

**ANALYSIS OF COLD WORKED HOLES FOR
STRUCTURAL LIFE EXTENSION**

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SUMMARY

Cold working holes for improved fatigue life of fastener holes are widely used on aircraft. This paper presents methods used by the authors to determine the percent of cold working to be applied and to analyze fatigue crack growth of cold worked fastener holes. An elastic, perfectly-plastic analysis of a thick-walled tube is used to determine the stress field during the cold working process and the residual stress field after the process is completed. The results of the elastic/plastic analysis are used to determine the amount of cold working to apply to a hole. The residual stress field is then used to perform damage tolerance analysis of a crack growing out of a cold worked fastener hole. This analysis method is easily implemented in existing crack growth computer codes so that cold worked holes can be used to extend the structural life of aircraft. Analytical results are compared to test data where appropriate.

INTRODUCTION

Cold working is the process of expanding holes in structural materials to produce residual compressive stresses which reduce the natural tendency of the structure to crack under cyclic load. Simply, the process is accomplished by pulling a tapered steel mandrel through a sleeved hole. The mandrel is tapered to a thickness which, when added to the split-sleeve thickness, causes enough expansion of the hole to produce compressive yielding at the hole. The steel sleeve is utilized as a "sacrificial lamb" to protect the surface of the hole from scoring and tool marks. The split in the sleeve allows for full expansion of the hole. The percentage of applied cold work is calculated by dividing the radial displacement of the hole due to the inserted mandrel, by the radius of the hole before cold working. The percentage of residual cold working is defined by the ratio of the increase of the radius of the hole after mandrel removal, to the radius of the hole prior to cold working.

This process imparts residual compressive stresses around the hole, which tend to impede the advance of fatigue cracking in a similar manner to the compressive residual stress in the plastic zone around a crack tip. In the case of cold working, the magnitude of this compressive stress approaches the yield stress of the material. Since equilibrium must be maintained, the induced compressive stress must be balanced by a residual tension stress field which typically affects free edges of parts or adjacent holes that may not be cold worked. During the course of normal maintenance, these areas are susceptible to nicks, scratches, and other tool marks that on their own are not too serious, but when coupled with the residual tension from cold working, can produce a new breeding ground for fatigue crack initiation and growth.

The first step in analyzing cold worked holes is to perform a stress analysis to determine the applied and residual stress fields. For this purpose the authors used as an analysis model the elastic, perfectly-plastic expansion of a thick-walled tube. Others [1] have proposed using three-dimensional finite element models to determine the stress field due to cold working. While this may give more accurate results, it is not considered practical for use in the production environment where it may be required to analyze hundreds of geometries. Once the applied stress and residual stress fields are determined, the information can be used to determine the amount of cold working to use for various edge distances. Validation of the stress analysis is performed by comparison to finite element analysis and measured strains in test specimens.

Knowing the residual stress field for the uncracked geometry, the stress intensity factors (SIF) for various crack lengths emanating from the cold worked hole can be determined using a Green's function. This approach for calculating stress intensity factors is sometimes referred to as an indirect method for SIF computation. It is a generally acceptable method in the presence of complicated stress distributions. The SIF for the cold worked hole can then be superpositioned with the SIF factor computed for each individual load cycle to determine crack growth during cyclic loading. Crack growth results are presented for constant amplitude and random cyclic loading conditions.

STRESS ANALYSIS OF COLD WORKED HOLES

Stress analyses of fastener hole cold-expansions were performed due to concern regarding the cold working of short edge-distance holes and to determine residual stress fields of cold worked holes. The primary concerns were fracture during cold working and the damage tolerance life of cold worked holes. Additionally, concerns arose about the residual tension stresses at the free edge nearest to cold-expanded holes. This tension field could accelerate crack growth from the edge to the hole if cracks or surface flaws were present or initiated on the free edge. This paper does not address the free edge residual tension stress issue. The purpose of this examination was to determine the induced and residual stress distribution through the ligament around cold worked holes.

Plasticity Model for Stresses in Cold Worked Holes

Examination of the cold working process began with a survey of documented experimental results showing the beneficial effects of cold working on fatigue life enhancement. Next, a classical analytical model was used to approximate the plastic deformation of a hole with applied internal pressure [2]. The expected induced and residual stress distribution through the ligament thickness can be seen in Figures 1 and 2, respectively. The constant k used to nondimensionalize the stresses in these plots is the value of the yield stress in pure shear. The Tresca yield criterion was adopted for this solution.

Mathematical plasticity calculations based on the model are made to determine the induced and residual stress field around cold worked holes. A diagram of the plasticity model geometric configuration is shown in Figure 3. The methodology approximates a cold worked hole geometry by a thick walled cylinder having an internal radius equal to the hole radius, a , and the distance to the nearest free edge equal to the external radius, b . The other parameters shown in Figure 3 are the plastic yield radius, c , and the reverse yield radius, ρ .

The equations for the mathematical model from Reference [2] which predict the induced stresses are:

$$\begin{aligned} \sigma_r &= -\frac{kc^2}{b} \left(\frac{b^2}{r^2} - 1 \right), & \sigma_\theta &= \frac{kc^2}{b^2} \left(\frac{b^2}{r^2} + 1 \right) & c \leq r \leq b \\ \sigma_r &= -k \left(1 - \frac{c^2}{b^2} + \ln \frac{c^2}{r^2} \right), & \sigma_\theta &= k \left(1 + \frac{c^2}{b^2} - \ln \frac{c^2}{r^2} \right) & a \leq r \leq c \end{aligned} \quad (1)$$

In these equations, σ_r is the induced radial stress, σ_θ is defined as the induced circumferential stress and k is the yield stress in pure shear. The zone from c to b is the elastic region while the zone from a to c is the plastic region. The variable c is thus defined as the yield radius. The internal pressure, p , is

$$p = k \left(1 - \frac{c^2}{b^2} + \ln \frac{c^2}{a^2} \right) \quad (2)$$

and the displacement at the inner radius due to the applied pressure is

$$u_a = a \left[\frac{\nu^2 p}{G(1+\nu)(b^2/a^2 - 1)} + (1-\nu) \frac{kc^2}{Ga^2} - (1-2\nu) \frac{p}{2G} \right] \quad (3)$$

The value of the plastic zone size, c , is still indeterminate from the above equations. The imposed radial displacement u_a is known from the cold working tool geometry so that c can be solved for in an iterative manner until the proper displacement at the inner radius is achieved.

Since the cold worked fastener holes were determined to undergo reverse yielding after removal of the mandrel, the residual stresses were determined by adding the following equations to those noted above.

$$\begin{aligned} \sigma_r' &= 2k \frac{\rho^2}{b} \left(\frac{b^2}{r^2} - 1 \right), & \sigma_\theta' &= -2k \frac{\rho^2}{b^2} \left(\frac{b^2}{r^2} + 1 \right) & \rho \leq r \leq b \\ \sigma_r' &= 2k \left(1 - \frac{\rho^2}{b^2} + \ln \frac{\rho^2}{r^2} \right), & \sigma_\theta' &= -2k \left(1 + \frac{\rho^2}{b^2} - \ln \frac{\rho^2}{r^2} \right) & a \leq r \leq \rho \end{aligned} \quad (4)$$

where the radius ρ is determined from the boundary condition $\sigma_r' = p$ at $r = a$. In these equations, the residual radial stress is defined as σ_r' , and σ_θ' represents the residual circumferential stress. For a more detailed discussion of this solution see Reference [2].

Model Validation

In order to validate the use of the plasticity model for decisions concerning cold working, comparisons to finite element models and USAF experimental results were made. First, a two dimensional non-linear analysis for investigation of a hole with 2.8 percent interference was performed by Crows [3]. Next, a three-dimensional NASTRAN lug model was developed for an investigation of cold working effects at Kelly AFB [4]. Finally, experimental data was obtained from strain gaging and cold working T-37 "302" attachment fitting lugs at Randolph AFB [5]. Graphical representations of these comparisons appear in Figures 4 through 7.

Figure 4 gives Crows' finite element results [3] for a 7075-T6 rectangular sheet with an interference fit hole. Results are given for the interference stress in both the hoop and radial directions together with the present solutions for a thick walled tube under applied pressure with the same b . It can be seen that a good agreement exists between them.

A three dimensional non-linear material NASTRAN analysis was performed to investigate to effects cold working at Kelly AFB[4]. The lug geometry analyzed had a hole radius of 0.438 inches and an edge distance of 1.093 inches. The material of the model was 7075-T6 die forging. Figure 5 shows comparisons for the residual hoop stresses through the ligament, i.e., the material from the edge of the hole to the free edge. This figure shows a favorable comparison between the results from the NASTRAN model and the plasticity model for 4.0 percent cold work. Figure 5 also shows that

the plasticity model is predicting reverse yielding while the NASTRAN results are not given at a fine enough resolution to determine if reverse yielding occurred.

Figures 6 and 7 show comparisons between the plasticity model and the USAF "302" lug experimental data [5]. The strain gage data was obtained from the free edge of the lugs. The geometry of the lugs is the same as the above NASTRAN model. Figure 6 shows the plasticity model to be an upper bound for the experimental data. The data in Figure 7 shows that the retained expansion can also be calculated accurately. The comparison of the plots supports the validity of the model for the ranges of cold work from 2.8 to 4.0 percent.

DAMAGE TOLERANCE ANALYSIS OF COLD WORKED HOLES

Damage tolerance analysis of cold worked holes is performed so that realistic crack growth lives for cold worked holes can reliably be predicted. To perform DTA of cold worked holes, the effect of the residual stress field on the stress intensity factor must be determined. After determining the SIF due to cold working, this value is superpositioned with the SIF due to the applied spectrum load. After superpositioning the SIFs, the growth increment due to the load cycle is calculated using standard linear elastic fracture mechanics (LEFM) crack growth rate models.

Determination of Stress Intensity Factors

The residual stress intensity factors were determined using Shaw's Green's function for a radially cracked hole[6] and the residual stress distribution as determined from the above equations. Shaw's Green's Function for a diametrically cracked hole is:

$$K_I = \frac{M_f}{\sqrt{\pi l}} \int_{-l}^l \sigma_x \left[\frac{l+x}{l-x} \right]^{1/2} dx \quad (5)$$

where M_f is a free surface correction given by

$$M_f = 1.0 + 0.12 \left(\frac{0.3 - l/R}{0.3} \right), \text{ for } 0 \leq \frac{l}{R} \leq 0.3 \quad (6)$$

and

$$M_f = 1.0, \text{ for } \frac{l}{R} > 0.3 \quad (7)$$

The stress intensity factor for one crack is given by Shah as

$$(K_I)_{\text{one crack}} = \sqrt{\frac{2R+l}{2R+2l}} (K_I)_{\text{two cracks}} \quad (8)$$

The ratio of the geometry factor for a Newman/Raju corner crack at a hole [7] to the NASA FLAGRO [8] solution for a radial through crack at a hole was used to account for the flaws being corner cracks rather than through-the-thickness cracks. The determination of the residual stress intensity factor was performed using a spread sheet. The stress intensity factor for a corner crack at a cold worked hole is:

$$K_{\text{cold}} = \frac{\beta_c}{\beta_t} (K_I)_{\text{one crack}} \quad (9)$$

where β_c is the Newman/Raju corner crack geometry factor and β_t is the geometry factor for a through crack. Figure 8 compares the stress intensity factor for a 0.31 inch open hole with a edge distance of 0.75 inches under 35 ksi far field stress to that of a 4.8 percent expanded cold worked hole in 7075-T6511 extrusion. It can be seen in this figure that the stress intensity for the cold worked hole is much less than the open hole case for a large portion of the crack length. The stress intensity factors for cold worked holes are larger than the stress intensity factor for an open hole at larger crack sizes due to residual tension stress at larger crack sizes.

Crack Growth Analysis Method

Spectrum fatigue crack growth predictions were made using the applied stress intensity range, ΔK , for each cycle in the flight by flight spectrum. The stress intensity factors were calculated by the superposition of the residual stress intensity factor due to cold working with the stress intensity from the cyclicly applied load. The calculated values of ΔK are used to determine the crack growth increment for that cycle. The method can be used with any of the standard crack growth rate equations. The authors modified a version of the Cracks III computer code[9] to perform the superposition on a cycle-by-cycle basis. The authors' work used the Walker equation for stress ratio adjustments in crack growth rates. The form of the Walker equation used is:

$$\frac{da}{dN} = C \left(\frac{\Delta K}{(1-R)^{1-M}} \right)^n \quad \text{for } R \geq 0.0 \quad (10)$$

and in the form

$$\frac{da}{dN} = C \left[(1 + R^2)^q \cdot K_{\max} \right]^n \quad \text{for } R < 0.0 \quad (11)$$

where R is the stress ratio, ΔK is the stress intensity range and C , M , n , q are material constants. The residual stress intensity is very important for determining the stress ratio adjustments in that most of the cycles have large negative stress ratios, and thus the crack growth rates are decreased significantly.

Validation of Crack Growth Analysis

The crack growth analyses of cold worked holes are validated with constant amplitude and random cycle loadings. The first case analyzed is a 0.31 inch hole in a 7075-T6511 aluminum alloy extruded panel, under constant amplitude far field stress of 35 ksi and an R ratio of 0.1 [10]. Figure 9 presents the test results for cold worked and non cold worked holes compared to the crack growth analysis. Figure 10 compares similar test results for $R=0.8$ to crack growth analysis results.

The final case analyzed was a 100 percent pin load 0.5616 inch fastener hole in 7075-T651 extrusion 1.067 inches wide and 0.09 inches thick [11]. The test spectrum was a cycle-by-cycle bearing stress spectrum for the T-37 horizontal stabilizer. The maximum spectrum bearing stress was 30.6 ksi. The fastener hole was expanded 3.64 percent. Crack growth retardation was predicted using the Wheeler plastic zone model [12]. The magnitude of the retardation parameter M was determined to be 4.0. Figure 11 presents the results of two tests specimens and the damage tolerance analysis.

DISCUSSION

While initially evaluating the effects of cold worked holes, the authors were concerned with the wide variation in applied expansion using standard tools. Due to hole and cold working tooling tolerances for a normal 0.25 inch hole, the applied expansion can vary from 2.5 percent to 6 percent. Figure 12 shows a calculation demonstrating how this variation in applied expansion can affect the damage tolerance life.

It is also possible to use this analysis to predict the amount of cold working required to stop a crack of a specified length from growing under spectrum loading. This is done by calculating the SIF for a cold worked hole with a crack length of x and super-positioning it with the SIF for a

cracked hole of length x at maximum spectrum stress. If the combined SIF is less than zero then no crack growth will occur. If the SIF is greater than zero, then the amount of cold working can be increased until the SIF becomes less than zero. Obviously there is a limit to the amount of cold working that may be applied.

In conclusion, a method for performing stress analysis and damage tolerance analysis of cold worked holes has been presented. The method has been validated to the extent possible with existing test data. The method is simple and does not require a nonlinear finite element analysis to obtain accurate results. The method lends itself to inclusion of computer programs to analyze damage tolerance lives of cold worked holes.

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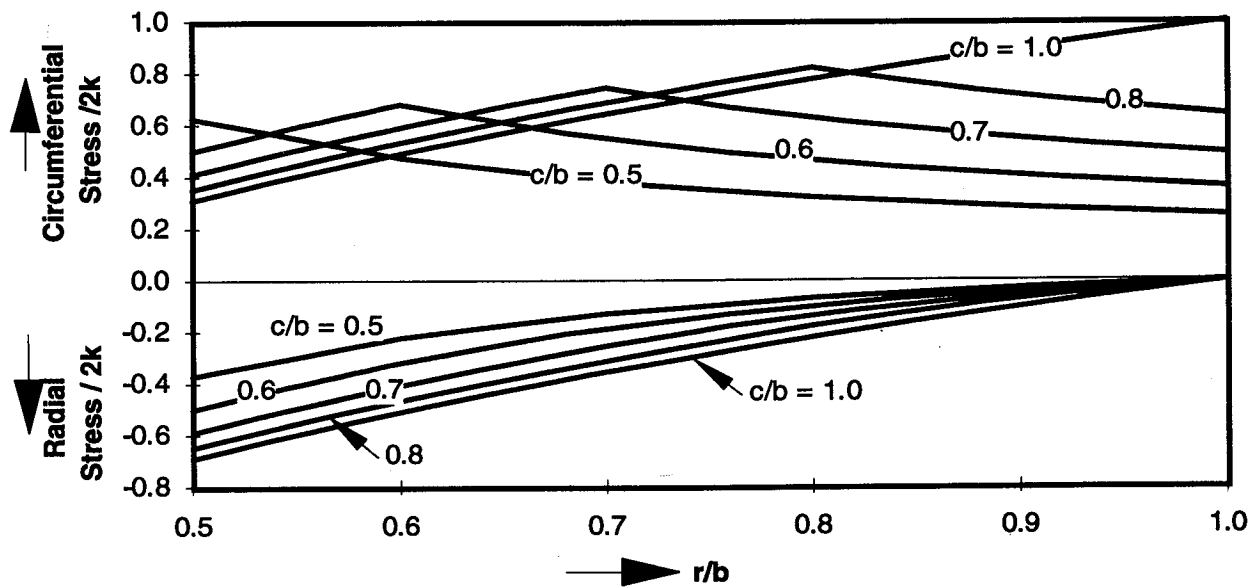


Figure 1. Typical nondimensional radial and circumferential stress in an elastic/plastic thick walled tube subjected to internal pressure.

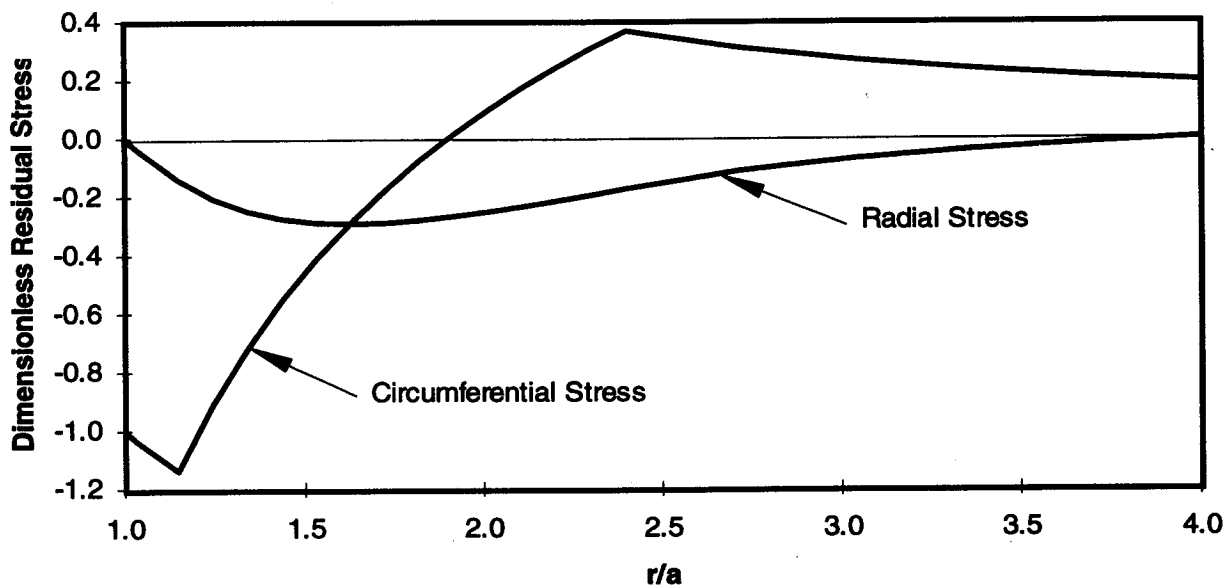


Figure 2. Typical residual stress distribution in a thick walled tube unloaded from an elastic/plastic state.

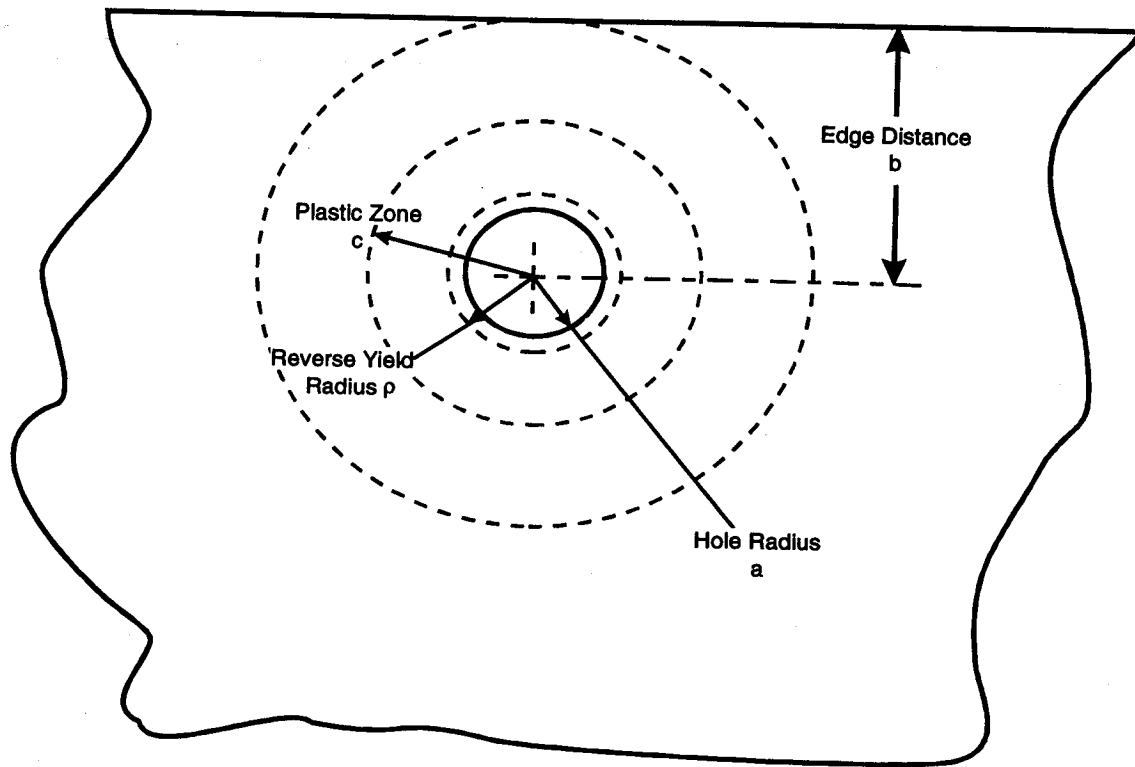


Figure 3. Idealized plasticity model geometric configuration.

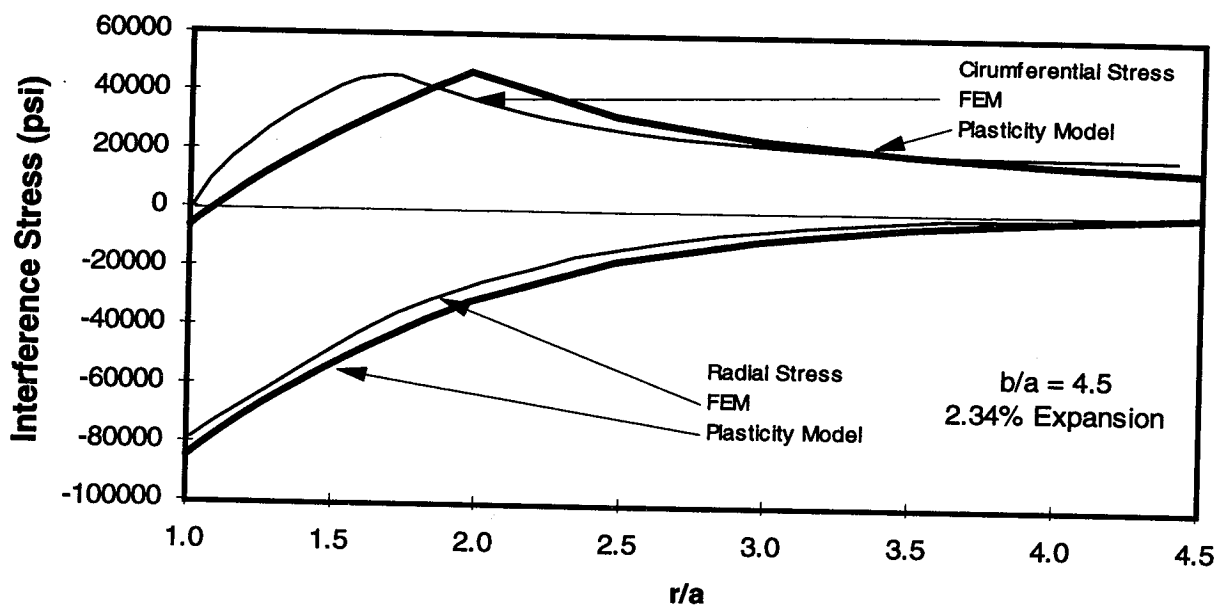


Figure 4. Comparison of results between plasticity model and finite element analysis[3].

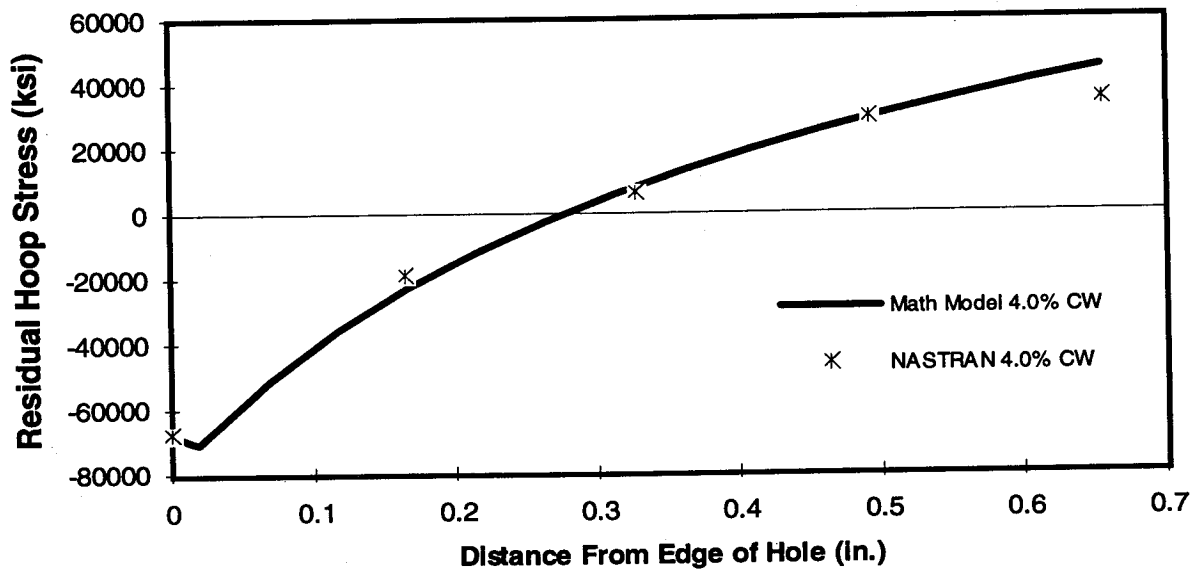


Figure 5. Comparison of results between plasticity model and USAF NASTRAN model of "302" lug.

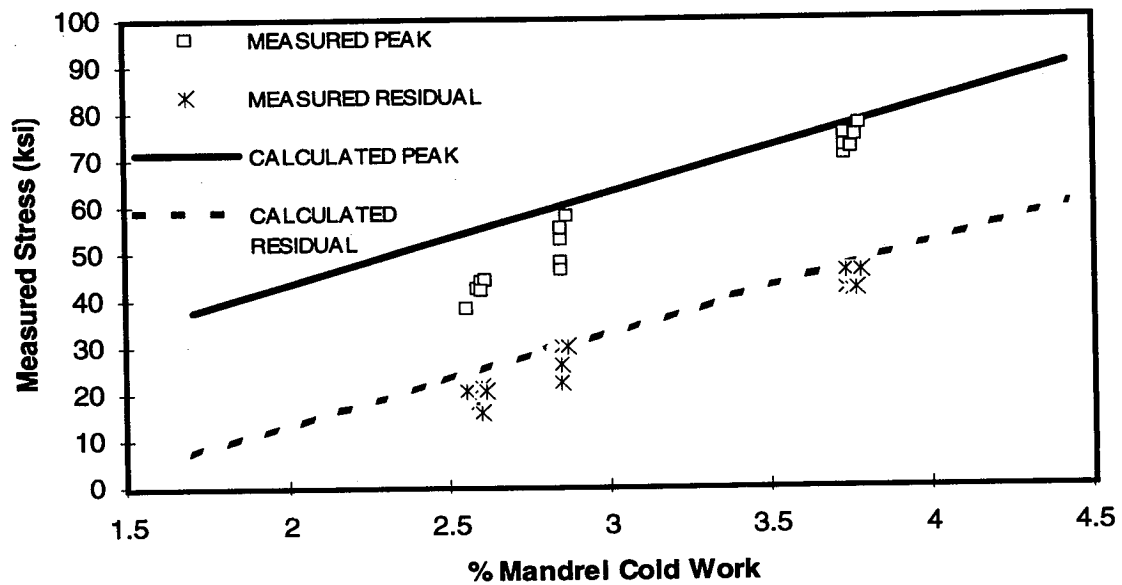


Figure 6. Comparison of stress results between plasticity model and USAF experimental results.

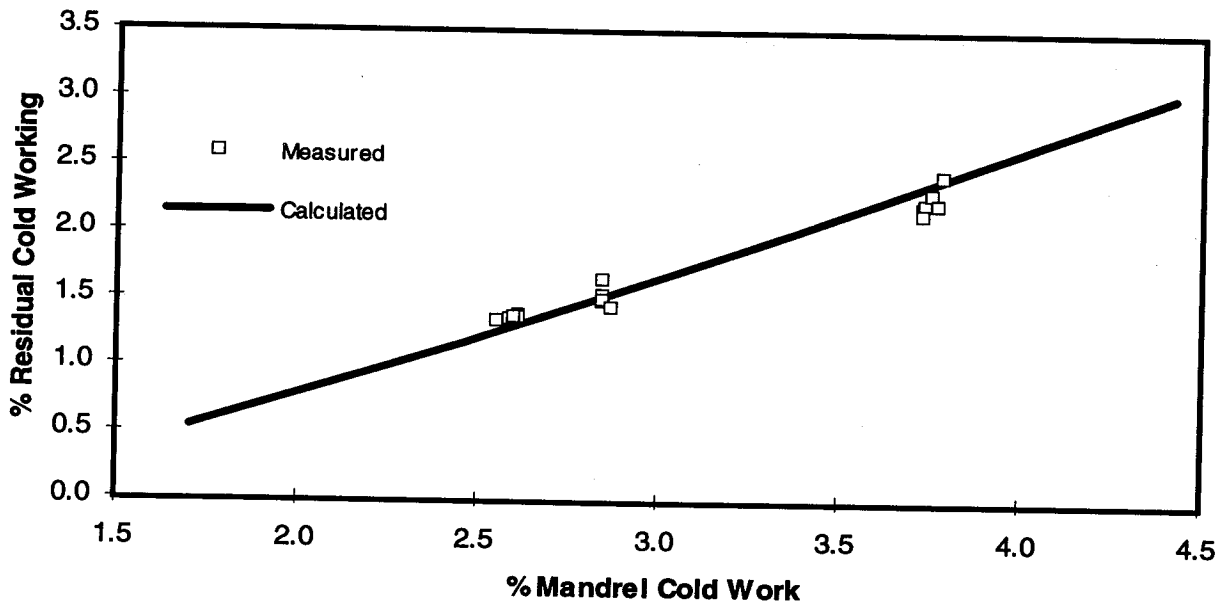


Figure 7. Comparison of percent residual cold working between plasticity model and USAF experimental results.

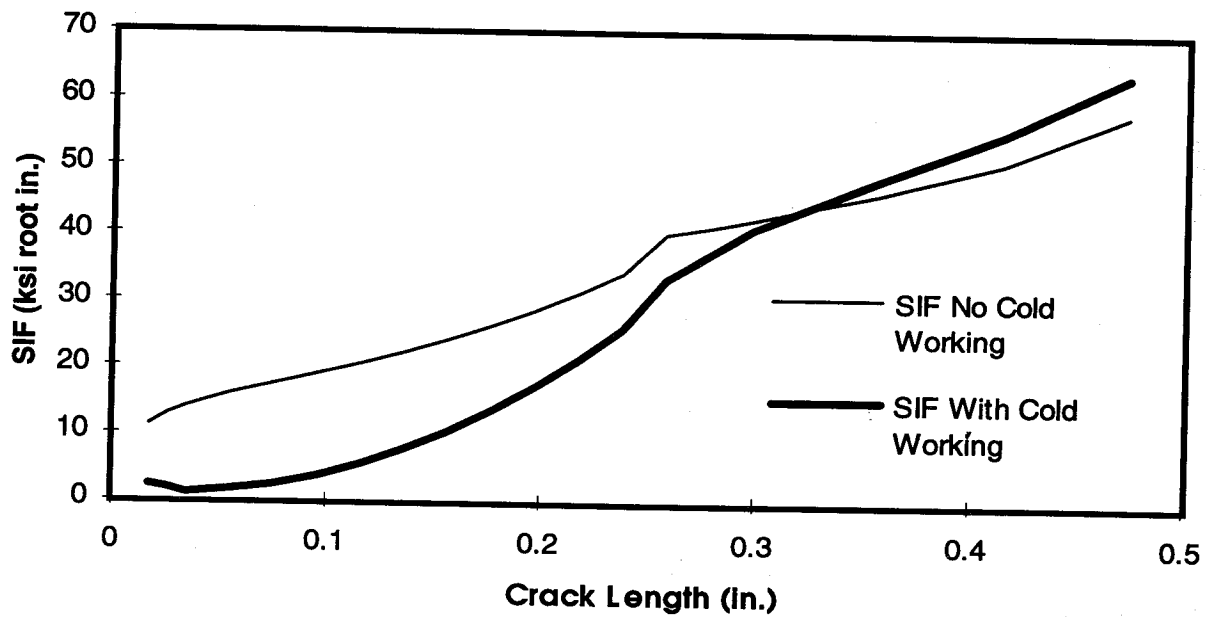


Figure 8. Comparison between the stress intensity factor for a cold worked hole and a normal hole.

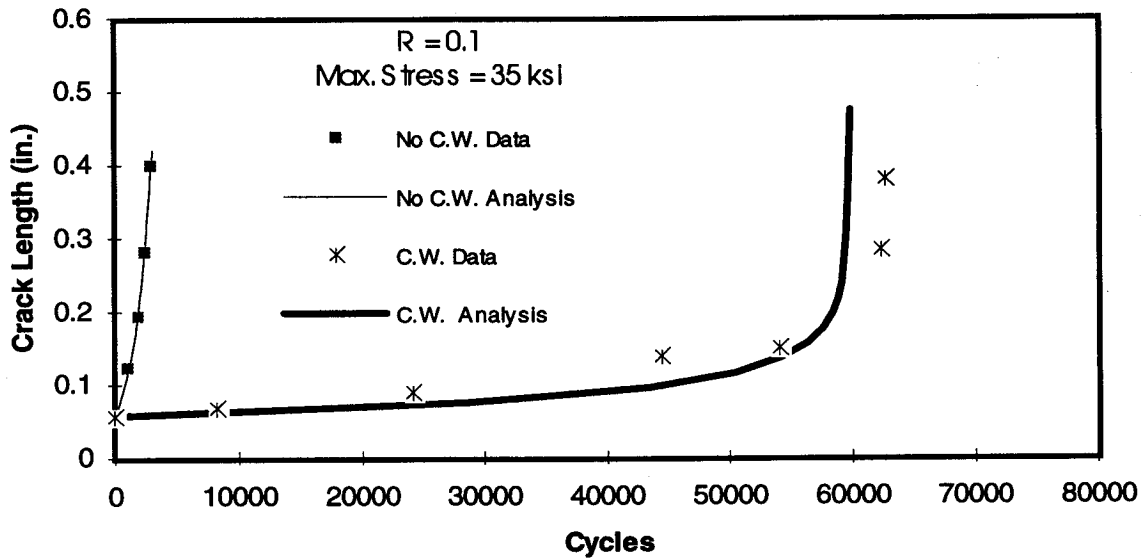


Figure 9. Comparison of constant amplitude damage tolerance analyses to test data for $R = 0.1$ and maximum stress of 35 ksi.

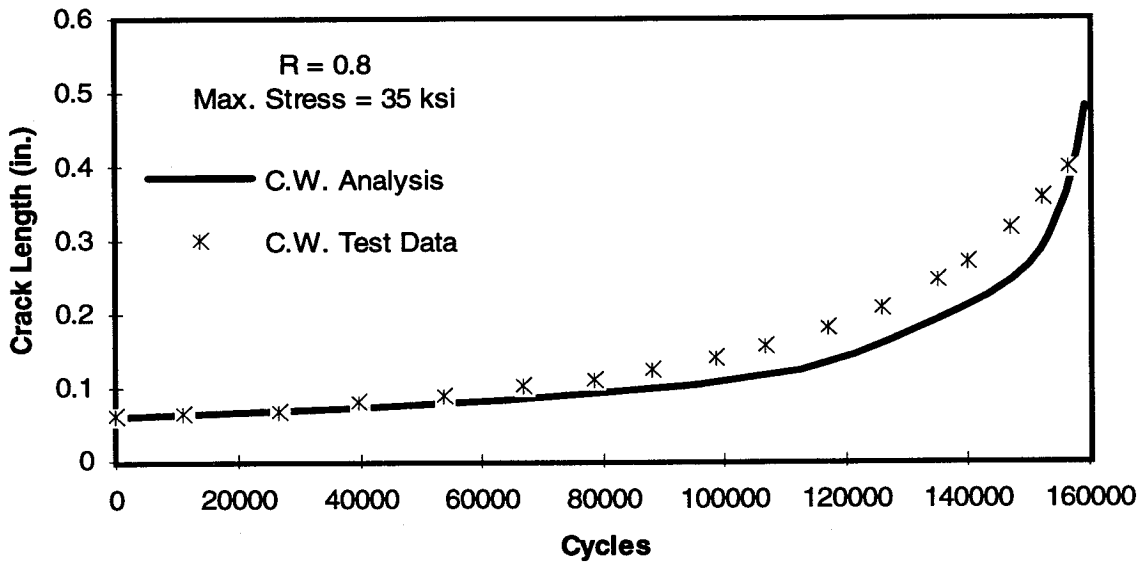


Figure 10. Comparison of constant amplitude damage tolerance analyses of a cold worked hole to test data for $R = 0.8$ and maximum stress of 35 ksi.

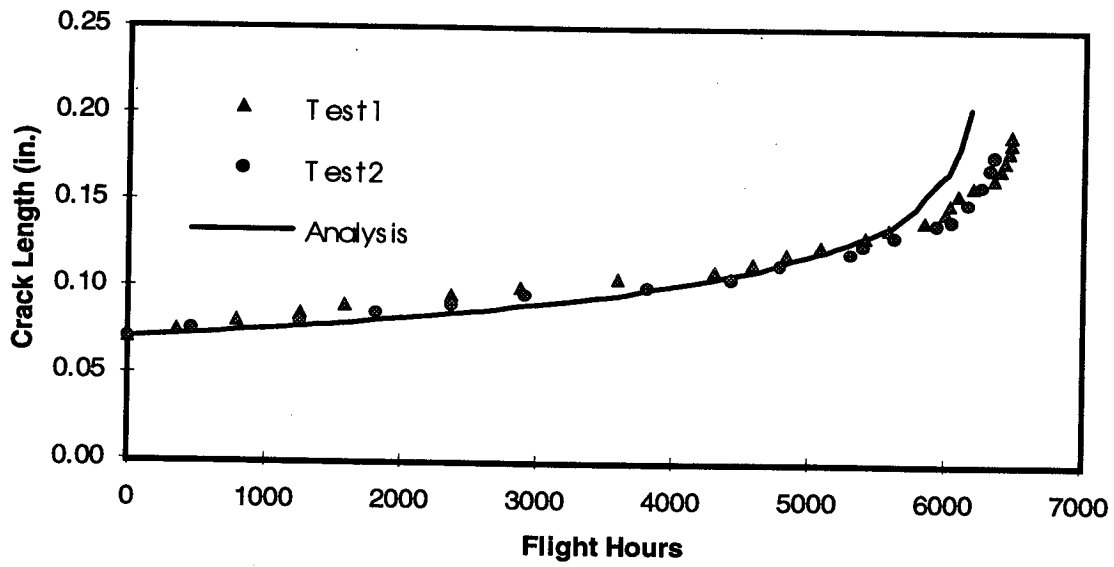


Figure 11. Spectrum crack growth analysis comparison of cold worked fastener hole to spectrum test data.

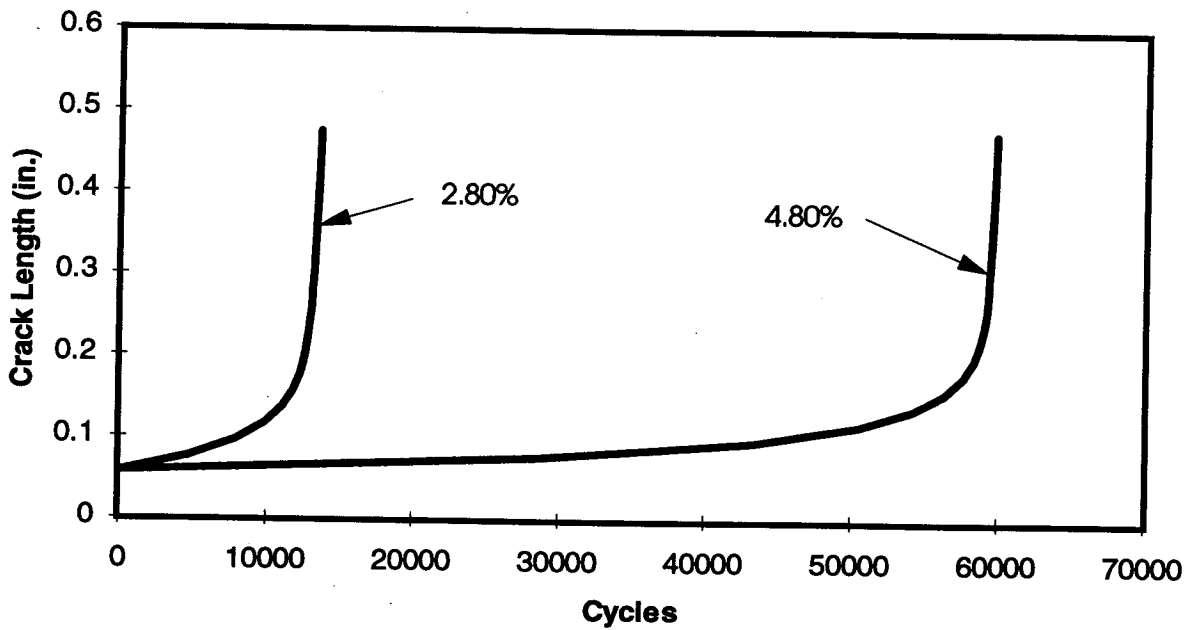


Figure 12. Variation in cold worked fastener hole damage tolerance life due to allowable tooling tolerance build up.