112591 **N95-19006**

1994

NASA/ASEE SUMMER FACULTY FELLOWSHIP PROGRAM

30929 p-6

MARSHALL SPACE FLIGHT CENTER
THE UNIVERSITY OF ALABAMA

SCATTERING OF POINT SOURCE ILLUMINATION BY AN ARBITRARY CONFIGURATION

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Monte Carlo Simulations [1], equivalent medium approaches [2, 3], and methods based on Boltzmann transport theory [4, 5] have been applied to study the scattering of light produced by lightning in a thundercloud. The Monte Carlo approach is the most straightforward but other approaches may yield better insight into the phenomena.

The cloud is assumed to consist of a uniform, homogeneous, random distribution of spherical water drops with an average radius of 10 μm and number density $\rho = 100$ cm^{-3}. In particular, light in the near infrared, $\lambda = 0.7774$ μm, is of interest. The equivalent medium approach, based on methods due to Twersky [6, 7], yields the coherently scattered fields. For the ratio of the average separation distance between water drops to the wavelength, the scattering is considered to be almost totally incoherent [8]. During the time interval of a typical lightning event, the movement of the water drops is negligible. For fixed configurations of scatterers, the distinction between coherent and incoherent scattering is not very clear in the literature. The best explanation in the author's estimation is given by Foldy [9], who admits that for a fixed configuration of scatterers, all of the scattering is strictly coherent. A rather artificial distinction is made. The following definition is unambiguous: the square magnitude of an average field $|\langle\psi\rangle|^2$, $\psi = \mathbf{E}$ or $\mathbf{H}$, is proportional to the coherent intensity; $V = \langle|\psi|^2\rangle - |\langle\psi\rangle|^2$ is proportional to the incoherent intensity. The function V is approximately given in terms of the square magnitude of the radiated field from a scatterer at $\mathbf{b}_s$ averaged over all possible configurations of the remaining scatterers,

$$\approx \rho\int d\mathbf{r}_s \left\langle |U_s|^2 \right\rangle_s \approx \rho\int d\mathbf{r}_s \left|\langle U_s\rangle\right|_s^2, \tag{1}$$

where $d\mathbf{r}_s$ is a volume element in local coordinates $\mathbf{r}_s = \mathbf{r} - \mathbf{b}_s$.

The problem of electromagnetic scattering of an incident plane wave by an arbitrary configuration of obstacles was solved by Twersky [10]. In this report, the results are extended to point source incidence corresponding to a Hertz dipole. Knowledge of the response of a fixed configuration of scatterers excited by a point source may provide insight to improve the accuracy of the values of bulk parameters for clouds which have been found using plane wave excitation.

As in [3], we transform to the frequency domain; time domain solutions are recovered by a Fourier integral. A dyadic formalism is used throughout. We will employ the following notation: ϕ, u, and ψ will denote the incident, scattered, and total fields respectively. Lower case letters will correspond to single scattering (an object in isolation); upper case letters will be used for multiply scattered fields. Functional dependencies in brackets imply plane wave incidence; the use of parentheses is reserved for fields corresponding to a point source. Modification from plane wave to point source excitation is accomplished by operating on the plane wave results with a Sommerfeld-type contour integral representation [11] of a spherical Hankel function.

Generally speaking, if a transverse wave has a direction $\hat{\mathbf{r}}$, its dyadic form will look like $\tilde{\mathbf{I}} - \hat{\mathbf{r}}\hat{\mathbf{r}}$, where $\tilde{\mathbf{I}}$ is the identity dyad. Incident fields will have the form

$$\tilde{\phi}[\mathbf{r},\hat{\mathbf{k}}] = (\tilde{\mathbf{I}} - \hat{\mathbf{k}}\hat{\mathbf{k}})\phi[\mathbf{r},\hat{\mathbf{k}}],\ \phi[\mathbf{r},\hat{\mathbf{k}}] = e^{i\mathbf{k}\cdot\mathbf{r}},\ \tilde{\phi}(\mathbf{r},\mathbf{r}') = \left(\tilde{\mathbf{I}} + \frac{\nabla\nabla}{k^2}\right)\phi(\mathbf{r},\mathbf{r}'),\ \phi(\mathbf{r},\mathbf{r}') = h_0^{(1)}(k|\mathbf{r}-\mathbf{r}'|), \tag{2}$$

where the first argument refers to the point of observation and the second is the direction of incidence or the source location. The plane wave can be taken as the incident electric or magnetic field. The function $\tilde{\phi}(\mathbf{r},\mathbf{r}')$ specifies the fields due a dipole located at $\mathbf{r}'$. For an electric or magnetic dipole with directions $\hat{\mathbf{p}},\hat{\mathbf{p}}_m$, we have [3]

$$\begin{aligned} \mathbf{E}^i &= \frac{ik^3}{4\pi\varepsilon_o}\tilde{\phi}(\mathbf{r},\mathbf{r}')\cdot\hat{\mathbf{p}},\ \mathbf{H}^i = \frac{k^3}{4\pi\omega\mu_o\varepsilon_o}\nabla h_o^{(1)}(|\mathbf{r}-\mathbf{r}'|)\times\hat{\mathbf{p}}, \\ \mathbf{E}^i_m &= -\frac{k^3}{4\pi\omega\varepsilon_o}\nabla h_o^{(1)}(|\mathbf{r}-\mathbf{r}'|)\times\hat{\mathbf{p}}_m,\ \mathbf{H}^i_m = \frac{ik^3}{4\pi}\tilde{\phi}(\mathbf{r},\mathbf{r}')\cdot\hat{\mathbf{p}}_m, \end{aligned} \tag{3}$$

where ε_0 and μ_0 are the permittivity and permeability of the medium external to the scatterers. Here, $\mathbf{E}^i$ and $\mathbf{H}^i$ are the electric and magnetic field vectors due to an electric dipole; the subscripts m denote the corresponding quantities due to a magnetic dipole. We will solve for electric fields when given an electric dipole for the source and work with magnetic fields for a magnetic dipole. The remaining fields may be found using Maxwell's eqs.

A dyadic version of the Helmholtz surface integral representation is given by [10]

$$\tilde{u} = \frac{k}{4\pi i}\int dS(\mathbf{r})\left\{\left[\hat{\mathbf{n}}\times\tilde{\phi}(\mathbf{r},\mathbf{r})\right]^T\cdot[\nabla\times\tilde{\psi}]-\left[\nabla\times\tilde{\phi}(\mathbf{r},\mathbf{r})\right]^T\cdot[\hat{\mathbf{n}}\times\tilde{\psi}]\right\} \equiv \left\{\tilde{\phi}(\mathbf{r},\mathbf{r}),\tilde{\psi}\right\}, \tag{4}$$

where ∇ operates on the variables associated with the vector $\mathbf{r}$ to a point on the surface of integration, T denotes the transpose, and $\hat{\mathbf{n}}$ is the outward normal. In (4), $\tilde{\psi} = \tilde{\psi}[\mathbf{r},\hat{\mathbf{k}}]$ or $\tilde{\psi}(\mathbf{r},\mathbf{r}')$ and $\tilde{u}$ can be $\tilde{u}[\mathbf{r},\hat{\mathbf{k}}]$ or $\tilde{u}(\mathbf{r},\mathbf{r}')$ depending on the initial excitation.

As $r \sim \infty$, we can write

$$\tilde{\phi}(\mathbf{r},\mathbf{r}) \sim \left(\tilde{\mathbf{I}}-\hat{\mathbf{r}}\hat{\mathbf{r}}\right)h_0^{(1)}(kr)e^{-ik\hat{\mathbf{r}}\cdot\mathbf{r}} \tag{5}$$

in (4) to obtain

$$\tilde{u} \sim h_0^{(1)}(kr)\tilde{g},\ \tilde{g} = \left\{\left(\tilde{\mathbf{I}}-\hat{\mathbf{r}}\hat{\mathbf{r}}\right)e^{-ik\hat{\mathbf{r}}\cdot\mathbf{r}},\tilde{\psi}\right\}. \tag{6}$$

Single scattering amplitudes for both kinds of excitations may be defined in this way. Sommerfeld's integral representation for $h_0^{(1)}$ is given by

$$\left(\tilde{\mathbf{I}}+\frac{\nabla\nabla}{k^2}\right)h_0^{(1)}(k|\mathbf{r}-\mathbf{r}|) = \frac{1}{2\pi}\int d\Omega(\hat{\mathbf{r}}_c)\left(\tilde{\mathbf{I}}-\hat{\mathbf{r}}_c\hat{\mathbf{r}}_c\right)e^{ik\hat{\mathbf{r}}_c\cdot(\mathbf{r}-\mathbf{r})}, \tag{7}$$

where $\hat{\mathbf{r}}_c = \hat{\mathbf{r}}_c(\theta_c,\varphi_c)$, $0 \le \varphi_c \le 2\pi$, and θ_c starts at zero and goes to $\theta_l - i\infty$, where θ_l is in an interval which guarantees the convergence of the integral. Substituting in (4), reversing the order of integrations, and recognizing $\tilde{g}$ from (6), yields the spectral representations

$$\tilde{u}\left[\mathbf{r},\hat{\mathbf{k}}\right] = \frac{1}{2\pi}\int d\Omega(\hat{\mathbf{r}}_c)e^{ik\hat{\mathbf{r}}_c\cdot\mathbf{r}}\tilde{g}\left[\hat{\mathbf{r}}_c,\hat{\mathbf{k}}\right],\ \tilde{u}(\mathbf{r},\mathbf{r}') = \frac{1}{2\pi}\int d\Omega(\hat{\mathbf{r}}_c)e^{ik\hat{\mathbf{r}}_c\cdot\mathbf{r}}\tilde{g}(\hat{\mathbf{r}}_c,\mathbf{r}'). \tag{8}$$

These representations are valid for r greater than the scatterer's projection on $\hat{\mathbf{r}}$.

Eq. (7) shows that a point source can be written as a superposition of plane waves. If the response of an object for plane wave excitation is known, its response to a point source can be obtained by superposition. Allowing the direction of incidence of a plane wave $\hat{\mathbf{k}}$ to be complex and regarding $\exp[-i\mathbf{k}\cdot\mathbf{r}']$ in (7) as a phase factor, we find the relation

$$\tilde{g}(\mathbf{r},\mathbf{r}') = \frac{1}{2\pi}\int d\Omega(\hat{\mathbf{k}})e^{-i\mathbf{k}\cdot\mathbf{r}'}\tilde{g}[\hat{\mathbf{r}},\mathbf{k}]. \tag{9}$$

An integral operator like (9) will be used to modify plane wave forms to point source excitation.

For spherical scatterers, explicit forms for the scattering amplitudes may be written in terms of Hansen's functions. The excitations may be written as

$$\begin{aligned}
\tilde{\phi}[\mathbf{r},\hat{\mathbf{k}}] &= \sum i^n\sqrt{n(n+1)}Q_n^m\left[\mathbf{M}^1_{mn}(k,\mathbf{r})\mathbf{C}_{-mn}(\hat{\mathbf{k}}) - i\mathbf{N}^1_{mn}(k,\mathbf{r})\mathbf{B}_{-mn}(\hat{\mathbf{k}})\right],\\
\tilde{\phi}(\mathbf{r},\mathbf{r}') &= \sum Q_n^m\left[\mathbf{M}^3_{mn}(k,\mathbf{r})\mathbf{M}^1_{-mn}(k,\mathbf{r}') + \mathbf{N}^3_{mn}(k,\mathbf{r})\mathbf{N}^1_{-mn}(k,\mathbf{r}')\right],\ r > r';\\
\sum &\equiv \sum_{n=1}^{\infty}\sum_{m=-n}^{n},\quad Q_n^m = \frac{(2n+1)(-1)^m}{n(n+1)},
\end{aligned} \tag{10}$$

where $\mathbf{C}$ and $\mathbf{B}$ are vector spherical harmonics. We use the definitions of these functions as given in [12] except that complex exponentials are used instead of even (cosine) and odd (sine) forms. The normalization of the spherical harmonics Y_n^m used here follows [11]. When $r < r'$, the corresponding vectors in the second eq. in (10) are interchanged.

The interior and radiated fields have the forms

$$\begin{aligned}
\tilde{\psi}_{in}[\mathbf{r},\hat{\mathbf{k}}] &= \sum\left\{\mathbf{M}^1_{mn}(K,\mathbf{r})\mathbf{a}_n^m[\hat{\mathbf{k}}] + \mathbf{N}^1_{mn}(K,\mathbf{r})\mathbf{b}_n^m[\hat{\mathbf{k}}]\right\},\\
\tilde{u}[\mathbf{r},\hat{\mathbf{k}}] &= \sum\left\{\mathbf{M}^3_{mn}(K,\mathbf{r})\mathbf{c}_n^m[\hat{\mathbf{k}}] + \mathbf{N}^3_{mn}(K,\mathbf{r})\mathbf{d}_n^m[\hat{\mathbf{k}}]\right\},\\
\tilde{\psi}_{in}(\mathbf{r},\mathbf{r}') &= \sum\left\{\mathbf{M}^1_{mn}(K,\mathbf{r})\mathbf{a}_n^m(\mathbf{r}') + \mathbf{N}^1_{mn}(K,\mathbf{r})\mathbf{b}_n^m(\mathbf{r}')\right\},\\
\tilde{u}(\mathbf{r},\mathbf{r}') &= \sum\left\{\mathbf{M}^3_{mn}(K,\mathbf{r})\mathbf{c}_n^m(\mathbf{r}') + \mathbf{N}^3_{mn}(K,\mathbf{r})\mathbf{d}_n^m(\mathbf{r}')\right\},
\end{aligned} \tag{11}$$

where $k = k(\varepsilon_0,\mu_0)$ and $K = K(\varepsilon,\mu)$ is the interior wave number. In general, the transition conditions at an interface, where the normal is denoted by $\hat{\mathbf{n}}$, require that

$$\hat{\mathbf{n}}\times\left(\tilde{\phi}+\tilde{u}\right) = \hat{\mathbf{n}}\times\tilde{\psi}_{in},\ \hat{\mathbf{n}}\times\nabla\times\left(\tilde{\phi}+\tilde{u}\right) = B\hat{\mathbf{n}}\times\nabla\times\tilde{\psi}_{in}, \tag{12}$$

where $B = \mu_0/\mu$ if $\tilde{\phi}$ represents an electric field and $\varepsilon_0/\varepsilon$ if $\tilde{\phi}$ is taken to be a magnetic field.

For a sphere of radius a, the scattering coefficients are given by

$$\begin{aligned}
\mathbf{a}_n^m[\hat{\mathbf{k}}] &= i^n\sqrt{n(n+1)}Q_n^m\mathbf{C}_{-mn}(\hat{\mathbf{k}})a_n, & \mathbf{a}_n^m(\mathbf{r}') &= Q_n^m\mathbf{M}^1_{-mn}(k,\mathbf{r}')a_n,\\
\mathbf{b}_n^m[\hat{\mathbf{k}}] &= i^{n-1}\sqrt{n(n+1)}Q_n^m\mathbf{B}_{-mn}(\hat{\mathbf{k}})b_n, & \mathbf{b}_n^m(\mathbf{r}') &= Q_n^m\mathbf{N}^1_{-mn}(k,\mathbf{r}')b_n,\\
\mathbf{c}_n^m[\hat{\mathbf{k}}] &= i^n\sqrt{n(n+1)}Q_n^m\mathbf{C}_{-mn}(\hat{\mathbf{k}})c_n, & \mathbf{c}_n^m(\mathbf{r}') &= Q_n^m\mathbf{M}^1_{-mn}(k,\mathbf{r}')c_n,\\
\mathbf{d}_n^m[\hat{\mathbf{k}}] &= i^{n-1}\sqrt{n(n+1)}Q_n^m\mathbf{B}_{-mn}(\hat{\mathbf{k}})d_n, & \mathbf{d}_n^m(\mathbf{r}') &= Q_n^m\mathbf{N}^1_{-mn}(k,\mathbf{r}')d_n,
\end{aligned} \tag{13}$$

where the constants a_n, b_n, c_n, and d_n are the appropriate set of Mie coefficients found in [3]. Series for the scattering amplitudes may be obtained by substituting large kr forms for the Hansen's functions in (11),

$$\begin{aligned}
\tilde{g}[\hat{\mathbf{r}},\hat{\mathbf{k}}] &= \sum n(n+1)Q_n^m\left[c_n\mathbf{C}_{mn}(\hat{\mathbf{r}})\mathbf{C}_{-mn}(\hat{\mathbf{k}}) + d_n\mathbf{B}_{mn}(\hat{\mathbf{r}})\mathbf{B}_{-mn}(\hat{\mathbf{k}})\right],\\
\tilde{g}(\hat{\mathbf{r}},\mathbf{r}') &= \sum\sqrt{n(n+1)}i^{-n}Q_n^m\left[c_n\mathbf{C}_{mn}(\hat{\mathbf{r}})\mathbf{M}^1_{-mn}(k,\mathbf{r}') + id_n\mathbf{B}_{mn}(\hat{\mathbf{r}})\mathbf{N}^1_{-mn}(k,\mathbf{r}')\right],
\end{aligned} \tag{14}$$

and may be used for the electric or magnetic fields given the same was chosen as the incidence after multiplication by appropriate constants. The remaining fields may be found by first interchanging those **M**'s and **N**'s in $\tilde{u}$ or **C**'s and **B**'s in $\tilde{g}$ which are functions of $\hat{\mathbf{r}}$. Solutions $\tilde{u}$ and $\tilde{g}$ for magnetic fields when $\tilde{\phi}$ is taken to be an electric field are then obtained by multiplication by $\gamma = -i\sqrt{\varepsilon_0/\mu_0}$; use $1/\gamma$ for the remaining situation.

Consider a fixed configuration of N scatterers. Each has its "center" located at $\mathbf{b}_s$, $s = 1,2,\ldots,N$. The total field for plane wave excitation may be written as

$$\tilde{\Psi}[\mathbf{r},\hat{\mathbf{k}}] = \tilde{\phi}[\mathbf{r},\hat{\mathbf{k}}] + \tilde{U}[\mathbf{r},\hat{\mathbf{k}}],\ \tilde{U}[\mathbf{r},\hat{\mathbf{k}}] = \sum_s \tilde{U}_s[\mathbf{r}_s,\hat{\mathbf{k}}]e^{ik\cdot\mathbf{b}_s} \sim h_0^{(1)}(kr)\tilde{G}[\mathbf{r},\hat{\mathbf{k}}],\ \mathbf{r}_s = \mathbf{r} - \mathbf{b}_s, \tag{15}$$

where $\tilde{U}_s$ is the scattered field from the scatterer at $\mathbf{b}_s$ as if it was located at the origin. Forms similar to those obtained for an isolated scatterer,

$$\tilde{U}_s[\mathbf{r}_s,\hat{\mathbf{k}}] = \frac{1}{2\pi}\int d\Omega(\hat{\mathbf{r}}_c)e^{ik\hat{\mathbf{r}}_c\cdot\mathbf{r}_s}\tilde{G}_s[\hat{\mathbf{r}}_c,\hat{\mathbf{k}}] \sim h_0^{(1)}(kr_s)\tilde{G}_s[\hat{\mathbf{r}},\hat{\mathbf{k}}],\ \tilde{G}_s[\hat{\mathbf{r}},\hat{\mathbf{k}}] = \left\{(\tilde{\mathbf{I}} - \hat{\mathbf{r}}\hat{\mathbf{r}})e^{-ik\hat{\mathbf{r}}\cdot\mathbf{r}},\tilde{U}_s[\mathbf{r},\hat{\mathbf{k}}]\right\}, \tag{16}$$

where $\hat{\mathbf{r}}_s \sim \hat{\mathbf{r}}$ as $r \sim \infty$, are also valid here. Taking phase differences into account, we have

$$\tilde{G}[\hat{\mathbf{r}},\hat{\mathbf{k}}] = \sum_s e^{ik(\hat{\mathbf{k}}-\hat{\mathbf{r}})\cdot\mathbf{b}_s}\tilde{G}_s[\hat{\mathbf{r}},\hat{\mathbf{k}}] \tag{17}$$

Twersky [10] obtained a coupled set of integral eqs. for $\tilde{G}_s[\hat{\mathbf{r}},\hat{\mathbf{k}}]$ in terms $\tilde{g}$,

$$\tilde{G}_s[\hat{\mathbf{r}},\hat{\mathbf{k}}] = \tilde{g}_s[\hat{\mathbf{r}},\hat{\mathbf{k}}] + \frac{1}{2\pi}\sum_{t\neq s}\int d\Omega(\hat{\mathbf{r}}_c)e^{ik(\hat{\mathbf{r}}_c-\hat{\mathbf{k}})\cdot\mathbf{b}_{st}}\hat{g}_s[\hat{\mathbf{r}},\hat{\mathbf{r}}_c]\cdot\tilde{G}_t[\hat{\mathbf{r}}_c,\hat{\mathbf{k}}],\ \mathbf{b}_{st} = \mathbf{b}_s - \mathbf{b}_t. \tag{18}$$

Recall that $\tilde{G}_s$ is related to $\tilde{U}_s$, which is written in local coordinates $\mathbf{r}_s$. Operating on (18) with the integral $(2\pi)^{-1}\int d\Omega(\hat{\mathbf{k}})\exp[-i\mathbf{k}\cdot(\mathbf{r}'-\mathbf{b}_s)]$ yields

$$\tilde{G}_s(\hat{\mathbf{r}},\mathbf{b}'_s) = \tilde{g}_s(\hat{\mathbf{r}},\mathbf{b}'_s) + \frac{1}{2\pi}\sum_{t\neq s}\int d\Omega(\hat{\mathbf{r}}_c)e^{ik\hat{\mathbf{r}}_c\cdot\mathbf{b}_{st}}\tilde{g}_s[\hat{\mathbf{r}},\hat{\mathbf{r}}_c]\cdot\tilde{G}_t(\hat{\mathbf{r}}_c,\mathbf{b}'_t),\ \mathbf{b}'_s = \mathbf{r}' - \mathbf{b}_s. \tag{19}$$

When the particle separations are large compared to wavelength, the integral in (19) may be evaluated asymptotically. A convenient method [10] to do this exists whenever the kernel, aside from the exponential, can be expanded in a series of vector spherical harmonics. Iterating while retaining an appropriate number of terms in the asymptotic series will generate a consistent asymptotic expansion for $\tilde{G}_s$. Exactly the same expansion as given in [10] will work for (19) except that $\tilde{G}[\,]$ is replaced by $\tilde{G}(\,)$. Retaining only the leading term in the integral, gives

$$\tilde{G}(\hat{\mathbf{r}},\mathbf{b}'_s) \sim \tilde{g}_s(\hat{\mathbf{r}},\mathbf{b}'_s) + \sum_{t\neq s} h_0^{(1)}(kb_{st})\tilde{g}_s[\hat{\mathbf{r}},\hat{\mathbf{b}}_{st}]\cdot\tilde{G}_t(\hat{\mathbf{b}}_{st},\mathbf{b}'_t). \tag{20}$$

A set of algebraic eqs. may be obtained by substituting (14) and

$$\tilde{G}_s(\hat{\mathbf{r}},\mathbf{b}'_s) = \sum\left[\mathbf{C}_{mn}(\hat{\mathbf{r}})\boldsymbol{C}^s_{mn}(\mathbf{b}'_s) + \mathbf{B}_{mn}(\hat{\mathbf{r}})\boldsymbol{D}^s_{mn}(\mathbf{b}'_s)\right] \tag{21}$$

into (19) and using orthogonality. Similar to [10], we obtain

$$
\begin{aligned}
\boldsymbol{C}^{s}_{mn} &= \sqrt{n(n+1)}\, i^{-n} c^{s}_{n} Q^{m}_{n} \mathbf{M}^{1}_{-mn}(k, \mathbf{b}'_{s}) + n(n+1) Q^{m}_{n} c^{s}_{n} \sum_{t \neq s} \sum_{q,p} \left[E^{qp}_{-mn}(\mathbf{b}_{st}) \boldsymbol{C}^{t}_{qp} + F^{qp}_{-mn}(\mathbf{b}_{st}) \boldsymbol{D}^{t}_{qp} \right], \\
\boldsymbol{D}^{st}_{mn} &= \sqrt{n(n+1)}\, i^{-n} d^{s}_{n} Q^{m}_{n} \mathbf{N}^{1}_{-mn}(k, \mathbf{b}'_{s}) + n(n+1) Q^{m}_{n} d^{s}_{n} \sum_{t \neq s} \sum_{q,p} \left[-F^{qp}_{-mn}(\mathbf{b}_{st}) \boldsymbol{C}^{t}_{qp} + E^{st}_{-mn}(\mathbf{b}_{st}) \boldsymbol{D}^{t}_{qp} \right]; \\
E^{qp}_{-mn}(\mathbf{b}_{st}) &= \frac{1}{2\pi} \int d\Omega(\hat{\mathbf{r}}_c) e^{ik\hat{\mathbf{r}}_c \cdot \mathbf{b}_{st}} \mathbf{C}_{-mn}(\hat{\mathbf{r}}_c) \cdot \mathbf{C}_{qp}(\hat{\mathbf{r}}_c) = \frac{1}{2\pi} \int d\Omega(\hat{\mathbf{r}}_c) e^{ik\hat{\mathbf{r}}_c \cdot \mathbf{b}_{st}} \mathbf{B}_{-mn}(\hat{\mathbf{r}}_c) \cdot \mathbf{B}_{qp}(\hat{\mathbf{r}}_c), \\
F^{qp}_{-mn}(\mathbf{b}_{st}) &= \frac{1}{2\pi} \int d\Omega(\hat{\mathbf{r}}_c) e^{ik\hat{\mathbf{r}}_c \cdot \mathbf{b}_{st}} \mathbf{C}_{-mn}(\hat{\mathbf{r}}_c) \cdot \mathbf{B}_{qp}(\hat{\mathbf{r}}_c) = -\frac{1}{2\pi} \int d\Omega(\hat{\mathbf{r}}_c) e^{ik\hat{\mathbf{r}}_c \cdot \mathbf{b}_{st}} \mathbf{B}_{-mn}(\hat{\mathbf{r}}_c) \cdot \mathbf{C}_{qp}(\hat{\mathbf{r}}_c),
\end{aligned}
\tag{22}
$$

where the scalars E and F can be evaluated in terms of special functions.

In a cubic cloud 10 km on a side, there are approximately 10^{20} drops. For such large systems, a simple approach as suggested in (20) may be appropriate. Sufficiently far from the source, each drop in a small sub-volume of the cloud experiences essentially the same initial excitation. All of the drops in this sub-volume can be considered as a "compound" scatterer. Its scattering amplitude may be determined by (17). It will then be possible to consider fewer particles making up the cloud. Presumably, the procedure of obtaining $\tilde{\mathsf{G}}$ from $\tilde{G}$, considering $\tilde{\mathsf{G}}$ as a "compound" $\tilde{G}$, and working with a distribution of "compound" scatterers can be repeated to obtain a result which is amenable to numerical computations.

The author would like to express his appreciation to W. Koshak, R. Blakeslee, H. Christian, and D. Phanord for their help and acknowledge the assistance of F. Six, L. Freeman, G. Karr, S. Novy and T. Shurtz.

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