# SLIGHTLY ANHARMONIC SYSTEMS IN QUANTUM OPTICS

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**Abstract**

We consider an arbitrary atomic system ($n$-level atom or many such atoms) interacting with a strong resonant quantum field. The approximate evolution operator for a quantum field case can be produced from the atomic evolution operator in an external classical field by a 'quantization prescription', passing the operator arguments to Wigner $D$-functions. Many important phenomena arising from the quantum nature of the field can be described by such a way.

## 1 Introduction

The behaviour of atomic systems interacting with a quantized electromagnetic field in a cavity has been studied for a long time. Even the simplest model of a single two-level system interacting with a single field mode in a lossless cavity, the Jaynes-Cummings model (JCM) [1], reveals interesting properties like collapses and revivals of atomic inversion oscillations, trapping states, Schrödinger Cats, etc. [1-3]. They stem from the nonlinear nature of the JCM.

For a linear system, the Hamiltonian is usually a linear function of generators of some representation of a finite-dimensional Lie algebra, and the Evolution operator (EO) belongs to the corresponding Lie group representation. This property is referred as a *Dynamical Symmetry.* The most familiar examples are the Harmonic oscillator and the spin rotation in an external magnetic field. Their dynamical groups are, correspondingly, $SU(1,1)$ and $SU(2)$. Dynamical symmetry results in an equidistant spectrum (or a spectrum consisted of several equidistant parts), coherent states (i.e., nonspreading wave packets) and many other attributes of 'harmonic behaviour' [4, 5].

Real systems are oftenly nonlinear. Interesting phenomena appear, if the nonlinearity is 'weak', i.e., the dynamics is 'almost harmonic' one. For instance, the JCM in the classic field limit is equivalent to the spin-1/2 rotation in the external field, that gives the simplest example of the dynamical symmetry (two-dimensional representation of $SU(2)$). JCM collapses and revivals appear in the case of a strong quantum field, when the system is slightly nonlinear.

JCM dynamics is very instructive and we may say more about it. The JCM possesses an exact solution and its EO can be found explicitly. It is $2\times 2$-matrix with coefficients depending

on photon operators, see, e.g. [3]. On the other hand, the number of excitation $N$ is a constant of motion in this model, and its EO can be written as a direct sum of infinitely many $2\times 2$-matrices corresponding to different values of $N$. Every matrix is similar to the spin-1/2 rotation around $x$-axis, but different blocks rotate with different frequencies, $\Omega_N$, called quantum Rabi frequencies. The spectrum consists from infinitely many pairs of levels separated by $2\Omega_N$, and anharmonicity results in a nonlinear dependence of quantum Rabi frequency on the excitation number, $\Omega_N = g\sqrt{N}$ ($g$ is a coupling constant). Therefore, the JCM can be considered as an interesting example of a nonlinear system, however possessing the exact dynamical symmetry.

Treating the field classically, we put $\Omega_N \to \overline{\Omega} \sim g\sqrt{\overline{n}}$, using the classical Rabi frequency rather than quantum ones. (Here $\overline{n}$ is the field intensity in units of photon number.) For a strong quantum field with Poisson photon distribution, different $\Omega_N$ contribute to dynamics, $\overline{n} - \sqrt{\overline{n}} < N < \overline{n} + \sqrt{\overline{n}}$. The difference in frequencies $g\sqrt{\overline{n}+\Delta n} - g\sqrt{\overline{n}-\Delta n}g \sim \sqrt{\Delta n}/\overline{n} \sim g$ becomes important for times $\sim g^{-1}$. This is just the JCM collapse time. It is in the range of modern experimental possibilities, both for Rydberg atom micromasers and for optical microcavities (see the references in the review [1]). The frequency of revivals is proportional to $g/\sqrt{\overline{n}}$. Thus, it is not surprising, that JCM collapses and revivals can be described in the frame of expansion over the inverse field intensity [3].

Natural generalizations of JCM involve more resonant levels and more atoms. Already a system of many two-level atoms (the Dicke model [6]) does not allow an exact solution. However, for very general class of atomic systems interacting with the quantum field under the Rotating Wave Approximation, the excitation number remains a constant of motion. If one of the field modes contains a lot of photons, we can neglect the other modes and develop a perturbation theory with the inverse excitation number as a small parameter. Precisely, the initial number of photons must be much larger then the maximum possible number of atomic excitations $\mathcal{A}$. This program has been realized at the level of wave functions for the Dicke model in Refs. [7] and for more general systems in Ref. [8].

Here, we shall find an explicit form of the EO for an arbitrary atomic system interacting with a strong resonant quantum field. The atomic operators for such a system form a finite-dimensional representation of some compact Lie algebra (see, e.g., [9]). For the case of identical atoms, the algebra depends on the number of levels while the representation depends on the number of atoms and symmetry properties of initial atomic state under the atomic permutations. For instance, the case of $A$ identical two-level atoms excited from a symmetric state leads to the $(A+1)$-dimensional representation of $SU(2)$. The exact EO can be written as a direct sum of finite dimensional-blocks with different excitation numbers. The dimension of blocks for the case $N > \mathcal{A}$ is determined by the atomic algebra representation, but the exact Hamiltonian in every block is a nonlinear function of the representation generators (see Eq. (9) below). However, we shall show that the dynamical symmetry can be restored in the zeroth- and the first-order approximations. It means that the evolution operator in every block can be well approximated by the operator from the corresponding Lie group representation. In the Dicke model case this approximate motion is a rotation of the collective atomic pseudospin (of the length $A/2$) around $x$-axis. Once again, the rotation frequencies in different blocks depend nonlinearly on the excitation number (see Eq. (10) below). Therefore, our zeroth-order approximation possesses a dynamical symmetry in the same sense as the exact JCM solution. The difference is that the motion in every block is described by the appropriate representation of atomic algebra instead of spinor $SU(2)$ representations for

JCM. Being restricted to JCM case our theory just reproduces its exact solution.

The work is organized as follows. In the next section we shall describe the model. In Sec. 3 the asymptotic evolution operator for an arbitrary atomic system interacting with a strong quantum field is found. It has a matrix form in the atomic basis with coefficients depending on the photon operators. Let us remind, that the evolution operator for the atomic system in an external classical field (*semiclassical EO*) is a matrix of a finite rotation from the atomic group representation. The corresponding matrix elements are calculated by the group representation theory (see, e.g., [10]).

Remarkably, the approximate 'quantum' EO can be obtained from the semiclassical one by a simple 'quantization prescription' (see the text before Eq. (23)), which introduces the operator arguments into the Wigner $D$-functions. Therefore, our results enable to write (without calculations) approximate matrix elements of the quantum evolution operator as far as the ones of the semiclassical EO are known.

We demonstrate the convenience of the form (23) for the EO in Sec. 4, where it is used to reproduce the wave functions found in Ref. [8] and to prove the approximate factorization of the system wave function for special initial conditions. Making use of Eq. (23) drastically simplifies the original proof of factorization [8]. The wave functions of Ref. [8] contain the information about collapses and revivals, trapping states and Schrödinger cats and provide the correct structure of the field quasiprobability distribution for the systems under study. Therefore, the proposed asymptotic form for the EO describes all these phenomena connected with the quantum nature of the field.

# 2 Description of the model

We shall work with the following Hamiltonian

$$\hat{H} = \omega(\hat{n} + \hat{h}) + \hat{V}, \quad \hat{V} = g(a\hat{X}_+ + a^\dagger \hat{X}_-). \tag{1}$$

Here, $a, a^\dagger, \hat{n} = a^\dagger a$ are the photon annihilation, creation and number operators, describing a cavity quantized field mode with the frequency $\omega$. $g$ is the coupling constant with resonant atoms placed into the cavity, (we consider the exact resonance case for simplicity). $\hat{h}$ is the bare atomic system Hamiltonian determining the configuration of atomic levels, $\hat{X}_\pm$ are atomic operators describing transitions between resonant levels and obeying commutation relations

$$[\hat{h}, \hat{X}_-] = -\hat{X}_-, \quad [\hat{h}, \hat{X}_+] = \hat{X}_+. \tag{2}$$

Hamiltonian (1) with conditions (2) describes quite general atomic system interacting with a resonant mode of quantized field under the Rotating Wave Approximation. The Dicke model corresponds to the particular case when an atomic system consists of $A$ two-level atoms [6]. Then the operators $\hat{h}, \hat{X}_\pm$ belong to the $(A+1)$-dimensional representation of $su(2)$ algebra and obey the additional commutation relation

$$[\hat{X}_+, \hat{X}_-] = 2\hat{h} \tag{3}$$

The simplest case of the JCM corresponds to the two-dimensional representation of $su(2)$.

We define the basis of the atomic algebra representation as

$$|k, \gamma\rangle_{at}, \quad 0 \le k \le \mathcal{A}, \quad \hat{h}|k, \gamma\rangle_{at} = (k - C)|k, \gamma\rangle_{at}, \tag{4}$$

where $k$ is a number of excitations in the atomic system, $\mathcal{A}$ is its maximal value, $C$ is a constant corresponding to the bottom energy level of the atomic system ($k=0$). $\gamma$ denotes all the other atomic indexes (atomic level populations), which are given by eigenvalues of the operators from the Cartan subalgebra. For example, in the case of $su(2)$ algebra we have the only Cartan operator $\hat{h}$ and there are no additional indexes. Then the maximum possible number of atomic excitations $\mathcal{A}$ is equal to the number of atoms $A$, and the bottom level of atomic energy is $C = A/2$. In the presence of additional indexes (say, for three-level atoms) the bare atomic levels may be degenerate and the dimension of the representation is then larger than $\mathcal{A}+1$.

It follows from Eqs. (2) that the excitation number operator commutes with the Hamiltonian

$$\hat{N} = a^{+}a + \hat{h}, \quad [\hat{N}, \hat{H}] = 0. \tag{5}$$

It is useful to introduce the basis

$$|N,k\rangle = |N-k\rangle_f \otimes |k\rangle_{at}, \quad 0 \le k \le \mathcal{A}, \quad \hat{N}|N,k\rangle = (N-C)|N,k\rangle, \tag{6}$$

where $|n\rangle_f$ is a Fock field states, $|k\rangle_{at}$ is a bare atomic state (4). For a fixed value of $N$, the Hamiltonian (1) in the basis (6) is a finite dimensional matrix. It's rows and columns are numerated by the indexes $k$ and $\gamma$. (In Eq. (6) and below, we omit the index $\gamma$.)

We shall explore the field phase operators defined as [11]

$$\begin{gathered} a^{\dagger} = \exp(-i\hat{\phi})\sqrt{\hat{n}+1}, \quad a = \sqrt{\hat{n}+1}\exp(i\hat{\phi}), \\ \exp(-i\hat{\phi})|n\rangle_f = |n+1\rangle_f \quad \exp(i\hat{\phi})|n\rangle_f = |n-1\rangle_f, \quad n > 0. \end{gathered} \tag{7}$$

These phase operators are unitary and provide the correct physical results for large $n$'s. The following commutation relations are valid:

$$f(\hat{n})\exp(-i\hat{\phi}) = \exp(-i\hat{\phi})f(\hat{n}+1), \quad \exp(i\hat{\phi})f(\hat{n}) = f(\hat{n}+1)\exp(i\hat{\phi}), \tag{8}$$

where $f(\hat{n})$ is an arbitrary function of the photon number operator (determined by its Taylor expansion). Eqs. (8) follow from similar relations for the operators $a, a^{\dagger}$.

# 3 The Evolution Operator for the strong field case

We consider here the case of initially strong field. Then, the total number of excitations is larger than the maximum possible atomic excitation number, i.e."$\hat{N} \sim \hat{n} \gg \hat{h}$", and we can build a perturbation theory with $\hat{N}^{-1}$ as a small parameter. It is convenient to divide the derivation into subsequent steps.

1. We eliminate $a, a^{\dagger}$ and $\hat{n}$ from the Hamiltonian (1) using Eqs. (7) and the excitation number conservation, $\hat{n} = \hat{N} - \hat{h}$,

$$\hat{V} = g\left(\sqrt{\hat{N}-\hat{h}+1}\,\exp(i\hat{\phi})\hat{X}_{+} + \hat{X}_{-}\exp(-i\hat{\phi})\sqrt{\hat{N}-\hat{h}+1.}\right) \tag{9}$$

2. The square roots in the last equation can be written as series of powers of the operator

$$(\hat{N}+1/2)^{-1} \equiv g^2/\hat{\Omega}^2, \quad \hat{\Omega}(\hat{N}) \equiv g\sqrt{\hat{N}+1/2.} \tag{10}$$

We get

$$\hat{V} = \hat{\Omega}\left[\hat{H}_0 + \frac{g^2}{\hat{\Omega}^2}\hat{H}_1 + \ldots\right], \tag{11}$$

$$\hat{H}_0 = \exp\left(i\hat{\phi}\right)\hat{X}_+ + \exp\left(-i\hat{\phi}\right)\hat{X}_-, \tag{12}$$

$$\hat{H}_1 = -1/2\{\hat{h}, \hat{H}_0\}_+, \ldots. \tag{13}$$

Here $\{,\}_+$ stands for anticommutator.

Our choice of a small parameter (Eq. (10) rather than $\hat{N}^{-1}$) is important, since it provides vanishing first order corrections to eigenfrequencies, as it has been shown in Ref. [7]. In fact, the first-order corrections are in such a way included into the zeroth-order ones, that improves the quality of the zeroth-order approximation. According to Eq. (11), the whole time scale is determined by the factor $\hat{\Omega}(\hat{N})$, which plays the role of the Rabi frequency for the problem under study.

3. It is convenient to use the following transformation

$$\hat{Q} = \exp\left[i\hat{\phi}(\hat{h} + C)\right]. \tag{14}$$

This operator is unitary on the states $n > \mathcal{A} \geq k$. Acting on the basis vectors it gives

$$\hat{Q}|k\rangle_{at} \otimes |n\rangle_f = \exp(i\hat{\phi}k)|k\rangle_{at} \otimes |n\rangle_f = |k\rangle_{at} \otimes |n-k\rangle_f = |N=n, k\rangle. \tag{15}$$

Since the operator $\hat{h}+C$ has an integer spectrum, the operator $\hat{Q}$ is a direct sum of different powers of the phase operator.

Directly from the definitions (14),(7) and from the commutators (2),(8) we find

$$f(\hat{n})\hat{Q}^{-1} = \hat{Q}^{-1}f(\hat{n} + \hat{h} + C), \quad \hat{Q}f(\hat{n}) = f(\hat{n} + \hat{h} + C)\hat{Q}, \tag{16}$$

$$\hat{Q}\hat{X}_+\hat{Q}^{-1} = \exp(i\hat{\phi})\hat{X}_+, \qquad \hat{Q}\hat{X}_-\hat{Q}^{-1} = \exp(-i\hat{\phi})\hat{X}_+. \tag{17}$$

4. Here we shall find the zeroth-order EO. (Wave functions in the second order for the Dicke model have been calculated in Ref. [7].) Applying the $Q$-transformation to the zeroth-order Hamiltonian and using Eqs. (16),(17) we have

$$\hat{V} \approx \hat{\Omega}(\hat{n}+\hat{h}) = \hat{Q}\,\hat{\Omega}(\hat{n}-C)\,\hat{H}_{cl}\hat{Q}^{-1}, \quad \hat{H}_{cl} \equiv \hat{X}_+ + \hat{X}_-. \tag{18}$$

Therefore, $Q$-transformation removes the nondiagonal photon operators from the zeroth-order Hamiltonian. It transforms the operator $\hat{H}_0$ into the purely atomic operator $\hat{H}_{cl}$, which has a sense of the atomic system Hamiltonian in an external constant classical field. Moreover, the $Q$-transformation removes the field operators from the coefficients $\hat{H}_0, \hat{H}_1, \ldots$ in all the orders. Simultaneously, the $Q$-transformation changes the expansion parameter, $\hat{\Omega}(\hat{N}) \to \hat{\Omega}(\hat{n}-C)$, transforming it into the function of the photon number operator. Therefore, the $Q$-transformation separates field and atomic variables in the expansion (11) in such a way, that atomic operators appear in the coefficients and the photon number operator is included into the expansion parameter. Thus, the calculation of high order corrections involve only the atomic operators.

5. Now we can calculate the matrix elements of the EO with the Hamiltonian (18)

$$\exp(-it\hat{V}) \approx \exp\left[-it\hat{\Omega}(\hat{N})\hat{H}_0\right] = \hat{Q}\exp\left[-\hat{\Omega}(\hat{n}-C)\hat{H}_{cl}\right]\hat{Q}^{-1} \tag{19}$$

between the atomic states (they are still operators in the field space). Taking into account the action of the $Q$-operator on the atomic states, we find

$$
\begin{aligned}
{}_{at}\langle k|\exp(-it\hat{V})|m\rangle_{at} &\approx e^{i\hat{\phi}k}\,{}_{at}\langle k|\exp\left[-it\hat{\Omega}(\hat{n}-C)\hat{H}_{cl}\right]|m\rangle_{at}\,e^{-i\hat{\phi}m} \\
&= {}_{at}\langle k|\exp\left[-it\hat{\Omega}_k\hat{H}_{cl}\right]|m\rangle_{at}\,\exp(i\hat{\phi}(k-m)),
\end{aligned}
\tag{20}
$$

where we have denoted

$$
\hat{\Omega}_k \equiv \hat{\Omega}(\hat{n}+k-C) = g\sqrt{\hat{n}+k-C+1/2}. \tag{21}
$$

Shifting $e^{i\hat{\phi}k}$ to the right in the last line of Eq. (20), we have used the commutator (7).

What profit have we got with Eq. (20)? We have separated the field phase operators, writing them on the right. The operator $\hat{\Omega}_k\hat{H}_{cl}$ contains the only field operator $\hat{n}$ which commutes with all the other ingredients and may be treated as a $Q$-number in the calculation of the exponent. Thus, we can reconstruct the quantum field EO if we know the atomic EO in the external classical field

$$
\hat{U}_{cl}(\omega) = \exp(-it\omega\hat{H}_{cl}). \tag{22}
$$

The matrix $\hat{U}_{cl}(\omega)$ is the rotation operator from the atomic group representation. It's matrix elements are known from the standard group representation theory. For the Dicke model case, the operator $\hat{H}_{cl} = \hat{X}_+ + \hat{X}_- = 2\hat{S}_x$ is just a generator of rotations around $x$-axis, and the matrix elements of $\exp(-it\omega 2\hat{S}_x)$ are the usual Wigner $D$-functions.

The quantum EO matrix elements can be produced from the matrix $\hat{U}_{cl}(\omega)$ by the following 'quantization prescription':

(i) to substitute the group parameter $\omega$ in the $(k,\gamma)$-th row of the matrix $\hat{U}_{cl}(\omega)$ by the operator $\omega \to \hat{\Omega}_k = \sqrt{\hat{n}+k-C+1/2}$, (here $k$ is the atomic excitation number in the row $(k,\gamma)$ );

(ii) to multiply every matrix element ${}_{at}\langle k,\gamma|\hat{U}|m,\gamma'\rangle_{at}$ from the right by the power of the phase operator $\exp(i\hat{\phi}(k-m))$.

Writing explicitly in Eq. (20) the additional atomic indexes $\gamma$, we have

$$
{}_{at}\langle k,\gamma|\hat{U}|m,\gamma'\rangle_{at} = {}_{at}\langle k,\gamma|\hat{U}_{cl}(\hat{\Omega}_k)|m,\gamma'\rangle_{at}\,\exp(i\hat{\phi}(k-m)). \tag{23}
$$

This equation is our principal result. It corresponds to the wave functions found in Ref. [8]. Being restricted to the case of a single two-level atom, it gives the exact JCM EO (see, e.g., [3]). The quantization prescription formulated above can be easily generalized to include detuning [12].

# 4 The wave function factorization

The most remarkable feature of dynamics with the EO (23) is an approximate wave function factorization for special initial states. Let the field initially be in a coherent state, $|in\rangle_f = |\alpha\rangle$, $\alpha \equiv \sqrt{\overline{n}}e^{i\phi}$, where $\overline{n}$ and $\phi$ are the initial photon number and the phase of the field. (The number $e^{i\phi}$ may not be confused with the phase operator $\exp(i\hat{\phi})$). The initial atomic state is taken as an eigenstate of the operator

$$
\hat{H}_{cl}(\phi) = e^{i\phi}\hat{X}_+ + e^{-i\phi}\hat{X}_- = e^{i\phi\hat{h}}\hat{H}_{cl}e^{-i\phi\hat{h}}, \quad \hat{H}_{cl}(\phi)|\underline{p(\phi)}\rangle_{at} = \lambda_p|\underline{p(\phi)}\rangle_{at}, \quad |\underline{p(\phi)}\rangle_{at} = e^{i\phi\hat{h}}|\underline{p}\rangle_{at}.
$$

Here, $\hat{H}_{cl}$ is determined by Eq. (18). We shall call $\lambda_p$ and $|\underline{p(\phi)}\rangle_{at}$ as semiclassical eigenvalues and eigenvectors. The vector $|\underline{p}\rangle_{at}$ is also an eigenvector of the semiclassical evolution operator (22)

$$\hat{U}_{cl}(\omega)|\underline{p}\rangle_{at} = \exp\left(-i\omega\lambda_p t\right)|\underline{p}\rangle_{at}. \tag{24}$$

It is known [13], that coherent states with large photon numbers are approximately eigenstates of the phase operators. More precisely,

$$\exp(ik\hat{\phi})|\alpha\rangle = \exp(ik\phi)|\alpha\rangle + O(k/\sqrt{\overline{n}}). \tag{25}$$

Now we act by the EO (23) to the initial state $|in\rangle = |\alpha\rangle \otimes |\underline{p}\rangle_{at}$. Multiplying the equation from the left by an arbitrary atomic vector $\langle k|_{at}$ and substituting the EO matrix element, Eq. (23) we have

$$\begin{aligned}
{}_{at}\langle k|\,\hat{U}\,|\alpha\rangle_f \otimes |\underline{p(\phi)}\rangle_{at} &\approx \sum_m e^{i\phi(k-C)}\,{}_{at}\langle k|\hat{U}_{cl}(\hat{\Omega}_k)|m\rangle_{at}\,{}_{at}\langle m|\underline{p}\rangle_{at} \otimes |\alpha\rangle_f \\
&= {}_{at}\langle k|e^{i\phi(k-C)}e^{-it\lambda_p\hat{\Omega}_k}|\underline{p}\rangle_{at} \otimes |\alpha\rangle_f \\
&= {}_{at}\langle k|\exp\left(-it\lambda_p g\sqrt{\hat{n}+\hat{h}+1/2}\right)|\underline{p(\phi)}\rangle_{at} \otimes |\alpha\rangle_f .
\end{aligned} \tag{26}$$

Here, in the first line, the phase factor coming from the action of the field phase operator $e^{i\hat{\phi}(k-m)}$ on the coherent state, Eq. (25) cancels the atomic phase factor ${}_{at}\langle m|\underline{p(\phi)}\rangle = e^{i\phi(m-C)}\,{}_{at}\langle m|\underline{p}\rangle_{at}$ to give $e^{i\phi(k-C)}$. In the second line we use the property (24), that is $\hat{U}_{cl}(\hat{\Omega}_k)|\underline{p}\rangle_{at} = e^{-it\lambda_p\hat{\Omega}_k}|\underline{p}\rangle_{at}$. We stress, that the operator-valued group parameter $\hat{\Omega}_k$ depends on the number of the row, so for rows with different $k$'s we must use different semiclassical EO's with different values of the group parameter. Finally, in the last line of Eq. (26) we substitute $e^{i\phi(k-C)}\,{}_{at}\langle k|\underline{p}\rangle_{at} = {}_{at}\langle k|\underline{p(\phi)}\rangle_{at}$ and ${}_{at}\langle k|e^{-it\lambda_p\hat{\Omega}_k} = {}_{at}\langle k|\exp(-it\lambda_p g\sqrt{\hat{n}+\hat{h}+1/2})$. Since ${}_{at}\langle k|$ is an arbitrary atomic state, we can write the wave function in the form

$$\hat{U}(t)|in\rangle \approx \exp\left[-it\lambda_p g\sqrt{\hat{n}+\hat{h}+1/2}\right]|in\rangle.$$

Introducing the notation $\overline{\Omega} \equiv g\sqrt{\overline{n}+1/2}$ we can approximate the square root as

$$\sqrt{\hat{n}+\hat{h}+1/2} \approx \sqrt{\hat{n}+1/2} + \frac{\hat{h}}{2\sqrt{\overline{n}+1/2}} + O\left(\hat{n}^{-3/2}\right), \quad \frac{1}{\sqrt{\hat{n}+1/2}} \approx \frac{g}{\overline{\Omega}} + O\left(\frac{1}{\overline{n}}\right)$$

and we find the factorized wave function

$$\begin{aligned}
|\Psi(t)\rangle &\cong |\,\Phi_p(t)\rangle \otimes |\,A_p(t)\rangle, \\
|\Phi_p(t)\rangle &= \exp\left(-itg\lambda_p\sqrt{\hat{n}+1/2}\right)|\alpha\rangle_f, \\
|A_p(t)\rangle &= \exp\left[-itg^2\lambda_p\hat{h}/(2\overline{\Omega})\right]|\underline{p}\rangle_{at},
\end{aligned} \tag{27}$$

This wave function has been found by a different way in Ref. [8] and used for the discussion of the trapping states, collapses, revivals and Schrödinger Cat states for the systems under study.

If different semiclassical eigenstates contribute to the initial state, the wave function is a superposition of corresponding factorized states (27). The evolution operator Eq. (23) describes the dynamics from any initial state, such that the initial photon number is much larger then the maximum number of atomic excitations.

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