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Noisy bases in Hilbert space: A new class of thermal coherent states and their properties

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Abstract

Coherent mixed states (or thermal coherent states) associated with the displaced harmonic oscillator at finite temperature, are introduced as a "random" (or "thermal" or "noisy") basis in Hilbert space. A resolution of the identity for these states is proved and used to generalise the usual coherent state formalism for the finite temperature case. The Bargmann representation of an operator is introduced and its relation to the P and Q representations, is studied. Generalised P and Q representations for the finite temperature case are also considered and several interesting relations among them are derived.

1. Introduction

Coherent states have played an important role in various areas of physics. They provide a non-orthonormal, over-complete basis in the Hilbert space, which however is very useful in many problems. In spite of the non-orthonormal nature of this basis, the resolution of the identity makes it practically usable in the sense that it can be used for the expansion of an arbitrary state in the coherent state basis.

In a previous publication [1] we have considered a generalisation of the ordinary coherent states into the so-called "coherent mixed states" or "thermal coherent states". They describe displaced harmonic oscillators at finite temperature T; or alternatively, mixtures of coherent states in thermal noise [2]. In contrast to the various types of coherent states considered in the literature which are pure states, our coherent mixed states are, as the name indicates, mixed states in general; and they are pure states only in the special case of zero temperature. They can be considered as a "noisy" or "random" basis in the Hilbert space. We prove that there exists a resolution of the identity for these states, and this makes possible an expansion of an arbitrary state in the coherent mixed state basis. The Q and P representations which are usually defined in terms of ordinary coherent states are generalised within our formalism.

The purpose of this paper is to expand our previous work and express it within the Bargmann representation [3]. This representation makes possible the exploitation of the powerful theory of analytic functions in the complex plane, within a quantum mechanical context. In section 2, we define the coherent mixed states in the Bargmann representation. For example, we derive a transformation which connects the Bargmann representation with the usual x- an p- representations. We also explain how an operator can be expressed in a differential form or in an integral form (i.e. as the kernel of an integral) in the Bargmann formalism. In section 3 we explain how our mixed coherent states can be considered as a "random" or "noisy" basis in the Hilbert space. In section 4 we explain the relation between the Bargmann representation of on operator and its P, Q, W (Wigner) representations. In section 5 we introduce generalised (finite temperature) P and Q representations and examine various relations among them. Known results [4, 5, 6] on P and Q representations are in this section generalised for the finite temperature P and Q representations. We conclude in section 6 with a discussion of our results.

2. Displaced oscillator at finite temperature in the Bargmann representation

We consider the Glauber coherent states

$$|z\rangle = D(z)\,|0\rangle = \exp\left(-\tfrac{1}{2}|z|^2\right)\sum_{N=0}^{\infty} z^N (N!)^{-1/2}\,|N\rangle$$

$$D(z) = \exp[za^+ - z^*a];\ [a,a^+] = I \tag{1}$$

$$\langle z|z'\rangle = \exp\left(-\tfrac{1}{2}|z|^2 - \tfrac{1}{2}|z'|^2 + z^*z'\right)$$

We introduce the Bargmann analytic representation by considering an arbitrary state

$$|f\rangle = \sum_{N=0}^{\infty} f_N|N\rangle = \sum_{N=0}^{\infty} f_N\,(N!)^{-1/2}\,(a^+)^N|0\rangle$$

$$\sum_{N=0}^{\infty}|f_N|^2 = 1 \tag{2}$$

$$\langle f^*| = \left(|f^*\rangle\right)^+ = \sum_{N=0}^{\infty} f_N\langle N|$$

and representing it with the analytical function

$$|f\rangle \longrightarrow f_B(z) = F_B(|f\rangle;z = \exp\left(\tfrac{1}{2}|z|^2\right)\langle z^*|f\rangle = \sum_{N=0}^{\infty} f_N.z^N(N!)^{-1/2} \tag{3}$$

Using the resolution of the identity

$$\int\frac{d^2z}{\pi}\,|z\rangle\langle z| = I;\ d^2z = d(\mathrm{Re}z)\,d\,(\mathrm{Im}z) = \frac{1}{2i}\,dzdz^* \tag{4}$$

we easily prove that the scalar product of two states $|f\rangle$, $|g\rangle$ can be expressed as

$$\langle f^*|g\rangle = \int f_B(z)\, g_B(z^*) \exp(-|z|^2) \frac{d^2z}{\pi} \tag{5}$$

The creation and annihilation operators are represented as

$$a \longrightarrow \frac{d}{dz} \tag{6}$$

$$a^+ \longrightarrow z$$

As an example we consider the coherent states $|A\rangle$ and the number eigenstates $|N\rangle$ which in the Bargmann representation are represented by the analytical functions:

$$|A\rangle \longrightarrow F_B(|A\rangle;z) = \exp\left[-\tfrac{1}{2}|A|^2 + Az\right]$$

$$|N\rangle \longrightarrow F_B(|N\rangle;z) = z^N (N!)^{-1/2} \tag{7}$$

We next introduce transformations that connect the Bargmann representation with the usual position and momentum representations denoted here with the indices x,p correspondingly

$$f_x(z_R) = \pi^{-3/4} \exp\left[-\tfrac{1}{2} z_R^2\right] \int_{\infty}^{\infty} dz_I \exp(-z_I^2)\, f_B(2^{1/2} z) \tag{8}$$

$$f_p(z_I) = \pi^{-3/4} \exp\left[-\tfrac{1}{2} z_I^2\right] \int_{\infty}^{\infty} dz_R \exp(-z_R^2)\, f_B(2^{1/2} z^*) \tag{9}$$

where z_R = Rez and z_I = Imz. The proof is based on equ (3) and the integral representations of the Hermite Polynomials:

$$H_N(x) = 2^N \pi^{-1/2} \int_{\infty}^{\infty} (x + it)^N \exp(-t^2)\, dt \tag{10}$$

An operator θ

$$\theta = \sum \theta_{NM} |N\rangle \langle M| \tag{11}$$

can be represented by the analytic function of two variables:

$$\theta \longrightarrow \theta_B(z_1, z_2^*) = B(\theta; z_1, z_2^*) = \exp\left[\tfrac{1}{2}|z_1|^2 + \tfrac{1}{2}|z_2|^2\right] \langle z_1^*|\theta|z_2^*\rangle$$

$$= \sum_{N,M} \theta_{NM} z_1^{\,N} (z_2^{\,*})^M \left[(N!)(M!) \right]^{-1/2} \tag{12}$$

We refer to this as the B-representation. It provides an "integral" representation of an operator in the Bargmann formalism. The operator is here represented by a kernel of an integral. The action of this operator on the (arbitrary) state $|f\rangle$ of equ (2) can be described by the integral

$$\theta|f\rangle \longrightarrow \int B\,(\theta;z_1,z'^{*})\; f_B\,(z')\; \exp\,(-|z'|^2)\; \frac{d^2z'}{\pi} \tag{13}$$

The B - representation of the product of two operators is given by

$$B\,(\theta_1\theta_2;z_1,z_2^{\,*}) = \int B\,(\theta_1;z_1,z_3^{\,*})\; B\,(\theta_2;z_3,z_2^{\,*})\; \exp\,(\,-|z_3|^2)\; \frac{d^2z_3}{\pi} \tag{14}$$

The creation and annihilation operators (already given in equ (6)) are here represented by the functions:

$$\begin{aligned} B\,(a;z_1,z_2*) &= z_2^{\,*} \exp\,(z_1z_2^{\,*}) \\ B\,(a^+;z_1,z_2^{\,*}) &= z_1 \exp\,(z_1z_2^{\,*}) \end{aligned} \tag{15}$$

The representations of equ (15) are consistent with these of equ (6). Indeed

$$\begin{aligned} &\int z_2^{\,*} \exp\,(z_1z_2^{\,*})\; \exp\,(-|z_2|^2)\; f\,(z_2)\; \frac{d^2z_2}{\pi} = \frac{d}{dz_1}\; f_B(z_1) \\ &\int z_1 \exp\,(z_1z_2^{\,*})\; \exp\,(-|z_2|^2)\; f\,(z_2)\; \frac{d^2z_2}{\pi} = z_1 f_B(z_1) \end{aligned} \tag{16}$$

Both equations can be proved using the fact that f(z) is a holomorphic function therefore

$$\frac{1}{2\pi i} \int \frac{f(z_2)}{z_2 - z_1}\, dz_2 = f_B(z_1) \tag{17}$$

Where the integral is taken around some suitable contour enclosing the point z_1 in an anticlockwise direction. Note that the trace of an operator can be expressed as

$$Tr(\theta) = \int \frac{d^2z}{\pi}\; \exp\,(-|z|^2)\; B\,(\theta;z,z^{*}) \tag{18}$$

and that the trace of the product of two operators can be expressed as:

$$\mathrm{Tr}\,(\Theta_1\Theta_2) = \int B\,(\Theta_1;z_1,z_2^*)\; B\,(\Theta_2;z_2,z_1^*)\; \exp\,(-|z_1|^2-|z_2|^2)\; \frac{d^2z_1}{\pi}\;\frac{d^2z_2}{\pi} \tag{19}$$

For later use we mention that the unit operator I is represented by the function

$$B\,(I;z_1,z_2^*) = \exp\,(z_1z_2^*) \tag{20}$$

and that the displacement operator is represented by the function

$$B\,(D(A);z_1,z_2^*) = \exp\left[-\tfrac{1}{2}|A|^2 - A^*z_2^* + z_1A + z_1z_2^*\right] \tag{21}$$

A density matrix ρ with eigenvalues p_N, eigenstates $|e^{(N)}\rangle$ and matrix elements with respect to the number eigenstates p_{NM}

$$\begin{aligned} &\rho = \sum p_N\; |e^{(N)}\rangle\langle e^{(N)}| = \sum p_{NM}|N\rangle\langle M| \\ &0 \le p_N \le 1 \\ &\sum p_N = 1 \end{aligned} \tag{22}$$

can be represented by the analytical function of two variables (equ (12)):

$$\rho_B(z_1,z_2^*) = \exp\left[\tfrac{1}{2}|z_1|^2 + \tfrac{1}{2}|z_2|^2\right] \langle z_1^*|\rho|z_2^*\rangle \tag{23}$$

$$= \sum p_N\; e^{(N)}{}_B\,(z_1)\left[e^{(N)}{}_B\,(z_2)\right]^* = \sum p_{NM}\;(z_1)^N(z_2^*)^M\left[(N!)(M!)\right]^{-\frac{1}{2}}$$

The displaced oscillator of finite temperature is represented by the density matrix ($k_B = h = \omega = 1$):

$$\rho(A;\beta) = D(A)\;\exp\,[-\beta a^+a]D^+\;(A)\;(1-e^{-\beta})$$

$$= \exp\left[-\beta(a^+-A^*)(a-A)\right]\;(1-e^{-\beta})$$

where β is the inverse temperature. We can easily prove that in limit $\beta \longrightarrow \infty$ $(T \longrightarrow o)$

$$\lim_{\beta\longrightarrow\infty} \rho\;(A;\beta) = |A\rangle\langle A| \tag{25}$$

The Bargmann representation of this density matrix can be found from equ (23). We prove:

$$\rho_B(A;\beta;z_1,z_2^*) = (1-e^{-\beta})$$

$$\times \exp\left[(1 - e^{-\beta})\ (A^* z_2^* + z_1 A - |A|^2) + z_1 z_2^* e^{-\beta}\right] \tag{26}$$

3. <u>Mixed states as "noisy bases" in Hilbert space:</u>

The simplest type of basis in a Hilbert space is the orthonormal $\{|u_N\rangle\}$ for which

$$\pi_N = |u_N\rangle \langle u_N| \tag{27}$$

$$\pi_N \pi_M = \delta_{NM} \pi_N \tag{28}$$

$$\sum \pi_N = 1 \tag{29}$$

The π_N are orthonormal projection operators. Equ (29) is important for two reasons. First, it is a proof of the completeness of the basis. And second, it can be used to expand an arbitrary state $|s\rangle$ as:

$$|s\rangle = \sum s_N |u_N\rangle \tag{30}$$

$$s_N = \langle u_N | s\rangle \tag{31}$$

This second point is very important from a practical point of view, because for some bases we might have an abstract proof of completeness, but not a resolution of the identity like (29); and then we do not know how to use this basis, in practice.

Another type of basis is provided by the coherent states, which is overcomplete and non-orthonormal:

$$\pi(A) = |A\rangle \langle A|| \tag{32}$$

$$\pi^2(A) = \pi(A) \tag{33}$$

$$\int \frac{d^2A}{\pi}\ \pi(A) = 1 \tag{34}$$

The $\pi(A)$ are still projection operators (describing pure states); but in this case they are non-orthonormal. And yet, the resolution of the identity (34) allows us to express an arbitrary state $|s\rangle$ as

$$|s\rangle = \int \frac{d^2A}{\pi}\ s(A)\ |A\rangle \tag{35}$$

$$s(A) = \langle A|s\rangle \tag{36}$$

Our proposal in this and in our previous work [1] is to use a set of mixed states as a basis in a Hilbert space. A mixed state described by a density matrix ρ with eigenstates $|e_N>$ and eigenvalues p_N

$$\rho = \sum_{N=0}^{\infty} p_N |e_N> <e_N|, \tag{37}$$

$$\sum_{N=0}^{\infty} p_N = 1; \quad 0 \le p_N \le 1, \tag{38}$$

represents a set of states $\{|e_N>\}$ with a probability distribution $\{p_N\}$. Therefore the idea of using mixed states as a basis replaces the "fixed" veetors which are usually used in a basis with "noisy vectors" (i.e. "random vectors").

The basis used in this paper is the set of all density matrices $\rho(A;\beta)$ of equ (24) for all complex values A and fixed (but arbitrary) value of β. In this case equ (37) becomes

$$\rho\ (A;\beta) = \sum p_N\ (\beta)\ |N;A> <N;A| \tag{39}$$

$$|N;A> \equiv D(A)|N> \tag{40}$$

$$p_N\ (\beta) = \left[1 - \exp\ (-\beta)\right] \exp\ (-\beta N) \tag{41}$$

We have proved in [1] that the $\rho(A;\beta)$ obey the resolution of the identity

$$\int \frac{d^2A}{\pi}\ \rho\ (A;\beta) = I \tag{42}$$

This is a significant relation for our purposes because it can be used to expand an arbitrary state $|s>$ as

$$|s> = \int \frac{d^2A}{\pi}\ \rho\ (A,\beta)|s> \tag{43}$$

The density matrices $\rho(A;\beta)$ have been expressed in the Bargmann representation in equ (26). The resolution of the identity (42) can be written in the Bargmann representation as:

$$\int \rho_B\ (A;\beta;z_1,z_2^*) \frac{d^2A}{\pi} = \exp\ (z_1 z_2^*) \tag{44}$$

where as explained in equ (18) the right hand side is the unit operator in this formalism.

4. B- representation and its relationship to P, Q, W representations:

There has been a lot of discussion in the literature (reviewed in [4, 5, 6]) on the Q, P and Weyl representations and the relationships among them. The purpose of this section is to examine the relationship of the

B-representation with the others; and also to present some basic material which will be used in the next section, where all these quantities will be generalised into their finite temperature equivalents.

The Q and P representations of an operator θ are defined as

$$Q\ (\theta;A) = \langle A|\theta|A\rangle \tag{45}$$

$$\theta = \int P(\theta;A)\pi(A)\frac{d^2A}{\pi} \tag{46}$$

where the $\pi(A)$ have been defined in (32).

The Q and P-representations are related with the Bargmann representation of equ (12) as follows:

$$Q(\theta;z) = \left[\exp\ (-|z|^2)B(\theta;z^*,z)\right] \tag{47}$$

$$P(\theta;A) = \exp\ (|A|^2)\int\frac{d^2z}{\pi}\ B(\theta;\ -\ z^*,z)\ \exp\ (Az^*\ -\ A^*z)\Bigg] \tag{48}$$

Equ (47) can be easily proved with the use of the definition (12); equ (48) is similar to the result given by Mehta [7].

We introduce the notation $\tilde{f}(w)$ for the two-dimensional Fourier transform of the function $f(z)$ defined as:

$$\tilde{f}(w) = \int d^2z\ \exp\left[i(w_Rz_R + w_Iz_I)\right]f(z) \tag{49}$$

where the indices R,I indicate the real and imaginary parts correspondingly. We can prove the following relations that express the Bargmann representation in terms of the P and Q representations:

$$B(\theta;z_1,z_2^*) = \int P\ (\theta;z)\ \exp\ (zz_1 + z^*z_2^*\ -\ |z|^2)\ \frac{d^2z}{\pi} \tag{50}$$

$$B(\theta;z_1,z_2^*) = e^{z_1z_2^*}\int\frac{d^2w}{(2\pi)^2}\tilde{Q}\ (\theta;w)\ \exp\left[-\ i\tfrac{1}{2}(wz_1 + w^*z_2^*)\right] \tag{51}$$

We next use the relation

$$\int\frac{d^2A}{\pi}\ f\ (A^*,A)\ k\ \exp\ (-k|A\ -\ B|^2) = \exp\left(\frac{1}{4k}\ \Delta_B\right)\ f\ (B^*,B)$$

$$\Delta_B = 4\ \frac{\partial^2}{\partial B\partial B^*}\ ,k>0 \tag{52}$$

which can be proved with a Fourier transform of both sides. Δ_B is the Laplacian in a two-dimensional space.

We next consider the Wigner function corresponding to an operator Θ.

$$\Theta_W (A) = \mathrm{Tr} \left[\Theta D(A)\right] \tag{53}$$

It is known [4] that

$$\Theta = \int \frac{d^2A}{\pi} \mathrm{Tr} \left[D^+(A)\Theta\right] D(A) = \int \frac{d^2A}{\pi} \Theta_W (-A) D (A) \tag{54}$$

For a density matrix ρ the definition (53) leads to the more familiar expression

$$\rho_W (A) = \mathrm{Tr} \left[\rho D(A)\right] = \int dx \langle -2^{-\frac{1}{2}} A_R + x \,|\rho|\, 2^{-\frac{1}{2}} A_R + x\rangle \exp\left[i\, x\, (2^{\frac{1}{2}} A_I)\right] \tag{55}$$

It is convenient for later purposes to make a trivial change of variables

from A to $\frac{1}{2}iw$ and denote the resulting function by $\tilde{W}(\Theta;w)$

$$\tilde{W} (\Theta;w) = \pi \mathrm{Tr} \left[\Theta D(\tfrac{1}{2}iw)\right] = \pi\Theta_W(\tfrac{1}{2}iw) \tag{56}$$

We shall also use its Fourier transform $W(z;\Theta)$ defined as the inverse of the transform given in equ (36). Using equs (54), (21) we prove:

$$B(\Theta;z_1,z_2^*) = \int \frac{d^2A}{\pi^2} \tilde{W} (\Theta;2iA) \exp \left[-\tfrac{1}{2}|A|^2 + Az_1 - A^*z_2^* + z_1z_2^*\right] \tag{57}$$

Using equs (19), (21), (56) we prove the inverse of this transform:

$$\tilde{W} (\Theta;2iA) = \frac{1}{\pi} \int d^2z_1\, d^2z_2\, B (\Theta;z_1,z_2^*)$$

$$\exp \left[-\tfrac{1}{2}|A|^2 - |z_1|^2 - |z_2|^2 - A^*z^*_1 + z_2A + z_2z_1^*\right] \tag{58}$$

5. Generalized P and Q representations for finite temperature

The formalism of P and Q representations is based on coherent states. Although they form an overcomplete basis, it is the fact that a resolution of the identity (equ(4)) is available, that makes them practically usable. The density matrices $\rho(A;\beta)$ provide a generalization of the coherent states and they also obey the resolution of the identity (42). It seems therefore natural to define generalized P and Q representations based on $\rho(A;B)$. More specifically we introduce:

$$Q\ (\Theta;A;\beta) = \mathrm{Tr}\left[\rho(A;\beta)\Theta\right] \tag{59}$$

$$\Theta = \int\frac{d^2A}{\pi}\ \rho\ (A;\beta)\ P\ (\Theta;A;\beta) \tag{60}$$

It is clear from equ (25) that in the limit T——>0 (β——>∞) they reduce to the ordinary Q and P representations.

From equs (59), (60) we easily get

$$Q\ (\Theta;A;\beta) = \int\frac{d^2B}{\pi}\ P(\Theta,B;\beta')\ \mathrm{Tr}\left[\rho(B,\beta')\ \rho\ (A,\beta)\right] \tag{61}$$

We next show

$$\mathrm{Tr}\left[\rho(B,\beta')\ \rho\ (A,\beta)\right] = 2\left[g(\beta,\beta')\right]^{-1}\exp\left[-2\ \frac{1}{g(\beta,\beta')}\ |A-B|^2\right] \tag{62}$$

where

$$g(\beta,\beta') = \frac{\sinh\ [\frac{1}{2}\ (\beta + \beta')]}{\sinh\ (\frac{1}{2}\ \beta)\ \sinh\ (\frac{1}{2}\ \beta')} \tag{63}$$

Combining equs (61), (62), (63) and taking into account equ (52) we get:

$$Q\ (\Theta;z;\beta) = \exp\left[\frac{1}{8}\ g\ (\beta,\beta')\ \Delta_z\right]\ P\ (\Theta;z;\beta') \tag{64}$$

Fourier transform of this equation gives

$$\tilde{Q}\ (\Theta;w;\beta) = \exp\left[-\frac{1}{8}\ g\ (\beta,\beta')\ |w|^2\right]\ \tilde{P}\ (\Theta;w;\beta') \tag{65}$$

In the special case $\beta = \beta'$ equs (63), (64), (65) give

$$g\ (\beta,\beta) = 2\ \coth\ (\tfrac{1}{2}\beta) \tag{66}$$

$$Q\ (\Theta;z;\beta) = \exp\left[\tfrac{1}{4}\ \coth\ (\tfrac{1}{2}\beta)\ \Delta_z\right]\ P\ (\Theta;z;\beta) \tag{67}$$

$$\tilde{Q}\ (\Theta;w;\beta) = \exp\left[-\tfrac{1}{4}\ \coth\ (\tfrac{1}{2}\beta)\ |w|^2\right]\ \tilde{P}\ (\Theta;w;\beta) \tag{68}$$

In the zero temperature limit

$$\lim\ g\ (\beta,\beta) = 2 \tag{69}$$

$\beta \longrightarrow \infty$
and equs (67), (68) reduce to

$$Q\ (\theta;z) = \exp\ (\tfrac{1}{4}\ \Delta_z)\ P\ (\theta;z) \tag{70}$$

$$\tilde{Q}\ (\theta;w) = \exp\ (-\tfrac{1}{4}\ |w|^2)\ \tilde{P}\ (\theta;w) \tag{71}$$

Equs (70), (71) are known in the literature [4, 5, 6]. Our contribution is to generalize them into equs (64), (65).

We next relate the P and Q representations to the Wigner function introduced in the previous section. Using equs (54), (55), (59), (60) we prove

$$Q\ (\theta;A;\beta) = \int \frac{d^2B}{(2\pi)^2}\ {}_2\tilde{W}\ (\theta;B)\ \exp\left[-\frac{1}{8}\ |B|^2\ \coth\ (\tfrac{1}{2}\beta)\right]$$

$$\times \exp\left[-i(A_RB_R + A_IB_I)\right] = \exp\left[\frac{1}{8}\ \coth\ (\tfrac{1}{2}\beta)\Delta_A\right]\ W\ (\theta;A) \tag{72}$$

$$P\ (\theta;A;\beta) = \int \frac{d^2B}{(2\pi)^2}\ {}_2\tilde{W}\ (\theta;B)\ \exp\left[\frac{1}{8}\ |B|^2\ \coth\ (\tfrac{1}{2}\beta)\right] \times$$

$$\times \exp\left[-\ i(A_RB_R + A_1B_1)\right] = \exp\left[-\ \frac{1}{8}\ \coth\ (\tfrac{1}{2}\beta)\Delta_A\right]\ W\ (\theta;A) \tag{73}$$

Note that from equs (72), (73) we can derive equ (67). The expressions (72), (73) are identical, apart from a minus sign. In this sense the Q-representation can be considered as the analytic continuation of the P-representation at "negative temperatures". In the zero temperature limit ($\beta \longrightarrow \infty$) the above equations reduce to

$$Q\ (\theta;A) = \exp\left[\frac{1}{8}\ \Delta_A\right]\ W\ (\theta;A) \tag{74}$$

$$P\ (\theta;A) = \exp\left[-\ \frac{1}{8}\ \Delta_A\right]\ W\ (\theta;A) \tag{75}$$

6. Conclusions

Generalizations of the original coherent states are usually based on replacing the Weyl group with another one (e.g. SU(2), SU(1,1) etc.). All these coherent states are pure states. In this paper and in ref [1] we have studied coherent mixed states associated with the displaced oscillator at finite temperature. We have shown that these states can be viewed as consisting a "random" (or "noisy" or "thermal") basis in the Hilbert Space. The fact that we were able to prove a resolution of the identity for these states, makes them practically usable.

All the calculations in this paper have been presented in the Bargmann representation. Relations between the Bargmann and the P, Q representations of an operator have been studied. A generalization of the P and Q representations for the finite temperature case, has been proposed and various relations among them have been studied.

From a practical point of view our coherent mixed states can be used for the description of coherent signals in thermal noise. There is a lot of activity in this area [8] and our work provides theoretical support to such studies.

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