NONCLASSICAL LIGHT IN INTERFEROMETRIC MEASUREMENTS

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Abstract

It is shown that the even and odd coherent light and other nonclassical states of light like superposition of coherent states with different phases may replace the squeezed light in interferometric gravitational wave detector to increase its sensitivity.

1   Introduction

The problem of detecting gravitational wave has been a subject of interest for many years [1]. Specially the quantum sensitivity of Michelson interferometric gravitational wave detection (GWD) has been discussed by Caves [2]. In Michelson interferometer, the light from an input laser beam splits through a 50-50 beam splitter (BS), bounces back and forth between two end mirrors of interferometer and recombines again at the BS. The intensity at one or both output ports of the interferometer provides informations about the difference between the two displacements of the end mirrors. The quantum mechanical treatment of the system shows that the vacuum fluctuations enter in to the interferometer from the unused port and result in a limit on the optimum power of the input laser, which comes out to be quite large and of no experimental interest. Caves [2] suggested that by squeezing the vacuum, the optimum power of the laser can be reduced considerably. Squeezed states [3] of an electromagnetic field are non-classical states in which the quantum fluctuations in one quadrature can be reduced below the standard quantum limit at the expanse of the increased fluctuations in the other quadrature such that the Heisenberg uncertainty principle remains valid.

It is also interesting to try to use the other non-classical light in the place of squeezed light and study its effect on the better sensitivity of the interferometer in GWD. The different superpositions of coherent states because of their non-classical nature are of our particular interest. Yurke and Stoler [4], have predicted that a coherent state propagating in a dispersive medium evolves into a superposition of two coherent states 180^0 out of phase. Another type of superposition of coherent states, namely, even and odd coherent states was introduced by Dodonov, Malkin, and Man'ko

[5]. Even coherent states are closely related to the squeezed vacuum states because they too are the superposition of even number of photons but with different coefficients. The non-classical properties of Yurke-Stoler coherent states and even and odd coherent states have been discussed in [6]. In Refs.[7] -[10], different theoretical possibilities regarding the generations of even and odd coherent states have been discussed. The properties of even and odd coherent states as a representatives of a set of nonclassical light states have been considered recently by Nieto and Truax [11].

In the following sections we will study the effects of the non-classical light on the optimal power of the input laser for interferometric GWD. The most general analysis of non-classical states in interferometry was done by Yurke, McCall and Klauder [12]. We will following the approach adopted by Ansari et al.[13], in which the noise error can be expressed as a product of two factors with tensorial-like structure, each of the factors being related to the geometry of an interferometer and light states correspondingly.

2 Nonclassical Light

In this section we will briefly discuss the properties of three types of superposition of coherent states, Yurke-Stoler coherent states (YS), even (ECS) and odd (OCS) coherent states.

2.1 Even and Odd Coherent States

The even and odd coherent states may be defined in the form [5]

$$| \beta_{\pm} >= N_{\pm}(| \beta > \pm | -\beta >),$$
(1)

where + sign is for ECS and - sign is for OCS. $| \beta >$ is a coherent state and the normalizing constants $N_{\pm}$ are

$$N_+ = \frac{e^{|\beta|^2/2}}{2\sqrt{\cosh | \beta |^2}},$$

$$N_- = \frac{e^{|\beta|^2/2}}{2\sqrt{\sinh | \beta |^2}}.$$
(2)

Also from Eq.(1), we can define the relations

$$a | \beta_+ > = \beta\sqrt{\tanh | \beta |^2} | \beta_- >,$$

$$a | \beta_- > = \beta\sqrt{\coth | \beta |^2} | \beta_+ > .$$
(3)

With the help of above equations we can easily evaluate the expectation values of first and higher order moments of annihilation and creation operators of even and odd coherent states. For example,

$$< a >_+ =< \beta_+ | a | \beta_+ >= \beta\sqrt{\tanh | \beta |^2} < \beta_+ | \beta_- >= 0,$$
(4)

as even and odd coherent states are orthogonal states. Similarly,

$$
\begin{aligned}
< a^\dagger a >_+ &= |\beta|^2 \tanh |\beta|^2, \\
< a^\dagger a >_- &= |\beta|^2 \coth |\beta|^2, \\
< a^2 >_\pm &= \beta^2, \\
< a^{\dagger 2} >_\pm &= \beta^{*2}.
\end{aligned}
\tag{5}
$$

2.2 Yurke-Stoler Coherent States

Yurke-Stoler (YS) coherent states are defined as [4],[6]

$$
| \beta >_{YS} = \frac{1}{\sqrt{2}} (| \beta > + e^{i\pi/2} | -\beta >).
\tag{6}
$$

In terms of number states these states can be defined as

$$
| \beta >_{YS} = \frac{e^{-|\beta|^2/2}}{\sqrt{2}} \sum_{n=0}^{\infty} \frac{\beta^n}{\sqrt{n!}} (1 + i(-1)^n) | n > .
\tag{7}
$$

The first order moments of YS coherent states are not equal to zero as in the case of ECS or OCS

$$
< a >_{YS} = -i\beta e^{-2|\beta|^2},
\tag{8}
$$

and second order moments are

$$
\begin{aligned}
< a^\dagger a >_{YS} &= |\beta|^2, \\
< a^2 >_{YS} &= \beta^2.
\end{aligned}
\tag{9}
$$

We will use different first and second order moments as given in Eqs.(4-9) in the following section, when we will discuss the important rcle played by nonclassical light for GWD.

3 Michelson Interferometer for GWD

Michelson interferometer is a two arms device at the end of which two mirrors are attached to strings, thus behaving as two pendula. The positions of the mirrors are controlled by the joint action of the restoring force and the radiation pressure [14]. We will suppose that in all process the dissipative and active effects are negligible and the conservation of energy is ensured.

There are two input field modes described by the operators $(a_i, a_i^\dagger)$ at the two ports of the interferometer. At the end mirrors M_i, the fields are defined by $(b_i, b_i^\dagger)$. The output fields at the two ports P_i are described by $(c_i, c_i^\dagger)$. The input fields are related with the fields at the mirrors through the relations

$$
\begin{aligned}
b &= Va, \\
b^\dagger &= a^\dagger V,
\end{aligned}
\tag{10}
$$

where

$$a = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix},$$

$$b = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix},$$

$$a^{\dagger} = \begin{pmatrix} a_1^{\dagger} & a_2^{\dagger} \end{pmatrix},$$

$$b^{\dagger} = \begin{pmatrix} b_1^{\dagger} & b_2^{\dagger} \end{pmatrix}. \tag{11}$$

Also

$$V = \Phi K, \tag{12}$$

with

$$\Phi = \begin{pmatrix} e^{i\phi_1} & 0 \\ 0 & e^{i\phi_2} \end{pmatrix},$$

$$K = \begin{pmatrix} \zeta_1 & \xi_2 \\ \xi_1 & \zeta_2 \end{pmatrix}. \tag{13}$$

In Eq.(13) ζ and ξ are the complex transitivity and reflectivity parameters of the BS arbitrarily oriented for the i-th field mode respectively and ϕC_i is the phase distance between BS and M_i.

The relations between the input field and the output fields at the two interferometric ports are of the form

$$\begin{aligned} c &= Ua, \\ c^{\dagger} &= a^{\dagger} U^{\dagger} \end{aligned} \tag{14}$$

with

$$U = -K^T \Phi^2 K = -V^T V, \tag{15}$$

where – sign in Eq.(15) corresponds to the phase change on reflection at the mirrors. Thus from the above equations we can define the relations between different fields by including all the informations about influence of the BS and the end mirrors M_i.

3.1 Sources of Noise

The accuracy with which the difference in displacement z can be measured is limited by the Heisenberg uncertainty principle. Following [2], we have two sources of errors namely radiation pressure error and photon counting noise. The standard quantum limit for a Michelson interferometer can be obtained by balancing these two sources of error. Radiation pressure error (PR) is due to the pressure exerted by the field on the mirrors and the photon counting noise (PC) is due to the fluctuations in the number of photons in the input field. So,

$$\Delta z = \sqrt{(\Delta z_{RP})^2 + (\Delta z_{PC})^2}, \tag{16}$$

where

$$(\Delta z_{RP})^2 = \sigma_{RP}^2 \left(\frac{\hbar\omega\tau}{mc}\right)^2 ,$$

$$(\Delta Z_{PC})^2 = \sigma_{PC}^2 \left(\frac{\partial(c^\dagger\sigma_3 c)}{\partial(\phi_2 - \phi_1)}\right)^{-2} . \tag{17}$$

Also

$$\sigma_{RP}^2 = \; < (b^\dagger\sigma_3 b)^2 > - < b^\dagger\sigma_3 b >^2 ,$$

$$\sigma_{PC}^2 = \; < (c^\dagger\sigma_3 c)^2 > - < c^\dagger\sigma_3 c >^2 . \tag{18}$$

In Eq.(17), τ is the observation time and m is the mass of the end mirrors. Here we consider that BS is attached to a large mass M $(M >> m)$, which remained fixed during the observation time. By using Eqs.(10-15), we can write

$$\sigma_{RP}^2 = (V^\dagger\sigma_3 V)_{ik}(V^\dagger\sigma_3 V)_{mn}T_{ikmn},$$

$$\sigma_{PC}^2 = (U^\dagger\sigma_3 U)_{ik}(U^\dagger\sigma_3 U)_{mn}T_{ikmn}, \tag{19}$$

with the summation over the repeated indices taken from 1 to 2 and

$$T_{ikmn} = < a_i^\dagger a_k a_m^\dagger a_n > - < a_i^\dagger a_k > < a_m^\dagger a_n > . \tag{20}$$

Eq.(20) allows us to study the use of different field modes from the input port. By using Eqs.(16-20), we can write

$$\Delta z = X_{ikmn}T_{ikmn} \qquad (ikmn = 1, 2), \tag{21}$$

where X_{ikmn} contains the geometrical and physical properties of the interferometer.

If we consider a 50-50 ideally thin BS which introduces a phase difference of $\pi/2$ between the reflected and the transmitted waves, then from Eq.(10) and (13), we can write

$$V^\dagger\sigma_3 V = \begin{pmatrix} o & i \\ -i & o \end{pmatrix} \tag{22}$$

and

$$U^\dagger\sigma_3 U = \begin{pmatrix} -\cos\phi & -\sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix}, \tag{23}$$

where $\phi = \phi_2 - \phi_1$. Also if the interferometer is operated in the dark fringe, then two arms of the interferometer can be adjusted such that $\phi = (2n + 1)\pi/2$. For dark fringe operation we get

$$X_{1212} = X_{2121} = -A^2 + B^2,$$

$$X_{1221} = X_{2112} = A^2 + B^2. \tag{24}$$

Also

$$A = \left(\frac{\hbar\omega\tau}{mc}\right),$$

$$B = \left(\frac{\partial I}{\partial Z}\right)^{-1} \tag{25}$$

and

$$I = <c^\dagger \sigma_3 c>,$$
$$Z = \phi \frac{c}{2\omega}. \tag{26}$$

The variable Z corresponds to the difference between the displacement of two end mirrors with respect to their mean position due to radiation pressure exerted by the input laser.

(i)   The corresponding field contributions can be found from Eq.(20). If we consider that the input field at port P_1 is a coherent light and from the second port is in even or odd coherent states, then the two fields are anticorrelated and the states of these fields can be written as

$$\mid \psi > = \mid \alpha, \beta_\pm > . \tag{27}$$

For the case of even coherent light we can write the coefficients T_{ikmn} as

$$T_{1111} = \alpha^2$$
$$T_{1122} = 0$$
$$T_{1212} = \alpha^2 \mid \beta \mid^2 e^{2i\theta_1}$$
$$T_{1221} = \alpha^2 \mid \beta \mid^2 tanh \mid \beta \mid^2 + \alpha^2$$
$$T_{2112} = \alpha^2 \mid \beta \mid^2 tanh \mid \beta \mid^2 + \mid \beta \mid^2 tanh \mid \beta \mid^2$$
$$T_{2121} = \alpha^2 \mid \beta \mid^2 e^{-2i\theta_1}$$
$$T_{2211} = 0$$
$$T_{2222} = \mid \beta \mid^4 - \mid \beta \mid^4 tanh^2 \mid \beta \mid^2 + \mid \beta \mid^2 tanh \mid \beta \mid^2, \tag{28}$$

where θ_1 is the phase of β and we have consider α to be real. Also for OCS we will get the same expressions as in the above equation except $tanh \mid \beta \mid^2$ should be replace by $coth \mid \beta \mid^2$.

(ii)   For the case of Yurke-Stoler coherent states from the second port and the coherent state from the first port we can define the states as

$$\mid \psi > = \mid \alpha, \beta_{YS} >, \tag{29}$$

and the new expressions for T_{ikmn} are

$$T_{1111} = \alpha^2$$
$$T_{1122} = 0$$
$$T_{1212} = \alpha^2 \mid \beta \mid^2 e^{2i\theta_2}(1 + e^{-4|\beta|^2})$$
$$T_{1221} = \alpha^2 \left[\mid \beta \mid^2 \left(1 - e^{-4|\beta|^2}\right) + 1\right]$$
$$T_{2112} = \mid \beta \mid^2 \left[\alpha^2 \left(1 - e^{-4|\beta|^2}\right) + 1\right]$$
$$T_{2121} = \alpha^2 \mid \beta \mid^2 e^{-2i\theta_2}(1 + e^{-4|\beta|^2})$$
$$T_{2211} = 0$$
$$T_{2222} = \mid \beta \mid^2, \tag{30}$$

where θ_2 is the phase of β in the case of YS coherent states. A comparison of Eqs.(29) and (31) shows the difference between different order correlations between the two types of the input fields from port P_2.

3.2 Optimum Input Laser Power

The general expression for $(\Delta z)^2$ by using Eqs.(21) and (25) becomes

$$(\Delta z)^2 = A^2(T_{1221} + T_{2112} - T_{1212} - T_{2121}) + B^2(T_{1221} + T_{2112} + T_{1212} + T_{2121}). \tag{31}$$

Minimizing the total error with respect to α^2 gives optimal value of α^2 (coherent field intensity from port P_1). In the presence of ordinary vacuum fluctuations from the second port, the optimum intensity of the input laser becomes [2]

$$(\alpha_{opt}^2)^o = \frac{mc^2}{2\hbar\omega^2\tau}. \tag{32}$$

Caves [2] showed that the optimal laser intensity can be reduced considerably if we squeezed the vacuum from the second port. We will analyze the situation when the squeezed vacuum is replaced by the nonclassical light as discussed before.

In the first case, we will study the effect of even and odd coherent states on the optimum value of α^2. Under the condition of $\alpha^2 >>| \beta |^2 \tanh | \beta |^2$, we get

$$(\alpha_{opt}^2)^{ev} = \sqrt{\frac{2 | \beta |^2 \tanh | \beta |^2 + 2 | \beta |^2 \cos 2\theta_1 + 1}{2 | \beta |^2 \tanh | \beta |^2 - 2 | \beta |^2 \cos 2\theta_1 + 1}}(\alpha_{opt}^2)^o, \tag{33}$$

and for OCS

$$(\alpha_{opt}^2)^{od} = \sqrt{\frac{2 | \beta |^2 \coth | \beta |^2 + 2 | \beta |^2 \cos 2\theta_1 + 1}{2 | \beta |^2 \coth | \beta |^2 - 2 | \beta |^2 \cos 2\theta_1 + 1}}(\alpha_{opt}^2)^o. \tag{34}$$

Thus for $\theta_1 = \pi/2$ and under the limit $1 <<| \beta |^2<< \alpha^2$, we get

$$(\alpha_{opt}^2)^{ev} = \frac{(\alpha_{opt}^2)^o}{2 | \beta |}. \tag{35}$$

Eq. (35) allows us an alternative way to reduce the optimum input laser power or to increase the sensitivity of interferometer by using even or odd coherent states from the second port of the interferoeter. As $| \beta |>>1$, from Eq.(35), we predict that the optimum value qf the input laser intensity can be reduced considerably if we apply even or odd coherent state from the second port.

When we apply Yurke-Stoler coherent states and for the choices of $\alpha^2 >>| \beta |^2$ and $\theta_2 = \pi/2$, we get the relation

$$(\alpha_{opt}^2)^{YS} = \sqrt{\frac{-2 | \beta |^2 e^{-4|\beta|^2} + 1}{4 | \beta |^2 + 1}}(\alpha_{opt}^2)^o. \tag{36}$$

Also in the limit of $1 <<| \beta |^2<< \alpha^2$, we will get the same expression as we get in the case of ECS or OCS, i.e.,

$$(\alpha_{opt}^2)^{YS} = \frac{(\alpha_{opt}^2)^o}{2 | \beta |}. \tag{37}$$

Eqs.(35) and (37) show that we get the same expressions for the optimum power of input laser for large $| \beta |$. Thus we predict an important property of the superposition of coherent states that differet superpositions of coherent states may play an important role in reducing the optimum power of input laser. In other words by applying these coherent states, better quantum sensitivity of interferometer can be achieved as compare to no field contribution from the second port.

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