FATIGUE AND DAMAGE TOLERANCE SCATTER MODELS

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SUMMARY

Effective Total Fatigue Life and Crack Growth Scatter Models are proposed. The first of them is based on the power form of the Wöhler curve, fatigue scatter dependence on mean life value, cycle stress ratio influence on fatigue scatter, and validated description of the mean stress influence on the mean fatigue life. The second uses in addition are fracture mechanics approach, assumption of initial damage existence, and Paris equation.

Simple formulae are derived for configurations of models. A preliminary identification of the parameters of the models is fulfilled on the basis of experimental data. Some new and important results for fatigue and crack growth scatter characteristics are obtained.

INTRODUCTION

The total fatigue life (from the initial condition to the failure) or the life between any two conditions of element damage (for example, from the moment when the damage is detectable to the moment when the damage becomes a critical one) are random values, and therefore they should not be described by mean values only and should be characterized by a complete probability distribution or, at least, by both mean values and standard deviations.

The scatter is usually so big that it cannot and should not be considered as an annoying thing that hinders the investigator to reveal the "pure" fatigue life characteristics (FLC). On the countrary, the scatter must be considered as a very important feature of the fatigue life which demands special and careful attention.

These considerations are especially referred to the structures of "high criticality" whose failure may lead to serious results. Indeed, the procedures of ensuring service safety should be based in this case on the "worst copy" characteristics which are much more or less (about 3 - 5 times or more) than the mean value. Such a large "scatter penalty" may turn the very hard activity for a 10-20 percent saving in mean fatigue life to an "absurd" work.

The conclusion is clear: a deep scientific study of scatter characteristics as a research target is needed. The reasons for scatter, the values of scatter, the influences on scatter; in other words, the mechanism of scatter forming should be known. On the basis of the abovementioned knowledge the phenomenological models of high probability should be created.

THE FORMING OF FATIGUE LIFE MODELS.

Let us name as Life Parameters (LP) such factors "included" into the relations of accepted form which simply give the necessary FLCs (the crack growth included). As an example of LPs one can consider the factors of analytical presentations of endurance curves in power or any other form, the factors of Paris equation, etc.

Let us formulate assumptions which form the basis of models to be proposed:

(i)Between the LPs and FLCs "quasi-deterministic" relations exist

(ii)The forms of the relations are similar for any copy of the airframe structure. The scatter of FLCs is connected therefore only with LPs scatter.

Using these assumptions we can obtain the solution to a complicated problem through answers on two more simple questions:

-what are the forms of equations;

-what kind of "satisfaction" of these equations by random factors is suitable.

The assumptions are naturally not exact. Indeed, the same "input" should not give the same "output" because "parasitic scatter" connected with many influences has not been taken into account. But if the LPs totality is chosen sufficiently this additional scatter can be considered as a negligible one.

THE SCATTER MODEL OF TOTAL FATIGUE LIFE (TFL) FOR PULSING LOADING.

The TFL means the number of cycles up to failure (or to the fatigue damage condition close to failure).

A simple and effective model is proposed based on the vast amount of experimental data. The collection of data of that kind began in the late 40s. Many investigations of test results developed in the USSR during 1948-1969 were fulfilled in TsAGI (ref.1 and 2). The test amount is illustrated in table 1. The example of test results interpretation is shown on figure 1; mean values of the fatigue life logarithm are plotted on the X-axis, its standard deviation values - on the Y-axis. The generalized data (figure 2) emphasize a common rule: the minimum scatter takes place in the vicinity of 10^4 cycles mean life; the scatter increases both by increasing and by decreasing the life from this value.

The same conclusions are made by A.M.Stagg as a result of his detailed investigations. Let us cite some words from his paper (ref.3):

"(1)The coefficient of variation increases with an increase in the life of a specimen above a value of $\log N$ of about 4.

(2)The coefficient of variation increases with a decrease in the value of $\log N$ below a value of 4. This trend is by no means as well supported as (1) above".

A significant experimental result was obtained in ref.4: a "quasi-interaction point" of fatigue curves exists (see figure 3 where both scales, X and Y, are logarithmic) for several aluminium alloy semi-finished products. This observation in addition to other experimental data mentioned above gives an important argument to the following scatter model formulation (ref. 5 and 6):

Every copy of an airframe element has an individual fatigue curve. The form of the relation between fatigue life and the maximum stress value σ_o in case of the pulsing loading is kept similar to the form of the mean curve. For example, it can be a curve of power form

$$N(\sigma_o/\sigma_o^*)^m = A^*, \tag{1a}$$

or be logarithmed as

$$\log N + m\log(\sigma_o/\sigma_o^*) = \log A^* = C^*. \tag{1b}$$

The parameters m and A^* (or C^*) are random and are altered from one copy of an airframe to another.

The stress value σ_o^* which corresponds to the "neck" of the "fan" formed by individual curves is apparently the material feature (figure 4). It can be shown (ref.5) that the C^* value which is equal to the fatigue life logarithm at the σ_o^* level does not correlate with the parameter m; this parameter evidently characterized the slope angle of fatigue curves as straight lines in double logarithmic scale. As the correlation factor r_{mC^*} is equal to zero the main equations for the mean life logarithm value $\overline{\log N}$ and for the standard deviation $S_{\log N}$ would be written as

$$\overline{\log N} = \overline{C}^* - \overline{m}\log(\sigma_o/\sigma_o^*) \tag{2}$$

$$S_{\log N} = \sqrt{d^2 + a^2(\overline{\log N} - b)^2}, \tag{3}$$

where $a^2 = D_m/\overline{m}^2$, $b = \overline{C}^*$, $d^2 = D_{C^*}$. A standard notation D for a dispersion is used.

Equation (3) describes (figure 5) the relation of the figure 2 type quite sufficiently. For example, the values $a \simeq 0.2$, $b \simeq 4.5$ and $d \simeq 0.1$ are valid for D16 aluminium alloy sheets.

GENERALIZATION OF THE TFL SCATTER MODEL IN THE CASE OF THE POSITIVE MEAN VALUE OF THE LOAD CYCLE (MVLC).

The form of the type (1) equation for this case may be accepted on the basis of a well confirmed approach to the evaluation of MVLC influence on mean TFL. The proposals of ref.7 and 8 known for many years and based on energetic considerations turned out to be valid.

There is a simple equation

$$\sigma_0^{eq} = \sigma_{max}(1 - R)^{\theta}, \tag{4}$$

which one can use to obtain a maximum value of a pulsing cycle load that corresponds to the same mean fatigue life but for the loading with an any positive MVLC. The parameters of this equation have the following sense: σ_{max} and σ_{min} are the maximum and the minimum values of the cycle load; $R = \sigma_{min}/\sigma_{max}$ is the stress ratio which may alter (for arbitrary positive MVLC) within $-1 \le R \le +1$ range. The parameter $0 \le \theta \le 1$ is a new fatigue life parameter.

The generalized equations similar to (1a) and (1b) would be written as

$$N(\sigma_{max}/\sigma_0^*)^m (1-R)^{m\theta} = A^* \tag{5a}$$

or, be logarithmed and inserted with a notation $\beta = m\theta$, as

$$logN + mlog(\sigma_0/\sigma_0^*) + \beta log(1-R) = logA^* = C^*. \tag{5b}$$

These equations are sufficiently valid (ref.9 and 10). An example of a straight regression line is given on figure 6 for coupon test data (aluminium alloy D16, sheet). The tests are fulfilled in a wide range of stress ratios. Each experimental point corresponds to mean life for the sample of at least five specimens (the total scope is 233 specimens). The mean values of the abovementioned parameters are: $m=4.66$; $\theta=0.575$; $\beta=2.68$.

The analysis shows that in the presence of a new fatigue life parameter β the form of equation (3) and the value of the parameter a must not be changed but the formulae for parameters b and d which are included in the equation (3) become more complicated and should be written as follows:

$$b = \overline{C}^* - \overline{\beta}(1 - r_{m\beta}(\gamma_\beta/\gamma_m)) log(1-R), \tag{6}$$

$$d^2 = D_{C^*} + D_\beta(1-r_{m\beta}^2) log^2(1-R) - 2r_{\beta C^*} \sqrt{D_\beta D_{C^*}}\, log(1-R). \tag{7}$$

The equation (7) includes a dispersion D_β and paired correlation factors $r_{m\beta}$ and $r_{\beta C^*}$; about the values of these factors some reasonable assumptions should be proposed.

Indeed, if the parameter θ is not random and its value is the same for any copy of the airframe structure; i.e $\theta = \overline{\theta} = const$, the correlation factor $r_{\beta C^*} = 0$ and (it can be simply shown) $r_{m\beta} = 1$. In other words the parameters β and C^* are practically independent, but parameters m and β are statistically "tied".

If the parameter β is random, i.e. it has an individual value for any copy of the airframe structure, one may suppose that the situation inverses, i.e. $r_{\beta C^*} \to 1$, but $r_{m\beta} \to 0$. An additional argument for that assumption is the result of its application in the equations (6) and (7); the formulae become graceful and symmetric as

$$b = \overline{C^*} - \bar{\beta} log(1-R), \quad (6a)$$

$$d = S_{C^*} - S_{\beta} log(1-R); \quad (7a)$$

here S_{C^*} and S_{β} are corresponding standard deviations.

Thus the generalized TFL scatter model is characterized by three fatigue life parameters: m, C^* and β; any of them should be defined by two values - mean and standard deviation.

As an example the identification of this model was done (figure 7) on the basis of aluminium alloy 2024 coupon test results (ref.11). One should take into account that the fatigue life experimental standard deviations are only estimates. The characteristics of fatigue life parameters are the following: $\bar{m}=4$; $\overline{C^*}=4.5$, $\bar{\beta}=2$; $S_m=0.22$, $S_{C^*}=0.046$ and $S_{\beta}=0.07$.

It is interesting to note that mean values of the parameters are practically the same as the analogic values for the aluminium alloy D16, but the standard deviation of the parameter m (it may be considered as a material parameter) is bigger ($S_m=0.8$) for the D16 alloy. As for approximately two times difference in C^* scatter ($S_{C^*}=0.046$ for the 2024 alloy coupons instead of $S_{C^*}=0.1$ for D16 alloy generalized data) the explanation is very simple: the coupons for laboratory tests are made, as usual, more carefully to avoid the "annoying scatter".

THE FATIGUE CRACK GROWTH SCATTER MODEL

The present-day methods of service safety insurance for structures of "high criticality" use the Damage Tolerance Principle; therefore the problem of fatigue crack growth scatter should attract extreme attention.

For the case of cyclic pulsing loading one may use (in the course of a preliminary approach) a suitable linear fracture mechanics model (ref.12) taking into account that for large cracks (which can be detected by simple visual methods) the distortion effect of the stress intensity coefficient due to the stress concentrator existence will be negligible.

An additional assumption is adopted that the total fatigue life is exhausted by the crack growth only. The growth begins from a conditionally existent "initial crack" (considered as a quantitative symbol of generic initial state); the value of its length l_{ini} may be obtained by an extrapolation procedure from real crack length's range.

Using the Paris equation one may obtain a very simple formula for the relation "crack length l - fatigue life N" as

$$l = l_{ini} / (1 - N / N_{asy})^{1/q} \quad (8)$$

where N_{asy} is the abscissa of the vertical asymptote (figure 8); n is the power factor of the Paris equation; $q=(n-2)/2$.

An important formula may be derived as

$$N_{asy}(\sigma_o/\sigma_o^*)^n = \tilde{N}^*_{asy}\left[l_{ini}/\bar{l}_{ini}\right]^{1/q}, \tag{9}$$

which expresses a random value N_{asy} through the individual (random and statistically independent) fatigue life parameters n, l_{ini} and the parameter $\tilde{N}^*_{asy}$ defined as a parameter N_{asy} for $l_{ini}=\bar{l}_{ini}$ and for stress σ_o which is equal to σ_o^* stress in the individual fatigue curves "fan neck". The value $\bar{l}_{ini}$ is the mean value of the "initial flaw".

If the value N_{asy} may be approximately considered as the total fatigue life, the similarity of (9) and (1a) equations is evident; the conclusion is that the fatigue curve parameter m and the Paris equation parameter n should be close to one another.

It is followed from equations (8) and (9) that the logarithm of the fatigue life up to any state which is characterized by a crack length $l>l_{ini}$ is defined as

$$log N(l)=a^*-qb_{ini}-n\,log(\sigma_o/\sigma_o^*)+log[1-10^{qb_{ini}}(\bar{l}_{ini}/l)^q], \tag{10}$$

where

$$a^*=log\tilde{N}^*_{asy}; \qquad b_{ini}=log(l_{ini}/\bar{l}_{ini}). \tag{11}$$

The crack growth duration ΔN in the crack detectable range may be derived by the eqauation (10) as $\Delta N = N_{asy} - N(l_{det})$ where l_{det} is the crack length which can be reliably detected by using check methods.

On the basis of the proposed model an important result is obtained: the correlation factor between ΔN and N_{asy} is not a constant value but it depends on loading conditions and other essential parameters. It can vary throughout the whole interval of its existence ($-1\le r\le 1$). In particular, if

-load level is high;

-the scatter characteristics of the initial "flaws" dimensions are low (i.e. the manufacturing process is highly stable);

-the mean value of the ratio $\Delta N / N_{asy}$ is large (i.e. the inspection methods allow technicians to detect the fatigue cracks of short lengths),

then ΔN and N_{asy} can correlate positively and strongly. These conditions usually take place in the laboratory sample tests.

However, the practical use of the Damage Tolerance Principle on full size airframes shows an opposite situation: the typical load levels in service are low, the manufacturing processes are ordinary, and the most often used flaw detection method is the visual inspection that allows inspectors to detect large cracks only. Under these conditions, the correlation coefficient can reduce to zero, and even turn out to be notably negative.

The analysis shows that a close connection exists between the relation of the crack growth duration scatter to the fatigue life scatter, on the one hand, and the relation of corresponding mean values, on the other hand, i.e. between $S_{log\Delta N} / S_{logN_{asy}}$ and $\overline{N}_{asy} / \overline{\Delta N}$.

It is very important from a practical point of view that the connection is approximately the same independently to the "initial flaw" scatter. An example is given (see figure 9) for two typical values [d=0.1 and d=0.15; see equation (3)] of minimum fatigue life scatter in the "fan neck" and for typical probabilistic characteristics of fatigue life parameters.

CONCLUSIONS

Simple and effective models of total fatigue life and crack growth scatter are proposed. Preliminary results are obtained which permit consideration of the proposals as acceptable. The identification of these models on the basis of experimental data will give much possibility for

-revealing, investigating and explaning the causes of scatter;

-predicting the scatter characteristics for materials and structures;

-obtaining generalized scatter characteristics for reliable standardization;

-forming effective approaches to the scatter "bearing process" control in order to minimize its value.

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Table 1.

Material	Number of samples		Total number of test objects	
	Coupons	Panels, structure elements	Coupons	Panels, structure elements
Aluminium alloy D16 (2024 analogy)	1063	364	9500	1200
Aluminium alloy B95 (7075 analogy)	535	81	4173	250
Steel 30ХГСА	652	360	4000	1100

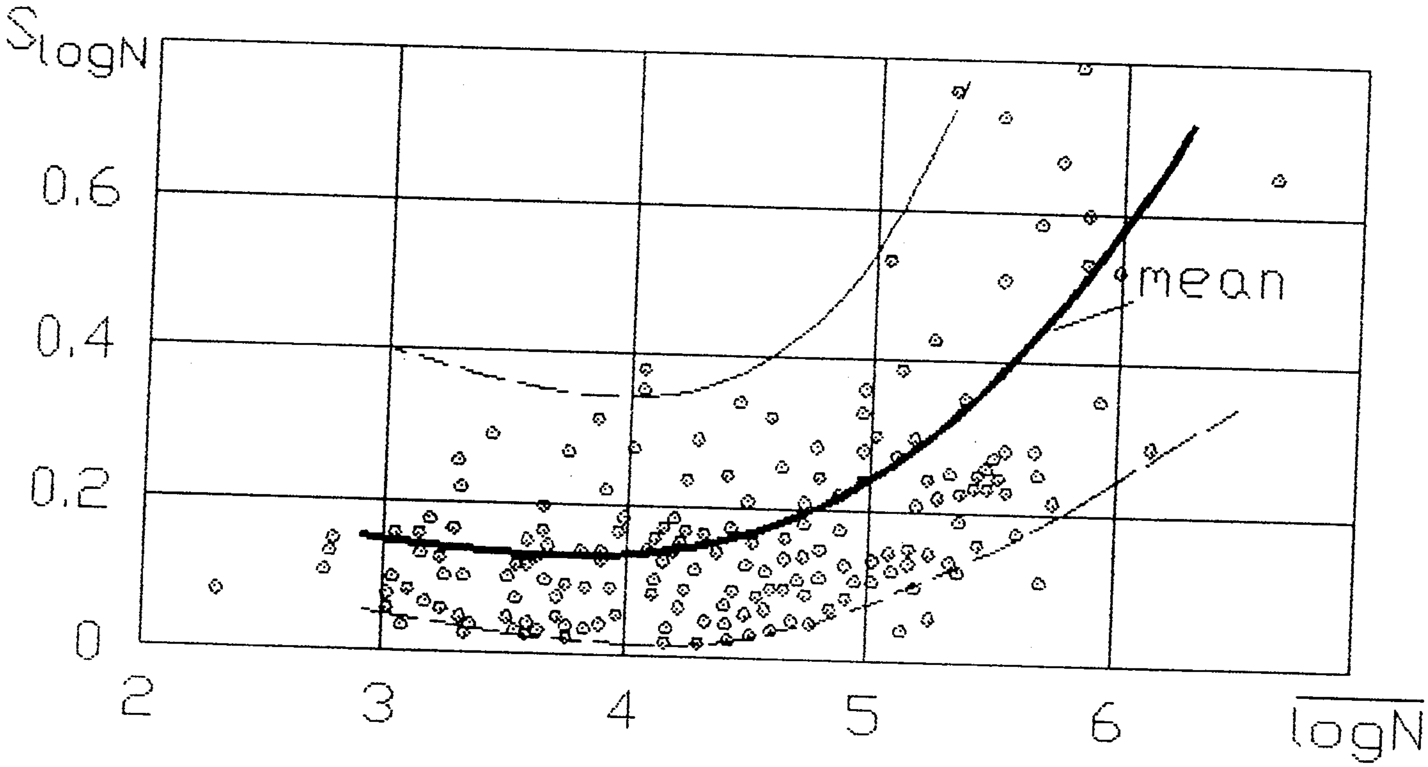

Figure 1. The example of scatter - mean life test results interpretation (ref.2).

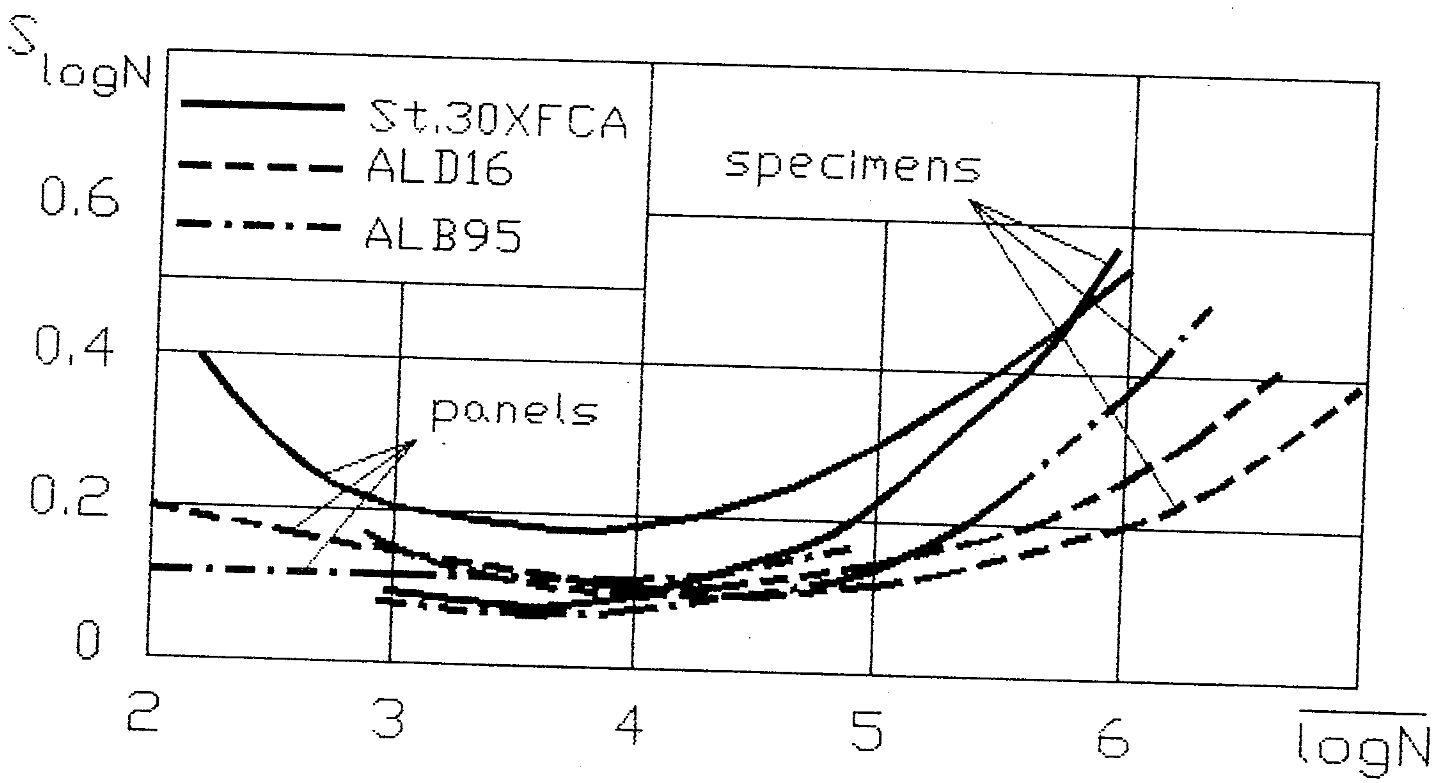

Figure 2. The generalized scatter - mean life plot (ref.1). Mean curves for different materials and test objects.

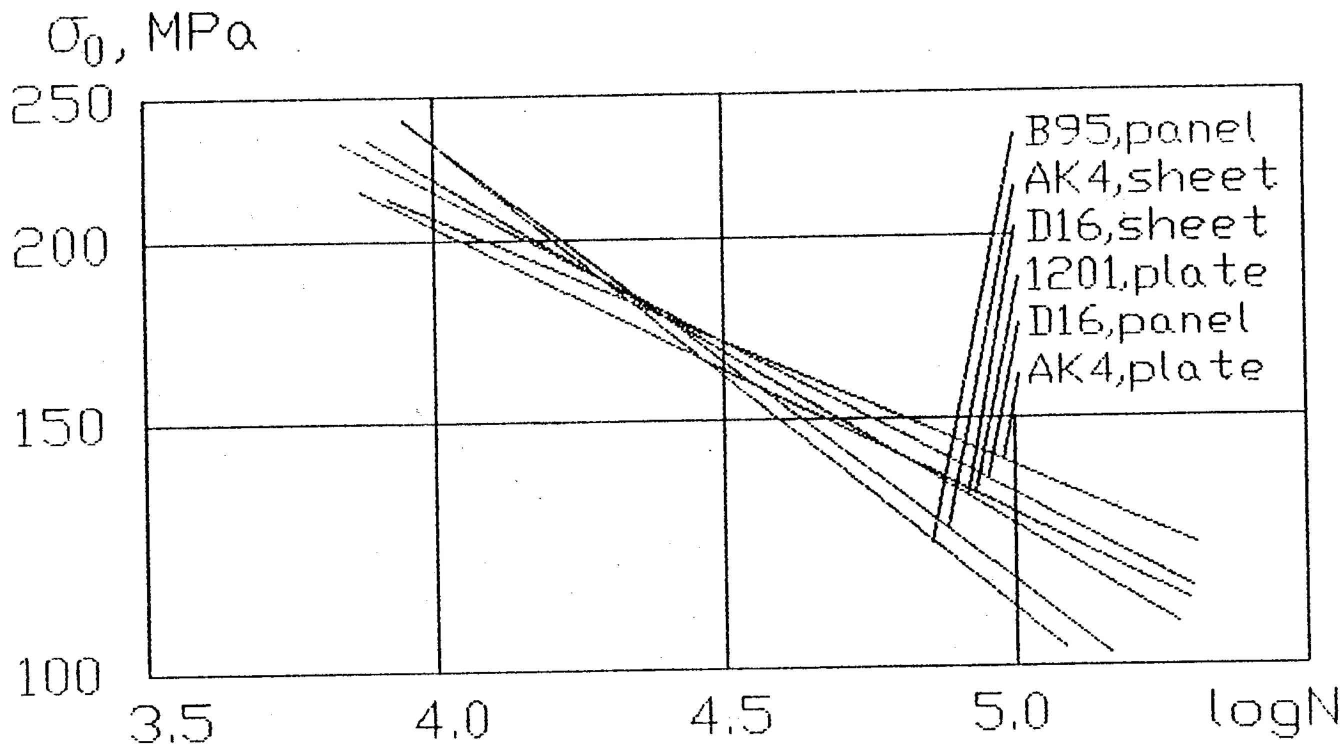

Figure 3. Fatigue curves for several aluminium alloys semi-finished products (ref.4).

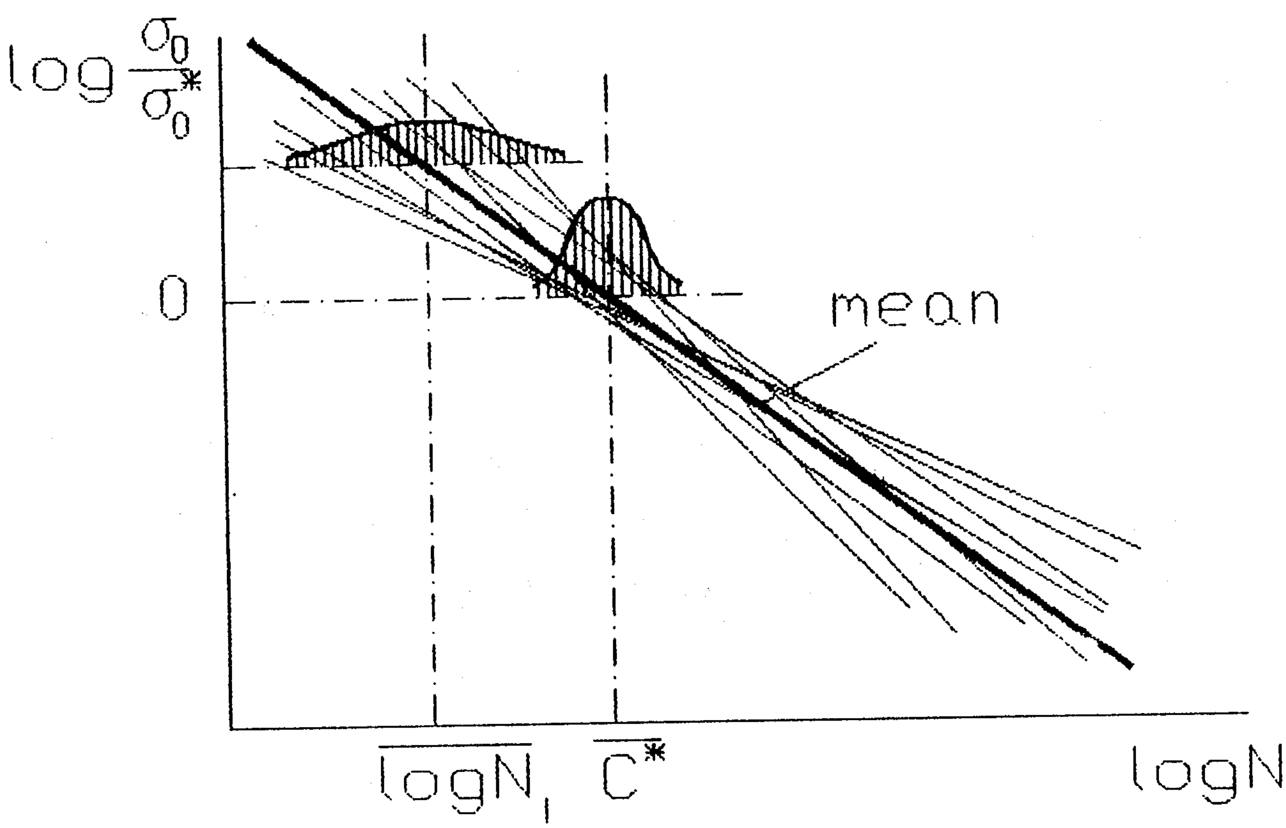

Figure 4. The "fan" of individual fatigue curves.

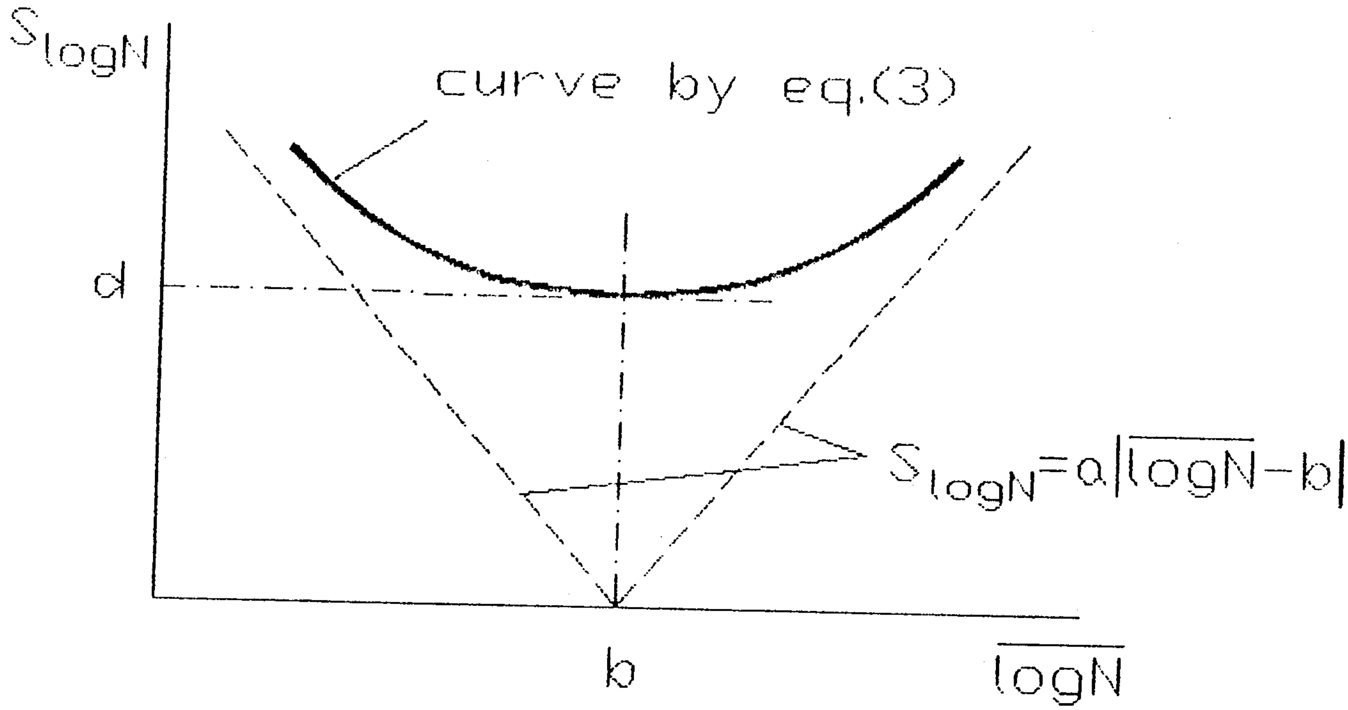

Figure 5. The theoretical "scatter - mean life" equation.

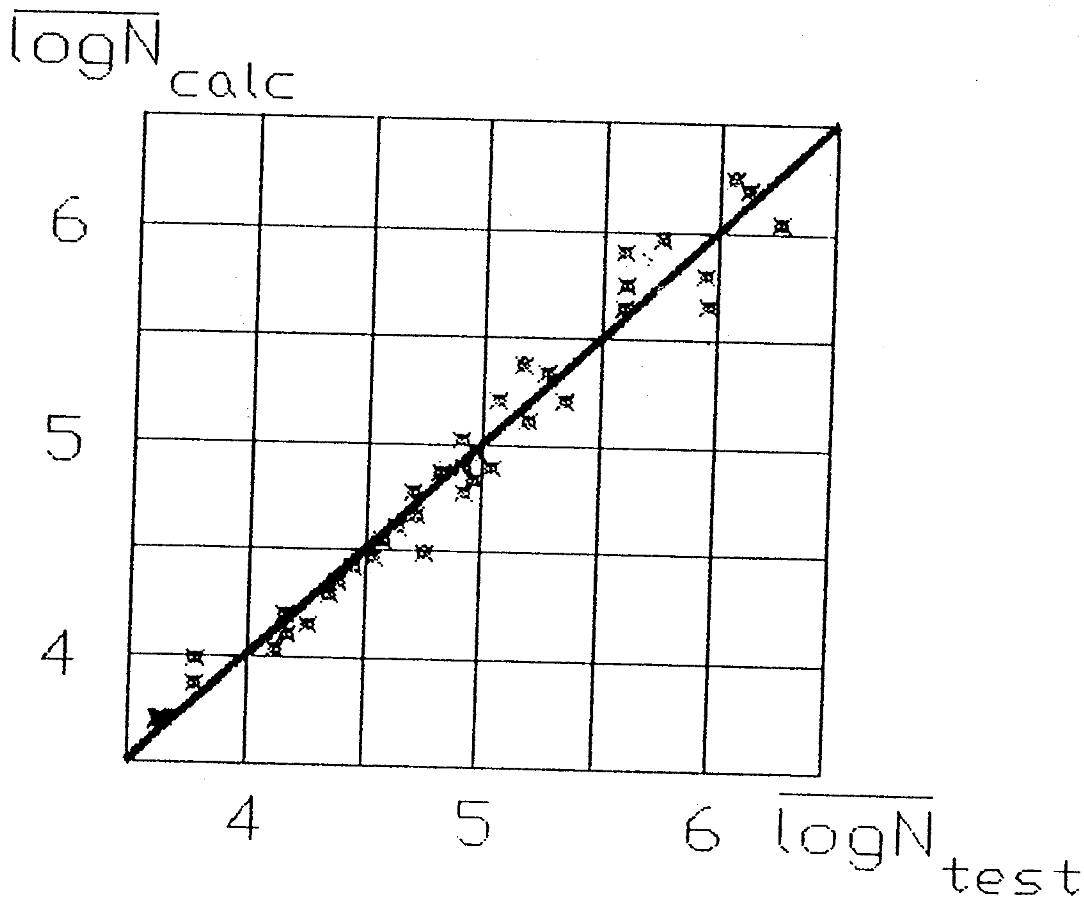

Figure 6. An example of equation (4) validation (ref.10).

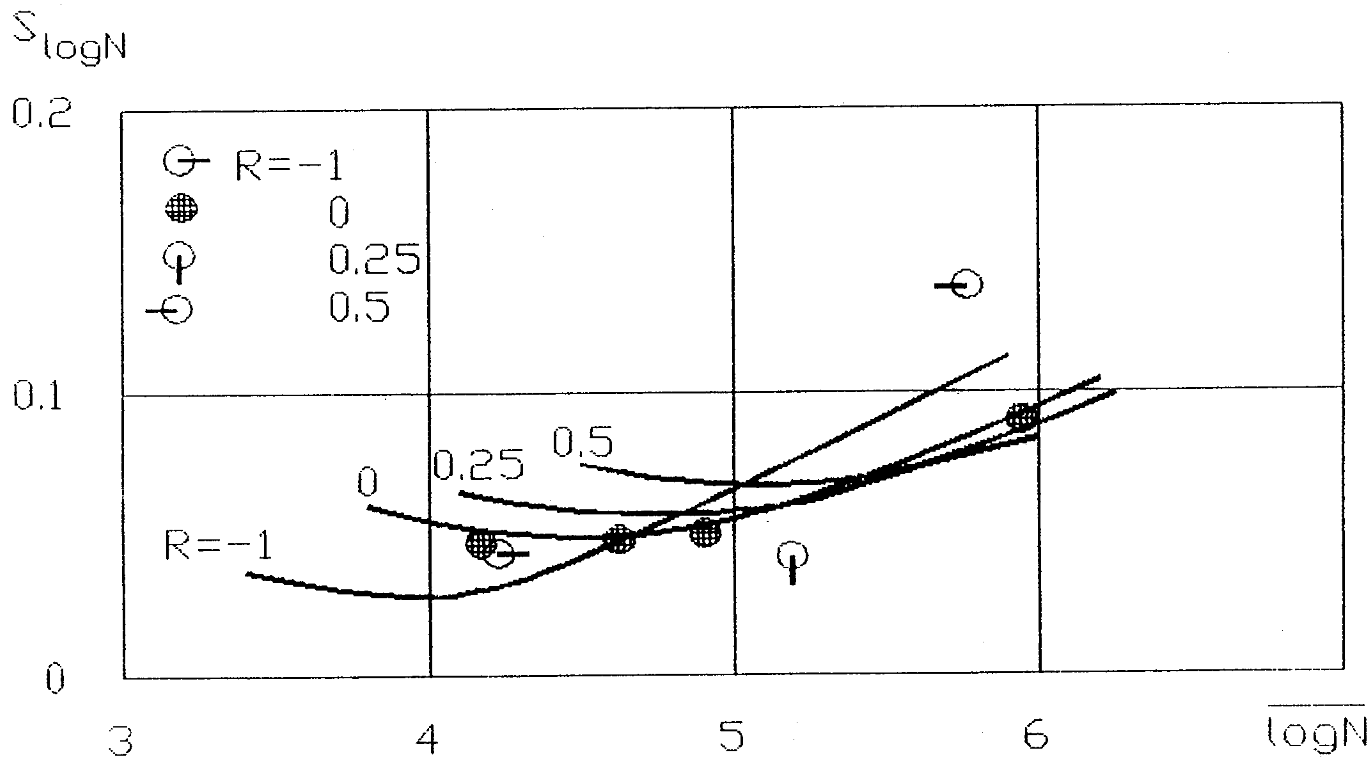

Figure 7. The influence of stress ratio on scatter characteristics. The experimental points (ref.11) and identified theoretical curves.

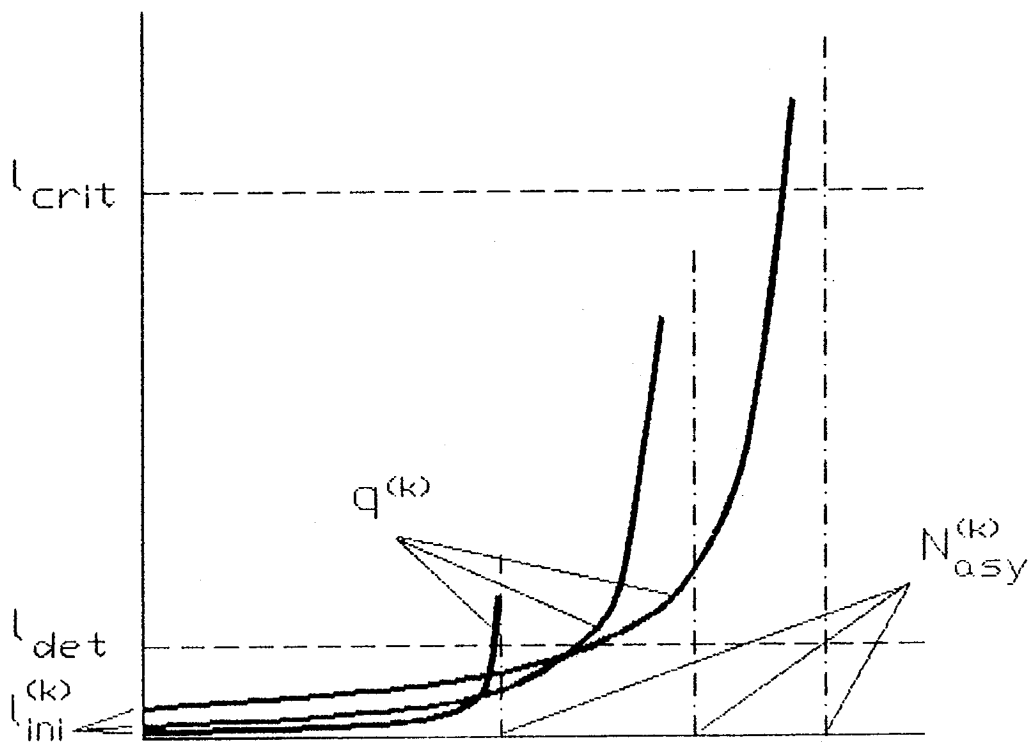

Figure 8. A crack growth scatter illustration.

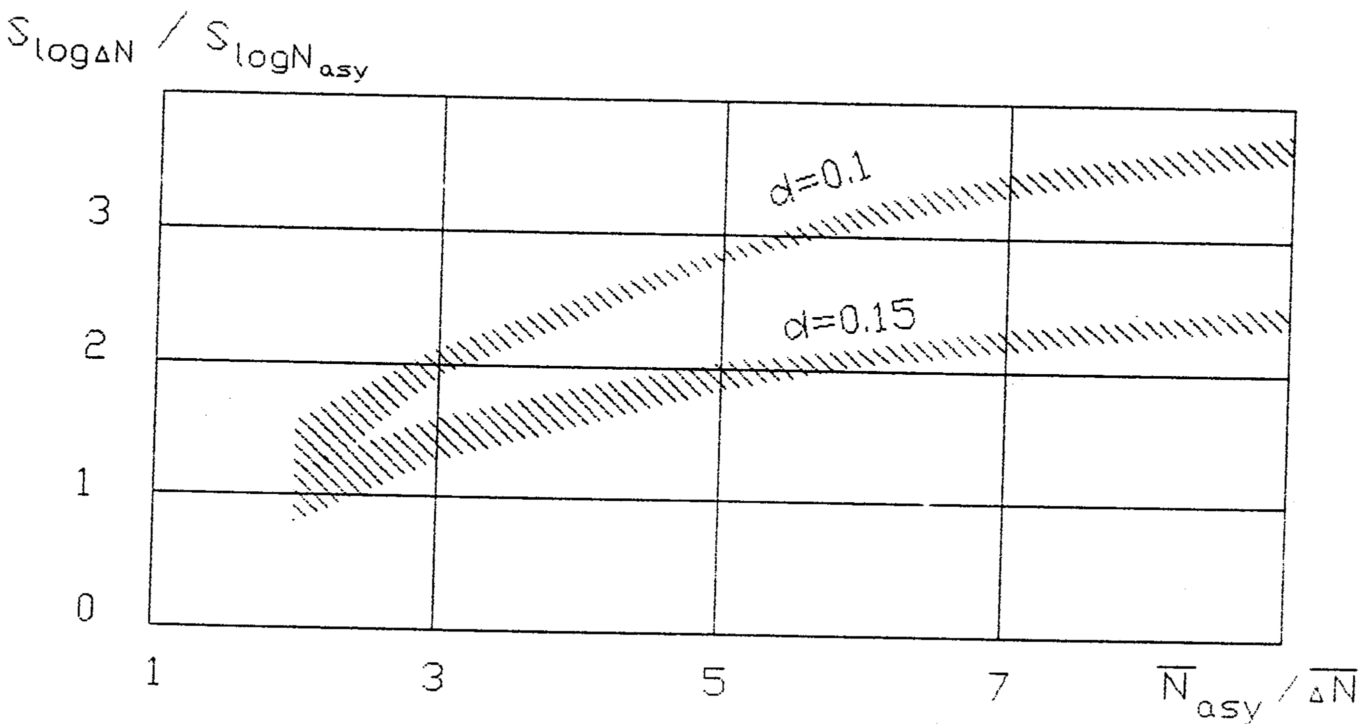

Figure 9. Relations between fatigue life and crack growth scatter values.