11-18
39428
26P

NASA Technical Memorandum 109166

Mathematical Modeling of a Class of Multibody Flexible Space Structures

A. G. Kelkar
NRC Fellow, Langley Research Center, Hampton, Virginia

December 1994

National Aeronautics and
Space Administration
Langley Research Center
Hampton, Virginia 23681-0001

Summary

A mathematical model for a general multibody flexible spacecraft is obtained. The generic spacecraft considered consists of a flexible central body to which a number of flexible multibody structures are attached. The coordinate systems used in the derivation allow effective decoupling of the translational motion of the entire spacecraft from its rotational motion about its center of mass. The derivation assumes that the deformations in the bodies are only due to elastic motions. The dynamic model derived is a closed-form vector-matrix differential equation. The model developed can be used for analysis and simulation of many realistic spacecraft configurations.

1 Introduction

A class of next-generation spacecraft is expected to inlude nonlinear, multibody, flexible space systems. Some of the current spacecraft can also be catagorized under this class. Examples of such systems are [1] : satellites with flexible appendages, such as solar arrays and antennas, space-shuttle with remote manipulator system (RMS), and flexible space platforms with multiple articulated payloads. Mathematical modeling of these systems is quite complex. This problem has been addressed in the existing literature (e.g., see [2]); however, in this paper, a different approach is taken to derive the equations of motion which yields compact closed-form equations of motion. The derivation uses modeling techniques similar to those used in the robotics literature (e.g., see [3], [4]). The formulation is relatively general and can be used for a large class of spacecraft.

First, for the sake of completeness, some of the mathematical aspects of modeling rotating systems are summarized in the section on mathematical preliminaries. Next, the kinematic equations, i.e., the position, velocity, and acceleration relations, for a particle mass of the system are obtained. Once the kinematic equations are derived, the dynamics of the system can be modeled by using various methods; for example , the Newtonian approach, the calculus of variations approach, and the Lagrangian approach. The equations of motion derived using any of these methods are equivalant; however, the Lagrange-Euler formulation is used here since it is an energy-based approach (i.e, it uses scalar formulation) and is easy to work with compared to

Newtonian approach which deals with vector quantities. It is assumed in the derivation that the bodies deform only due to the elastic motion. However, any other deformations such as thermal deformations can be easily included in the formulation with some modifications in the potential energy function.

In deriving the kinematic equations for the chain of multiple flexible bodies, the coordinate systems become an important element of the derivation. A large part of kinematics deals with the coordinate transformations used to represent the position and orientation of the body. In view of this, we will begin with a study of the operations of translation and rotation, and the transformations which are used to represent these motions.

2 Mathematical Preliminaries

2.1 Rotations

Consider a rigid body as shown in Figure 1, to which a body-axis system, i.e., a body-fixed coordinate frame, (X_b, Y_b, Z_b), is attached. Let (X_o, Y_o, Z_o) represent some fixed reference frame whose origin is concentric with the body axes system. Our aim is to relate the coordinates of a point P on the body in the (X_b, Y_b, Z_b) frame to those in the (X_o, Y_o, Z_o) frame. Let i_b, j_b, k_b denote orthonormal basis vectors in the body frame and i_o, j_o, k_o denote the orthonormal basis vectors in the fixed frame. Then, the position vector of point P, $\overline{r}_p$, in the body frame is given as

$$\overline{r}_p^b = r_{bx} i_b + r_{by} j_b + r_{bz} k_b \tag{1}$$

and in the fixed frame is given by

$$\overline{r}_p^o = r_{ox} i_o + r_{oy} j_o + r_{oz} k_o \tag{2}$$

Since $\overline{r}_p^b$ and $\overline{r}_p^o$ are representations of the same vector $\overline{r}_p$, the relation between the components of $\overline{r}_p$ in two different systems can be obtained as follows.

$$r_{bx} = i_b \cdot \overline{r}_p^b = i_b \cdot \overline{r}_p^o \tag{3}$$

Similarly,

$$r_{by} = j_b \cdot \overline{r}_p^b = j_b \cdot \overline{r}_p^o \tag{4}$$

and

$$r_{bz} = k_b \cdot \overline{r}_p^b = k_b \cdot \overline{r}_p^o \tag{5}$$

Equations (3)-(5) can be rewritten in a compact form as

$$\begin{bmatrix} r_{bx} \\ r_{by} \\ r_{bz} \end{bmatrix} = R_o^b \begin{bmatrix} r_{ox} \\ r_{oy} \\ r_{oz} \end{bmatrix}$$

or, notationally,

$$\overline{r}_p^b = R_o^b \overline{r}_p^o \tag{6}$$

where

$$R_o^b = \begin{bmatrix} i_b \cdot i_o & i_b \cdot j_o & i_b \cdot k_o \\ j_b \cdot i_o & j_b \cdot j_o & j_b \cdot k_o \\ k_b \cdot i_o & k_b \cdot j_o & k_b \cdot k_o \end{bmatrix} \tag{7}$$

Similarly, the coordinates of $\overline{r}_p$ in the fixed frame can be expressed in terms of the coordinates in the body frame as

$$\overline{r}_p^o = R_b^o \overline{r}_p^b \tag{8}$$

where

$$R_b^o = \begin{bmatrix} i_o \cdot i_b & i_o \cdot j_b & i_o \cdot k_b \\ j_o \cdot i_b & j_o \cdot j_b & j_o \cdot k_b \\ k_o \cdot i_b & k_o \cdot j_b & k_o \cdot k_b \end{bmatrix} \tag{9}$$

Since dot products are commutative, from equations (7) and (9), we can see that

$$R_b^o = R_o^{b\,T} = R_o^{b\,-1} \tag{10}$$

and

$$R_o^b = R_b^{o\,T} = R_b^{o\,-1} \tag{11}$$

Then,

$$R_o^b R_b^o = R_b^{o\,T} R_b^o = R_o^{b\,T} R_o^b = I_3 \tag{12}$$

where I_3 is the 3×3 identity matrix. The transformations, R_b^o and R_o^b, are called 'orthogonal' transformations. These transformations are also referred to as 'orthonormal' transformations since all column vectors in the transformation matrix are unit vectors in addition to being orthogonal. Thus, the transformation matrices can be used to relate the representations of the same vector in two different coordinate reference frames. Note that the transformations do not change the vector itself but only its representation. Another important thing to be noted is that the rows of R_b^o are the direction cosines of i_o, j_o, and k_o, respectively, in the body coordinate frame.

Matrices R_o^b and R_b^o can also be interpreted as rotation matrices wherein (i_b, j_b, k_b) are orthonormal basis vectors in the final direction of the (i_o, j_o, k_o) axis system after rotations about the selected coordinate axes. The properties of rotation matrices are developed in the next section.

2.2 Basic Rotation Matrices

When the rotation matrix represents a change in orientation about any one of the principal coordinates of the reference frame, X, Y, or Z, it is called as the 'basic' rotation matrix. So, if a new coordinate system, say $\mathcal{B}(\overline{u}, \overline{v}, \overline{w})$, is obtained by rotation α of the old coordinate system , say $\mathcal{A}(u, v, w)$, about X axis, then the basic rotation matrix associated with this rotation is given by, $R_{\mathcal{B}}^{\mathcal{A}} = R_{X,\alpha}$. Similarly, the basic rotation matrices associated with rotations, β about Y axis and γ about Z axis, are given by $R_{Y,\beta}$ and $R_{Z,\gamma}$, respectively. Referring to Figures (2a)-(2c), the basic rotation matrices can be written as

$$R_{X,\alpha} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\alpha & -sin\alpha \\ 0 & sin\alpha & cos\alpha \end{bmatrix} \tag{13}$$

$$R_{Y,\beta} = \begin{bmatrix} cos\beta & 0 & sin\beta \\ 0 & 1 & 0 \\ -sin\beta & 0 & cos\beta \end{bmatrix} \tag{14}$$

$$R_{Z,\gamma} = \begin{bmatrix} cos\gamma & -sin\gamma & 0 \\ sin\gamma & cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{15}$$

The reason these matrices are called basic rotation matrices is because any finite arbitrary rotation can be achieved by a composition of these matrices. However, since the finite rotations are not commutative, the order of multiplication of these matrices during composition is very important.

2.3 Composite Rotations

As stated previously, any arbitrary finite rotation can be achieved by a composition of the basic rotation matrices, i.e., by following a sequence of basic rotations. In obtaining the composite rotation matrix there are three different possibilities. The successive rotations can take place either about the prinicipal axes of the fixed reference frame, or it can take place about the principal axes of the rotating frame itself, or a combination of both. The following procedure can be followed to obtain a composite rotation matrix.

When the rotation occurs about any principal axis of a fixed reference frame, premultiply the last resultant rotation matrix by the corresponding basic roation matrix and, when the rotation occurs about any principal axis of rotating reference frame itself, postmultiply the last resultant rotation matrix by corresponding basic roation matrix.

Let us suppose that the two axes systems, $OXYZ$ (fixed) and $oxyz$ (rotating), are initially coincident. Then the rotation matrix will just be an identity matrix, I_3. Now suppose that $oxyz$ undergoes the following sequence of rotations. It rotates about OX axis through an angle α and then rotates through an angle ϕ about ox axis. The composite rotation matrix in this case will then be given by

$$R = R_{X,\alpha} I_3 R_{x,\phi} = R_{X,\alpha} R_{x,\phi} = R_{X,(\alpha+\phi)} = R_{x,(\alpha+\phi)} \tag{16}$$

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\alpha & -sin\alpha \\ 0 & sin\alpha & cos\alpha \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\phi & -sin\phi \\ 0 & sin\phi & cos\phi \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos(\alpha+\phi) & -sin(\alpha+\phi) \\ 0 & sin(\alpha+\phi) & cos(\alpha+\phi) \end{bmatrix} \tag{17}$$

2.4 Rotation About an Arbitrary Axis

In many cases, the rotations of the body-fixed coordinate system take place about an axis other than the principal axes of the fixed frame or the rotating frame. In the case when the rotation takes place about an arbitrary axis, the rotation matrix can be obtained as follows. Referring to Figure 3, let $\overline{v} = \{v_x, v_y, v_z\}^T$ be the unit vector along the axis of rotation and ψ be the angle through which the rotation takes place. Now obtain the rotation matrix for following sequence of operations. Rotate $\overline{v}$ about OX by angle α which will bring vector $\overline{v}$ in the XZ plane. Then rotating about OY axis by $-\beta$ will align $\overline{v}$ vector with OZ axis. Then rotate about OZ or $\overline{v}$ by angle ψ and then reverse the rotation order to bring vector $\overline{v}$ back to its original position. This sequence of rotation will lead to the following composition of basic rotation matrices.

$$R = R_{X,-\alpha} R_{Y,\beta} R_{Z,\psi} R_{Y,-\beta} R_{X,\alpha} \tag{18}$$

Noting that,

$$sin\alpha = \frac{v_y}{\sqrt{v_y^2 + v_z^2}} \quad cos\alpha = \frac{v_z}{\sqrt{v_y^2 + v_z^2}} \tag{19}$$

$$sin\beta = v_x \quad cos\beta = \sqrt{v_y^2 + v_z^2} \tag{20}$$

the rotation matrix can be rewritten as [3]

$$R = \begin{bmatrix} v_x^2(1 - cos\psi) + cos\psi & v_x v_y(1 - cos\psi) - v_z sin\psi & v_x v_z(1 - cos\psi) + v_y sin\psi \\ v_x v_y(1 - cos\psi) + v_z sin\psi & v_y^2(1 - cos\psi) + cos\psi & v_y v_z(1 - cos\psi) - v_x sin\psi \\ v_x v_z(1 - cos\psi) - v_y sin\psi & v_y v_z(1 - cos\psi) + v_x sin\psi & v_z^2(1 - cos\psi) + cos\psi \end{bmatrix} \tag{21}$$

In mathematical notation, these 3×3 rotational matrices are said to belong to $\mathcal{SO}(3)$ space. The notation $\mathcal{SO}(3)$ stands for Special Orthogonal group of order 3 [4].

2.5 Properties of Rotational Matrices

The rotational matrices have some special properties which play an important role in the mathematical modeling of the system. These properties are listed below.

1) As shown previously, $R^T = R^{-1}$, i.e., $RR^T = I_3$.

2) The columns of R represent the unit vectors along the principal axes of the rotated coordinate frame with respect to the reference frame unit vectors.

3) Since $R^T = R^{-1}$ the rows of rotation matrix represent the unit vectors along principal axes of reference frame with respect to rotating frame.

4) Any row (column) of rotation matrix is orthonormal to any other row (column). This is the direct consequence of properties 1 and 2.

5) If $\overline{a}$, $\overline{b} \in \mathcal{R}^3$, where $\mathcal{R}^3$ is 3−dimensional Euclidean space, then $R(\overline{a} \times \overline{b}) = R\overline{a} \times R\overline{b}$ where symbol $\times$ denotes vector cross product. (Note that this equality is valid only for orthogonal rotational matrices)

2.6 Skew-Symmetric Matrices and Cross Product Operator

A skew-symmetric matrix, $\mathcal{S}$, has the property: $s_{ii} = 0$ and $s_{ij} = -s_{ji}$ for $i \neq j$. Then, an immediate consequence of this property is given by: $\mathcal{S} + \mathcal{S}^T = 0$. These matrices play an important role in the computation of vector-matrix operations involving vectors belonging to $\mathcal{R}^3$ space and matrices belonging to $\mathcal{RO}(3)$ space. To illustrate, consider a vector cross product $\overline{a} \times \overline{b}$ which can be written in terms of vector-matrix multiplication as $\mathcal{S}(\overline{a})\overline{b}$, where $\mathcal{S}(\cdot)$ is referred as the 'cross product operator matrix', and is given by

$$\mathcal{S}(\overline{a}) = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix} \tag{22}$$

This cross product operator matrix has some important properties which are discussed below. For the sake of simplicity, we will use the following simplified notation in the remainder of the paper.

$$\mathcal{S}(\cdot) = (\tilde{\cdot}) \tag{23}$$

2.7 Properties of Cross Product Operator Matrix

1) Using property (5) of rotational matrices and the definition of $\mathcal{S}(\cdot)$, it can be shown [4] that for any vector, $\overline{v} \in \mathcal{R}^3$ and $R \in \mathcal{RO}(3)$,

$$R\mathcal{S}(\overline{v})R^T = \mathcal{S}(R\overline{v}) \tag{24}$$

2) If $R_{\overline{v},\theta}$ represents the rotation about an axis aligned with vector $\overline{v}$ by angle θ then the derivative

of R with respect to θ is given by [4]

$$\frac{dR_{\overline{v},\theta}}{d\theta} = \mathcal{S}(\overline{v})R_{\overline{v},\theta} \tag{25}$$

3) As an obvious consequence of the cross product property, we obtain another property: $\mathcal{S}(\overline{v}_1)\overline{v}_2 = -\mathcal{S}(\overline{v}_2)\overline{v}_1$, where $\overline{v}_1$ and $\overline{v}_2$ are 3-vectors.

3 Mathematical Modeling

The objective of this section is to derive the equations of motion for nonlinear, multibody, flexible, spacecraft in the most general framework possible.

3.1 Modeling Considerations

The focus configuration of a generic spacecraft under consideration has a branched geometry with a relatively large central body. The system under consideration can be schematically represented by the configuration shown in Figure 4. It is assumed that all bodies in the system are flexible. The deformations in the bodies are assumed to be due to elastic motions only; however, any other deformations such as, due to thermal effects, can also be modeled if required. The system model under consideration has cluster configuration. It consists of one central body attached to various appendage-bodies to form a branch geometry. For the purpose of derivation the following notations are used. Let each body be denoted by B_{ij} where, the first subscript indicates the branch the body belongs to and the second subscript indicates the body number in that particular branch. Since the number and the locations of various bodies are arbitrary the system configuration is highly general.

3.2 Coordinate Systems

Referring to Figure 5, X_I, Y_I, Z_I is the inertial coordinate system; X_{cm}, Y_{cm}, Z_{cm} is the coordinate system with the origin fixed at the center of mass of the entire spacecraft and is aligned with the inertial frame; X_c, Y_c, Z_c is the coordinate frame attached to the central body with the origin attached to the center of mass, and X_{ij}, Y_{ij}, Z_{ij} represent the local coordinate system attached to the ij-th body with the origin located at the point of connection between $i(j-1)$-th body and ij-th body. The motion of each local coordinate system origin, O_{ij}, is defined with respect to the previous local coordinate frame. In order to derive dynamical equations, it is necessary to obtain expressions for the kinematic quantities, i.e., the position, velocity, and acceleration of the spacecraft.

3.3 Kinematics of a Spacecraft

Consider a spacecraft with its instantaneous center of mass located at O_{cm} (Figure 5). Then, the vector, $\overline{r}_{cm}$, determines the inertial position of the spacecraft. In order to decouple the translational motion of the entire spacecraft from the rotational motion about its center of mass, for the reasons that will be apparent later, the orientation of the frame X_{cm}, Y_{cm}, Z_{cm} is assumed to be same as X_I, Y_I, Z_I, i.e. the rotation matrix, R^I_{cm}, will be an identity matrix. The vector $\overline{r}_{cm}$ can be expressed in terms of orbital elements [2]. Vectors $\overline{a}_c$ and $\overline{\rho}_c$ represent displacement of the center of mass of the central body due to rigid and elastic motions, respectively. Vector $\overline{r}_c$ then represents the vector sum of these two vectors, i.e., $\overline{r}_c = \overline{a}_c + \overline{\rho}_c$. In addition to the translational motion of the entire spacecraft, the spacecraft can undergo the rotational motion about its own center of mass. This motion can be characterized by the rotational transformation, R^{cm}_c, between X_{cm}, Y_{cm}, Z_{cm} and X_c, Y_c, Z_c. Since, X_{cm}, Y_{cm}, Z_{cm} and X_I, Y_I, Z_I are aligned, R^{cm}_c describes the orientation of the spacecraft with respect to the inertial frame also. R^{cm}_c is generally obtained by using Euler rotations. Other rigid body degrees of freedom arise from the interconnections between different bodies of the spacecraft, each of which can be described by the transformation of the type, $R^{i(j-1)}_{ij}$, between any two consecutive body frames in the chain. O_{ij} represents the origin of the ij-th body frame and its position with respect to $i(j-1)$-th body frame is defined by vector $\overline{s}_{ij}$. Also, each $\overline{s}_{ij}$ is the vector sum of $\overline{a}_{ij}$ and $\overline{\rho}_{ij}$, where $\overline{a}_{ij}$ and $\overline{\rho}_{ij}$ represent rigid body and elastic displacements of ij-th frame, respectively. First, the equations for position, velocity, and acceleration will be obtained for a representative particle mass dm in the ij-th body, i.e., the j-th body in the i-th branch. Referring to Figure 6, the position vector of a particle mass dm in ij-th body, in the local reference frame (i.e, ij-th frame), is given by

$$\overline{u}^{ij}_{dm} = \overline{d}^{ij}_{dm} + \overline{\rho}^{ij}_{dm} \tag{26}$$

where $\overline{d}^{ij}_{dm}$ and $\overline{\rho}^{ij}_{dm}$ represent rigid and elastic displacements of mass dm in ij-th frame. The position vector of dm with respect to $i(j-1)$-th frame is then given by

$$\overline{u}^{i(j-1)}_{dm} = \overline{s}_{ij} + R^{i(j-1)}_{ij}\overline{u}^{ij}_{dm} \tag{27}$$

Finally, the position vector of dm in the inertial frame of reference is given by

$$\overline{r}_{dm} = \overline{r}_{cm} + R^I_{cm}\overline{r}_c + \left(\sum_{k=1}^{k=j-1} R^I_{ik}\overline{s}_{ik}\right) + R^I_{ij}\overline{u}^{ij}_{dm} \tag{28}$$

where R^I_{ij} is given by

$$R^I_{ij} = R^I_{cm}R^{cm}_c R^c_{i1}\cdots R^{i(j-1)}_{ij} \tag{29}$$

The velocity of the particle mass dm is given by taking the time derivative of equation (28).

$$\begin{aligned}
\overline{v}_{dm} = \dot{\overline{r}}_{dm} &= \dot{\overline{r}}_{cm} + \frac{d}{dt}(R^I_{cm}\overline{s}_c) + \frac{d}{dt}(\sum_{k=1}^{k=j-1} R^I_{ik}\overline{s}_{ik}) + \frac{d}{dt}(R^I_{ij}\overline{u}^{ij}_{dm}) \\
&= \overline{v}_{cm} + R^I_{cm}\dot{\overline{s}}_c + \sum_{k=1}^{k=j-1}(\dot{R}^I_{ik}\overline{s}_{ik}) + \sum_{k=1}^{k=j-1}(R^I_{ik}\dot{\overline{s}}_{ik}) \\
&+ \dot{R}^I_{ij}\overline{u}^{ij}_{dm} + R^I_{ij}\dot{\overline{u}}^{ij}_{dm}
\end{aligned} \tag{30}$$

Now using properties 2 and 3 (Section 2.5) of the cross product operator, noting that $R^I_{cm} = I_3$, and using the notation (23), the time derivatives in equation (30) can be evaluated as follows.

$$\dot{\overline{s}}_c = \dot{\overline{a}}_c + \dot{\overline{\rho}}_c = \tilde{\overline{\omega}}_c\overline{a}_c + \sum_k \Phi_{ck}\dot{q}_{ck} = -\tilde{\overline{a}}_c\overline{\omega}_c + \sum_k \Phi_{ck}\dot{q}_{ck} \tag{31}$$

where, $\overline{\rho}_c = \sum_k \Phi_{ck}q_{ck}$, Φ_{ck} is the mode shape matrix of the central body, and q_{ck} are the generalized coordinates. Similarly,

$$\dot{\overline{s}}_{ij} = \dot{\overline{a}}_{ij} + \dot{\overline{\rho}}_{ij} = \tilde{\overline{\omega}}_{ij}\overline{a}_{ij} + \sum_k \Phi_{ijk}\dot{q}_{ijk} = -\tilde{\overline{a}}_{ij}\overline{\omega}_{ij} + \sum_k \Phi_{ijk}\dot{q}_{ijk} \tag{32}$$

where, $\dot{\overline{\rho}}_{ij} = \sum_k \phi_{ijk}\dot{q}_{ijk}$ and Φ_{ijk}, q_{ijk} being the k-th mode shape matrix and generalized coordinate for ij-th body, respectively.

$$\begin{aligned}
\dot{R}^I_{ij}\overline{s}_{ij} &= \dot{R}^{cm}_c R^c_{i1}\cdots R^{i(j-1)}_{ij}\overline{s}_{ij} + R^{cm}_c\dot{R}^c_{i1}\cdots R^{i(j-1)}_{ij}\overline{s}_{ij} \\
&+ \cdots + R^{cm}_c R^c_{i1}\cdots \dot{R}^{i(j-1)}_{ij}\overline{s}_{ij} \\
&= (R^{cm}_c R^c_{i1}\widetilde{\cdots R^{i(j-1)}_{ij}}\overline{s}_{ij})\overline{\omega}_c + R^{cm}_c(R^c_{i1}\widetilde{\cdots R^{i(j-1)}_{ij}}\overline{s}_{ij})\overline{\omega}_{i1}
\end{aligned}$$

$$
\begin{aligned}
& + \cdots + R_c^{cm} R_{i1}^c \cdots R_{i(j-1)}^{i(j-2)} (\widetilde{R_{ij}^{i(j-1)} \overline{s}_{ij}}) \overline{\omega}_{ij} && (33) \\
\dot{R}_{ij}^I \overline{u}_{dm}^{ij} &= (\widetilde{R_c^{cm} R_{i1}^c \cdots R_{ij}^{i(j-1)} \overline{u}_{dm}^{ij}}) \overline{\omega}_c + R_c^{cm} (\widetilde{R_{i1}^c \cdots R_{ij}^{i(j-1)} \overline{u}_{dm}^{ij}}) \overline{\omega}_{i1} \\
& + \cdots + R_{cm}^I R_c^{cm} R_{i1}^c \cdots R_{i(j-1)}^{i(j-2)} (\widetilde{R_{ij}^{i(j-1)} \overline{u}_{dm}^{ij}}) \overline{\omega}_{ij} && (34) \\
R_{ij}^I \dot{\overline{u}}_{dm}^{ij} &= R_{ij}^I (\dot{\overline{d}}_{dm}^{ij} + \overline{\rho}_{dm}^{ij}) \\
&= R_{ij}^I (\tilde{\overline{\omega}}_{ij} \overline{d}_{dm}^{ij} + \sum_k \Phi_{ijk}^{dm} \dot{q}_{ijk}) \\
&= -R_{ij}^I \tilde{\overline{d}}_{dm}^{ij} \overline{\omega}_{ij} + R_{ij}^I \sum_k \Phi_{ijk}^{dm} \dot{q}_{ijk} && (35)
\end{aligned}
$$

Substituting equations (33) thru (35) in equation (30) we get

$$
\begin{aligned}
\overline{v}_{dm} &= \overline{v}_{cm} - \tilde{\overline{a}}_c \overline{\omega}_c + \sum_k \Phi_{ck} \dot{q}_{ck} \\
& - R_c^{cm} [\sum_{j=1} R_{ij}^c \tilde{\overline{a}}_{ij} \overline{\omega}_{ij}] + \sum_{j=1}^{j} R_{ij}^{cm} (\sum_k \Phi_{ijk} \dot{q}_{ijk}) \\
& + \sum_j (\widetilde{R_c^{cm} R_{i1}^c \cdots R_{ij}^{i(j-1)} \overline{s}_{ij}}) \overline{\omega}_c + R_c^{cm} [\sum_{j=1}^{j} (\widetilde{R_{ij}^c \overline{s}_{ij}})] \overline{\omega}_{i1} + \cdots \\
& + R_{i(z-1)}^{cm} (\widetilde{\sum_{j=z}^{j} R_{ij}^{i(z-1)} \overline{s}_{ij}}) \overline{\omega}_{iz} + \cdots + R_{i(j-1)}^{cm} (\widetilde{R_{ij}^{i(j-1)} \overline{s}_{ij}}) \overline{\omega}_{ij} \\
& + (\widetilde{R_c^{cm} R_{i1}^c \cdots R_{ij}^{i(j-1)} \overline{u}_{dm}^{ij}}) \overline{\omega}_c + R_c^{cm} (\widetilde{R_{i1}^c \cdots R_{ij}^{i(j-1)} \overline{u}_{dm}^{ij}}) \overline{\omega}_{i1} \\
& + \cdots + R_c^{cm} R_{i1}^c \cdots R_{i(j-1)}^{i(j-2)} (\widetilde{R_{ij}^{i(j-1)} \overline{u}_{dm}^{ij}}) \overline{\omega}_{ij} \\
& - R_{ij}^I \tilde{\overline{d}}_{dm}^{ij} \overline{\omega}_{ij} + R_{ij}^I \sum_k \Phi_{ijk}^{dm} \dot{q}_{ijk} && (36)
\end{aligned}
$$

Simplifying and regrouping equation (36) gives

$$
\begin{aligned}
\overline{v}_{dm} &= \overline{v}_{cm} + [-\tilde{\overline{a}}_c + \sum_j (\widetilde{R_c^{cm} R_{i1}^c \cdots R_{ij}^{i(j-1)} \overline{s}_{ij}}) + (\widetilde{R_c^{cm} R_{i1}^c \cdots R_{ij}^{i(j-1)} \overline{u}_{dm}^{ij}})] \overline{\omega}_c \\
& + [-R_{i1}^{cm} \tilde{\overline{a}}_{i1} + R_c^{cm} (\sum_{j=1}^{j} (\widetilde{R_{ij}^c \overline{s}_{ij}})) + R_c^{cm} (\widetilde{R_{ij}^c \overline{u}_{dm}^{ij}})] \overline{\omega}_{i1} + \cdots \\
\cdots & + [-R_{iz}^{cm} \tilde{\overline{a}}_{iz} + R_{i(z-1)}^{cm} (\widetilde{\sum_{j=z}^{j} R_{ij}^{i(z-1)} \overline{s}_{ij}}) + R_{i(z-1)}^{cm} (\widetilde{R_{ij}^{i(z-1)} \overline{u}_{dm}^{ij}})] \overline{\omega}_{iz} \\
& + \cdots + [-R_{ij}^{cm} \tilde{\overline{a}}_{ij} + R_{i(j-1)}^{cm} (\widetilde{R_{ij}^{i(j-1)} \overline{s}_{ij}}) + R_{i(j-1)}^{cm} (\widetilde{R_{ij}^{i(j-1)} \overline{u}_{dm}^{ij}})] \overline{\omega}_{ij} \\
& - R_{ij}^I \tilde{\overline{d}}_{dm}^{ij} \overline{\omega}_{ij} + \sum_k \Phi_{ck} \dot{q}_{ck} + \sum_{j=1}^{j} R_{ij}^{cm} (\sum_k \Phi_{ijk} \dot{q}_{ijk}) + R_{ij}^I \sum_k \Phi_{ijk}^{dm} \dot{q}_{ijk} && (37)
\end{aligned}
$$

Equation (37) can be rewritten in a compact form as

$$\overline{v}_{dm} = \overline{v}_{dm}^{trans} + \overline{v}_{dm}^{rot} = \overline{N}\dot{\overline{p}} \tag{38}$$

where $\overline{v}_{dm}^{trans}$ and $\overline{v}_{dm}^{rot}$ are the contributions to the velocity vector of particle mass dm from the translational and rotational-plus-flexible motion of the spacecraft, respectively, and

$$\begin{aligned}
\overline{v}_{dm}^{trans} &= \overline{v}_{cm} && (39)\\
\overline{v}_{dm}^{rot} &= N\dot{p} && (40)\\
\overline{N} &= [I_3 \quad N] && (41)\\
\dot{\overline{p}} &= \{\overline{v}_{cm} \quad \dot{p}\}^T && (42)
\end{aligned}$$

In equations (39)-(42), N is $3 \times n$ matrix, n is the total number of rotational-plus-flexible degrees of freedom, and p is $n \times 1$ vector of generalized velocities corresponding to these degrees of freedom. Matrix N has the following form:

$$N = [N_{rigid} \quad N_{flex}] \tag{43}$$

where, N_{rigid} is $3\times3(lm+1)$ matrix related to the rigid degrees of freedom and N_{flex} is $3\times s(lm+1)$ matrix related to the flexible degrees of freedom. For the sake of notational simplicity, and without loss of generality, it is assumed that each chain structure has m bodies and that each body has s flexible degrees of freedom. Then vector $\dot{p}$ has the form

$$\dot{p} = [\overline{\omega}_c, \overline{\omega}_{11}, \cdots \overline{\omega}_{1m}, \cdots \overline{\omega}_{ij}, \cdots \overline{\omega}_{lm}, \dot{q}_{c1}, \cdots \dot{q}_{cs}, \dot{q}_{111}, \cdots \dot{q}_{ijk}, \cdots \dot{q}_{lms}]^T \tag{44}$$

The matrices N_{rigid} and N_{flex} are given as follows.

$$N_{rigid} = [N_c, N_{i1}, \cdots, N_{iz}, \cdots, N_{ij}] \tag{45}$$

where

$$\begin{aligned}
N_c &= -\tilde{\overline{a}}_c + \sum_j (\widetilde{R_c^{cm} R_{i1}^c \cdots R_{ij}^{i(j-1)} \overline{s}_{ij}}) + (\widetilde{R_c^{cm} R_{i1}^c \cdots R_{ij}^{i(j-1)} \overline{u}_{dm}^{ij}}) && (46)\\
N_{i1} &= [-R_{i1}^{cm}\tilde{\overline{a}}_{i1} + R_c^{cm}(\sum_{j=1}^{j}(\widetilde{R_{ij}^c \overline{s}_{ij}})) + R_c^{cm}(\widetilde{R_{ij}^c \overline{u}_{dm}^{ij}})] && (47)
\end{aligned}$$

$$
\begin{aligned}
N_{iz} &= [-R_{iz}^{cm}\tilde{\overline{a}}_{iz} + R_{i(z-1)}^{cm}(\widetilde{\sum_{j=z}^{j} R_{ij}^{i(z-1)}\overline{s}_{ij}}) + R_{i(z-1)}^{cm}(\widetilde{R_{ij}^{i(z-1)}\overline{u}_{dm}^{ij}})] && (48)\\
N_{ij} &= [-R_{ij}^{cm}\tilde{\overline{a}}_{ij} + R_{i(j-1)}^{cm}(\widetilde{R_{ij}^{i(j-1)}\overline{s}_{ij}}) + R_{i(j-1)}^{cm}(\widetilde{R_{ij}^{i(j-1)}\overline{u}_{dm}^{ij}})] && (49)\\
& && (50)
\end{aligned}
$$

and

$$
N_{flex} = \left[\sum_k \Phi_{ck}, \quad \cdots, R_{ij}^{cm}\sum_k \Phi_{ijk} \quad R_{ij}^{I}\sum_k \Phi_{ijk}^{dm}\right] \tag{51}
$$

Having obtained the expression for velocity, the kinetic energy for the particle mass dm is given by

$$
dT_{dm} = \frac{1}{2}\overline{v}_{dm}^{T}\overline{v}_{dm}dm \tag{52}
$$

Then the kinetic energy for the entire spacecraft can be obtained by integrating equation (52), i.e.,

$$
T = \int_{\Omega} dT_{dm} = \frac{1}{2}\int_{\Omega}\overline{v}_{dm}^{T}\overline{v}_{dm}\mu d\Omega \tag{53}
$$

where, μ is the mass density, $\overline{v}_{dm}$ is as given in equation (37), and Ω denotes the spatial domain of integration. Substituting equation (52) into equation (53), we get

$$
\begin{aligned}
vT &= \frac{1}{2}\int_{\Omega}(\overline{N}\dot{\overline{p}})^{T}\overline{N}\dot{\overline{p}}\mu d\Omega\\
&= \frac{1}{2}\int_{\Omega}\dot{\overline{p}}^{T}(\overline{N}^{T}\overline{N})\dot{\overline{p}}\mu d\Omega && (54)
\end{aligned}
$$

Equation (54) can be rewritten in a compact form as

$$
T = \frac{1}{2}\dot{\overline{p}}^{T}M(\overline{p})\dot{\overline{p}} \tag{55}
$$

where, $M(\overline{p})$ is the mass-inertia matrix of the system and is given by

$$
M(\overline{p}) = \int_{\Omega}(\overline{N}^{T}\overline{N})\mu d\Omega \tag{56}
$$

$M(\overline{p})$ is symmetric and positive definite matrix.

In practice, for the most spacecraft applications, a control engineer needs to know only rotational dynamic model of the spacecraft since the translational motion of the spacecraft, as a whole, is controlled only periodically by separate thruster-jets whenever reboosting of orbit is necessary. In such a case, the kinetic energy of the spacecraft is given by

$$
\begin{aligned}
T_{rot} &= \frac{1}{2}\int_{\Omega} \overline{v}_{dm}^{rot\,T}\,\overline{v}_{dm}^{rot}\mu d\Omega \\
&= \frac{1}{2}\int_{\Omega} (N\dot{p})^T N\dot{p}\mu d\Omega \\
&= \frac{1}{2}\int_{\Omega} \dot{p}^T (N^T N)\dot{p}\mu d\Omega
\end{aligned} \tag{57}
$$

and the inertia matrix is then given by

$$
M(p) = \int_{\Omega} (N^T N)\mu d\Omega \tag{58}
$$

$M(p)$ is also symmetric and positive definite matrix. Using (57) and (58), T_{rot} can be rewritten in compact form as

$$
T_{rot} = \frac{1}{2}\dot{p}^T M(p)\dot{p} \tag{59}
$$

3.4 Potential Energy

The potential energy of the system could be due to many sources, such as elastic displacement, thermal deformation, etc. The deformations due to thermal effects is not considered here, however, it can be easily included in the formulation if desired. Thus, it is assumed that the potential energy has contribution only from the elastic strain energy. Also, it assumed that the materials under consideration are isotropic in nature and that they obey Hook's law.

For the isotropic materials obeying Hook's law, the strain energy differential is given as

$$
\delta V = \int_{\Omega} \sigma^T \delta\epsilon d\Omega \tag{60}
$$

which can be rewritten as

$$
\delta V = \int_{\Omega} \Psi d\Omega \tag{61}
$$

where Ψ is the strain energy density and has the form

$$
\Psi = \sigma_{xx}\epsilon_{xx} + \sigma_{yy}\epsilon_{yy} + \ldots + \sigma_{yz}\epsilon_{yz} \tag{62}
$$

Now, for materials obeying Hook's law, following equality holds.

$$\sigma^T = E\epsilon \tag{63}$$

The strain-displacement relation is given by

$$\epsilon = \mathcal{D}u \tag{64}$$

where, u is the general displacement vector and $\mathcal{D}$ is the differential operator defined by relations

$$\epsilon_{ij} = \frac{1}{2}\left[u_{i,j} + u_{j,i} + \sum_{k=1}^{3} u_{k,i}^{i,j}\right] \qquad \{i,j = 1,2,3\} \tag{65}$$

The vector u can be expressed in terms of modal coordinates as

$$u = \Phi q \tag{66}$$

Now, from equation (64)

$$\delta\epsilon = \mathcal{D}\Phi\delta q \tag{67}$$

Substituting in δV, get

$$\begin{aligned}
\delta V &= \int_\Omega \sigma^T \delta\epsilon d\Omega \\
&= \int_\Omega \epsilon^T E\mathcal{D}\Phi\delta q d\Omega \\
&= \int_\Omega q^T(\mathcal{D}\Phi)^T E\mathcal{D}\Phi\delta q d\Omega \\
&= q^T \int_\Omega (\mathcal{D}\Phi)^T E\mathcal{D}\Phi d\Omega\delta q \\
&= q^T K\delta q
\end{aligned} \tag{68}$$

where, K is called the stiffness matrix of the system and is given by

$$K = \int_\Omega (\mathcal{D}\Phi)^T E\mathcal{D}\Phi d\Omega \tag{69}$$

The potential energy of the system is, then given by

$$V = \frac{1}{2}q^T K q \tag{70}$$

3.5 Equations of Motion

As previously stated, without the loss of generality, only the rotational motion of the spacecraft is considered in deriving the dynamic model of the spacecraft. If necessary, the translational motion can be easily included in the equations of motion by using appropriate kinetic energy term in the Lagrangian , i.e., by using T instead of T_{rot} and using $\overline{p}$ instead of p as a displacement vector.

Using equations (59) and (70) the Lagrangian of the system is defined as

$$L = T_{rot} - V \tag{71}$$

For the purpose of convenience, L can be rewritten in the indicial notation as

$$L = T_{rot} - V = \frac{1}{2}\sum_{i,j} M_{ij}\dot{p}_i\dot{p}_j - V(q) \tag{72}$$

The Euler-Lagrange equations for the system can then be derived from

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{p}_k}\right) - \frac{\partial L}{\partial p_k} = F_k \tag{73}$$

where, F_k are generalized forces from non-conservative force field. Evaluating the derivatives,

$$\frac{\partial L}{\partial \dot{p}_k} = \sum_j M_{kj}\dot{p}_j \tag{74}$$

and

$$\begin{aligned} \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{p}_k}\right) &= \sum_j M_{kj}\ddot{p}_j + \sum_j \dot{M}_{kj}\dot{p}_j \\ &= \sum_j M_{kj}\ddot{p}_j + \sum_{i,j} \frac{\partial M_{kj}}{\partial p_i}\dot{p}_i\dot{p}_j \end{aligned} \tag{75}$$

Also

$$\frac{\partial L}{\partial p_k} = \frac{1}{2}\sum_{i,j} \frac{\partial M_{ij}}{\partial p_k}\dot{p}_i\dot{p}_j - \frac{\partial V}{\partial p_k} \tag{76}$$

Thus the Euler-Lagrange equations can be written

$$\sum_j M_{kj}\ddot{p}_j + \sum_{i,j}\{\frac{\partial M_{kj}}{\partial p_i} - \frac{1}{2}\frac{\partial M_{ij}}{\partial p_k}\}\dot{p}_i\dot{p}_j - \frac{\partial V}{\partial p_k} = F_k \qquad k = 1, ..., n \tag{77}$$

By interchanging the order of summation and taking advantage of symmetry, it can be seen that

$$\sum_{i,j}\{\frac{\partial M_{kj}}{\partial p_i}\}\dot{p}_i\dot{p}_j = \frac{1}{2}\sum_{i,j}\{\frac{\partial M_{kj}}{\partial p_i} + \frac{\partial M_{ki}}{\partial p_j}\}\dot{p}_i\dot{p}_j \tag{78}$$

Hence

$$\sum_{i,j}\{\frac{\partial M_{kj}}{\partial p_i} - \frac{1}{2}\frac{\partial M_{ij}}{\partial p_k}\}\dot{p}_i\dot{p}_j = \sum_{i,j}\frac{1}{2}\{\frac{\partial M_{kj}}{\partial p_i} + \frac{\partial M_{ki}}{\partial p_j} - \frac{\partial M_{ij}}{\partial p_k}\}\dot{p}_i\dot{p}_j \tag{79}$$

The terms

$$C_{ijk} = \frac{1}{2}\{\frac{\partial M_{kj}}{\partial p_i} + \frac{\partial M_{ki}}{\partial p_j} - \frac{\partial M_{ij}}{\partial p_k}\} \tag{80}$$

are known as Christoffel symbols. Note that, for each fixed k, we have $C_{ijk} = C_{jki}$. Also

$$\frac{\partial V}{\partial p_k} = K_{kj}q_j \tag{81}$$

Finally, then Euler-Lagrange equations of motion can be written as

$$\sum_j M_{kj}\ddot{p}_j + \sum_{i,j} C_{ijk}\dot{p}_i\dot{p}_j + D_{kj}\dot{p} + K_{kj}p_j = F_k, \qquad (k = 1, 2, ..., n) \tag{82}$$

where D is the inherent structural damping matrix and Dp is the vector of nonconservative forces.

In the equations (82), there are four types of terms. The first type of terms involve the second type derivative of the generalized coordinates. The second are quadratic terms in the first derivatives of p, where the coefficients may depend on p. These terms can be further classified into two types. Terms involving a product of the type $\dot{p}^2$ are called *centrifugal* terms, while those involving a product of the type $\dot{p}_i\dot{p}_j$ where $i \neq j$ are called Coriolis terms. The third type are the ones which involve only the first derivative of the generalized coordinates and they are the dissipative forces due to the inherent damping. The fourth type of terms involve only p but not its derivatives. These arise from differentiating the potential energy. In the matrix-vector notation, the equations (82) are written in a compact form as

$$M(p)\ddot{p} + C(p, \dot{p})\dot{p} + Dp + Kp = F \tag{83}$$

The k, j-th element of the matrix $C(p, \dot{p})$ is defined as

$$\begin{aligned} c_{kj} &= \sum_{i=1}^{n} c_{ijk}(p)\dot{p}_i \\ &= \sum_{i=1}^{n}\frac{1}{2}\{\frac{\partial M_{kj}}{\partial p_i} + \frac{\partial M_{ki}}{\partial p_j} - \frac{\partial M_{ij}}{\partial p_k}\}\dot{p}_i \end{aligned} \tag{84}$$

Now, an important property of systems whose equations of motion are given by (83), is derived next.

Theorem.- The matrix $S(p, \dot{p}) = \dot{M}(p) - 2C(p, \dot{p})$ is skew symmetric.

Proof.- The kj-th component of the time derivative of the inertia matrix, $\dot{M}(p)$ is given by the chain rule as

$$\dot{M}_{kj} = \sum_{i=1}^{n} \frac{\partial M_{kj}}{\partial p_i} \dot{p}_i$$

Therefore, the kj-th component of $S = \dot{M} - 2C$ is given by

$$\begin{aligned} S_{kj} &= \dot{M}_{kj} - 2C_{kj} \\ &= \sum_{i=1}^{n} \left[\frac{\partial M_{kj}}{\partial p_i} - \left\{ \frac{\partial M_{kj}}{\partial p_i} + \frac{\partial M_{ki}}{\partial p_j} - \frac{\partial M_{ij}}{\partial p_k} \right\} \right] \dot{p}_i \\ &= \sum_{i=1}^{n} \left[\frac{\partial M_{ij}}{\partial p_k} - \frac{\partial M_{ki}}{\partial p_j} \right] \dot{p}_i \end{aligned} \tag{85}$$

Since the inertia matrix is symmetric, i.e., $M_{ij} = M_{ji}$, it follows from (85) by interchanging the indices k and j that

$$S_{jk} = -S_{kj}$$

This completes the proof.

3.6 Conclusions

A generic mathematical model for a class of multibody flexible spacecraft was developed. A judicious choice of coordinate systems was made which allowed decoupling of translational and rotational dynamics of the spacecraft. This is a useful way of modeling spacecraft dynamics since, depending on the application, one can choose to use only rotational, or translational, or a complete rotational-plus-translational dynamic model of the spacecraft. Although only rotational model was presented, the Lagrangian formulation was done for a complete rotational-plus-translational dynamic model of the spacecraft. In the derivation of potential energy, an assumption was made that potential energy terms are only due to elastic deformations; however, as stated previously, the potential energy contributions from any other sources of deformations, such as thermal deformations, can also be included in the same way as elastic deformations. The model developed can be used for multibody spacecraft such as space-based manipulators, multipayload platforms, satellites with flexible appendages, and many more. With minor modifications, the model can be used even for terrestrial robots. In summary, most spacecraft models can be obtained as special cases of the model developed in this paper.

References

[1] *Jane's Space Directory*, Jane's Data Division, Jane's Information Group Limited, Sentinel House, 163 Brighton Road, Coulsdon, Surrey CR5 2NH, UK, 1993.

[2] Ng, A. C. and Modi, V. J.: 'A Formulation for Studying Dynamics of Interconnected Bodies With Application', Proc. AIAA/AAS Astrodynamics Conference, August 15-17, 1988, Paper No. 88-4303-CP, pp. 660-668.

[3] Fu, K. S., Gonzalez, R. C., and Lee, C. S. G.: *Robotics: Control, Sensing, Vision, and Intelligence*, McGraw Hill, New York, 1987.

[4] Spong, M. W. and Vidyasagar, M.: *Robot Dynamics and Control*, John Wiley and Sons, New York, 1989.

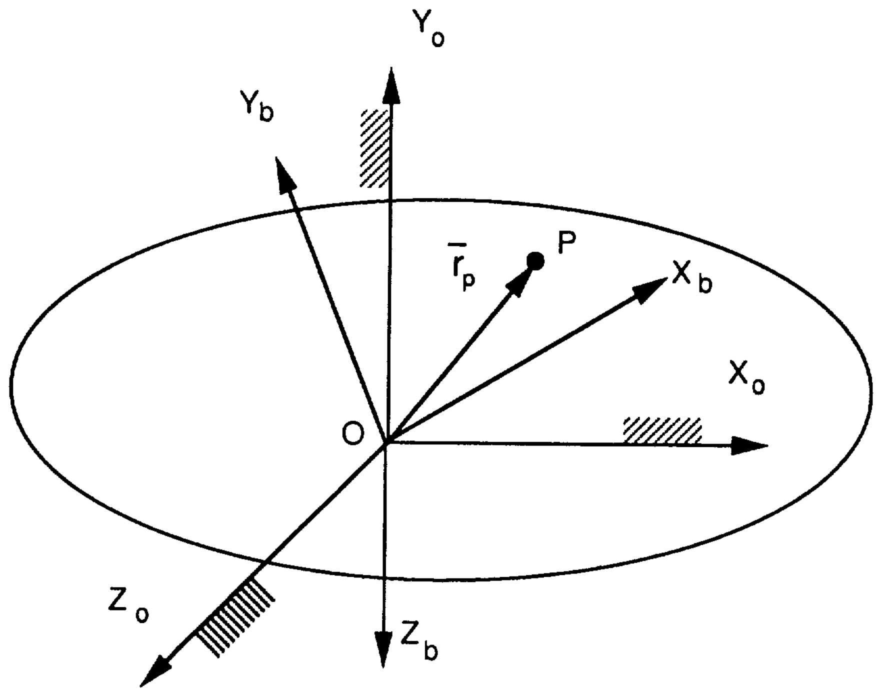
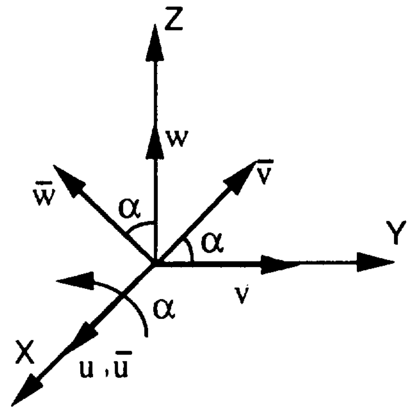
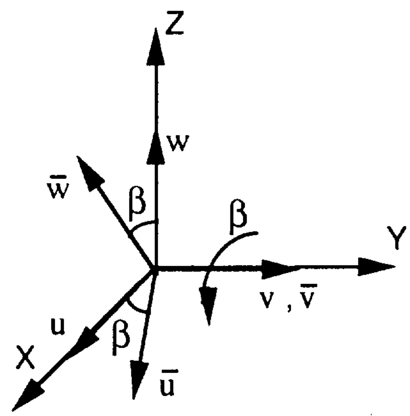


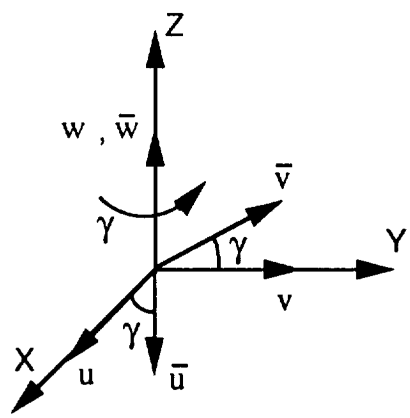
Figure 1. Fixed reference and body axes systems



(a) X - rotation



(b) Y - rotation



(c) Z - rotation

Figure 2. Basic rotations

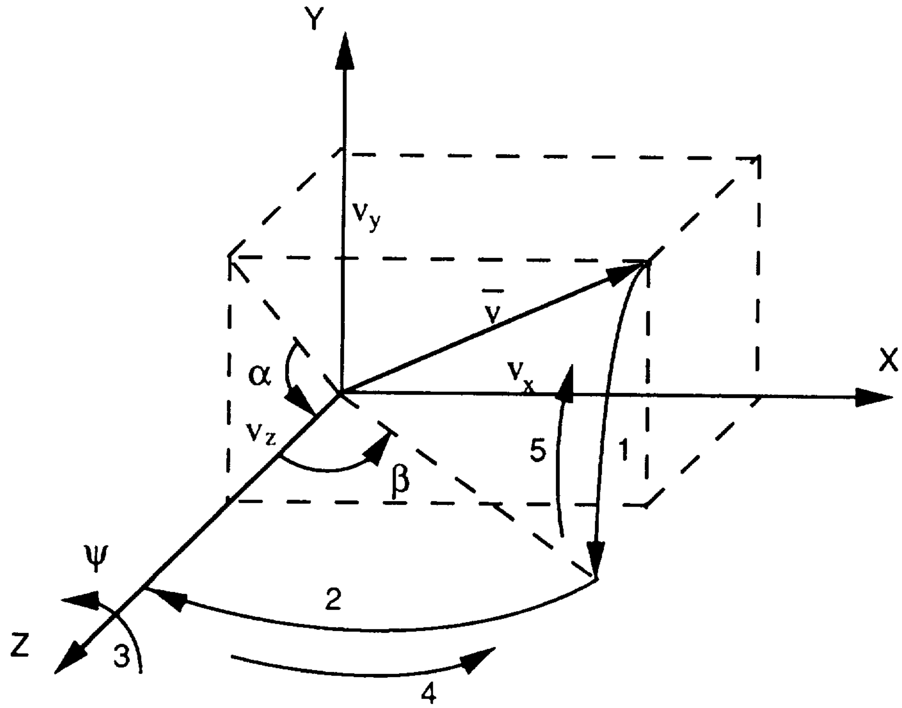


Figure 3. Rotation about an arbitrary axis

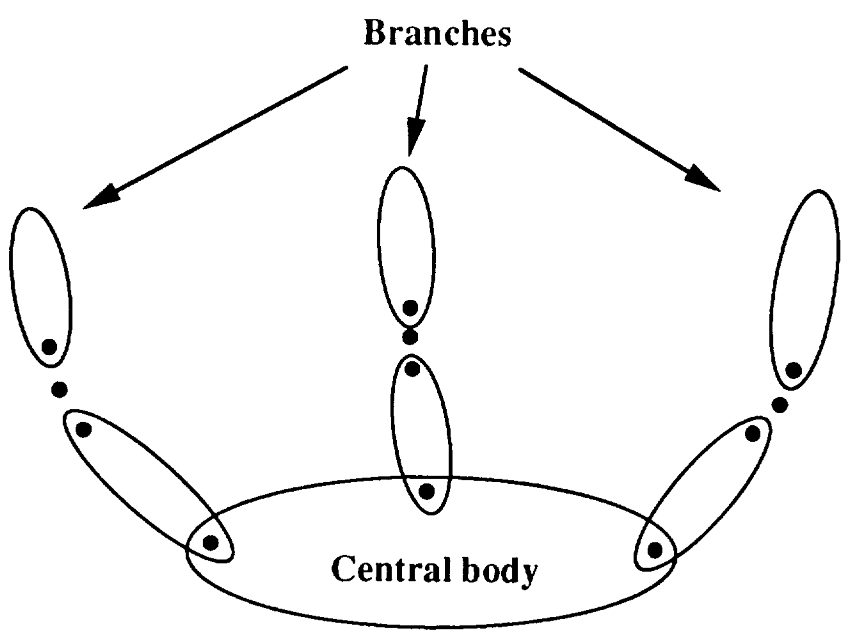


Figure 4. Schematic of a multibody spacecraft

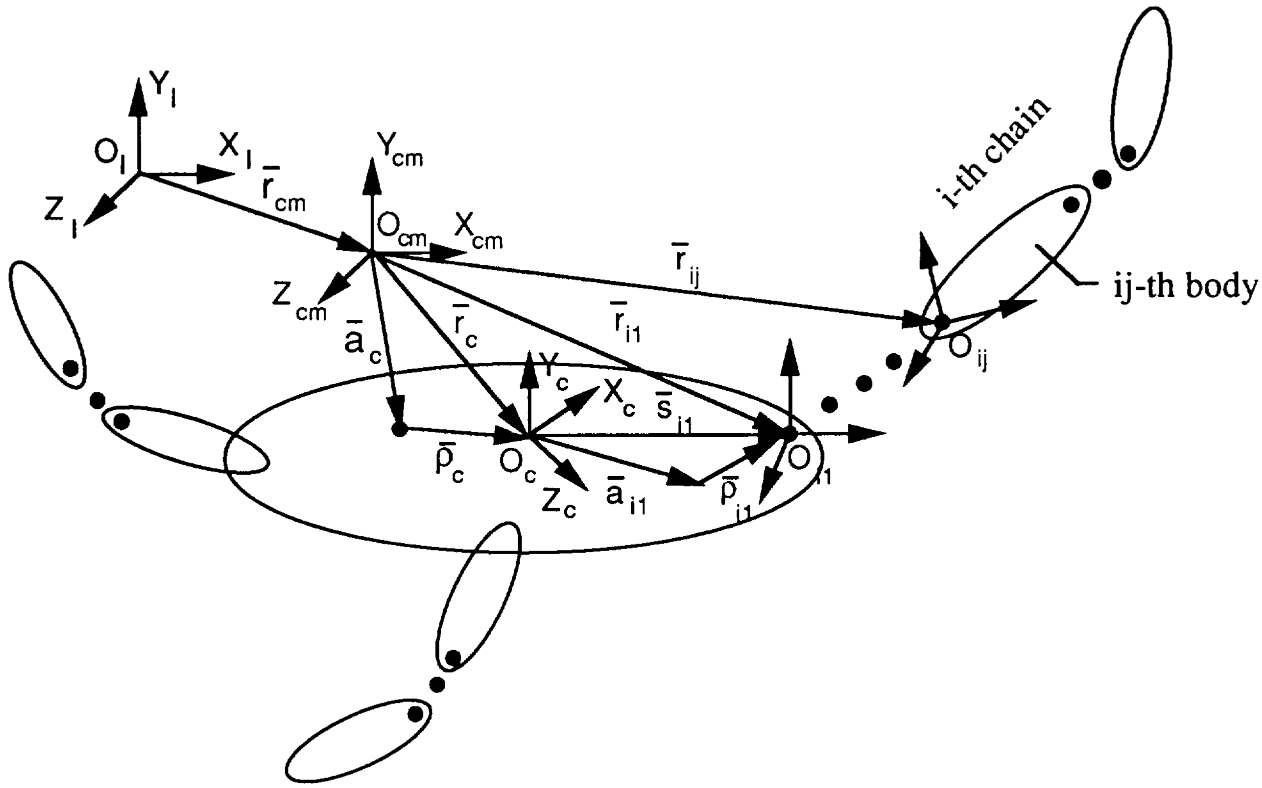


Figure 5. Coordinate systems

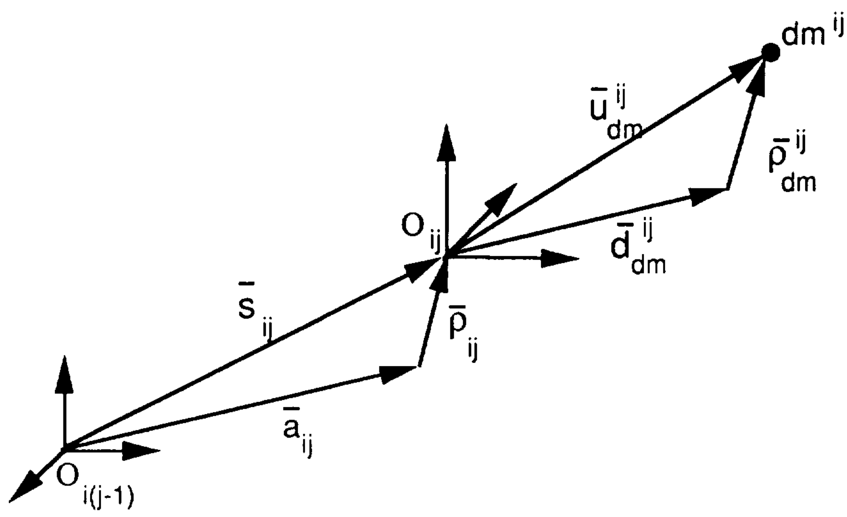


Figure 6. Position vector of particle mass dm in (ij)-th body

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	December 1994	Technical Memorandum

4. TITLE AND SUBTITLE
Mathematical Modeling of a Class of Multibody Flexible Space Structures

5. FUNDING NUMBERS
505-64-52-01

6. AUTHOR(S)
Atul G. Kelkar

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
NASA Langley Research Center
Hampton, VA 23681-0001

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)
National Aeronautics and Space Administration
Washington, DC 20546-0001

10. SPONSORING / MONITORING AGENCY REPORT NUMBER
NASA TM-109166

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION / AVAILABILITY STATEMENT
Unclassified-Unlimited
Subject Category 18

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

A mathematical model for a general multibody flexible spacecraft is obtained. The generic spacecraft considered consists of a flexible central body to which a number of flexible multibody structures are attached. The coordinate systems used in the derivation allow effective decoupling of the translational motion of the entire spacecraft from its rotational motion about its center of mass. The derivation assumes that the deformations in the bodies are only due to elastic motions. The dynamic model derived is a closed-form vector-matrix differential equation. The model developed can be used for analysis and simulation of many realistic spacecraft configurations.

14. SUBJECT TERMS
Spacecraft Modeling; Multibody Spacecraft; Spacecraft Dynamics; and Nonlinear Spacecraft

15. NUMBER OF PAGES
24

16. PRICE CODE
A03

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
unclassified	unclassified		