Fermi National Accelerator Laboratory

IS A MASSIVE TAU NEUTRINO
JUST WHAT
COLD DARK MATTER NEEDS?

Scott Dodelson,[1] Geza Gyuk,[1,2] and Michael S. Turner[1,2,3]

[1]*NASA/Fermilab Astrophysics Center*
Fermi National Accelerator Laboratory, Batavia, IL 60510-0500

[2]*Department of Physics*
The University of Chicago, Chicago, IL 60637-1433

[3]*Department of Astronomy & Astrophysics*
Enrico Fermi Institute, The University of Chicago, Chicago, IL 60637-1433

ABSTRACT

The cold dark matter (CDM) scenario for structure formation in the Universe is very attractive and has many successes; however, when its spectrum of density perturbations is normalized to the COBE anisotropy measurement the level of inhomogeneity predicted on small scales is too large. This can be remedied by a tau neutrino of mass 1 MeV − 10 MeV and lifetime 0.1 sec − 100 sec whose decay products include electron neutrinos because it allows the total energy density in relativistic particles to be doubled without interfering with nucleosynthesis. The anisotropies predicted on the degree scale for "τCDM" are larger than standard CDM. Experiments at $e^{\pm}$ colliders may be able to probe such a mass range.

The cold dark matter (CDM) scenario for the formation of structure in the Universe (i.e., galaxies, clusters of galaxies, superclusters, and so on) is both well motivated and very successful [1]. The CDM model begins with a flat Universe with most of its mass in slowly moving ("cold") particles such as axions or neutralinos and a baryon fraction compatible with primordial nucleosynthesis ($\sim 5\%$). The density perturbations that seed structure formation are the nearly scale-invariant perturbations that arise from quantum fluctuations during inflation [2]. Standard CDM has one free parameter: the normalization of the spectrum. The recent COBE detection [3] of anisotropy in the temperature of the cosmic background radiation (CBR) now provides the normalization by fixing the amplitude of perturbations on very large scales ($\lambda \sim 10^3 h^{-1}$ Mpc). But, when the spectrum is so normalized, the level of inhomogeneity predicted on small scales ($\lambda \lesssim 10 h^{-1}$ Mpc) is too large by a factor of two (the Hubble constant $H_0 = 100 h \, \mathrm{km\,s^{-1}\,Mpc^{-1}}$). Just how serious this problem is remains to be seen.

A number of "fixes" have been proposed. They involve changing either the initial power spectrum or the energy content of the Universe. The simplest way of changing the power spectrum is "tilt" [4] (i.e., deviation from scale invariance) which reduces the power on small scales; in fact, several plausible models of inflation predict a tilted spectrum. However, this solution seems to lead to insufficient power on intermediate scales ($\lambda \sim 30 h^{-1}$ Mpc $- 100 h^{-1}$ Mpc). Changing the energy content changes the transfer function, which relates the primeval power spectrum to that today [5]. For example, mixed dark matter (MDM) [6] (65% CDM + 30% hot dark matter in the form of 7 eV neutrinos + 5% baryons) or ΛCDM [7] (80% vacuum energy + 15% CDM + 5% baryons) both reduce the power on small scales.

The fix advocated here involves increasing the energy density in relativistic particles, which also modifies the transfer function. In the standard scenario the radiation content at late times ($t \gg 1 \sec$) consists of photons and three massless neutrino species with slightly lower temperature $T_\nu = (4/11)^{1/3} T_\gamma$, accounting for a total radiation energy density $\rho_{\mathrm{rad}} = g_* \pi^2 T_\gamma^4 / 30$. Here $g_* = 2 + 2(7/8)(4/11)^{4/3} N_\nu$ counts the number of effectively massless degrees of freedom and is equal to 3.36 for $N_\nu = 3$. In our "τCDM" model g_* will be about a factor of two larger.

To see why this helps, consider the scale λ_{EQ}, which crossed the horizon at the time the energy density in matter was equal to that in radiation. On scales much greater than λ_{EQ} the transfer function is unity because pertur-

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bations on these scales crossed the horizon during the matter-dominated era and began growing as soon as they did. On scales much smaller than $\lambda_{\rm EQ}$ the transfer function decreases as λ^2 because perturbations on these scales crossed the horizon before matter-radiation equality and did not begin growing until the matter-dominated era. Thus, $\lambda_{\rm EQ}$ controls the shape of the transfer function; it is given by, $\lambda_{\rm EQ} \sim (R_0/R_{\rm EQ})t_{\rm EQ}$ (R_0 is the scale factor today; $R_{\rm EQ}$ and $t_{\rm EQ}$ are the scale factor and age of the Universe at matter-radiation equality). Since the energy density in matter red shifts as R^{-3} and that in radiation as R^{-4}, $R_0/R_{\rm EQ} = \rho_{\rm matter}/\rho_{\rm rad} \simeq 2.4\Omega_0 h^2 \times 10^4/(g_*/3.36)$ (Ω_0 is the ratio of the energy density to the critical density; $\Omega_0 = 1$ for a flat Universe). The age at matter-radiation equality $t_{\rm EQ} \sim m_{\rm Pl}/g_*^{1/2}T_{\rm EQ}^2$, and so

$$\lambda_{\rm EQ} \sim 10\,{\rm Mpc}(\Omega_0 h^2)^{-1}(g_*/3.36)^{1/2}. \tag{1}$$

Increasing g_* increases $\lambda_{\rm EQ}$; since COBE fixes the power spectrum on scales $\lambda \gg \lambda_{\rm EQ}$, doing so reduces the power on small scales (see Fig. 1).

How much additional radiation is needed? Since cosmological distances scale as the inverse of the Hubble constant, the shape of the power spectrum is set by $(h^{-1}\,{\rm Mpc})/\lambda_{\rm EQ} \propto \Gamma \equiv (\Omega_0 h)(g_*/3.36)^{-1/2}$. The best fit to all the data seems to require a "shape parameter" $\Gamma \simeq 0.3$ [8], whereas $\Gamma \simeq 0.5$ for the canonical values of $\Omega_0 = 1.0$, $g_* = 3.36$, and $h = 0.5$.[1] For a given Hubble constant and shape parameter, the number of equivalent neutrino species called for is $N_\nu = 20(0.3/\Gamma)^2\Omega_0^2(h/0.5)^2 - 4.3$. To achieve $\Gamma = 0.3$ with $h = 0.5(0.4)$ requires $N_\nu = 16(8.5)$.

Therein lies the rub: Primordial nucleosynthesis restricts the equivalent number of massless neutrino species to be less than 3.4 ($g_* \le 3.54$) [9]. The argument is well known: More relativistic degrees of freedom increase the expansion rate during nucleosynthesis and lead to an earlier freeze out of the neutron fraction at a higher value; this results in too much ^{4}He if $N_\nu > 3.4$.

One way around this has been discussed previously: a late-decaying, massive particle species, e.g., the once viable 17 keV neutrino [10]. Since the energy density in a massive particle species grows relative to radiation as the scale factor, a massive particle whose energy density at the epoch of nucleosynthesis is negligible can produce a significant amount of radiation if it

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[1]In order to insure a sufficiently aged Universe with $\Omega_0 = 1$, h must lay at the lower end of current measurements, $h \sim 0.5$. Of course, another fix is to posit a very low value of the Hubble parameter, $h = 0.3$, which leads to a very old Universe, $t_0 \simeq 22\,{\rm Gyr}$.

decays late enough, in the case of the 17 keV neutrino, $t \sim 10^7$ sec [11].

Our proposal is very different and involves a subtle aspect of nucleosynthesis. We recently examined the effect of an unstable tau neutrino on nucleosynthesis for decay modes including $\nu_\tau \rightarrow \nu_e + \phi$ (ϕ is an effectively noninteracting scalar particle) [12]. The decay products of a massive tau neutrino can have more energy density than a massless neutrino species; for most decay modes this leads to an unacceptable increase in ^{4}He. However, for the $\nu_e \phi$ decay mode there is a competing effect: Some decay-produced ν_e's are captured by neutrons, reducing the neutron fraction and suppressing ^{4}He production [13]. This opens the door for adding relativistic degrees of freedom—which we've seen can help CDM—without overproducing ^{4}He. The effect is even more pronounced for the $\nu_\tau \rightarrow \nu_e + \nu_e \bar{\nu}_e$ mode.

The key to all of this is the fact that the decay-produced ν_e's resemble a bath of neutrinos with temperature lower than that at which the neutron fraction freezes out (~ 1 MeV), and thus, their interactions with nucleons tend to reduce the neutron fraction. The rate for neutron-proton interconversion ($n + \nu_e \leftrightarrow p + e^-$, etc.) varies as T^5 and so the typical energy of a captured neutrino is $5T$, or 5 MeV at freeze out; decay-produced electron neutrinos have energy $m_\nu/2$ (lower for the $3\nu_e$ mode). Thus, the effective temperature of decay-produced neutrinos will be lower than 1 MeV provided $m_\nu \lesssim 10$ MeV. Conversely, for masses greater than 10 MeV, decay-produced neutrinos increase the neutron fraction as they appear hotter. This simple explanation is borne out by detailed calculations [12].

Next, consider the lifetime dependence. Decays much earlier than 1 sec occur before freeze out, when the neutron fraction is close to its equilibrium value and so there is little effect. The dilution effect of the expansion decreases the capture probability of a ν_e, and so few ν_e's produced much later than 1 sec are captured. Furthermore, the energy density of the massive tau neutrino grows relative to the radiation as the scale factor, and for lifetimes greater than 10 sec increases the expansion significantly rate, tending to increase ^{4}He production. For long lifetimes the increase in expansion rate wins out and leads to higher ^{4}He production; for lifetimes around a few seconds the two competing effects cancel; for short lifetimes the effect of neutron captures wins out leading to lower ^{4}He production. For a lifetime of ~ 0.3 sec the reduction in ^{4}He is maximized and can be as large $\Delta Y_P \sim 0.10$ [12].

There are three ways to take advantage of the "nucleosynthesis physics" described above to increase the radiation content in the Universe. The first

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involves a tau neutrino of mass $1\,\mathrm{MeV}-10\,\mathrm{MeV}$ and lifetime $0.1\,\mathrm{sec}-\mathrm{few\,sec}$. Tau-neutrino decays can reduce the neutron fraction enough to make room for up to the equivalent of 16 additional neutrino species (see Fig. 2). For lifetimes this short the energy density of the tau neutrino does not become significant before it decays, and the extra energy density in relativistic species must arise from additional massless degrees of freedom. Many extensions of the standard model predict new massless degrees of freedom (see below). (While we have not explored masses less than $1\,\mathrm{MeV}$ for technical reasons [12], Fig. 2 suggests that a massive muon neutrino that decays to electron neutrinos might also allow for many additional massless species.)

The second possibility involves a tau neutrino of lifetime $10\,\mathrm{sec}-50\,\mathrm{sec}$ and is probably the most attractive. Here, not only do tau-neutrino decays inhibit ^{4}He production but they also produce the additional energy density in relativistic particles. That energy density, expressed in equivalent number of neutrino species, is roughly [14]

$$\Delta N_\nu \simeq 0.4(rm_\nu/\,\mathrm{MeV})(\tau_\nu/\,\mathrm{sec})^{1/2}, \tag{2}$$

where r is the freeze-out abundance of the massive tau neutrino relative to that of a massless neutrino species ($r \sim 0.1-1$ for $m_\nu = 1\,\mathrm{MeV}-10\,\mathrm{MeV}$) and the total equivalent number of massless neutrinos $N_\nu = 2 + \Delta N_\nu$.

In Fig. 3 we show the regions of the tau-neutrino mass-lifetime plane that are excluded by primordial nucleosynthesis and contours of the energy density of the tau-neutrino's decay products (based on our numerical calculations [12]). We exclude a mass/lifetime if for no value of the baryon-to-photon ratio the light-element abundances satisfy: $\mathrm{D/H} \geq 10^{-5}$; $(\mathrm{D}+^3\mathrm{He})/\mathrm{H} \leq 1 \times 10^{-4}$; $Y_P \leq 0.24$; $^7\mathrm{Li/H} \leq 1.4 \times 10^{-10}$. As discussed earlier, this excludes long lifetimes. Since ΔN_ν grows with lifetime, the boundary of the forbidden region is most interesting. Since it depends most sensitively on the limit to Y_P (and to a lesser degree on ^{7}Li) we also show the boundary for the relaxed constraints, $Y_P \leq 0.25$ and $^7\mathrm{Li/H} \leq 2 \times 10^{-10}$. For a Dirac neutrino $\Delta N_\nu \simeq 5(7)$ $(\nu_e\phi)$ and $8(10)$ $(3\nu_e)$ are the maximum permitted (the number in parenthesis for the relaxed constraints). The ΔN_ν's for a Majorana neutrino are about 2 neutrino species smaller as their abundance is smaller [12].

The final possibility is a variation on the previous one. While a massive tau neutrino provides a neat way of simultaneously suppressing ^{4}He production and producing additional radiation, a species with similar decay modes

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and a higher abundance could do even better. As noted earlier, for a lifetime of a few seconds the effects of decay-produced ν_e's and increased expansion rate cancel and increasing r does not affect ^{4}He production—but does increase ΔN_ν, cf. Eq. (2). Thus for $\tau \sim 0.1\,\mathrm{sec}-\mathrm{few}\,\mathrm{sec}$ ΔN_ν can be increased almost without limit. While r is not too different from one for a tau neutrino of mass $1\,\mathrm{MeV}-10\,\mathrm{MeV}$, it could be larger for a species with more degrees of freedom or weaker interactions. To illustrate, for a hypothetical $5\,\mathrm{MeV}$ species with $r = 2$, $\Delta N_\nu = 10$ $(\nu_e\phi)$, 17 $(3\nu_e)$ as large as is permitted.

Next, we consider a few of the consequences of τCDM. The level of inhomogeneity on small scales can be quantified by *rms* mass fluctuation in spheres of radius $8h^{-1}\,\mathrm{Mpc}$, σ_8. For COBE-normalized, standard CDM $\sigma_8 \simeq 1.2$; for mixed dark matter $\sigma_8 \simeq 0.75$; and for τCDM model with $\Gamma = 0.3$, $\sigma_8 \simeq 0.67$. The level of inhomogeneity predicted in τCDM on small scales is about a factor of two smaller than CDM and slightly lower than MDM. Since the *rms* fluctuation in galaxy counts is unity on the $8h^{-1}\,\mathrm{Mpc}$ scale, light is a "biased" tracer of mass in both τCDM and MDM, with bias factor $b \equiv 1/\sigma_8 \simeq 1.5$. On the other hand, as can be seen in Fig. 1, τCDM has more power than MDM on very small scales and so forming objects such as quasars at red shifts $z \sim 4-5$ should be less problematic.

We have computed the angular power spectrum of CBR anisotropy, $C_l = \langle |a_{lm}|^2 \rangle$ $(\delta T = \sum a_{lm} Y_{lm})$, and it is larger than standard CDM (see Fig. 4). To understand why, recall that the extra degrees of freedom delay matter-radiation equality. Last scattering occurs at the time of recombination $(T \sim 1/3\,\mathrm{eV})$, and in τCDM this happens closer to—or perhaps even in—the radiation-dominated era. As a result, the gravitational potential photons move through as they travel from the last-scattering surface is not constant, which leads to larger CBR anisotropy [15]. The quantity $l(l+1)C_l$ roughly corresponds to the expected variance of the CBR temperature difference measured between directions separated by angle $\theta \sim (200/l)^\circ$. For $l \sim 100-300$ this quantity is about a factor of 1.6 larger than standard CDM, which implies predicted temperature anisotropies on the degree scale that are a factor of $\sqrt{1.6}$ larger. While the situation is far from settled, there are experiments with detections that are larger than the CDM prediction [16].

In passing we mention two more consequences. First, a tau neutrino of mass $1\,\mathrm{MeV}-10\,\mathrm{MeV}$ can resurrect a theory that predicts too many massless degrees of freedom to be consistent with the "standard nucleosynthesis bound," cf. Fig. 2. Second, the cosmic sea of ν_e's is predicted to be very

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different: a nonthermal spectrum with higher energy per neutrino.

Finally, we turn to model building and verification. Within the framework of standard electroweak interactions the decay mode $3\nu_e$ is highly forbidden, as it involves flavor-changing neutral currents, and of course the $\nu_e\phi$ mode does not exist. In simple extensions of the standard model both modes can arise. For example, in models with family symmetry ϕ is a massless Nambu-Goldstone boson associated with the breakdown of a family symmetry (at energy scale f) and $\tau_\nu \sim 8\pi f^2/m_\nu^3 \sim 10\sec(f/10^9\,\text{GeV})^2(m_\nu/10\,\text{MeV})^{-3}$. In these models there can be many such Nambu-Goldstone bosons which contribute additional massless degress of freedom [17]. The $3\nu_e$ decay mode can be mediated by a heavy boson with mass M; the lifetime is then $\tau_\nu \sim 0.1\sec(m_\nu/10\text{MeV})^{-5}(M/M_W)^4\sin^{-2}\theta$. We note though that in many models this boson would also couple to the tau lepton and violate the stringent bounds on $\tau \to 3e$.

The best laboratory upper limits to the tau-neutrino mass are: 31 MeV by the ARGUS Collaboration and 32.6 MeV by the CLEO Collaboration [18]. They are based upon end-point studies of the five-pion decay mode of the tau lepton. It is possible that the mass sensitivity could be extended to the 5 MeV − 10 MeV range by looking at tau decay modes with multiple Kaons in the final state [19]; moreover, experiments done at B-factories may well be able to reach mass sensitivities of 5 MeV − 10 MeV [19].

To summarize, a tau neutrino of mass 1 MeV − 10 MeV and lifetime 10 sec − 100 sec whose decay products include electron neutrinos can significantly increase the energy density in relativistic particles without upsetting the successful predictions of primordial nucleosynthesis. Alternatively, a tau neutrino in the same mass range, but with lifetime 0.1 sec − 1 sec, allows many more massless degrees of freedom leading to the same end. The higher level of energy density in relativistic particles modifies the CDM transfer function fixing the nagging problem that CDM has with excessive power on small scales. The level of CBR anisotropy predicted in the τCDM model is slightly larger than CDM on the degree scale. Most importantly, perhaps, prospects for testing this hypothesis at $e^\pm$ colliders appear promising.

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Figure Captions

Figure 1: The power spectra for CDM models normalized to COBE: standard CDM ($h = 0.5$); mixed dark matter ($h = 0.5$); and τCDM ($h = 0.45$ and $g_* = 7.5$). The data points are from the IRAS 1.2Jy survey [20]. Wavenumber and wavelength are related by $k \equiv 2\pi/\lambda$.

Figure 2: Contours of the additional number of massless species that can be tolerated without violating the constraints to the light-element abundances (Dirac tau neutrino and $3\nu_e$ decay mode). [Note, because our calculations do not include inverse tau-neutrino decays, our results *may* be unreliable in the lower left-hand corner, $\tau \lesssim (\text{MeV}/m_\nu)^2$ sec; see Ref. [12].]

Figure 3: The equivalent number of massless neutrino species ΔN_ν produced by the decay of a massive tau neutrino and the excluded regions of the mass-lifetime plane for a massive Dirac tau neutrino and the $\nu_e\phi$ and $3\nu_e$ modes. The solid contours correspond to $\Delta N_\nu = 4, 6, 8, 10$ from left to right. The excluded regions are to the right of the broken lines (heavy for $Y_P \leq 0.24$; light for $Y_P \leq 0.25$).

Figure 4: The COBE-normalized angular power spectrum for standard CDM (with $h = 0.5$) and τCDM (with $h = .45$ and $\Delta N_\nu = 10$). For both, $\Omega_B h^2 = 0.0125$.
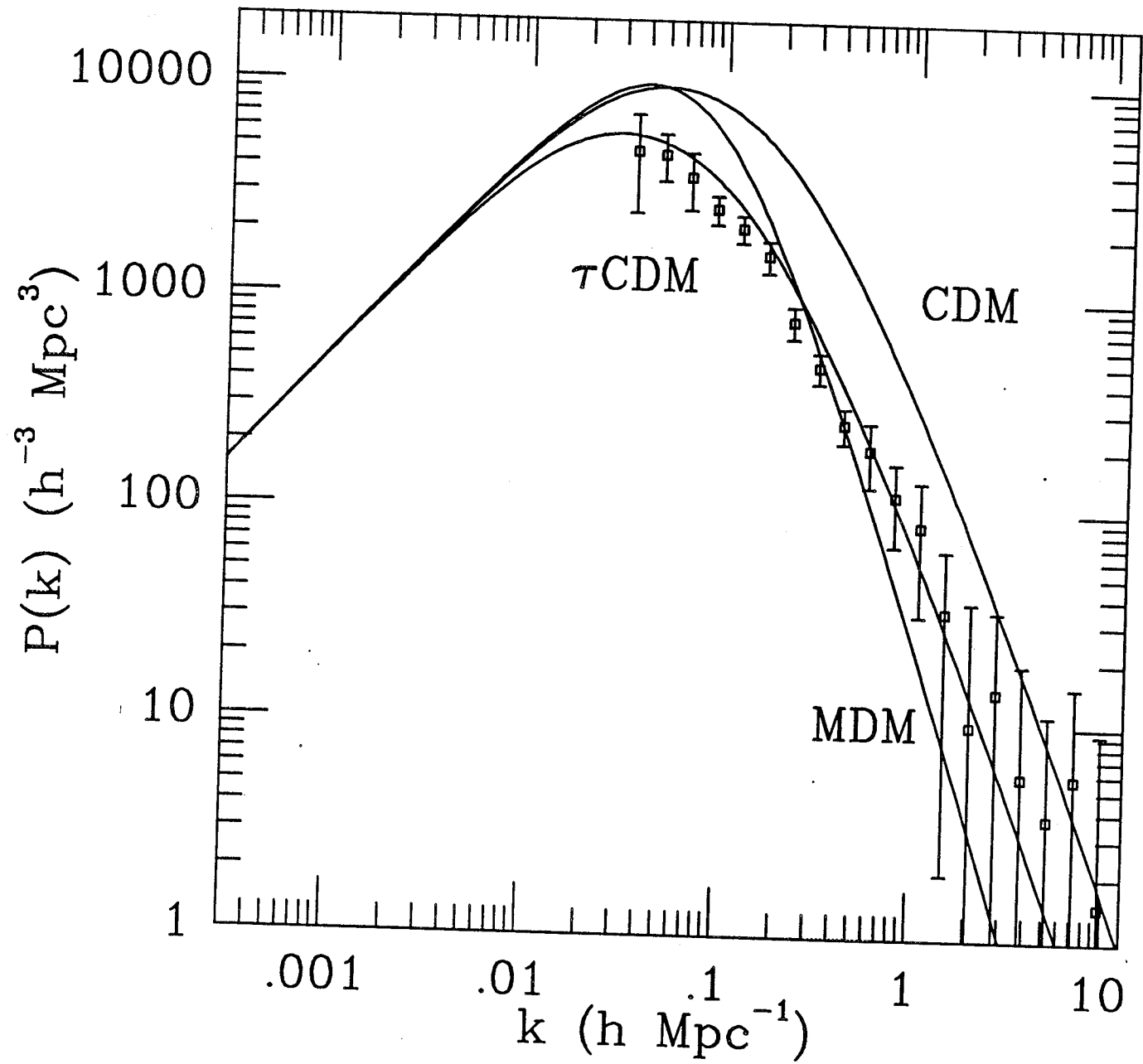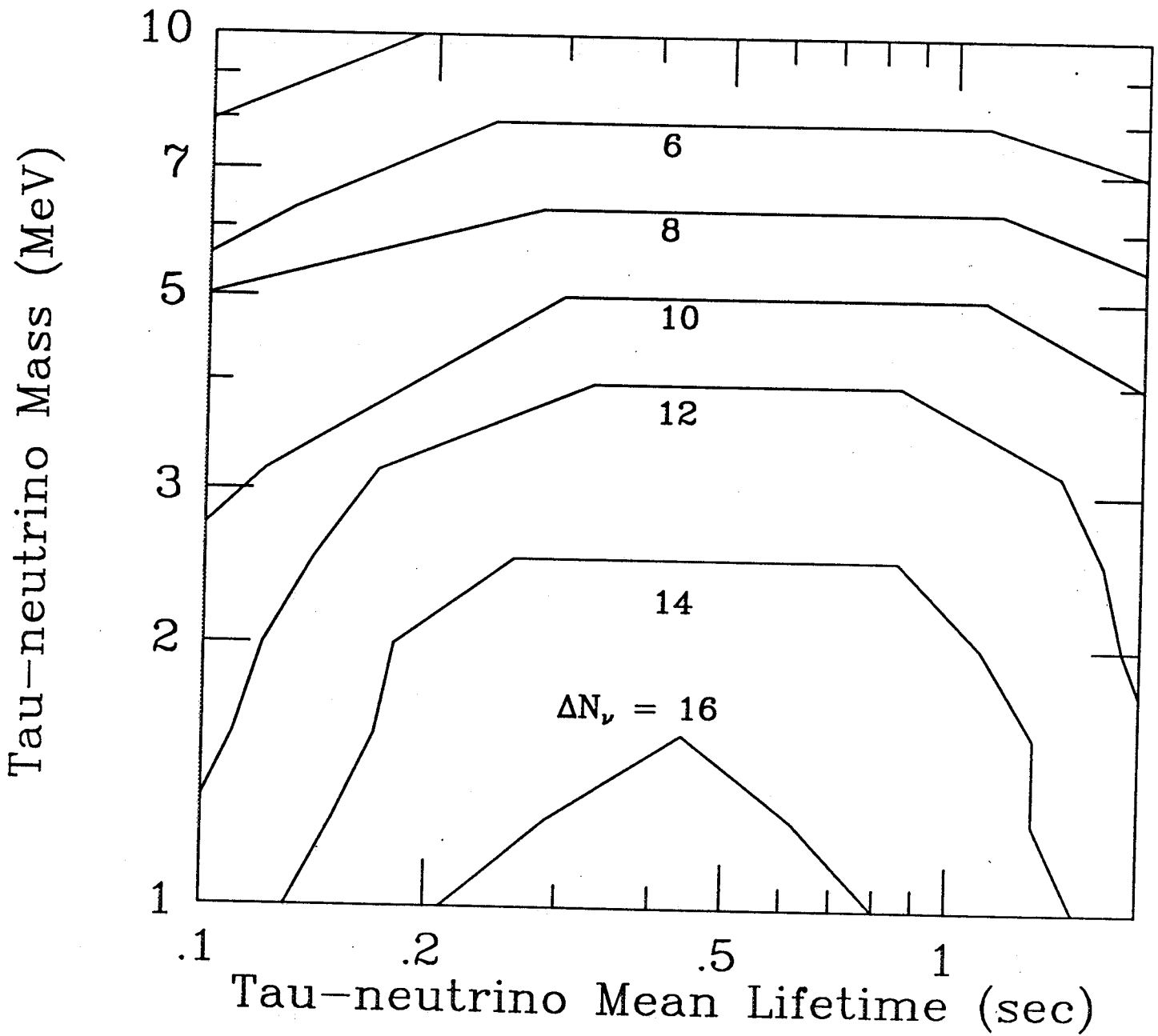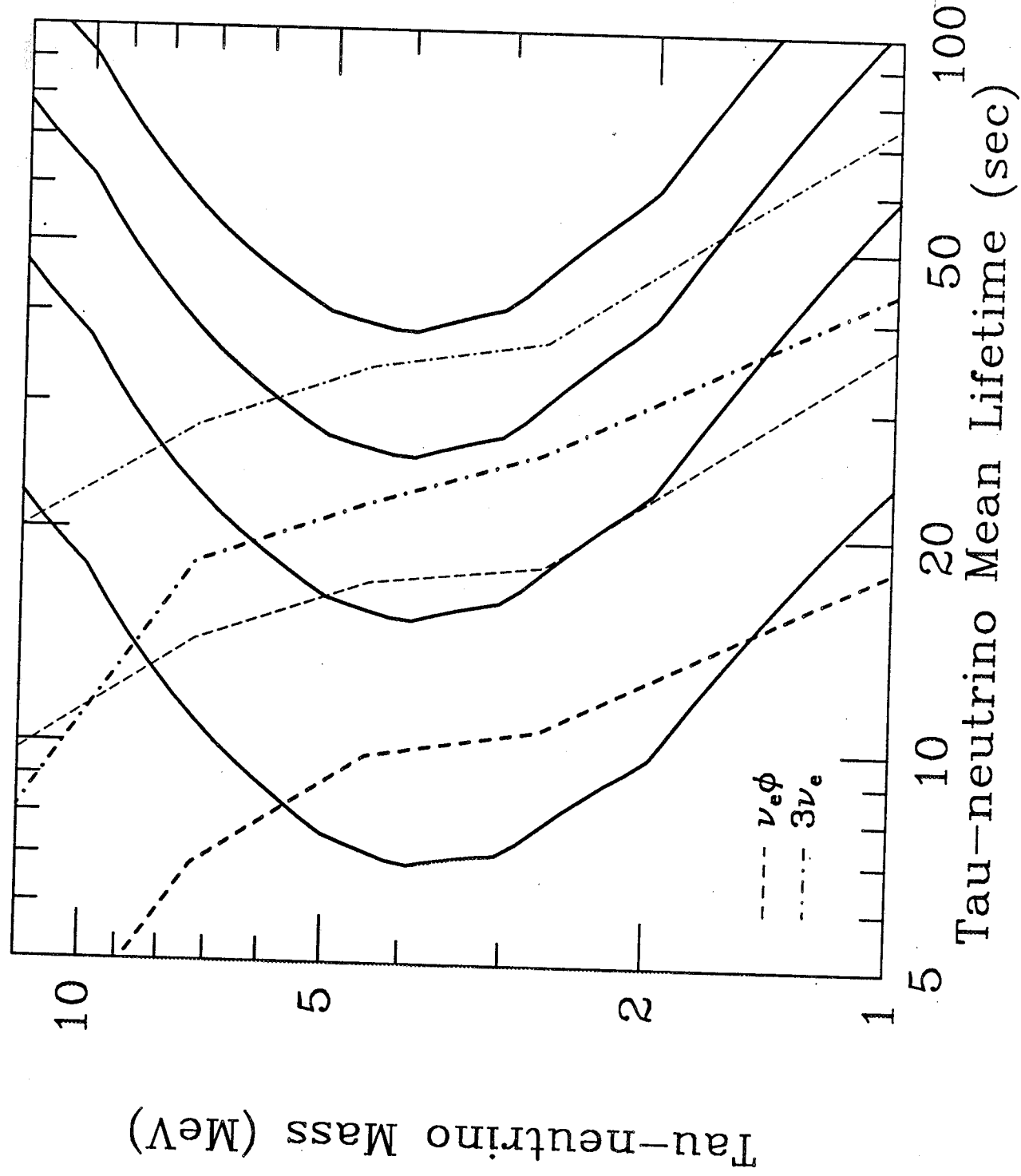
Figure
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Figure 2

Figure 3

Tau-neutrino Mass (MeV)

Tau-neutrino Mean Lifetime (sec)

$\nu_e\phi$

$3\nu_e$

Figure 4