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Shannon-Wehrl entropy for cosmological and black-hole squeezing

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Abstract

We discuss the Shannon-Wehrl entropy within the squeezing vocabulary for the cosmological and black hole particle production.

Models and concepts of quantum optics have been applied to quantum cosmology (cosmological particle production) already in the seventies [1]. More recently Grishchuk and Sidorov [GS] [2] used the formalism of squeezed states to discuss the spectrum of relic gravitons from inflation, the spectrum of primordial density perturbations, and even the Hawking radiation of Schwarzschild black holes. Apparently there is no new physics entailed [3]. However, the squeezing language may well be more effective in characterizing those physical processes which are of basic theoretical interest. Therefore many authors started to use this language in the cosmological context.

Here we apply the Shannon-Wehrl entropy (S_{sw}) [4] to the squeezing approach of [GS] [5].

In quantum optics S_{sw} is known as an important parameter which is employed to distinguish among various types of coherent states, measuring the relative degree of squeezing with respect to pure coherent states for which $S_{sw} = 1$ is minimum [6]. It is defined as follows

$$S_{sw} = -\frac{1}{\pi}\int d^2\alpha Q(\alpha)\ln Q(\alpha) \tag{1}$$

where $Q(\alpha)$ is the Q representation of the density operator satisfying the normalization condition $\frac{1}{\pi}\int d^2\alpha Q(\alpha) = 1$.

The calculation of S_{sw} for various types of states is not difficult [7]. Here we quote two results of which we shall make use in the following. For the one-mode squeezed states

$$S_{sw} = 1 + \frac{1}{2}\ln(\sinh^2 r - |\epsilon|^2) \tag{2}$$

where ϵ is the coherent percent component of the squeezed state. $\epsilon = 0$ means the squeezed vacuum state. For the thermal states the S_{sw} parameter may be written

$$S_{sw} = 1 - \ln(1 - \xi) \tag{3}$$

where ξ is the inverse of the Boltzmann modal factor $\xi = \exp(-\beta\hbar\omega)$.

Let us pass now to the squeezing approach of [GS]. The main idea is that gravitons created from zero point fluctuations of an initial vacuum cosmological state are at present in an one-mode squeezed quantum state as the result of the parametric amplification due to the interaction with the variable gravitational background. The squeezing coefficient r is a function of the cosmological evolution. Most authors [8] use an expansion in three stages: inflationary (i), radiation-dominated (r), and matter-dominated (m), with the transitions between stages considered in the 'sudden' approximation in which the kinematic effects of the transitions are neglected. Thus the Universe remains in the same quantum state as before transitions [9], and only a redistribution (squeezing) of the quasiparticles takes place. The parametric amplification occurs mainly at the inflationary stage, where the variation of the background is most rapid. The squeezing coefficient can be obtained from the ratios of the dimensionless scale factors at the returning (either at r-stage or m-stage) and exit of a given mode out of the Hubble sphere at the i-stage, as follows

$$\exp r = a(\eta_{ret})/a(\eta_{ex}) \tag{3}$$

According to [GS] r increased from $r \sim 1$ up to $r \sim 100$ for waves with present-day frequencies ranging from $\nu \approx 10^{-8} - 10^{-16}$Hz, which were amplified at the inflationary stage only. For waves in the range $\nu \approx 10^{-16} - 10^{-18}$Hz, the squeezing parameter may reach a value of 120 due to the additional amplification at the matter-dominated transition. We see that cosmological squeezing is about two orders of magnitude bigger than ordinary laboratory squeezing. This is indicative of the huge mean number of quasiparticles in every mode. Making use of the numerical values of the cosmological squeezing coefficient we can plot the S_{sw} entropic parameter for the one-mode squeezed graviton states according to Eq.(2).

Fig. 1

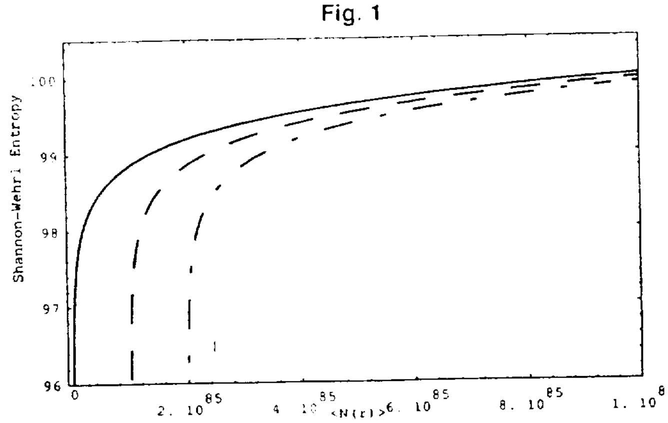

Fig. 1: Shannon-Wehrl entropy for graviton squeezed states with different coherent components ϵ (full line $\epsilon = 0\%$, slash line $\epsilon = 10\%$, slash-dot line $\epsilon = 20\%$).

The non-zero coherent component we allowed for would correspond to possible deviations of the initial quantum state of gravitons from the vacuum state [10].

In the case of Schwarzschild black holes a two-mode squeezing comes into play for any type of radiation detected at asymptotic infinity. However due to the special causal structure of the black hole spacetime, the asymptotic observations are limited to one mode only. Under such conditions the detected states turn into thermal ones. Actually, Hawking radiation is a distorted blackbody radiation, but we can consider it as an effective thermal one [11]. Therefore we plotted S_{sw} according to Eq.(3), with γ in the effective Boltzmann factor defined by

$$\frac{1}{\exp(\gamma)-1}=\frac{\Gamma(\omega)}{\exp(\beta_h \hbar\omega)-1} \tag{4}$$

where β_h is the horizon inverse temperature parameter, and $\Gamma(\omega)$ is the penetration factor of the curvature and angular momentum barrier around the black hole.

Fig. 2

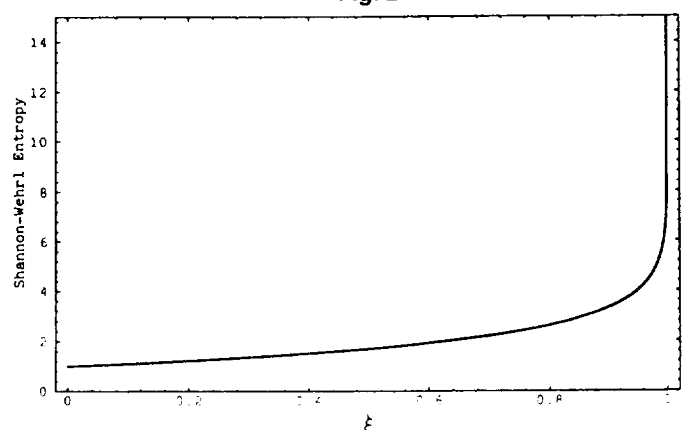

Fig. 2: Shannon-Wehrl entropy for the 'thermal' radiation of a $M = 10^{17}$ g Schwarzschild black hole as a function of the inverse of the effective Bolzmann factor.

We chose the mass of the black hole so that no massive particles are emitted. This would better correspond to the analogy with quantum optics.

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References

[1] Ya.B. Zel'dovich, JETP Lett. **12**, 307 (1970); Ya.B. Zel'dovich and A.A. Starobinsky, Sov. Phys. JETP **34**, 1159 (1972); B.L. Hu, Ph.D. Thesis, Princeton University, 1972; L. Grishchuk, Sov. Phys. JETP **40**, 409 (1975); B.K. Berger, Phys. Rev. D **11**, 2770 (1975)

[2] L.P. Grishchuk and Y.V. Sidorov, Phys. Rev. D **42**, 3413 (1990)

[3] A. Albrecht et al., Imperial College preprint TP/92-93/21 (1992)

[4] A. Wehrl, Rep. Math. Phys. **16**, 353 (1979)

[5] For more details see: H. Rosu and M. Reyes, IFUG preprint IFUG/94/11

[6] E.H. Lieb, Commun. Math. Phys. **62**, 35 (1978)

[7] A. Orłowski, Phys. Rev. A **48**, 727 (1993)

[8] M. Gasperini and M. Giovannini, Phys. Rev. D **47**, 1519 (1993); for a generalization to an arbitrary sequence of stages see: M.R. Garcia Maia, Phys. Rev. D **48**, 647 (1993)

[9] L.M. Krauss and M. White, Phys. Rev. Lett. **69**, 869 (1992)

[10] L.P. Grishchuk, Class. Quantum Grav. **10**, 2449 (1993)

[11] J.D. Bekenstein, Phys. Rev. Lett. **70**, 3680 (1993)