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COHERENT CONTROL IN SIMPLE QUANTUM SYSTEMS

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Abstract

Coherent dynamics of two-, three-, and four-level quantum systems, simultaneously driven by concurrent laser pulses of arbitrary and different forms, is treated by using a nonperturbative group-theoretical approach. The respective evolution matrices are calculated in an explicit form. General aspects of controllability of few-level atoms by using laser fields are treated analytically.

1. INTRODUCTION

We analyze the general aspects of the problem of dynamical coherent control of atomic populations in the framework of semiclassical approach when an external (laser) field is considered as an inexaustible energy reservoir and only the atomic behaviour is considered to be controllable. The natural controllers are strengths, frequencies and phases of the components of a polychromatic laser field driving simultaneously different atomic transitions. An initial atomic state may be also considered as a controller.

The dynamics of any nonstationary quantum system is described by the time evolution equation

$$i \frac{\partial}{\partial t} U(t,0) = H(t) \; U(t,0) \; , \quad U(0,0) = I \; , \; \hbar = 1 \qquad (1)$$

with the Hamiltonian

$$H(t) = \sum_{k=1}^{m} h_k(t) \; H_k \; . \qquad (2)$$

which depends on time explicitely. It is not an easy task to solve the eq.1 analitically even for a two-level system exposed to an arbitrary varying external field. Usualy used adiabatic and weak-field approximations as well as the approximation with slowly varying amplitudes breaks down if we deal with short and/or intense laser pulses. The well-known Floke method is useful only in the case of periodic excitations. The high level of control of atomic populations by using designed laser fields calls for a new nonperturbative technique that should be, on the one hand, rather general to enable one to treat uniformely the atomic dynamics under various excitation conditions, and on the other hand, it should be able to result in explicit solutions of the evolution equation (1).

The group-theoretical technique will be applied to solve this task for two-, three-, and four-level atoms exposed to an arbitrary time-varying polychromatic laser field. It is based on the concept of dynamical symmetry of nonstationary quantum processes [1-3].

After solving the eq.1 for a given model system, the problem of finite control [4] is to find such values of controllers $\{h_j(t), j=1,\dots,m\}$ that enables us to transform the system from an initial state to the desired final state at the target time $T > 0$. Such a transition is governed by the equation

$$| s(T) \rangle = U(T,0) \; | s(0) \rangle. \tag{3}$$

Generally speaking, the solution of this task is not unique. In addition one can put the task of optimal control which is to find such values of controllers that minimizes a given loss functional or maximizes a desired quality criterion [5].

2. GENERAL BACKGROUND

A single atom with N nondegenerate and nonequidistant levels

$$H_o \; |s_m\rangle = w_m \; |s_m\rangle, \quad m = 1,2,\dots,N, \tag{4}$$

interacts with amplitude- modulated components of a polychromatic field

$$E = (1/2) \sum_{l>}^{N} \sum_{k}^{N} E_{kl} \, (t) \exp \, (i \; w_{kl} \; t) \; + \text{c.c.} \tag{5}$$

by such a way that only one (k, l)-component of the field E_{kl} is in resonance with the (k, l)- atomic transition. If one deals with the electric dipole transitions

$$H_{int} = -E(t)\ d(t)\ ,$$

then by the parity reasons the energetic matrix $H_0 + H_{int}$ turns out to be tridiagonal. However, we will treat the general case (when all the N(N-1)/2 atomic transitions are driven independently on each other) to be able to include into consideration, if necessary, other interactions that may couple nonadjacent levels. The complex variable $E_{kl}(t)$ measures amplitude, phase, and polarization of the respective component.

Writing the state vector of a N-level atom in the form

$$|s(t)\rangle = \sum_{m=1}^{N} c_m(t)\ |s_m\rangle \exp\left(-i\ w_m\ t\right), \tag{6}$$

one can obtain from the nonstationary Scrodinger equation the following set of equations for the probability amplitudes in the rotating wave approximation

$$i\dot{s}_k(t) = \sum_{k<l}^{N} h_{kl}(t)\ s_l(t) + c.c., \tag{7}$$

where

$$h_{kl}(t) = -\ (1/2)\ E_{kl}(t)\ d_{kl}(t). \tag{8}$$

The resonant Hamiltonian has the following structure

$$H_{nn}(t) = 0, \quad H_{kl}(t) = h_{kl}(t) = H^{*}_{lk}(t)\ ,\ k \neq l. \tag{9}$$

Finally, the probability of finding our N-level system on level n at time t is expressed in terms of the evolution-matrix elements and the initial populations P_m as follows

$$P_n(t) = \sum_{m=1}^{N} |U_{nm}(t)\ |^2\ P_m(0). \tag{10}$$

3. COHERENT CONTROL ON THE SU(2) DYNAMICAL GROUP

A quantum system with the SU(2) dynamical symmetry is a fundamental model in the semiclassical theory of field-matter interactions. The hermitean Hamiltonian of such an arbitrarily driven system can be cast in the form

$$H(t) = h_0(t)\, R_0 + h^*(t)\, R_- + h(t)\, R_+ \tag{11}$$

with the generators satisfying the commutation relations

$$[R_+,\ R_-] = 2R_0,\ \ [R_0,\ R_{+,-}] = \pm R_{+,-}\ , \tag{12}$$

where c-number parameters h_0 and h are assumed to be arbitrary analytic functions. Writing the evolution operator in the factorized Wei-Norman form [6]

$$U = \exp\left(g_0 - i\int_0^t h_0(\tau)\, d\tau\right) R_0 \exp g_- R_- \exp g_+ R_+, \tag{13}$$

one can obtain the governing equation for $g = \exp(g_0/2)$

$$\ddot{g} - \left(\frac{\dot{h}}{h} + i\, h_0\right)\dot{g} + |h|^2 g = 0,\quad g(0) = 1,\ \dot{g}(0) = 0. \tag{14}$$

Once the eq.14 is solved all the other SU(2) group parameters are found in quadratures [6,2,3] It follows from the unitariry of (13) the simple conservation laws

$$|g|^2 - g_-\, g_+ = 1,\quad |g|^2\,(1 + |g_+|^2) = 1. \tag{15}$$

In the two-dimensional representation we get the expression

$$U^{(1/2)} = \begin{bmatrix} g & [2i(\dot{g})^* + h_0 g^*]/2h^* \\ (2i\,\dot{g} - h_0 g)/2h & g^* \end{bmatrix} \tag{16}$$

which is the time-evolution matrix for a two-level system driven by an external field with arbitrary amplitude and frequency modulation. In terms of populations the complete formal solution of the problem of two-level control is given by the formula

$$|g(T)|^2 = [2P_1(0) - 1]^{-1}\,[P_1(T) + P_1(0) - 1], \tag{17}$$

where $P_1(0)$ and $P_1(T)$ are the input and the output values of the first level population, respectively. Since the evolution is unitary one has $P_1(t) + P_2(t) = 1$.

4. POPULATION CONTROL OF A RESONANTLY DRIVEN THREE-LEVEL SYSTEM

Let us consider the coherent dynamics of a three-state system with an arbitrary level configuration simultaneously driven in resonance by three lasers with arbitrary and different time dependences of their amplitudes. Choosing the phases in an appropriate way we can write the system Hamiltonian in the matrix form with the following nonvanishing elements

$$H_{12} = H_{21} = h_{12}(t), \quad H_{23} = H_{32} = h_{23}(t), \quad H_{13} = -H_{31} = -ih_{13}(t).$$

If we write this matrix in terms of the generators of the standard representation of the SU(2) group, then the time-evolution matrix is simply the three-dimensional representation of this group [7]

$$U^{(1)} = \begin{bmatrix} u^2 & \sqrt{2}uv & v^2 \\ -\sqrt{2}uv^* & |u|^2-|v|^2 & \sqrt{2}u^*v \\ (v^*)^2 & -\sqrt{2}u^*v^* & (u^*)^2 \end{bmatrix}, \qquad (18)$$

where $u \equiv g$ and $v^* \equiv -(2i\dot{g} - h_0 g)/2h$. The governing equation for the driven three-level system is given by

$$\ddot{g} - [(\dot{h}_{12}-i\dot{h}_{23})(h_{12}-ih_{23})^{-1} + ih_{13}]\dot{g} + (1/4)(h_{12}^2 + h_{23}^2)\, g = 0.$$

The complete formal solution of the problem of three-level population control is given by the formula

$$|g(T)|^2 = \frac{(P_1(0)-P_2(0))(P_1(T)-P_2(0)) - (P_3(0)-P_2(0))(P_3(T)-P_2(0))}{(P_1(0)-P_3(0))(P_1(0)+P_3(0)-2P_2(0))}.$$

The useful relation exists between two variables u and v

$$|u(t)|^4 - |v(t)|^4 = [P_1(0) - P_3(0)]^{-1}\, [P_1(t) - P_3(t)].$$

Using the eqs.10 and (18) we can now easily find simple exact conditions for complete transfer of population from one level to another and for complete return of population to an initial level. Let at $t=0$ level 1 is fully populated, i.e. $P_1(0)= 1$, $P_2(0)= P_3(0)= 0$, then the population is transfered completely to the level 3 at time t, i.e. $P_3(t)= 1$, $P_1(t) = P_2(t)= 0$, if $g(t) = 0$. For the same initial conditions the population returns to the first level at time t, i.e. $P_1(t) = 1$, $P_2(t)= P_3(t)= 0$, when $|g(t)| = 1$.

5. POPULATION DYNAMICS OF A RESONANTLY DRIVEN FOUR-LEVEL SYSTEM

The resonant Hamiltonian of a four-level system simultaneously driven by six lasers with arbitrary and different time dependences of amplitudes is given by the eq.9 with N=4. If the phases are choosen in such a way that all the Rabi frequencies (8) are purely imagine ones then the four-level Hamiltonian generates the following 4x4-matrix realization of the SO(4) algebra

$$R_1^{32} = R_2^{13} = R_3^{21} = -R_1^{23} = -R_2^{31} = -R_3^{12} = J_1^{14} = J_2^{24} = J_3^{34} = -J_1^{41} = -J_2^{42} = -J_3^{43} = 1,$$

with zero remaining matrix elements and with the following commutation relations

$$[R_j, R_k] = e_{jkl} R_l, \; [J_j, J_k] = e_{jkl} R_l, \; [R_j, J_k] = e_{jkl} J_l.$$

One can rearrange this basis by such a way to obtain from the matrices R_j and J_k two mutually commuting sets of matrices $A_{o,+,-}$ and $B_{o,+,-}$ with the commutation relations (12). Now we can rewrite our Hamiltonian in the form

$$H(t) = \sum_{m=o,+,-} a_m(t) A_m + b_m(t) B_m , \qquad (19)$$

where the following short-hand notations are introduced

$$a_o \equiv i(h_{12} - h_{34}), \; 2a_- \equiv ih_{23} + h_{13} - ih_{14} + h_{24},$$

$$2a_+ \equiv ih_{23} - h_{13} - ih_{14} - h_{24}, \quad b_o \equiv i(h_{12} + h_{34}), \qquad (20)$$

$$2b_- \equiv ih_{23} + h_{13} + ih_{14} - h_{24}, \quad b_+ \equiv ih_{23} - h_{13} + ih_{14} + h_{24}.$$

Since H(t) exactly equals to the sum of the two SU(2) Hamiltonians, commuting with each other, we may use all the results obtained for the SU(2) dynamical symmetry. Thus the system posesses SU(2) ⊕ SU(2) dynamical symmetry [8] and the evolution operator can be written in the factorized form $U = U_g U_f$, where

$$U_g = \exp(g_0 A_0) \exp(g_- A_-) \exp(g_+ A_+),$$
$$U_f = \exp(f_0 B_0) \exp(f_- B_-) \exp(f_+ B_+) \qquad (21)$$

The governing equations for our four-level system take the form

$$\ddot{g} - \dot{a}_+ a_+^{-1}\,\dot{g} + (1/2)(i\dot{a}_0 - ia_0\,\dot{a}_+ a_+^{-1} + 2a_- a_+ + (1/2)a_0^2)g = 0,$$

$$\ddot{f} - \dot{b}_+ b_+^{-1}\,\dot{f} + (1/2)(i\dot{b}_0 - ib_0\,\dot{b}_+ b_+^{-1} + 2b_- b_+ + (1/2)b_0^2)f = 0,$$

with the variables $g \equiv \exp(g_0/2)$ and $f \equiv \exp(f_0/2)$ respectively. In an explicit form the evolution matrices are the following

$$U_g = g^{-1}\begin{vmatrix} g\,\mathrm{Re}g & g\,\mathrm{Im}g & \mathrm{Re}g_- & -\mathrm{Im}g_- \\ -g\,\mathrm{Im}g & g\,\mathrm{Re}g & \mathrm{Im}g_- & \mathrm{Re}g_- \\ -\mathrm{Re}g_- & -\mathrm{Im}g_- & g\mathrm{Re}g & -g\,\mathrm{Im}g \\ \mathrm{Im}g_- & -\mathrm{Re}g_- & g\,\mathrm{Im}g & g\,\mathrm{Re}g \end{vmatrix},$$

$$U_f = f^{-1}\begin{vmatrix} f\,\mathrm{Re}f & f\,\mathrm{Im}f & \mathrm{Re}f_- & \mathrm{Im}f_- \\ -f\,\mathrm{Im}\,f & f\,\mathrm{Re}f & \mathrm{Im}f_- & -\mathrm{Re}f_- \\ -\mathrm{Re}f_- & -\mathrm{Im}f_- & f\,\mathrm{Re}f & f\,\mathrm{Im}f \\ -\mathrm{Im}f_- & \mathrm{Re}f_- & -f\,\mathrm{Im}f & f\,\mathrm{Re}f \end{vmatrix},$$

where $g_- = g\,(2i\dot{g} - a_0\,g)/2a_+$ and $f_- = f\,(2i\dot{f} - b_0\,f)/2b_+$.

Let at $t = 0$ the first level is occupied with probability one, i.e. $P_1(0) = 1$, $P_2(0) = P_3(0) = P_4(0) = 0$. Then the exact condition for complete transfer of population to the upper level is given by

$$|g^{-1}(\mathrm{Re}f\,\mathrm{Im}g_- + \mathrm{Re}g_-\,\mathrm{Im}f) - f^{-1}(\mathrm{Re}g\,\mathrm{Im}f + \mathrm{Re}f_-\,\mathrm{Im}g_-)|^2 = 1 \quad (22)$$

and the condition for the first level to be completely depleted of population is

$$\mathrm{Re}g\,\mathrm{Im}f - \mathrm{Im}g\,\mathrm{Im}f - (gf)^{-1}(\mathrm{Re}g_-\,\mathrm{Re}f_- - \mathrm{Im}g_-\,\mathrm{Im}f_-) = 0. \quad (23)$$

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