure 1. This path, labeled "Fiber-Matrix Section", includes both fiber and matrix material in the cross-section. The temperatures were taken along the X-Coordinate direction indicated in figure 1.

For the models with one fiber through the thickness, the fibers are centered in the thickness direction within the composite. For the models with three and five fibers through the thickness and constantly spaced fibers, the vertical distance between each layer of fibers is assumed constant, and the fibers are centered within each fiber/matrix layer. For the models with linear fiber spacing, the vertical spacing between each layer of fibers differs from the one above or below it by a constant, and the fibers are concentrated towards the low temperature (top) face of the model (see fig. 3 for a schematic).

DISCUSSION OF RESULTS

The first case that was examined was the simple case of a plate with only one fiber through the thickness. This simple case was examined in order to determine the ability of the boundary element method to capture the local variations in the through the thickness temperature profile resulting from temperatures being specified on the top and bottom surfaces of the composite plate. The temperature profile is plotted in terms of temperature versus distance from the bottom face of the model (normalized by dividing by the model thickness). Figure 4 shows the through the thickness temperature profile for the "Fiber/Matrix Section" discussed above. The curve labeled "Fiber Element" is the curve computed using the BEST-CMS formulation where the fiber is modeled using line elements. The curve labeled "Initial HOTFGM Model" comes from (ref. 2), and is the curve computed using HOTFGM. Comparing the two curves, both curves predict a non-linear temperature profile, with HOTFGM predicting a piecewise linear temperature distribution and BEST-CMS predicting a more continuous temperature distribution. The nonlinearity is to be expected, due to the fact that in opposition to continuum theory, in which the composite is assumed to be homogeneous on the macroscopic level with "smeared" properties, in both types of analysis shown here the local variations in microstructure are taken into account even on the macroscopic level. In this particular case, where the fiber conductivity is fifty times greater than that of the matrix, it is expected that the temperature variation in the fiber would be much different than that in the matrix. The two curves are approximately one-half cycle "out of phase" due to the fact that in the HOTFGM model the fiber is assumed to be located at the bottom of each layer of fibers and its surrounding matrix, while in the boundary element models the fiber is centered within the layer, since the fibers are not allowed to intersect the matrix outer surface.

To examine the reason why the BEST-CMS curve predicts a continuous curve while HOTFGM predicts a piecewise linear curve, two boundary element models were constructed in which the fibers were explicitly modeled using quadrilateral surface elements instead of line elements. One model, for which an example is shown in figure 5 (the example being for the case with three fibers through the thickness), the fibers were assumed to extend to the outer surface of the model, which is the actual case in the composite. This condition is different from the assumption of the BEST-CMS fiber element formulation which assumes that the fiber ends can not be free surfaces, and are embedded slightly within the outer surface of the matrix. The curve computed using the model with explicitly modeled fibers is labeled "Quad Element Fibers - Fibers at End" in figure 4. To provide an additional check on whether the fiber end assumption is truly critical to the discrepancy between the BEST-CMS and analytical curves, an additional boundary element model with the fibers again explicitly modeled using quadrilateral surface elements was constructed, but in this case the fibers were embedded slightly within the outer surface of the matrix, in order to simulate the conditions imposed by the fiber element formulation. The curve computed using this model is labeled "Quad Element Fibers - Fibers Embedded" in figure 4.

Examining the curves in figure 4 computed using these two models, the model where the fibers extend all the way to the outer surface of the matrix yields the piecewise linear temperature profile predicted by HOTFGM. The model in which the fibers were embedded slightly within the outer surface of the matrix, on the other hand, predicts the same continuous temperature variation as that predicted by the fiber element model, and matches the fiber element results almost exactly. These results seem to indicate that the assumption in the fiber element formulation that the fiber ends are not free surfaces are the major cause of the qualitative discrepancy between the fiber element results and the HOTFGM results. Furthermore, these findings indicate that results obtained using the fiber element model should be averaged across the region containing fiber material and should only be utilized to obtain general trends.

One difference that was found between results obtained using the boundary element model with the explicitly modeled fibers (extended to the outer surface) and the resutls obtained using HOTFGM was the slope of the curve in the regions of the cross section composed of matrix material. One possible cause of this discrepancy is the fact that in HOTFGM, the fibers are assumed to have a square cross section, while in the boundary element models a circular cross section was utilized (an assumption in the fiber element formulation). The square cross section assumed in HOTFGM results in a slightly smaller fiber "diameter" than the actual fiber diameter, which is based on a circular fiber, in order to maintain a constant fiber volume fraction. To test the effects of varying the fiber shape, a boundary element model was constructed in which the fiber, while still assumed to be centered in the layer of fiber and matrix, was explicitly modeled using quadrilateral elements but with a square cross section (appropriately adjusting the fiber dimensions in order to maintain a constant fiber volume fraction). The profile computed using this model is shown in figure 6 (labeled "Quad Element Fibers - Square Fibers"), along with the profile computed by BEST-CMS using quadrilateral elements for the fiber and a circular fiber cross section and the results computed using HOTFGM.

As can be seen in the figure, the results obtained using the square fiber cross section also exhibit the piecewise linear character predicted by the circular fiber model. Quantitatively, the results computed using the two boundary element models only differ slightly as a result of the different fiber dimensions resulting from the variation in fiber cross section. However, the slope of the line in the region of the material cross section composed of matrix material predicted by the boundary element model with the square fiber matches that predicted by HOTFGM (which also assumes a rectangular fiber cross section) almost exactly. These results seem to indicate that varying the fiber cross sectional shape causes a slight variation in results, but that the difference is not extremely significant.

To examine the effect on the thermal profiles resulting from increasing the number of fibers through the thickness, boundary element models (utilizing fiber elements for the fibers) containing three and five fibers through the thickness were constructed and analyzed. The temperature profiles along the "Fiber/Matrix Section" for the model with three fibers through the thickness is plotted in figure 7, and the profile for the model with five fibers through the thickness is plotted in figure 8. In both cases, results from (ref. 2) obtained using HOTFGM are also plotted for comparison.

As can be seen in both figures, both the BEST-CMS results obtained using fiber elements and the HOTFGM results display a nonlinear temperature profile, and the BEST-CMS results again exhibit a more continuous temperature distribution than is seen in the HOTFGM results. To provide a point of comparison, for the case with three fibers through the thickness a boundary element model was constructed with the fibers explicitly modeled using quadrilateral elements and extending to the outer surface of the material. The analysis results computed using this model are also plotted in figure 7 (labeled "Quad Element Model" in the figure). Comparing the results obtained using the explicitly modeled fibers to the HOTFGM results, the comparison is quite good, except for the two profiles being one-half cycle out of phase due to the fact that in HOTFGM the fibers are placed at the bottom of the fiber/matrix layer, while in BEST-CMS the fibers are centered within the fiber/matrix layer.

One important point to note in both figures 7 and 8 is that the nonlinearity in the profiles resulting from coupling the local microstructure into the global response is still present. Again continuum theory, which assumes smeared homogenous properties, would predict a purely linear temperature profile. However, as the number of fibers through the thickness is increased, the variation from linearity is reduced in both the profiles predicted by BEST-CMS and the HOTFGM results. This behavior is to be expected, as when the number of fibers through the thickness is increased to a sufficiently high level, the material could indeed be assumed to be macroscopically homogenous, and the assumptions of continuum theory would be valid. While in the two cases examined here that point has not yet been reached, the trend towards obtaining a more linear temperature profile as the number of fibers through the thickness is increased is definitely present.

In actual functionally graded materials, the goal is to explicitly tailor the microstructure in order to obtain an optimal response. One way in which the microstructure can be tailored is in varying the spacing of the fibers within the material. In this particular study, the effects of changing the fiber spacing from a constant spacing to a linear spacing were examined. As discussed before, to impose the linear fiber spacing the spacing between two layers of fibers was set to vary from the spacing above or below it by a constant. Additionally, the fibers were concentrated towards the low temperature end of the composite. The temperature profiles for the models with linear fiber spacing computed by BEST-CMS, using fiber elements, are plotted in figure 9 for the model with three fibers through the thickness and in figure 10 for the model with five fibers through the thickness. In both cases, the profiles computed using constant fiber spacing are also plotted for comparison.

Examining the figures, in both cases the temperature profiles computed for the linear fiber spacing configuration exhibit lower overall temperature levels than are seen for the models with constant fiber spacing. Even for the model with five fibers through the thickness, in which the variation of the profile from linearity is significantly reduced (as compared to the models with one and three fibers through the thickness), varying the fiber spacing still has a noticeable effect on the temperature profile. The lower temperature levels resulting from the linear fiber spacing would likely result in lower overall stress levels in the material. The results thus seem to indicate that for plates with a relatively small number of fibers through the thickness, concentrating the fibers towards the low temperature end of the composite would most likely improve the response of the material (assuming lowering the stress level is a desired response). An important point to note is that by using continuum theory, with its smeared homogenous properties, the effects of varying the local structure would not be captured (even with the nonconstant fiber spacing, a linear temperature profile would again be predicted), and the advantages of using a functionally graded material could not be determined. Although not shown here, similar results were computed in (ref. 2) using HOTFGM. The results computed using the boundary element method thus serve to further validate the methodology incorporated into HOTFGM.

The final set of analyses that was conducted involved examining the effects on the through the thickness temperature profile of varying the thermal conductivity ratio between the fiber and matrix. As has been noted above, one likely cause of the nonlinearties seen in the previous thermal profiles is the large difference in thermal conductivities between the fiber and the matrix. To examine this hypothesis, a boundary element model was constructed with one fiber through the thickness (using fiber elements to model the fibers) where the fiber conductivity was reduced to five times that of matrix (from 50), and another model was constructed in which the fiber conductivity was again reduced to 2.2 times that of the matrix.

The thermal profiles (through the Fiber/Matrix cross section) computed using the two new sets of thermal conductivity ratios are plotted in figure 11 along with the original case where the fiber conductivity was 50 times higher than that of the matrix. As can be seen from the figure, as the conductivity ratio between the fiber and matrix is reduced, the thermal profile shows a smaller variation from linearity. These results make sense, since as the thermal conductivity of the fiber is reduced to be closer to that of the matrix, the material becomes more like a homogeneous material (at least in a thermal sense), and the assumptions of continuum theory become more applicable (at least for the thermal portion of the analysis).

To check that the trends towards a more linear profile seen in figure 11 are representative of the actual material behavior, and not just an artifact of the BEST-CMS fiber end assumption (fiber ends are not free surfaces) inherent in the fiber element formulation, boundary element models in which the fibers were explicitly modeled (with a circular cross section) using quadrilateral elements were constructed and analyzed for each of the three conductivity ratios examined in figure 11. These new results are shown in figure 12. As can be seen in the figure, the results obtained using explicitly modeled fibers show even more clearly that as the conductivity ratio between the fiber and matrix is reduced, the thermal profile shows a smaller variation from linearity. These results show that by explicitly tailoring the material properties of the fiber and matrix, the local response of the material can be varied significantly. Additionally, similar results were obtained in (ref. 2) using HOTFGM, which again helps to verify that HOTFGM accurately represents the material response for this class of materials.

To examine whether the trends observed above would still be valid as the number of fibers through the thickness is increased, a boundary element model with three constantly spaced fibers through the thickness was constructed using fiber elements and analyzed for each of the conductivity ratios described above. The thermal profiles computed for these analyses are plotted in figure 13. As can be seen in the figure, once again as the thermal conductivity ratio is reduced, the profiles show a smaller variation from linearity. These results verify that by varying the thermal conductivity ratio between the fiber and matrix, significantly different thermal responses, which would lead to strong variations in the resulting stress states, are obtained.

CONCLUSIONS

The boundary element method has been utilized to simulate the through the thickness temperature profile of a representative functionally graded material with varying numbers of fibers through the thickness and varying fiber spacing conditions. The results computed using the computer code BEST-CMS verified that the local microstructure should be coupled to the global response when a large temperature gradient is applied to a composite plate with a relatively small number of fibers through the thickness. The results also indicated that varying the microstructural geometry or constituent properties could significantly affect the results.

Additionally, by selective comparison to results computed using the analytical HOTFGM theory, the boundary element results verified that HOTFGM gives favorable results, and that the assumptions included in this theory are valid for this class of problems. This verification also shows the ability of the boundary element method to serve as a useful tool for verifying analytical theories.

For future work, the ability of BEST-CMS to compute accurate interior stresses for composite materials under thermo-mechanical loading will be refined and improved, so that stress profiles of models subjected to a thermal gradient can also be computed. Additionally, in order to take advantage of the ability of the boundary element method to model composites with complex interior architectures with a minimum of meshing required, techniques to analyze woven and braided composites will be developed.

ACKNOWLEDGMENTS

The authors would like to thank Dr. Steve Arnold of NASA Lewis Research Center for originating the idea for this study, and for many helpful discussions and assistance. The authors would also like to thank Dr. Arnold, as well as Dr. Pappu L.N. Murthy and Mr. Ho-Jun Lee for their helpful suggestions and discussions relating specifically to this paper. Finally, Dr. Marek-Jerzy Pindera and Dr. Jacob Aboudi of the University of Virginia must be thanked for providing additional information regarding the HOTFGM theory.

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TABLE I.—Constituent Properties of SiC/Ti-15-3 Used in Boundary Element Analysis

Material	E, GPa	α, 10–6 m/m/°C	κ, W/m–°C
SiC	414.0	4.9	400.0
Ti–15–3	100.0	9.6	8.0

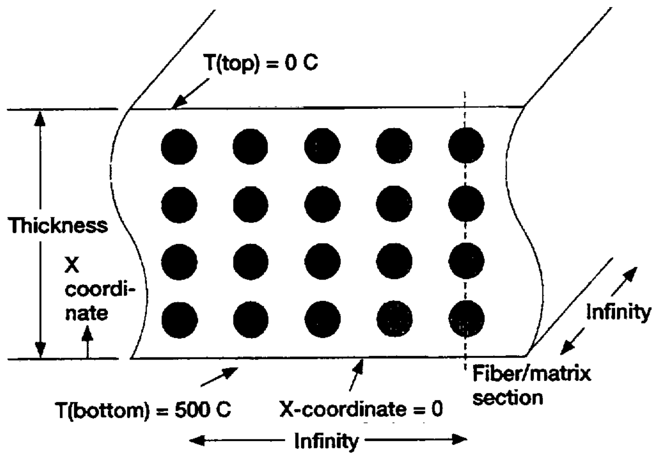

Figure 1.—Schematic of composite plate with a small number of fibers through the thickness subject to a temperature gradient.

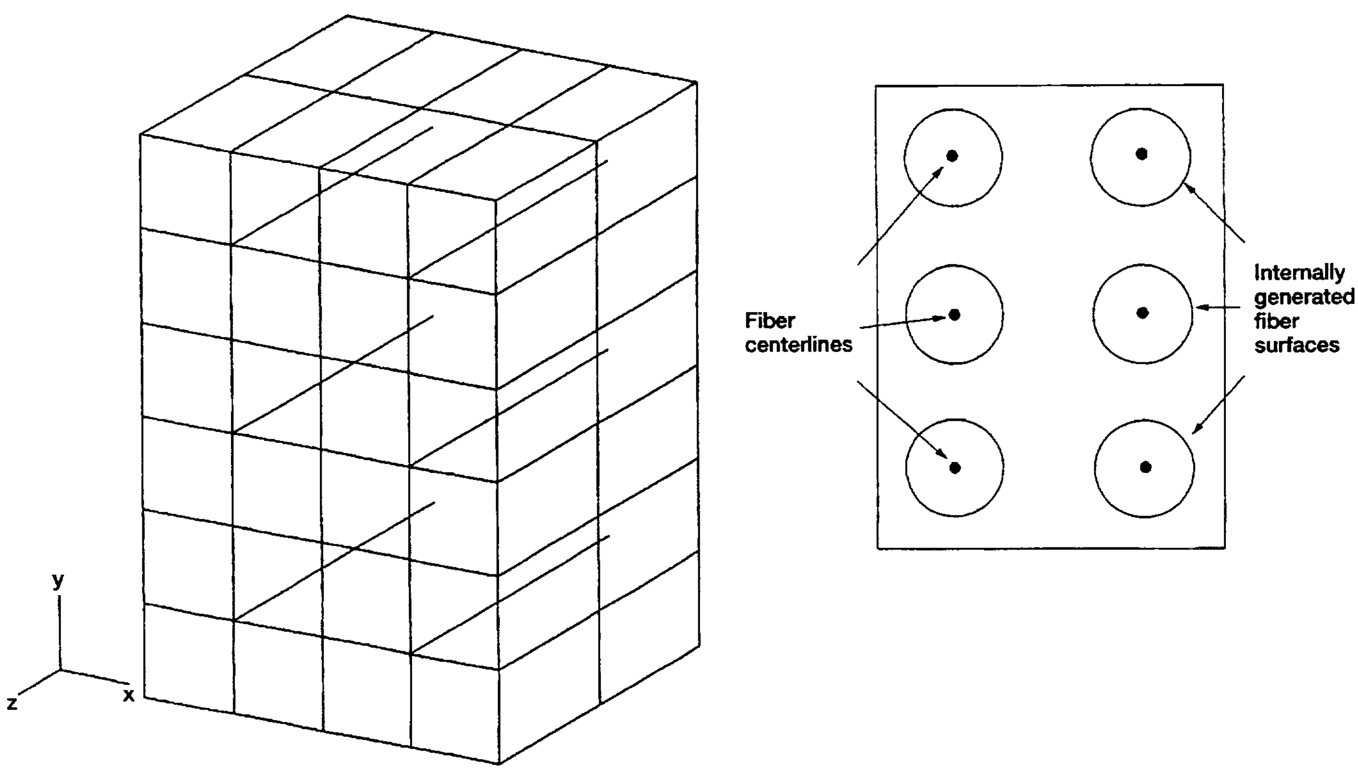

Figure 2.—Example boundary element model of composite plate with three fibers through the thickness.

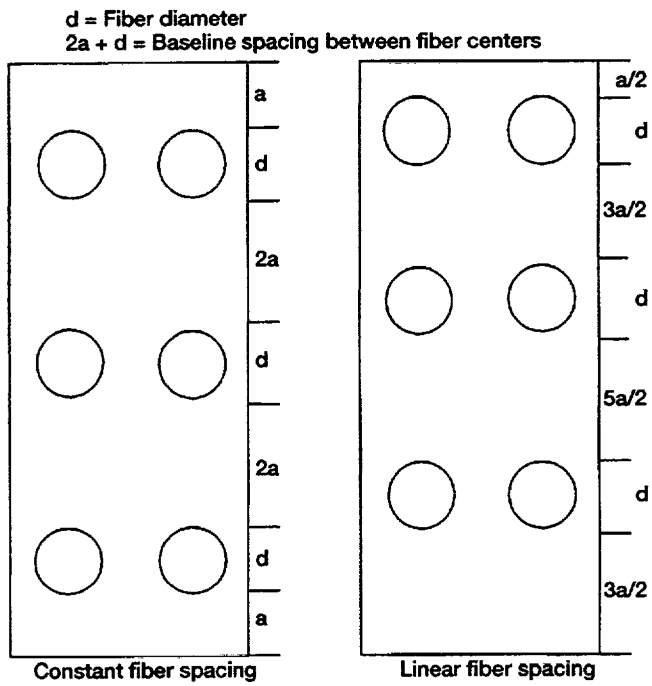

Figure 3.—Schematic of dimensions of composite plate with constant and linear fiber spacing.

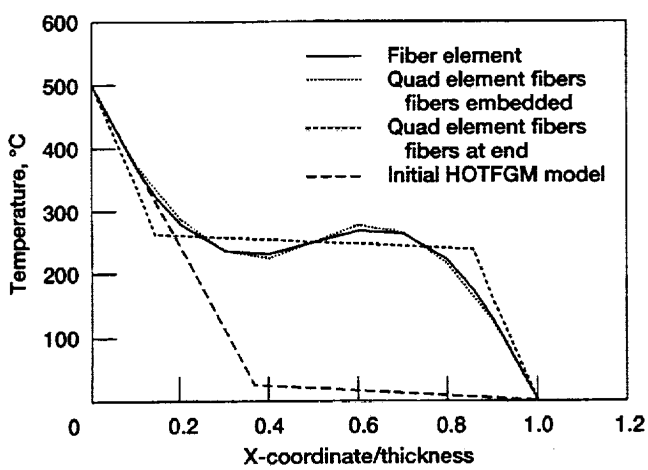

Figure 4.—Through the thickness temperature profile for SiC/Ti-15-3 with one fiber through the thickness.

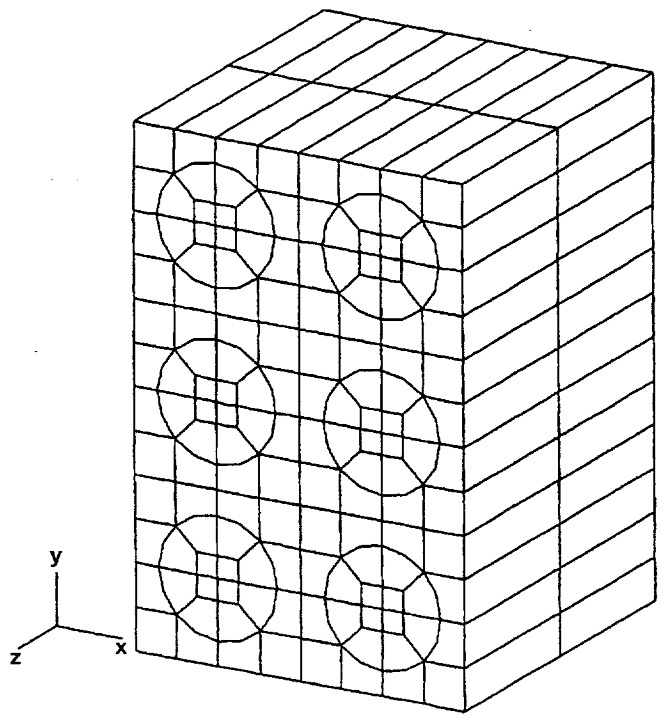

Figure 5.—Example boundary element model of composite plate with three fibers through the thickness and fibers explicitly modeled using surface elements.

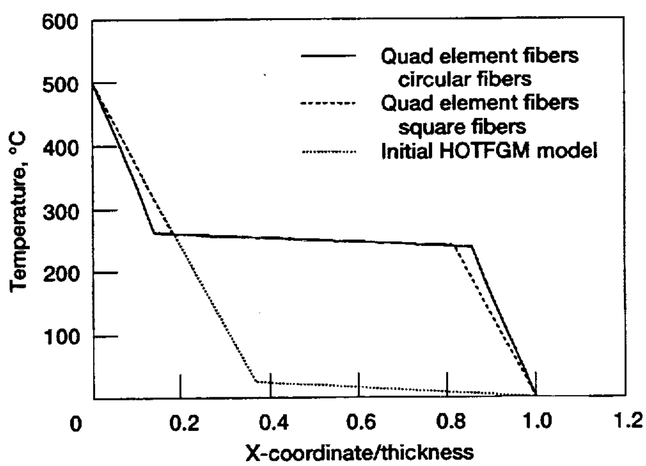

Figure 6.—Through the thickness temperature profile for SiC/Ti-15-3 with one fiber through the thickness and varied fiber cross sectional shape.

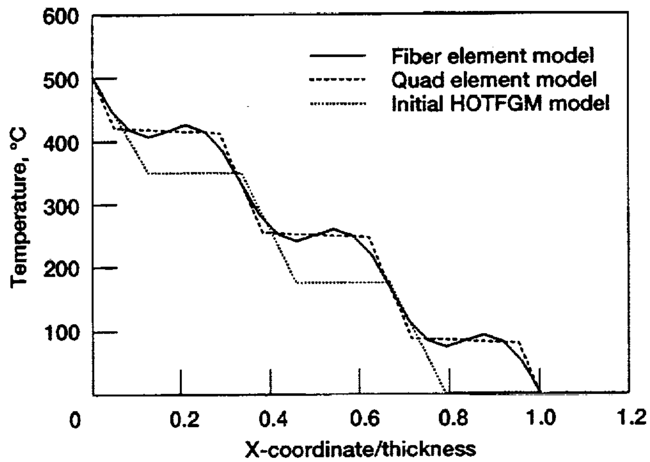

Figure 7.—Through the thickness temperature profile for SiC/Ti-15-3 with three fibers through the thickness and constant fiber spacing.

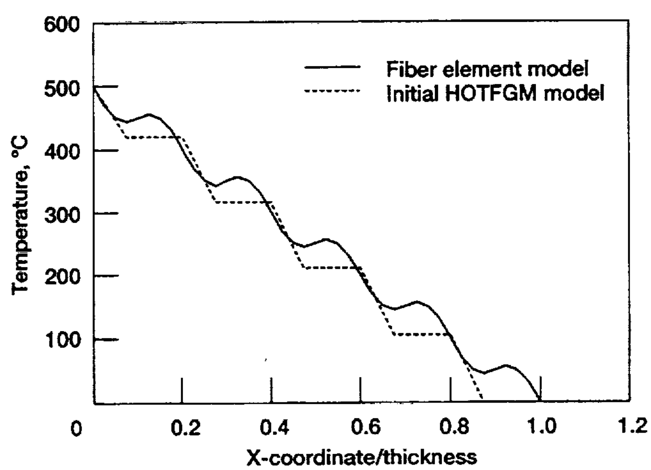

Figure 8.—Through the thickness temperature profile for SiC/Ti-15-3 with five fibers through the thickness and constant fiber spacing.

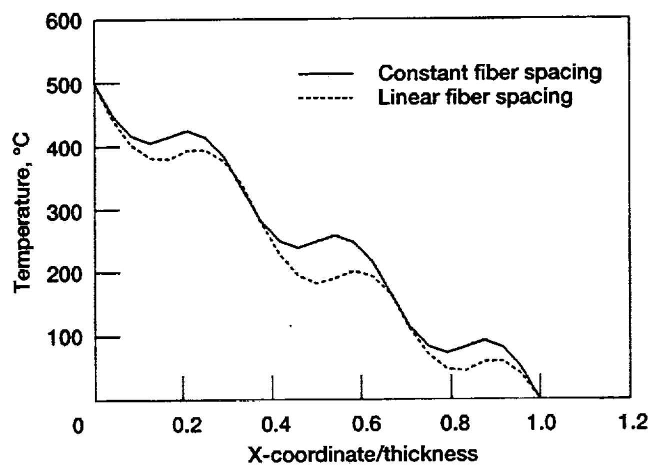

Figure 9.—Through the thickness temperature profile for SiC/Ti-15-3 with three fibers through the thickness and linear fiber spacing.

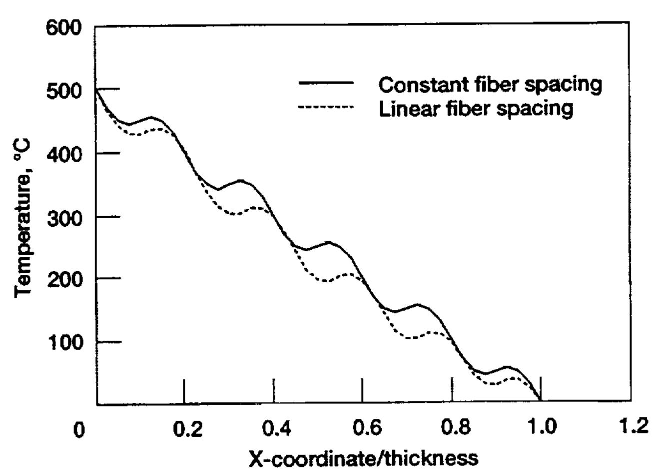

Figure 10.—Through the thickness temperature profile for SiC/Ti-15-3 with five fibers through the thickness and linear fiber spacing.

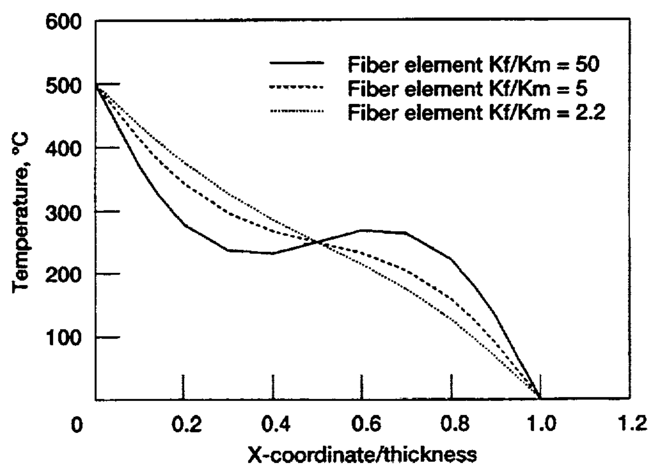

Figure 11.—Effect of fiber/matrix thermal conductivity ratio on the through the thickness temperature profile for SiC/Ti-15-3 with one fiber through the thickness and fibers modeled using fiber elements.

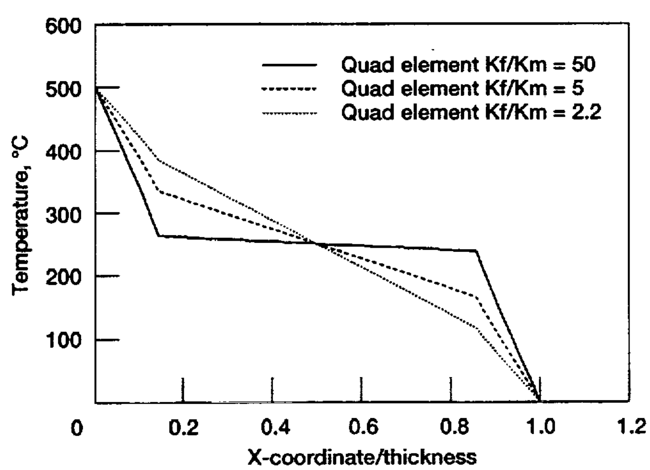

Figure 12.—Effect of fiber/matrix thermal conductivity ratio on the through the thickness temperature profile for SiC/Ti-15-3 with one fiber through the thickness and fibers modeled using quadrilateral elements.

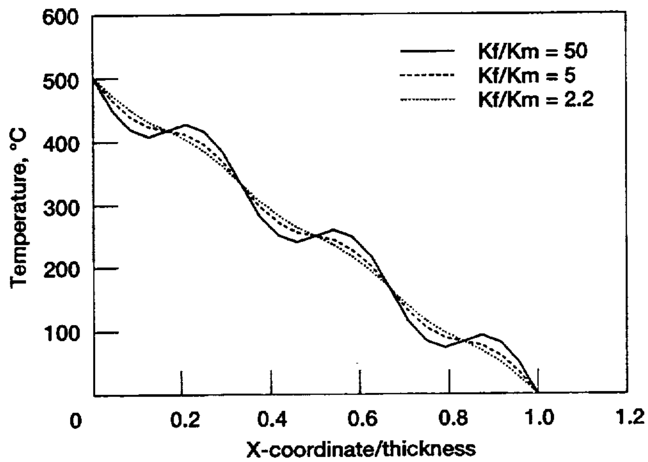

Figure 13.—Effect of fiber/matrix thermal conductivity ratio on the through the thickness temperature profile for SiC/Ti-15-3 with three fibers through the thickness and constant fiber spacing.

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY (*Leave blank*)	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	December 1994	Technical Memorandum

4. TITLE AND SUBTITLE

Thermal Analysis of a Functionally Graded Material Subject to a Thermal Gradient Using the Boundary Element Method

5. FUNDING NUMBERS

WU–505–63–12

6. AUTHOR(S)

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7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

National Aeronautics and Space Administration
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Cleveland, Ohio 44135–3191

8. PERFORMING ORGANIZATION REPORT NUMBER

E–9277

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

National Aeronautics and Space Administration
Washington, D.C. 20546–0001

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

NASA TM–106801

11. SUPPLEMENTARY NOTES

Prepared for the International Conference on Composites Engineering sponsored by the International Community on Composites Engineering, New Orleans, Louisiana, August 28–31, 1994. Responsible person, Robert K. Goldberg, organization code 5210, (216) 433–3330.

12a. DISTRIBUTION/AVAILABILITY STATEMENT

Unclassified - Unlimited
Subject Category 39

This publication is available from the NASA Center for Aerospace Information, (301) 621–0390.

12b. DISTRIBUTION CODE

13. ABSTRACT ***(Maximum 200 words)***

The boundary element method is utilized in this study to conduct thermal analysis of functionally graded composites, materials in which the internal microstructure or properties are explicitly tailored in order to obtain an optimal response, on the micromechanical (constituent) scale. A unique feature of the boundary element formulations used here is the use of circular shape functions to convert the two-dimensional integrations of the composite fibers to one-dimensional integrations. Using the computer code BEST-CMS, the through the thickness temperature profiles are computed for a representative material with varying numbers of fibers and fiber spacing in the thickness direction. The computed temperature profiles are compared to those obtained using an alternate analytical theory which explicitly couples the heterogeneous microstructure to the global analysis. The boundary element results compared favorably to the analytical calculations, with discrepancies that are explainable based on the boundary element formulation. The results serve both to demonstrate the ability of the boundary element method to analyze these types of materials, and to verify the accuracy of the analytical theory.

14. SUBJECT TERMS

Boundary element method; Composite materials, Metal matrix composites; Finite element method

15. NUMBER OF PAGES

15

16. PRICE CODE

A03

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified	Unclassified	

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Space Administration
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