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ON PARASUPERSYMMETRIC OSCILLATORS AND RELATIVISTIC VECTOR MESONS IN CONSTANT MAGNETIC FIELDS

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Abstract

Johnson-Lippmann considerations on oscillators and their connection with the minimal coupling schemes are visited in order to introduce a new Sakata-Taketani equation describing vector mesons in interaction with a constant magnetic field. This new proposal, based on a specific parasupersymmetric oscillatorlike system, is characterized by real energies as opposed to previously pointed out relativistic equations corresponding to this interacting context.

1. Relativistic descriptions of free vector mesons

Free vector mesons can be described through many (well known) equations, f.i.

- the KEMMER equation[1]

$$\left(\beta^{\mu} p_{\mu} - 1\right)\psi = 0$$

where the (10×10) matrices β_{μ} generate the Kemmer algebra

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$$\beta_\mu \beta_\nu \beta_\lambda + \beta_\lambda \beta_\nu \beta_\mu = g_{\mu\nu} \beta_\lambda + g_{\lambda\nu} \beta_\mu .$$

- the SAKATA-TAKETANI equation[2]

$$i \frac{\partial}{\partial t} \varphi = \left[(I \otimes \sigma_2) + \frac{\vec{p}^2}{2} I \otimes (\sigma_2 + i \sigma_1) - i S_j S_l p_j p_l \otimes \sigma_1 \right] \varphi$$

$$\equiv H_{ST} \varphi$$

where the (6×6) matrices are direct products of $D^{(1)}$ and Pauli matrices. Notice that, in the two above equations, we take as units the rest mass, the velocity of light and the Dirac constant. Our choice is also to use the metric tensor

$$G = \{ g^{\mu\nu} \mid g^{00} = -g^{ii} = 1 \} .$$

The Kemmer equation reduces to the Sakata-Taketani one when one considers the (six) physical components, only. Namely, the Hamiltonian form of the equation (1.1) together with the initial condition write

$$i \frac{\partial}{\partial t} \psi = \left(\left[\beta_0 , \beta_j \right] p_j + \beta_0 \right) \psi \equiv H_\kappa \psi , \tag{1.3a}$$

$$\left(H_\kappa \beta_0 - 1 \right) \psi = 0 . \tag{1.3b}$$

One can then shows that, through the action of the transformation $S = 1 + \beta_j \beta_0^2 p_j$, the above system becomes

$$\left(\beta_0^2 - 1 \right) \psi' = 0 , \tag{1.4a}$$

$$i \frac{\partial}{\partial t} \psi' = H_{ST} \psi' \tag{1.4b}$$

if

$$\psi' = S \psi = \begin{pmatrix} \varphi \\ 0 \end{pmatrix} . \tag{1.5}$$

2. Relativistic descriptions for vector mesons interacting with constant magnetic fields

The corresponding equations hold for vector mesons interacting with constant magnetic fields directed along the x_3-axis, i.e.

- in the KEMMER case [3]

$$\left(\beta^{\mu}\pi_{\mu} - 1 + (1-\beta_5^2)\, e\, B\, S_3\right)\psi = 0 \tag{2.1}$$

where

$$\pi_{\mu} = p_{\mu} - e\, A_{\mu}\,, \tag{2.2}$$

$$A_0 = 0\,,\quad A_1 = -\frac{B}{2}x_2\,,\quad A_2 = \frac{B}{2}x_1\,,\quad A_3 = 0\,, \tag{2.3}$$

$$\beta_5 = \frac{i}{4}\varepsilon_{\mu\nu\rho\sigma}\beta^{\mu}\beta^{\nu}\beta^{\rho}\beta^{\sigma}\,,\quad \varepsilon_{0123} = 1\,, \tag{2.4}$$

$$S_3 = i\left[\beta_1\,,\beta_2\right]; \tag{2.5}$$

- in the SAKATA-TAKETANI case [4]

$$i\frac{\partial}{\partial t}\varphi = \left[(I \otimes \sigma_2) + \frac{\vec{\pi}^2}{2} I \otimes (\sigma_2 + i\,\sigma_1) - i\, S_j\, S_l\, \pi_j\, \pi_l \otimes \sigma_1 + e\, B\, S_3 \otimes \sigma_2\right]\varphi\,. \tag{2.6}$$

The eigenvalues E corresponding to the physical components write [3,4] in both cases

$$E^2 = 1 + e\,B\,(n+\frac{1}{2}) + 2\,e\,B\,S\,,\quad S = 0\,,\pm 1\,,\quad n = 0\,,1\,,2\,,\cdots\,, \tag{2.7}$$

if we limit ourselves to the so-called perpendicular part (i.e. in the plane $(x_1\,,x_2)$) . So, for the particular values $n = 0$ and $S = -1$, we obtain

$$E^2 = 1 - e\,B \tag{2.8}$$

which could, for sufficiently large magnetic fields, lead to complex energies. This is an old problem [5] and we propose to solve it by investigating a very recent tool : the so-called "parasupersymmetric oscillators"

3. Parasupersymmetry and the corresponding new Sakata-Taketani equation

The nonrelativistic limit corresponding to the interacting Sakata-Taketani or Kemmer Hamiltonians (2.1) and (2.6) is

$$H_{NR} = \frac{1}{2}\left(\pi_1^2 + \pi_2^2\right) + e\,B\,S_3 . \tag{3.1}$$

Taking

$$\omega = e\,B , \tag{3.2a}$$

$$a = \frac{1}{\sqrt{2\,e\,B}}\left(\pi_1 + i\,\pi_2\right) \tag{3.2b}$$

$$a^\dagger = \frac{1}{\sqrt{2\,e\,B}}\left(\pi_1 - i\,\pi_2\right), \tag{3.2c}$$

we get

$$H_{NR} = \frac{\omega}{2}\{a\,,a^\dagger\} + \omega\,S_3 = \frac{\omega}{2}\{a\,,a^\dagger\} + \omega\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \tag{3.3}$$

and

$$E_{NR} = \omega\left(n + \frac{1}{2}\right) + \omega\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \tag{3.4}$$

These are the RUBAKOV-SPIRIDONOV parasupersymmetric Hamiltonian and spectrum for an oscillatorlike interaction[6]. A specific feature of this parasupersymmetric model is the existence of negative eigenvalues. This evidently leads to complex relativistic energies for sufficiently large magnetic fields and confirms the Tsai results[5].

We propose here to eliminate this defect by using another parasupersymmetric model : the BECKERS-DEBERGH parasupersymmetric oscillator [7] characterized by positive energies, only. More precisely, the BECKERS-DEBERGH spectrum corresponds to

$$E_{NR} = \omega(n+\frac{1}{2}) + \frac{\omega}{2}\begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \tag{3.5}$$

Thus, the remaining point is to construct a Sakata-Taketani Hamiltonian whose nonrelativistic limit would lead to such a spectrum. We take [8]

$$\begin{aligned} H_{ST} = &(I \otimes \sigma_2) + \frac{\pi_1^2+\pi_2^2}{2} I \otimes (\sigma_2 - i\sigma_1) + \frac{i}{2}(\pi_1^2+\pi_2^2) S_3^2 \otimes \sigma_1 \\ &-(\pi_1^2+\pi_2^2)\lambda + (\Pi_1^2+\Pi_2^2)\lambda - \frac{i}{2}(\pi_1\Pi_1 - \pi_2\Pi_2)(S_1^2 - S_2^2) \otimes \sigma_1 \\ &-\frac{i}{2}(\pi_1\Pi_2 - \pi_2\Pi_1)\{S_1, S_2\} \otimes \sigma_1 + eB\eta, \end{aligned} \tag{3.6}$$

where λ, η are undetermined and

$$\Pi_a = p_a + eA_a, \quad a = 1, 2, \tag{3.7}$$

In order to solve our problem, we have to impose different conditions like

$$\{\lambda, (S_1^2 - S_2^2) \otimes \sigma_1\} = \{\lambda, \{S_1, S_2\} \otimes \sigma_1\} = 0, \tag{3.8a}$$

$$-\frac{i}{2}\{(S_1^2 - S_2^2) \otimes \sigma_1, \eta\} + 2(\{S_1, S_2\} \otimes \sigma_1)\lambda - \frac{i}{2}\{S_1, S_2\} \otimes \sigma_3 = 0, \tag{3.8b}$$

$$-\frac{i}{2}\{\{S_1, S_2\} \otimes \sigma_1, \eta\} - 2((S_1^2 - S_2^2) \otimes \sigma_1)\lambda + \frac{i}{2}(S_1^2 - S_2^2) \otimes \sigma_3 = 0, \tag{3.8c}$$

in order to the eliminate terms like $(\pi_1^2+\pi_2^2)^2, \cdots$.

We then obtain

$$\lambda = \begin{pmatrix} a_1 & 0 & 0 & -\frac{i}{4} & a_3 & 0 \\ 0 & a_1 & 0 & -a_3 & -\frac{i}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & a_2 \\ \frac{i}{4} & a_3 & 0 & -a_1 & 0 & 0 \\ -a_3 & \frac{i}{4} & 0 & 0 & -a_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\eta = \begin{pmatrix} \alpha_1 & -2ia_1 & 0 & \alpha_4 & \alpha_3 & 0 \\ 2ia_1 & \alpha_1 & 0 & -\alpha_3 & \alpha_4 & 0 \\ 0 & 0 & \alpha_5 & 0 & 0 & \alpha_2 \\ 4ia_3-\alpha_4 & \alpha_3 & 0 & -\alpha_1 & 2ia_1 & 0 \\ -\alpha_3 & 4ia_3-\alpha_4 & 0 & -2ia_1 & -\alpha_1 & 0 \\ 0 & 0 & \alpha_6 & 0 & 0 & -\alpha_5 \end{pmatrix},$$

together with constraints like

$$a_1^2 - a_3^2 = -\frac{1}{16}, \quad a_3\alpha_3 = a_1\alpha_1, \quad \cdots .$$

Taking now

$$a_1 = 0, \quad a_2 = -\frac{i}{2}, \quad a_3 = \frac{1}{4},$$

$$\alpha_1 = \frac{i}{\omega}\sqrt{3}, \quad \alpha_2 = i, \quad \alpha_3 = 0, \quad \alpha_4 = -\frac{i}{\omega} + \frac{i}{2}, \quad \alpha_5 = \frac{i}{\omega}, \quad \alpha_6 = \frac{i}{\omega},$$

we finally have

$$E_{NR} = \omega\left(n' + \frac{1}{2}\right) + \frac{\omega}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \equiv E_{NR}^{B.D.} \tag{3.9a}$$

with

$$n' \equiv n_1 + n_2 . \tag{3.9b}$$

Of course, in these developments, we have exploited Johnson-Lippmann considerations[9] relating the motion in a constant magnetic field with oscillatorlike interactions and implying in particular that the eigenvalues of the operators $\frac{1}{2}(\pi_1^2+\pi_2^2)$ and $\frac{1}{2}(\Pi_1^2+\Pi_2^2)$ are $\omega(n_1+\frac{1}{2})$ and $\omega(n_2+\frac{1}{2})$, respectively. As a result, a Sakata-Taketani type Hamiltonian avoiding the complexity of the energies is

$$
\begin{aligned}
H_{ST} = {} & (I \otimes \sigma_2) + i(I \otimes \sigma_3) + i(\sqrt{3}-1)S_3^2 \otimes \sigma_3 \\
& + \frac{i}{2} I \otimes (\sigma_1 - i\sigma_2) + \frac{i}{2} S_3^2 \otimes (\sigma_1 + i\sigma_2) \\
& + \frac{1}{4}(\pi_1^2+\pi_2^2)\big(-i S_3 \otimes \sigma_1 + i S_3^2 \otimes \sigma_1 + I \otimes (\sigma_2 - i\sigma_1)\big) \\
& + \frac{1}{4}(\Pi_1^2+\Pi_2^2)\big(i S_3 \otimes \sigma_1 + i S_3^2 \otimes \sigma_1 + I \otimes (\sigma_2 - i\sigma_1)\big) \\
& - \frac{i}{2}(\pi_1\Pi_1 - \pi_2\Pi_2)(S_1^2 - S_2^2) \otimes \sigma_1 - \frac{i}{2}(\pi_1\Pi_2 + \pi_2\Pi_1)\{S_1, S_2\} \otimes \sigma_1 \\
& + eB\Big(\frac{i}{2} I \otimes (\sigma_1 + i\sigma_2) + \frac{1}{2} S_3^2 \otimes \sigma_2\Big).
\end{aligned}
\tag{3.10}
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