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1. Introduction

In recent years, active and passive control of sound and vibration in aeroelastic structures have received a great deal of attention due to many potential applications to aerospace and other industries. There exists a great deal of research work done in this area. Most recent advances in the control of sound and vibration can be found in the conference proceedings [1, 2]. In this report we will summarize our research findings supported by the NASA grant NAG-1-1175.

The problems of active and passive control of sound and vibration has been investigated by many researchers for a number of years. However, few of the articles are concerned with the sound and vibration with flow-structure interaction. Experimental and numerical studies on the coupling between panel vibration and acoustic radiation due to flow excitation have been done by Maestrello and his associates at NASA/Langley Research Center (see e.g. [3,4]). Since the coupled system of nonlinear partial differential equations is formidable, an analytical solution to the full problem seems impossible. For this reason, we have to simplify the problem to that of the nonlinear panel vibration induced by a uniform flow or a boundary-layer flow with a given wall pressure distribution. Based on this simplified model, we have been able to consider the control and stabilization of the nonlinear panel vibration, which have not been treated satisfactorily by other authors. Although the sound radiation has not been included, the vibration suppression will clearly reduce the sound radiation power from the panel. The major research findings will be presented in the next three sections. In Section two we shall describe our results on the boundary control of nonlinear panel vibration, with or without flow excitation. Sections three and four are concerned with some analytical and numerical results in the optimal control of the linear and nonlinear panel vibrations, respectively, excited by the flow pressure fluctuations. Finally, in Section five, we draw some conclusions from our research findings.

2. Boundary Control of Nonlinear Panel Vibration

Consider a rectangular panel whose mid-plane is bounded by $0 \leq x \leq a$ and $0 \leq y \leq b$. A spatially uniform air-flow passes over the panel with a time-dependent mean flow velocity $U(t)$, (see Fig. 1). For a long span $b \gg 1$, when the transverse deflection w is

uniform in the y -direction, a one dimensional structural model for the panel vibration is used, (see Fig. 2). At a high flow speed, the linear piston theory for the aerodynamic forces is assumed to be valid. Then the panel vibration is described by the following nonlinear integro-differential equation:

$$m \frac{\partial^2 w}{\partial t^2} + [P(t) - N(t)] \frac{\partial^2 w}{\partial x^2} + D \frac{\partial^4 w}{\partial x^4} + f(w) = 0, \quad (2.1)$$

where m is the mass density of the panel, P is the compressive in-plane load, $D = Eh^3/12(1 - \nu^2)$, E , h and ν denote the Young's modulus, the panel thickness and the Poisson's ratio, respectively. The additional tension due to the panel stretching is given by the integral:

$$N = (Eh/2a) \int_0^a \left(\frac{\partial w}{\partial x} \right)^2 dx, \quad (2.2)$$

and the aerodynamic force $f(w)$ can be expressed as

$$f(w) = \frac{\rho U^2}{M_c} \left(\frac{\partial w}{\partial x} + \frac{1}{U} \frac{\partial w}{\partial t} \right), \quad (2.3)$$

where ρ is the fluid density and M_c is the flow Mach number. Let K and V denote, respectively, the kinetic energy and the potential energy defined by

$$K_t(w) = \frac{1}{2} m \int_0^a \left(\frac{\partial w}{\partial t} \right)^2 dx, \quad (2.4)$$

$$V_t(w) = \frac{1}{2} \int_0^a \left\{ D \left(\frac{\partial^2 w}{\partial x^2} \right)^2 + \left(\frac{1}{2} N - P \right) \left(\frac{\partial w}{\partial x} \right)^2 \right\} dx, \quad (2.5)$$

so that the total energy is given by

$$E_t = K_t + V_t. \quad (2.6)$$

To release the compressive force P , we first apply a tensile force Q at $x = a$. Then the net compressive force becomes

$$R = (P - Q - N) \quad (2.7)$$

and the modified equation (2.1) takes the form

$$m \frac{\partial^2 w}{\partial t^2} - R \frac{\partial^2 w}{\partial x^2} + D \frac{\partial^4 w}{\partial x^4} = f(w). \quad (2.8)$$

If we multiply Eq. (2.7) by $\dot{w} = \frac{\partial w}{\partial t}$ and integrate the resulting equation over $0 \leq x \leq a$, it yields the following energy equation:

$$\begin{aligned} \dot{E}_t(w) = \frac{\partial}{\partial t} E_t(w) &= \left\{ D \left(\frac{\partial^2 w}{\partial x^2} \right) \left(\frac{\partial \dot{w}}{\partial x} \right) - \left(D \frac{\partial^3 w}{\partial x^3} - R \frac{\partial w}{\partial x} \right) \dot{w} \right\} \Big|_0^a \\ &- \int_0^a f(w) \dot{w} dx - \dot{P} \int_0^a \left(\frac{\partial w}{\partial x} \right)^2 dx. \end{aligned} \quad (2.9)$$

Since the left end is clamped, we have

$$\frac{\partial w}{\partial x} = 0 \quad \text{and} \quad \frac{\partial^3 w}{\partial x^3} = 0 \quad \text{at} \quad x = 0. \quad (2.10)$$

At the right end, $x = a$, in addition to Q , we apply a twist moment M and a vertical point-force F so that the boundary condition takes the form (see Fig. 3)

$$D \frac{\partial^2 w}{\partial x^2} = M \quad \text{and} \quad \left(D \frac{\partial^3 w}{\partial x^3} - R \frac{\partial w}{\partial x} \right) = -F. \quad (2.11)$$

The objective is to choose the control (Q, M, F) properly to stabilize the system.

By introducing the scalings $\hat{x} = x/a$, $\hat{t} = t(D/ma^4)^{1/2}$ and $\hat{w} = w/h$ etc., the initial-boundary value problem for the boundary control can be written in the following dimensionless form:

$$\left\{ \begin{array}{l} \frac{\partial^2 \hat{w}}{\partial \hat{t}^2} - \hat{R} \frac{\partial^2 \hat{w}}{\partial \hat{x}^2} + \frac{\partial^4 \hat{w}}{\partial \hat{x}^4} = \hat{f}(\hat{w}), \quad 0 < \hat{x} < 1, \\ \hat{w} = \hat{w}_0 \quad \text{and} \quad \frac{\partial \hat{w}}{\partial \hat{t}} = \hat{w}_1 \quad \text{at} \quad \hat{t} = 0, \\ \hat{w} = 0 \quad \text{and} \quad \frac{\partial \hat{w}}{\partial \hat{x}} = 0 \quad \text{at} \quad \hat{x} = 0, \\ \frac{\partial^2 \hat{w}}{\partial \hat{x}^2} = -\hat{M} \quad \text{and} \quad \left(\frac{\partial^3 \hat{w}}{\partial \hat{x}^3} - \hat{R} \frac{\partial \hat{w}}{\partial \hat{x}} \right) = \hat{F} \quad \text{at} \quad \hat{x} = 1, \end{array} \right. \quad (2.12)$$

where

$$\hat{R} = (\hat{N} + \hat{Q} - \hat{P})$$

with $\hat{P} = P/a^2$, $\hat{Q} = Q/a^2$ and

$$\begin{aligned} \hat{N} &= \alpha \int_0^1 \left(\frac{\partial \hat{w}}{\partial \hat{x}} \right)^2 d\hat{x}, \quad \alpha = Eh^3/2a^2, \\ \hat{f}(\hat{w}) &= -(\beta \frac{\partial \hat{w}}{\partial \hat{x}} + \gamma \frac{\partial \hat{w}}{\partial \hat{t}}). \end{aligned}$$

The remaining non-dimensional quantities are given by: $\beta = \rho a^3 U^2 / DM_c$, $\gamma = \rho a^2 U / M_c (DM)^{1/2}$, $\hat{w}_i = w_i/h$, $i = 0, 1$; $\hat{M} = Ma^2/Dh$ and $\hat{F} = Fa^3/Dh$.

Next we consider the case of free vibration without flow excitation, where the compressive force P is static. For simplicity, from now on, we shall drop the caret symbol over the dimensionless variables. For no flow, we have $\hat{f} = 0$ so that Eq. (2.12) yields

$$\left\{ \begin{array}{l} \frac{\partial^2 w}{\partial t^2} - R \frac{\partial^2 w}{\partial x^2} + \frac{\partial^4 w}{\partial x^4} = 0, \quad 0 < x < 1, \\ w = w_0 \text{ and } \frac{\partial w}{\partial t} = w_1 \text{ at } t = 0, \\ w = 0 \text{ and } \frac{\partial w}{\partial x} = 0 \text{ at } x = 0, \\ \frac{\partial^2 w}{\partial x^2} = -M \text{ and } \left(\frac{\partial^3 w}{\partial x^3} - R \frac{\partial w}{\partial x} \right) = F \text{ at } x = 1, \end{array} \right. \quad (2.13)$$

where

$$\begin{aligned} R &= \Delta P + N \text{ with } \Delta P = (Q - P), \\ N &= \alpha \int_0^1 \left(\frac{\partial w}{\partial x} \right)^2 dx. \end{aligned}$$

The corresponding kinetic energy K and potential energy V are

$$\begin{aligned} K &= K_t(w) = \frac{1}{2} \int_0^1 \dot{w}^2 dx, \\ V &= P_t(w) = \frac{1}{2} \int_0^1 \left\{ \left(\frac{\partial^2 w}{\partial x^2} \right)^2 + \left(\Delta P + \frac{1}{2} N \right) \left(\frac{\partial w}{\partial x} \right)^2 \right\} dx. \end{aligned}$$

It can be shown that, in order to have a positive-definite V , we require $\Delta P + 1 > 0$ or Q must be chosen so that

$$(P - Q) \leq (1 - \delta) < 1 \text{ for any } \delta > 0. \quad (2.14)$$

For $P < 1$, we simply taken $Q = 0$, or no tensile control is needed in this case. One notes that $P_{cr} = 1$ is the critical buckling load for a simply supported panel.

In view of the boundary conditions in Eq. (2.13) and the fact $f = \dot{P} = 0$, the energy equation (9) yields

$$\dot{E}_t(w) = F\dot{v}_1(t) - M\dot{\theta}_1(t), \quad (2.15)$$

where $\dot{v}_1(t) = \dot{w}(t, 1)$ and $\dot{\theta}_1(t) = \frac{\partial \dot{w}}{\partial x}(t, 1)$. For the energy E to decay, we will choose a feedback control pair of the form $F = g(\dot{v}_1)$ and $M = h(\dot{\theta}_1)$ such that $\dot{E}_t \leq 0$. An obvious choice is to assume g and h being linear,

$$F = -\mu\dot{v}_1 \text{ and } M = \nu\dot{\theta}_1, \quad (2.16)$$

where μ and ν are some positive constants. They can be regarded as the damping coefficients against the right end's transverse and rotational motions, correspondingly. A substitution of (2.16) into (2.15) gives

$$\dot{E}_t = -\mu\dot{v}_1^2 - \nu\dot{\theta}_1^2 \leq 0 \quad (2.17)$$

This shows that, if the sub-critical load condition (2.14) is satisfied, the linear control (2.15) will results in a decay in energy. In fact the rate of decay is exponential. To verify this fact, we need to introduce a perturbed energy E_t^ε as follows

$$E_t^\varepsilon(w) = E_t(w) + \varepsilon G_t(w), \quad (2.18)$$

where $0 < \varepsilon < 1$ and

$$G_t(w) = \int_0^1 x \left(\frac{\partial w}{\partial t} \right) \left(\frac{\partial w}{\partial x} \right) dx. \quad (2.19)$$

First, it is easy to show that

$$|G_t(w)| \leq E_t(w)$$

So that Eq. (2.18) leads to

$$(1 - \varepsilon)E_t \leq E_t^\varepsilon \leq (1 + \varepsilon)E_t, \quad (2.20)$$

or, E_t and E_t^ε , as far as the exponential decay is concerned, are equivalent. Next, by differentiating Eq. (2.19), invoking Eq. (2.13) and some mathematical inequalities, we can derive the following inequality:

$$\begin{aligned} \dot{E}_t^\varepsilon &= \dot{E}_t + \varepsilon \dot{G}_t \\ &\leq -\varepsilon E_t - \left[\mu - \frac{1}{2}(1 + \mu^2)\varepsilon \right] \dot{v}_1^2 - \nu(1 - \nu\varepsilon)\dot{\theta}_1^2. \end{aligned}$$

If we choose ε in the range $0 < \varepsilon < \varepsilon_0 \leq 1$ with $\varepsilon_0 = \min\{1, 2\mu/(1 + \mu^2), 1/\nu\}$, and make use of (2.20), it follows from the above inequality that

$$\dot{E}_t^\varepsilon \leq -\frac{\varepsilon}{(1 + \varepsilon)} E_t^\varepsilon,$$

which implies

$$E_t^\varepsilon \leq E_0^\varepsilon e^{-\lambda t}$$

or, by noting (2.20)

$$E_t \leq \left(\frac{1+\varepsilon}{1-\varepsilon}\right)E_0 e^{-\lambda t}, \lambda = \varepsilon/(1+\varepsilon).$$

Thus we have shown that the free vibration can be exponentially stabilized by a small boundary damping together with a tensile control Q , which is necessary only if the compressive force P exceeds a critical level specified by (2.14).

Now we consider the case of flow-induced vibration. Dropping the caret symbol over the dimensionless quantities again, under flow excitation the controlled problem (2.12) reads as follows:

$$\left\{ \begin{array}{l} \frac{\partial^2 w}{\partial t^2} - R \frac{\partial^2 w}{\partial x^2} + \frac{\partial^4 w}{\partial x^4} + \beta \frac{\partial w}{\partial x} + \gamma \frac{\partial w}{\partial t} = 0, \quad 0 < x < 1, \\ w = w_0 \text{ and } \frac{\partial w}{\partial t} = w_1 \text{ at } t = 0, \\ w = 0 \text{ and } \frac{\partial w}{\partial x} = 0 \text{ at } x = 0, \\ \frac{\partial^2 w}{\partial x^2} = -M \text{ and } \left(\frac{\partial^3 w}{\partial x^3} - R \frac{\partial w}{\partial x}\right) = F \text{ at } x = 1, \end{array} \right. \quad (2.21)$$

where $R = (N + Q - P)$, β and γ are time-dependent. By making use of Eq. (2.21) and integrating by parts, we can obtain the energy equation

$$\begin{aligned} \dot{E}_t &= \Delta \dot{P} \int_0^1 \left(\frac{\partial w}{\partial x}\right)^2 dx - \beta(t) \int_0^1 \dot{w} \left(\frac{\partial w}{\partial x}\right) dx - \gamma(t) \int_0^1 \dot{w}^2 dx \\ &+ F \dot{v}_1(t) - M \dot{\theta}_1(t), \end{aligned} \quad (2.22)$$

where $\Delta \dot{P} = (\dot{Q} - \dot{P})$.

First we choose the control law (2.16) and (2.14) for (P, F, M) as in the previous case. Then, by applying some integral inequalities, Eq. (2.22) yields the following inequality:

$$\begin{aligned} \dot{E}_t &\leq (\Delta \dot{P} + \beta^2/2\gamma) \int_0^1 \left(\frac{\partial w}{\partial x}\right)^2 dx - \frac{\gamma}{2} \int_0^1 \dot{w}^2 dx \\ &- \mu \dot{v}_1^2 - \nu \dot{\theta}_1^2, \end{aligned}$$

which is negative if

$$\Delta \dot{P} + \beta^2/2\gamma < 0.$$

Thus it is sufficient to set

$$\dot{Q} \leq (\dot{P} - \beta^2/2\gamma - \delta) \text{ for any } \delta > 0. \quad (2.23)$$

Then

$$\begin{aligned}\dot{E}_t &\leq -\frac{\gamma}{2} \int_0^1 \dot{w}^2 dx - \delta \int_0^1 \left(\frac{\partial w}{\partial x}\right)^2 dx \\ &\quad - \mu \dot{v}_1^2 - \nu \dot{\theta}_1^2.\end{aligned}\tag{2.24}$$

Let $E_t^\varepsilon = E_t + \varepsilon G_t$ as before. In this case, one can verify that

$$\begin{aligned}\dot{E}_t^\varepsilon &\leq -\varepsilon E_t - \frac{\gamma}{2}(1 - \gamma\varepsilon/2\rho) \int_0^1 \dot{w}^2 dx - \delta \int_0^1 \left(\frac{\partial w}{\partial x}\right)^2 dx \\ &\quad - \left[\mu - \frac{1}{2}(1 + \mu^2)\varepsilon\right] \dot{v}_1^2 - \nu(1 - \nu\varepsilon) \dot{\theta}_1^2 \\ &\leq -\varepsilon E_t,\end{aligned}\tag{2.25}$$

provided that $0 < \varepsilon < \varepsilon_1 \leq 1$ and

$$\varepsilon_1 = \min\{1, 2\mu/(1 + \mu^2), 1/\nu, \max_{t \geq 0}[2\beta(t)/\gamma(t)]\}.\tag{2.26}$$

As in the free-vibration case, by noting (2.20), we conclude that

$$E_t \leq \left(\frac{1 + \varepsilon}{1 - \varepsilon}\right) E_0 e^{-\lambda t}, \quad \text{with } \lambda = \varepsilon/(1 + \varepsilon),$$

which shows the exponential decay of energy by the given boundary control.

We remark that

1. By a more careful examination of the conditions (2.14), (2.16) and (2.26) for stability, they imply that the flow velocity can only have weak fluctuation which tends to zero quickly as $t \rightarrow \infty$.
2. In lieu of (2.27), the exponential stability holds if

$$\max_{t \geq 0}\{|\Delta \dot{P}| + \beta^2/2\gamma\} = \varepsilon_2 < 1.\tag{2.27}$$

This condition means that both the rate of change of the difference ($Q - P$) and the flow velocity are small. Since the governing equation holds for a high speed flow, the condition is unrealistic.

3. In general, the boundary control as discussed here may not be sufficient to stabilize the system. A more active control is required for this purpose.

To illustrate the analytical results, we consider the controlled system (2.21) in the special form:

$$\left\{ \begin{array}{l} \frac{\partial^2 w}{\partial t^2} - R \frac{\partial^4 w}{\partial x^2} + \beta \frac{\partial w}{\partial x} + \gamma \frac{\partial w}{\partial t} = 0, \\ w(0, x) = \frac{1}{2}(1 - \cos 2\pi x) \text{ and } \frac{\partial w}{\partial t}(0, x) = 0, \\ w = 0 \text{ and } \frac{\partial w}{\partial x} = 0 \text{ at } x = 0, \\ \frac{\partial^2 w}{\partial x^2} = -\nu \frac{\partial^2 w}{\partial t \partial x} \text{ and } \frac{\partial^3 w}{\partial x^3} - R \frac{\partial w}{\partial x} = \nu \frac{\partial w}{\partial t} \text{ at } x = 1, \end{array} \right. \quad (2.28)$$

where

$$R = 6(1 - \nu_0^2) \int_0^1 \left(\frac{\partial w}{\partial x} \right)^2 dx. \quad (2.29)$$

The associated energy of the system is given by

$$E(t) = \frac{1}{2} \int_0^1 \left\{ \left(\frac{\partial w}{\partial t} \right)^2 + \frac{1}{2} R \left(\frac{\partial w}{\partial x} \right)^2 + \left(\frac{\partial^2 w}{\partial x^2} \right)^2 \right\} dx. \quad (2.30)$$

To show the effect of boundary damping on the panel vibration, the system energy was computed based on a modal expansion. Since the boundary condition in (2.28) is non-standard, the eigen-function expansion for the static problem cannot be used. Instead we approximate the deflection $w(t, x)$ by a truncated Fourier series as follows:

$$w(t, x) \sim \frac{1}{2} a_0(t) + \sum_{n=1}^N \{ a_n(t) \cos 2n\pi x + b_n(t) \sin 2n\pi x \}, \quad (2.31)$$

where $N > 1$ is a fixed integer. When the above series is substituted into Eqs. (2.28), we obtain a coupled system of $(2N + 1)$ ordinary differential equations of the form:

$$\begin{aligned} \dot{a}_n(t) &= f_N(a_0, a_1, \dots, a_N; b_1, \dots, b_N), a_n(0) = \alpha_n, n = 0, 1, \dots, N, \\ \dot{b}_n(t) &= g_N(a_0, a_1, \dots, a_N; b_1, \dots, b_N), b_n(0) = \beta_n, n = 1, 2, \dots, N, \end{aligned}$$

The above system was solved numerically by the 4th-order Runge-Kutta method for $N = 2, 3, 4, 5$. We found that for $N = 5$, the modal amplitudes a_n and b_n are numerically negligible for $n > 3$. Thus all the results to be shown were obtained by truncating the series (2.25) at $N = 3$. The corresponding system energy $E(t)$ given by (2.24) was evaluated at $\nu_0 = 0.33$ and various other parameter values. The numerical results were summarized and displayed in Figs. 4–10, which exhibit the evolutions of the vibrational

energy levels with and without boundary damping (control). In the case of free vibration ($\beta = \gamma = 0$), Fig. 4 shows that, when $\mu = \nu = 0$ (without control) the energy is at the constant level of 130 and, when $\mu = \nu = 1.5$ (with control), it does indeed decay exponentially, in agreement with the theoretical prediction. For the ease of visualization, in the sequent figures, the results, with flow excitations, for the uncontrolled and the controlled cases were plotted separately. In Fig. 5 with flow parameters $\beta = 50$ and $\gamma = 0$ (without aerodynamic damping), the energy level oscillates periodically when there is no boundary damping. By contrast, given the control parameters $\mu = 2.5$ and $\nu = 1.5$. Fig. 6 shows the exponential decay of the system energy. Corresponding to Figs. 5 and 6, similar results are displayed in Figs. 7 and 8 when the flow damping parameter value was changed slightly from $\gamma = 0$ to $\gamma = 0.05$. The most drastic effect of the boundary damping is shown by Figs. 9 and 10 where the flow speed is high ($\beta = 500$) but the flow damping $\gamma = 0$. Without control, the energy level oscillates very rapidly, as seen from Fig. 9. This highly oscillatory state of energy can be reduced to an over-damped state, as depicted in Fig. 10, by introducing a boundary damping with control parameters $\mu = 14$ and $\nu = 6$.

3. Optimal Control of Linear Panel Vibration

Let us consider a viscous flow past over the elastic panel. The flow is governed by the well know Navier-Stokes equation:

$$\frac{\partial \bar{u}}{\partial t} + (\bar{u} \cdot \nabla) \bar{u} = -\frac{1}{\rho} \nabla p + \nu \Delta \bar{u}, \quad (3.1)$$

where the notations are standard. For a slightly compressible flow, the continuity equation reads

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \bar{u}) = 0. \quad (3.2)$$

The panel is regarded as an elastic plate with thickness h , Young's modulus E and Poisson's ratio γ . Under a uniform tension with $T > 0$ (or compression with $T < 0$) and the fluctuating wall pressure, the vertical displacement ζ of the plate satisfies the following equation:

$$\rho_w \frac{\partial^2 \zeta}{\partial t^2} = T \Delta \zeta - D \Delta^2 \zeta + p_w + q(\bar{x}, t) \quad (3.3)$$

where ρ_w is the plate density, p_w the wall pressure fluctuation, and q is the applied force as the active control. The constant D is the stiffness of the plate defined by

$$D = \frac{Eh^3}{12(1 - \gamma^2)}. \quad (3.4)$$

According to the boundary-layer theory, given an upstream velocity field \bar{U} , the flow near the plate can be determined by the Prandtl's approximation. In particular, if the panel is located on the $x - y$ plane, the pressure gradient $\frac{\partial p}{\partial z}$ across the boundary-layer is nearly constant, where $\bar{x} = (x, y, z)$. Suppose that the mean flow outside the boundary-layer is parallel to the plate so that $\bar{U} = (U_\infty, 0, 0) + \bar{U}_1(\bar{x}, t)$, where \bar{U}_1 is a small perturbation. To derive the equations for acoustic quantities \bar{u}_1, p_1 and ρ_1 , we let

$$\bar{u} = \bar{u}_0 + \bar{u}_1, p = p_0 + p_1 \quad \text{and} \quad \rho = \rho_0 + \rho_1 \quad (3.5)$$

where \bar{u}_0, p_0 and ρ_0 are flow variables associated with the mean flow. As in the stability analysis, we introduce a parallel flow approximation. Then, in view of (3.5), one obtains the acoustic equations from (3.1) and (3.2) by linearization:

$$\frac{\partial \bar{u}_1}{\partial t} + (\bar{u}_0 \cdot \nabla) \bar{u}_1 = -\frac{1}{\rho_0} \nabla p_1, \quad (3.6)$$

$$\frac{\partial \rho_1}{\partial t} + \nabla \cdot (\rho_0 \bar{u}_1 + \rho_1 \bar{u}_0) = 0. \quad (3.7)$$

For an isentropic flow, ρ_1 and p_1 are related by

$$\rho_1 = p_1/c^2, \quad (3.8)$$

where c is the speed of sound for the unperturbed flow. Aside from a static displacement, the vibration of the panel is described by the perturbation w of equation (3.3) as follows:

$$\rho_w \frac{\partial^2 w}{\partial t^2} = T \Delta w - D \Delta^2 w + f + q(\bar{x}, t), \quad (3.9)$$

where $f = \tilde{p}_w$ is the fluctuating part of the wall pressure excited by the unsteady boundary-layer flow. The coupling of the acoustic equations (3.6) and (3.7), and the plate equation is through the boundary conditions. For the plate equation (3.9), since

it is simply supported by a periodic structure, we need only to analyze the problem over a fundamental domain $0 \leq x \leq a, 0 \leq y \leq b$ and impose the boundary conditons:

$$w(x, y, t) = 0 \text{ at } x = 0, a; y = 0, b. \quad (3.10)$$

Since the pressure gradient $\frac{\partial p}{\partial z} = 0$ across the boundary, the wall pressure \tilde{p}_w can be determined from the perturbed potential flow field \bar{U}_1 through an approximate Euler's equation, that is,

$$\tilde{p}_w = F(\bar{U}_1). \quad (3.11)$$

To counter this excitation, a control force $q(x, y, t)$ was introduced in (3.9). The objective of the active control is to minimize the average vibrational energy and the control cost:

$$J(q) = \frac{1}{2T} \int_0^T \int_D \left\{ \alpha \left(\frac{\partial w}{\partial t} \right)^2 + \beta (\Delta w)^2 + \gamma |\nabla w|^2 + k q^2 \right\} dt dx dy, \quad (3.12)$$

where the time T may be infinite, \mathcal{D} is the basic domain $\{0 \leq x \leq a, 0 \leq y \leq b\}$; α, β, γ and k are positive constants. In the language of the optimal control of a distributed parameter system, the equation (3.9) is known as the equation of state and $J(q)$ defined by (3.12), the objective or cost functional. Here the physical problem of vibrational control reduces to an optimization problem: Given the wall pressure excitation \tilde{p}_w , find an optimal control $q^*(x, y, t)$ from a certain admissible class Q of functions which minimizes the objective functional $J(q)$, that is,

$$J(q^*) = \min\{J(q), q \text{ in } Q\}. \quad (3.13)$$

To obtain an analytical solution, we consider the case of simply supported boundary conditions:

$$w(x, y, t) = 0 \text{ at } x = 0, a; y = 0, b,$$

$$\frac{\partial^2 w}{\partial x^2}(x, y, t) = 0 \text{ at } x = 0, a \text{ and } \frac{\partial^2 w}{\partial y^2}(x, y, t) = 0 \text{ at } y = 0, b. \quad (3.14)$$

The initial conditions are given by

$$w(x, y, 0) = w_0(x, y), \quad \frac{\partial w}{\partial t}(x, y, 0) = w_1(x, y). \quad (3.15)$$

It is well known that the set of functions

$$\varphi_{mn}(x, y) = 2 \sin \frac{m\pi}{a}x \sin \frac{n\pi}{b}y, \quad m, n = 1, 2, \dots \quad (3.16)$$

are orthogonal eigenfunctions associated with the plate equation (3.9) and the corresponding eigenvalues are

$$\lambda_{mn} = T\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right] + D\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]^2. \quad (3.17)$$

In terms of the above eigenfunctions, we can expand the displacement w , the wall pressure f and the control as follows:

$$w(x, y, t) = \sum_{m,n=1}^{\infty} w_{m,n}(t)\varphi_{mn}(x, y), \quad (3.18)$$

$$f(x, y, t) = \sum_{m,n=1}^{\infty} f_{m,n}(t)\varphi_{mn}(x, y) \quad (3.19)$$

and

$$q(x, y, t) = \sum_{m,n=1}^{\infty} q_{m,n}(t)\varphi_{mn}(x, y), \quad (3.20)$$

where the coefficients w_{mn} etc. are computed by

$$w_{mn} = \langle w, \varphi_{mn} \rangle = \int_0^a \int_0^b w(x, y, t)\varphi_{mn}(x, y) dx dy,$$

and so on. A substitution of the expansions (3.18)-(3.20) into the equations (3.9), (3.15) and (3.12) yields the following uncoupled system of equations:

$$\begin{cases} \rho_w \ddot{w}_{mn} + \lambda_{mn} w_{mn} = f_{mn}(t) + q_{mn}(t), \\ w_{mn}(0) = w_{0,mn}, \quad \dot{w}_{mn}(0) = w_{1,mn} \end{cases} \quad (3.21)$$

and

$$J(q) = \sum_{m,n=1}^{\infty} J_{mn}(q), \quad (3.22)$$

where

$$J_{mn}(q) = \frac{1}{2T} \int_0^T \{\alpha \dot{w}_{mn}^2(t) + \mu_{mn} w_{mn}^2(t) + k q_{mn}^2(t)\} dt \quad (3.23)$$

for $m, n = 1, 2, \dots$. Since the modes are uncoupled, if the cost J_{mn} for each mode is minimized, so does the total cost J .

For a given (m, n) -mode, dropping all the subscripts, we are led to consider the so-called “linear regulator” problem in optimal control: Find the control q in the equation

$$\begin{cases} \rho \ddot{w} + \lambda w = f(t) + q(t) , \\ w(0) = w_0, \dot{w}(0) = w_1 , \end{cases} \quad (3.24)$$

which minimizes

$$J(q) = \frac{1}{2T} \int_0^T \{ \alpha \dot{w}^2 + \mu w^2 + kq^2 \} dt , \quad (3.25)$$

where μ_{mn} is given as in λ_{mn} with D and T replaced by β and γ , respectively. By the method of adjoint state,^[5] for the cost to be minimal, the state w and its adjoint v must satisfy the optimality system:

$$\begin{cases} \rho \ddot{w} + \lambda w = f(t) + \frac{1}{k}(\alpha w - v) , \\ w(0) = w_0, \dot{w}(0) = w_1 , \end{cases} \quad (3.26)$$

and

$$\begin{cases} \rho \ddot{v} + \lambda v = (\alpha \lambda + \mu)w , \\ v(T) = \dot{v}(T) = 0 . \end{cases} \quad (3.27)$$

The optimal control q^* is given by $q = \frac{1}{k}(\alpha w - v)$. One notes that, due to the coupling between w and v , the above system (3.26)–(3.27) is a two-point boundary-value problem. Numerically it can be solved by the shooting method. Some numerical results for the original modal equations (3.21) and (3.23) have been obtained.

For example, we choose $a = 4\pi, b = \pi, w_0 = w_1 \equiv 0$ and $T = 4$, and set

$$f_{mn}(t) = \frac{1}{(m^2 + n^2)} \cos(m^2 + n^2)^{1/2}t, \quad m, n = 1, 2, \dots$$

All the parameters are taken to be one except for β , which is zero. The maximal amplitude of vibration under a optimal control has been computed and some results, corresponding to 4 modes ($m + n = 4$), are shown in Fig. 11 to Fig. 13. In the above figures, the solid curves represent the controlled amplitudes, which are in contrast with the uncontrolled ones. It is seen that the control is very effective in reducing the vibration amplitudes. For an independent interest, the controlled mode shape at $t = 4$

is plotted as shown in Fig. 14.

4. Optimal Control of Nonlinear Panel Vibration

Similar to Eq. (2.1), under a tension T and without flow excitation, a simplified model of the control of nonlinear panel vibration, in lieu of Eq. (3.9), is given by

$$m \frac{\partial^2 w}{\partial t^2} = (T + N) \frac{\partial^2 w}{\partial x^2} - D \frac{\partial^4 w}{\partial x^4} + f(x, t) + q(x, t), 0 < x < a, \quad (4.1)$$

with $w(x, 0) = g(x)$, and $\frac{\partial w}{\partial t}(x, 0) = h(x)$, where, as before,

$$N = \frac{Eh}{2a} \int_0^a \left(\frac{\partial w}{\partial x} \right)^2 dx. \quad (4.2)$$

For a simply supported beam, we have

$$w = 0 \text{ and } \frac{\partial^2 w}{\partial x^2} = 0 \text{ at } x = 0, a \quad (4.3)$$

while, for a clumped beam, the following holds

$$w = 0 \text{ and } \frac{\partial w}{\partial x} = 0 \text{ at } x = 0, a. \quad (4.4)$$

Other boundary conditions are possible. In the subsequent analysis, we consider the simply supported case (4.3) only. The objective is to choose an optimal control $q^*(x, t)$ which minimizes the following cost function:

$$J(q) = \frac{1}{2T_0} \int_0^{T_0} \int_0^a \left\{ \alpha \left(\frac{\partial w}{\partial t} \right)^2 + \beta \left(\frac{\partial^2 w}{\partial x^2} \right)^2 + w^2 + kq^2 \right\} dt dx. \quad (4.5)$$

where α , β and k are positive constants.

The necessary condition for minimum is the vanishing of the variation δJ of J , or

$$\delta J(q) = \frac{1}{T} \int_0^T \int_0^a \left\{ \alpha \left(\frac{\partial w}{\partial t} \right) \left(\frac{\partial \delta w}{\partial t} \right) + \beta \left(\frac{\partial^2 w}{\partial x^2} \right) \left(\frac{\partial^2 \delta w}{\partial x^2} \right) + w \delta w + kq \delta q \right\} dt dx = 0, \quad (4.6)$$

where δw and δq denote the variations of w and q , respectively. Note that, by taking the variation of equations (4.1) and (4.3), we can relate δw to δq by the variational equation:

$$\begin{aligned} M \delta w = \rho \frac{\partial^2 \delta w}{\partial t^2} &= (T + N_t) \frac{\partial^2 \delta w}{\partial x^2} + D \frac{\partial^4 \delta w}{\partial x^4} - (\delta N_t) \frac{\partial^2 w}{\partial x^2} = \delta q, \\ \delta w &= \frac{\partial}{\partial t} \delta w = 0 \text{ at } t = 0, \\ \delta w &= \frac{\partial^2}{\partial x^2} \delta w = 0 \text{ at } x = 0, a, \end{aligned} \quad (4.7)$$

where

$$\begin{aligned}\delta N_t &= \gamma \int_0^a \left(\frac{\partial w}{\partial x}\right) \left(\frac{\partial \delta w}{\partial x}\right) dx, \\ \text{with } \gamma &= \frac{Eh}{a}.\end{aligned}\tag{4.8}$$

Upon eliminating δq from Eq. (4.6) by Eq. (4.7) and noting Eq. (4.1), it yields

$$\delta J = \frac{1}{T} \int_0^T \int_0^a \left\{ \left(-\alpha \frac{\partial^2 w}{\partial t^2} + \beta \frac{\partial^4 w}{\partial x^4} + w\right) \delta w + kqM\delta w \right\} dt dx = 0,$$

which implies, after integrating by parts several times, that the optimal control q must satisfy the following equation.

$$\begin{aligned}M^* q &= m \frac{\partial^2 q}{\partial t^2} - (T + N) \frac{\partial^2 q}{\partial x^2} + D \frac{\partial^4 q}{\partial x^4} + \gamma \frac{\partial^2 w}{\partial x^2} \int_0^a q \frac{\partial^2 w}{\partial x^2} dx \\ &= \frac{1}{k} \left(\alpha \frac{\partial^2 w}{\partial t^2} - \beta \frac{\partial^4 w}{\partial x^4} - w \right) \equiv Aw,\end{aligned}\tag{4.9}$$

and the terminal - boundary condition:

$$\begin{aligned}q &= 0 \text{ and } \frac{\partial q}{\partial t} + \frac{\partial}{\rho k} \frac{\partial w}{\partial t} = 0 \text{ at } t = T, \\ q &= \frac{\partial^2 q}{\partial x^2} = 0 \text{ at } x = 0, a.\end{aligned}\tag{4.10}$$

The equations (4.1) and (4.9) together with the conditions (4.3) or (4.4) and (4.10) form the optimality system. A peculiar feature of the system is that the state w is coupled to the control q which satisfies a terminal condition. This has caused some difficulty in computing the optimal solution.

To solve the system, we proceed by applying the Galerkin method of approximation. For the simply supported case (4.3), we may use the following set of admissible function

$$\varphi_n(x) \sqrt{\frac{2}{a}} \sin \frac{n\pi}{a}, n = 1, 2, \dots\tag{4.11}$$

as an orthonormal basis. Now we expand w and q as follows:

$$\begin{aligned}w(x, t) &= \sum_{n=1}^{\infty} w_n(t) \varphi_n(x), \\ q(x, t) &= \sum_{n=1}^{\infty} q_n(t) \varphi_n(x).\end{aligned}\tag{4.12}$$

Substituting the series (4.12) into equs. (4.1) and (4.9) yields

$$m\ddot{w}_n(t) + [(T + Q) + D\lambda_n^2]\lambda_n^2 w_n = f_n(t) + q_n(t), \quad (4.13)$$

$$\begin{aligned} m\ddot{q}_n(t) + [(T + Q) + D^2\lambda_n^2]\lambda_n^2 q_n \\ + \gamma \left(\sum_{m=1}^{\infty} \lambda_m^2 w_m q_m \right) \lambda_n^2 w_n = \frac{1}{k} (\alpha \ddot{w}_n - \beta \lambda_n^4 w_n - w_n), n = 1, 2, \dots, \end{aligned}$$

$$w_n(0) = g_n, \dot{w}_n(0) = h_n, \quad (4.14)$$

$$q_n(T) = 0, \dot{q}_n(T) + \frac{\partial}{\partial k} \dot{w}_n(T) = 0,$$

where $\dot{w}_N = \frac{d}{dt} w_n$, $f_n = \int_0^a f \varphi_n dx$, etc; $\lambda_n = \frac{n\pi}{a}$ and

$$Q = \sum_{m=1}^{\infty} \lambda_m^2 w_m^2.$$

By a modal truncation, the above infinite coupled system can be reduced to a finite system as follows:

$$m\ddot{w}_{N,n} + [(T + Q_N) + D\lambda_n^2]\lambda_n^2 w_{N,n} = f_n + q_n, \quad (4.15)$$

$$\begin{aligned} m\ddot{q}_{N,n} + [(T + Q_N) + D\lambda_n^2]\lambda_n^2 q_{N,n} + \lambda_n^2 w_{N,n} \left(\sum_{m=1}^N \lambda_m^2 w_{N,m} q_{N,m} \right) \\ = \frac{1}{k} [\alpha \ddot{w}_{N,n} - (1 + \beta \lambda_n^4) w_{N,n}], n = 1, 2, \dots, N, \end{aligned}$$

which are subject to the conditions as in (14), where Q_N is N -term truncation of Q .

The truncated system (4.15) can be solved numerically. The numerical solution of the truncated problem poses two technical difficulties: the large scale in computation and resulting two-point boundary values given in time domain. As a result, we can only handle a small set of modal equations. To be computationally efficient, we adopted the so called shooting method. The method consists of solving the boundary value problem as an initial value problem by assigning the missing initial data and then adjusting the data by iterations until the end point conditions are met. The iteration procedure is based on a fixed point algorithm in locating the zeros of a function.

The numerical computation has been carried out in a SUN workstation. The results confirmed the theoretical prediction that the feedback control can drastically reduce the panel fluttering and it is more effective in suppressing the lower frequency vibrations. As

an illustration, by suitably choosing the parameter values, some results for a three mode calculation are shown in Figs. 11–13 under a time-harmonic wall pressure fluctuation. For the given numerical results, we set $N = 3$, $f_i(t) = (\sin t/2 + \cos t/2)/3i$, and the parameter values: $m = 1$, $\alpha = 0.1$, $\beta = 1$, $\gamma = 0.01$, $D = 0.02$, $k = 1$. The domain of computation is given by $0 \leq x \leq 4$ and $0 \leq t \leq 4$. The amplitudes of the first three modes Z_1, Z_2 and Z_3 are plotted in Figs. 11–13, where the solid curves depict the amplitudes for the controlled system in contrast with the chain-like curves for the uncontrolled case. The reduction in the modal vibration amplitudes is very remarkable. Consequently, if we consider the sound generated by the vibrating panel, the sound radiation power or the aero-acoustic noise level will be reduced drastically by the active control.

5. Conclusions

Several problems in the control of panel vibration due to the flow excitation were studied. The main research results are presented in the previous three sections. The problems were solved analytically and numerically. Based on these results, we can draw the following conclusions:

In the boundary control of nonlinear panel vibration, by means of the energy method and some mathematical inequalities, the boundary stabilization of a vibrating nonlinear elastic panel was studied. The panel is clamped at one edge and free to vibrate at the other edge. In general, the panel is subject to a compressive in-plane loading combined with an aerodynamic forcing. Without any control, the panel would flutter due to flow induced instability. To stabilize the panel, a boundary control was introduced as the combination of a bending moment, a vertical point force and a tensile force applied to the free edge. Two cases, corresponding to the absence and the presence of an aerodynamic loading, were treated separately. For no flow, this is the case of free vibration. Even though the energy of the uncontrolled system is conserved, with initial disturbance, the system may buckle or sustain a persistent large-amplitude oscillation. To render the energy an exponential decay, it was found sufficient to apply a tensile force, if necessary, to reduce the net force to a subcritical level and, at the same time, to introduce a boundary damping. The damping mechanism consists of a pair of frictional force and

torgue, which are linearly proportional to the transverse and the angular velocities of the right edge, respectively. Therefore, if the compressive force is subcritical, the passive control in the form of a boundary damping suffices to stabilize the system. In an analogous situation, the result seems to be in agreement with the experimental evidence that a boundary damping is effective in suppressing the panel vibration [6]. In the presence of unsteady flow and compressive force, the panel is subject to flow excitation. If the flow velocity is oscillatory and decays rapidly, it is possible to stabilize the panel by applying a time varying tensile force $Q(t)$ together with a boundary damping as before. But the control force Q must follow the flow fluctuation closely. For this reason, such scheme is not robustic. However, for slowly varying compressive force and, at the same time, small flow parameter β^2/γ , the system can be stabilized as in the free-vibration case. It is believed that, when the flow parameter is large, the boundary control may not be sufficient for stabilization and a stronger mode of control, such as a distributed control, will be required for this purpose. The preliminary results of this results were summarized in a paper which was published in Recent Advances in Active Control of Sound and Vibration, Volume 2, [7]. The full paper [8] containing the analytical and numerical results was accepted by the Journal of Sound for publication and Vibration and will appear shortly.

In the control of both linear and nonlinear panel vibrations, we investigated some optimal control problems with and without flow excitations. For the linear problems, the vibration control of a simply - supported rectangular plate was treated. The control objective is to minimize the objective function, which is the sum of the vibrational energy and the cost of control. For the nonlinear control problem, a simplified panel equation similar to the boundary control problem was used. By the optimal control theory for distributed parameter systems, the problems can be solved by deriving the optimality equations for the adjoint states. These equations coupled with the dynamical equations for the panels must be solved to yield the optimal control forces. For approximate solutions, we adopted the Galerkin method or the modal expansion to reduce the governing partial differential equations to a finite system of ordinary differential equations by truncation. For the linear elastic plate, the control of each modal ampli-

tude reduced to the so-called linear regular problem which can be solved with relative ease. For the nonlinear control problem, the modal equations and their adjoint-state equations are nonlinearly coupled and the solutions are difficult to obtain. However numerical solutions were carried out. The numerical results show clearly that the control is very effective in suppressing the panel vibration and the sound generation by the elastic panel. The results will be summarized in one or two papers to be submitted for publication.

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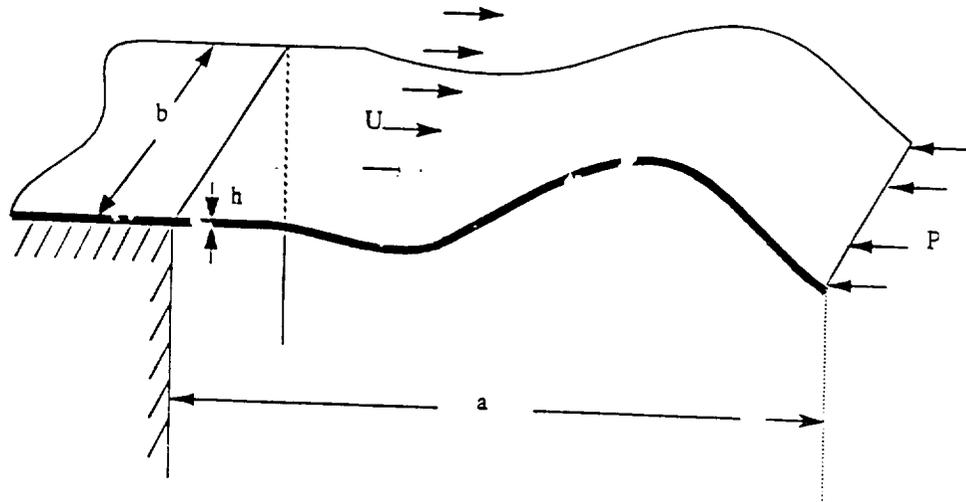


Figure 1: Panel Vibration with Flow Excitation.

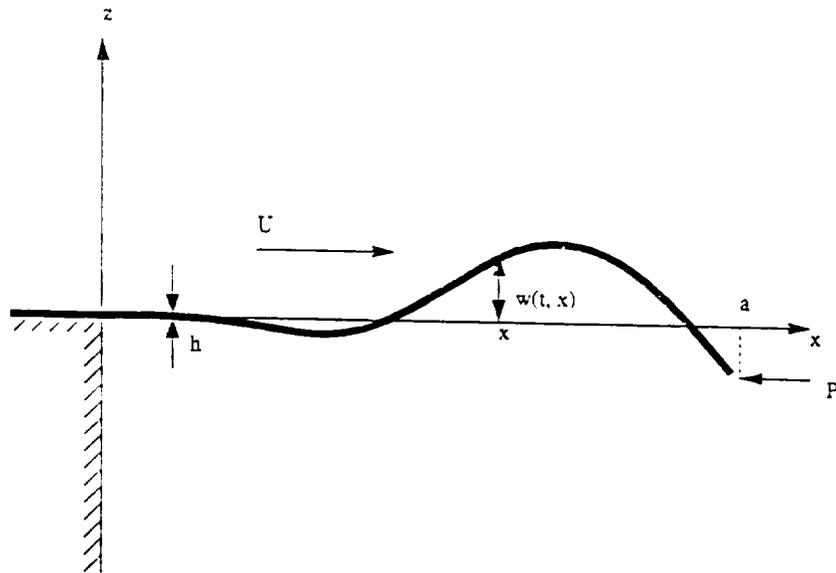


Figure 2: Schematic Diagram of Panel Vibration with Flow Excitation.

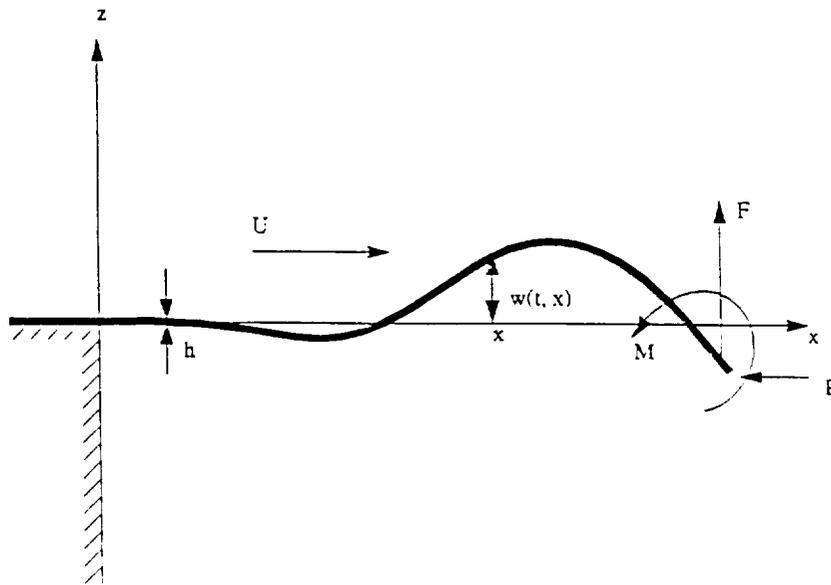


Figure 3 : Schematic Diagram of Controlled Panel with Flow Excitation.

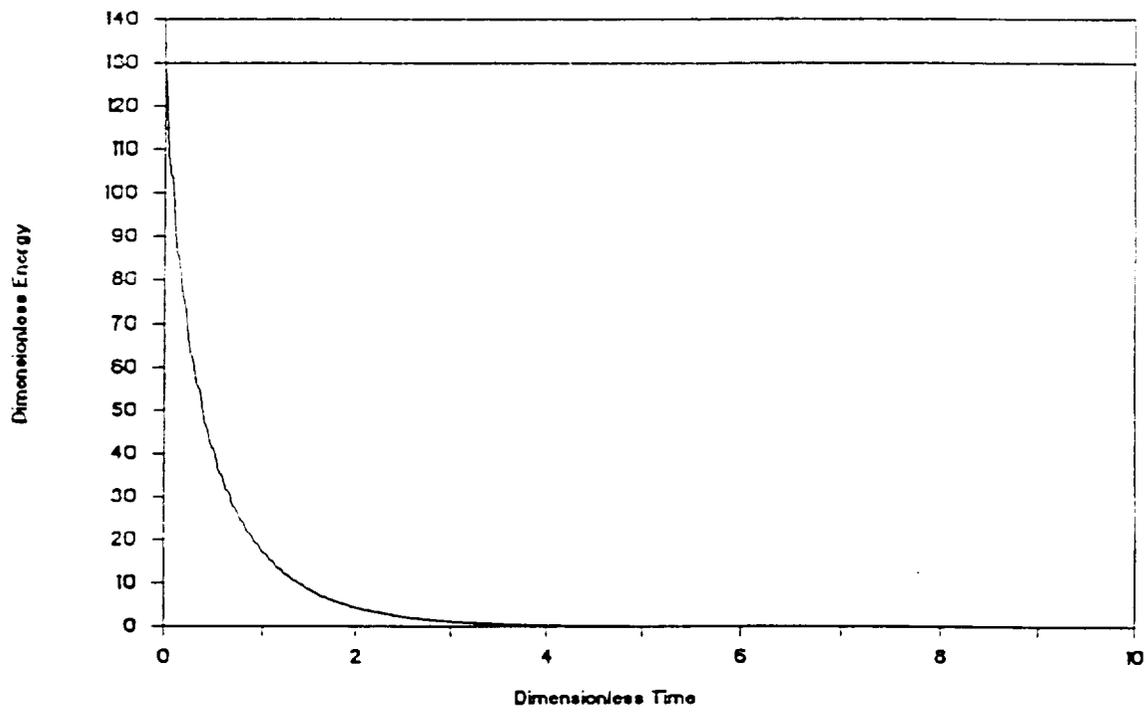


Figure 4: flow parameters: $\beta = 0, \gamma = 0$, uncontrolled and controlled with $\mu = 1.5, \nu = 1.5$

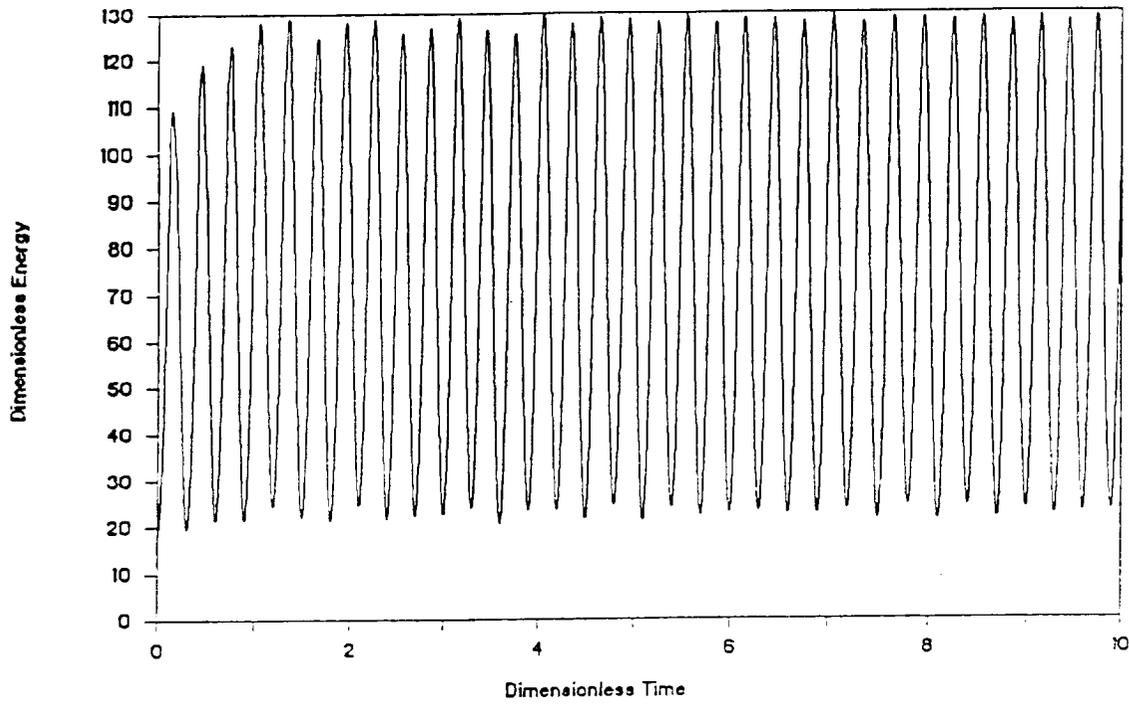


Figure 5: flow parameters: $\beta = 50, \gamma = 0$, uncontrolled

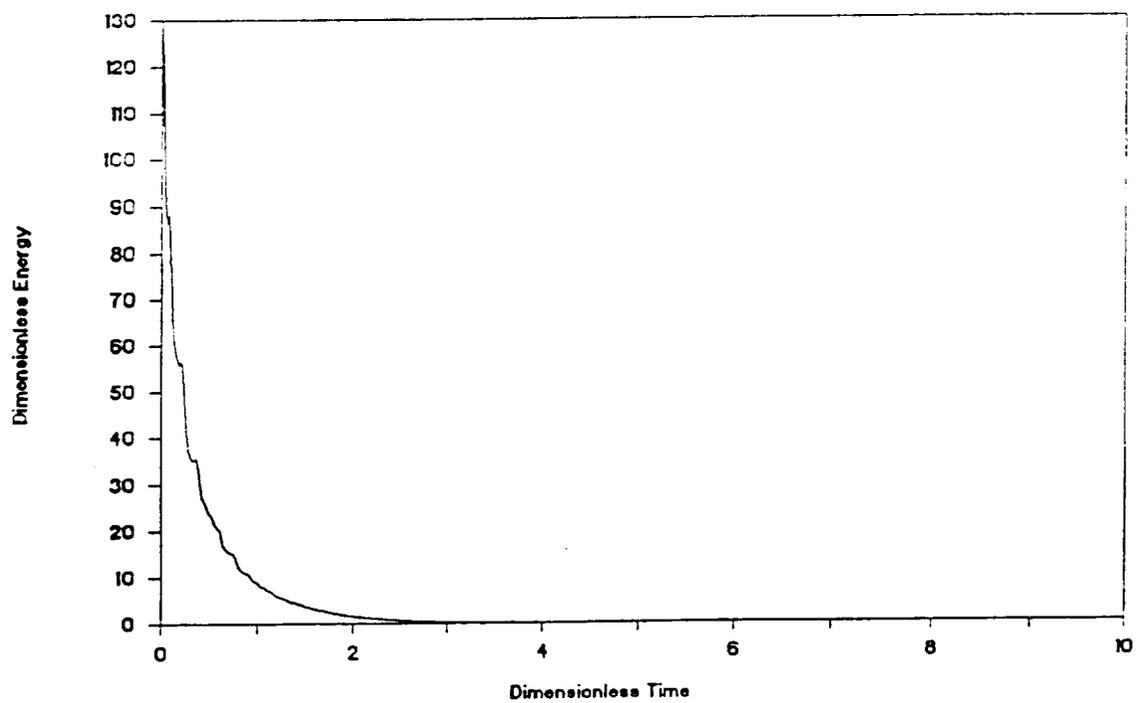


Figure 6: flow parameters: $\beta = 50, \gamma = 0$, control parameters: $\mu = 2.5, \nu = 1.5$

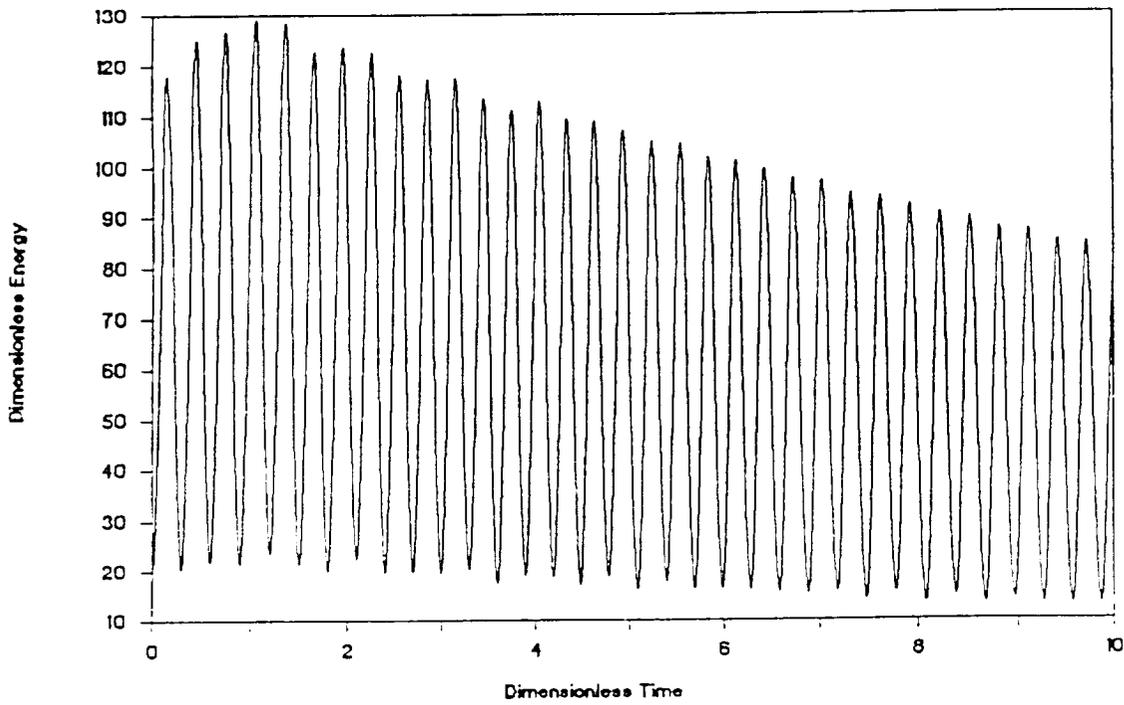


Figure 7: flow parameters: $\beta = 50, \gamma = 0.05$, uncontrolled

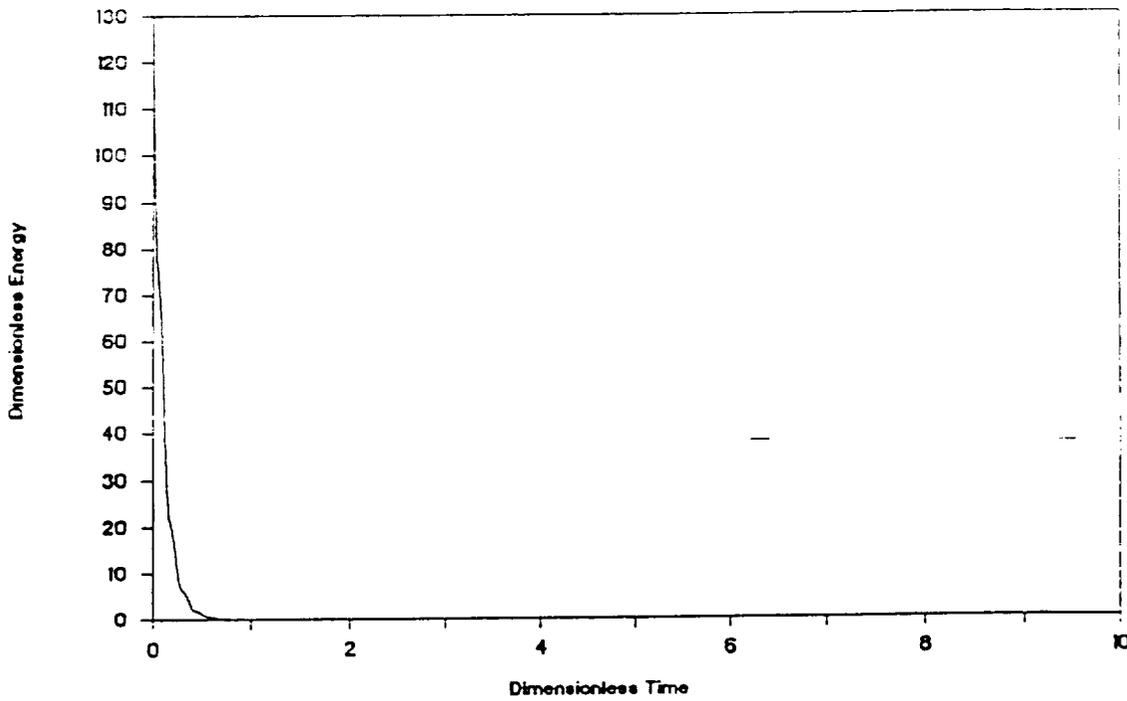


Figure 8: flow parameters: $\beta = 50, \gamma = 0.05$, control parameters: $\mu = 1.5, \nu = 1.5$

Time History of Energy

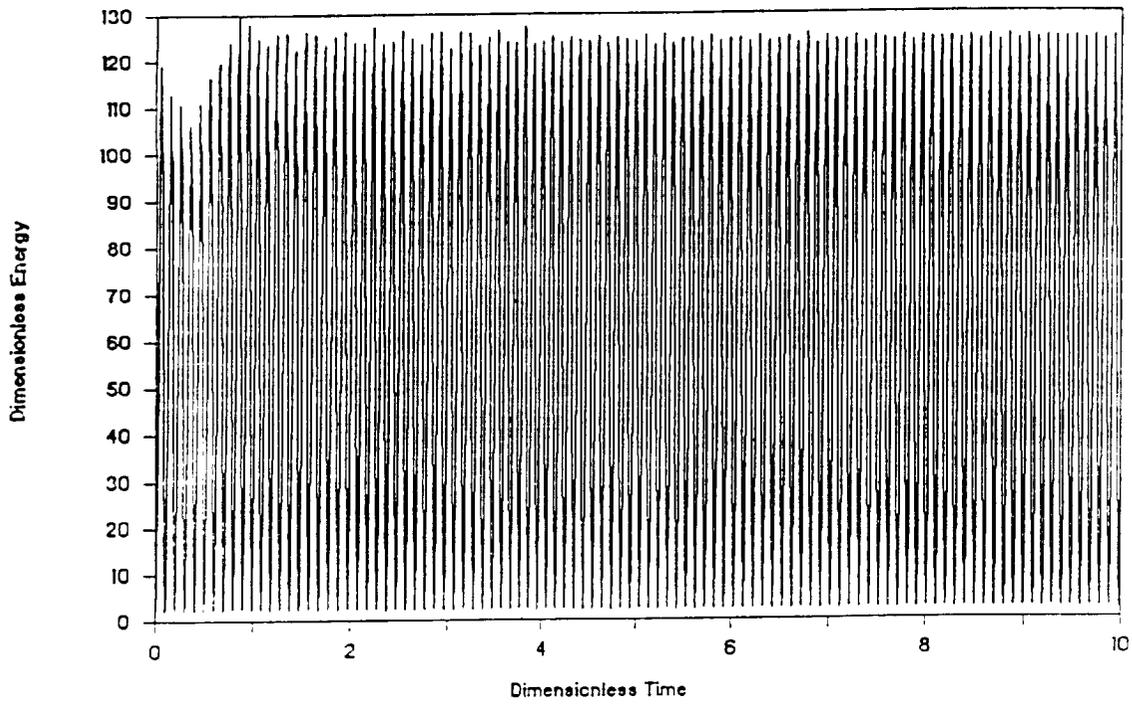


Figure 9: flow parameters: $\beta = 500, \gamma = 0$, uncontrolled

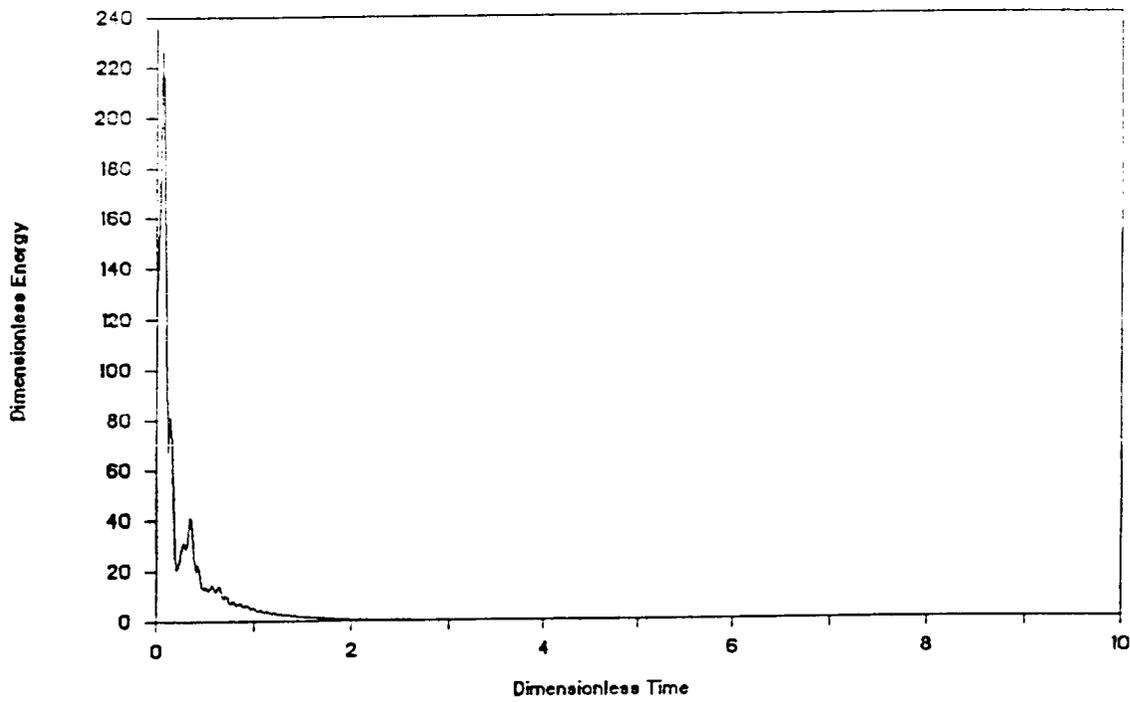


Figure 10: flow parameters: $\beta = 500, \gamma = 0$, control parameters: $\mu = 13.0, \nu = 6.0$

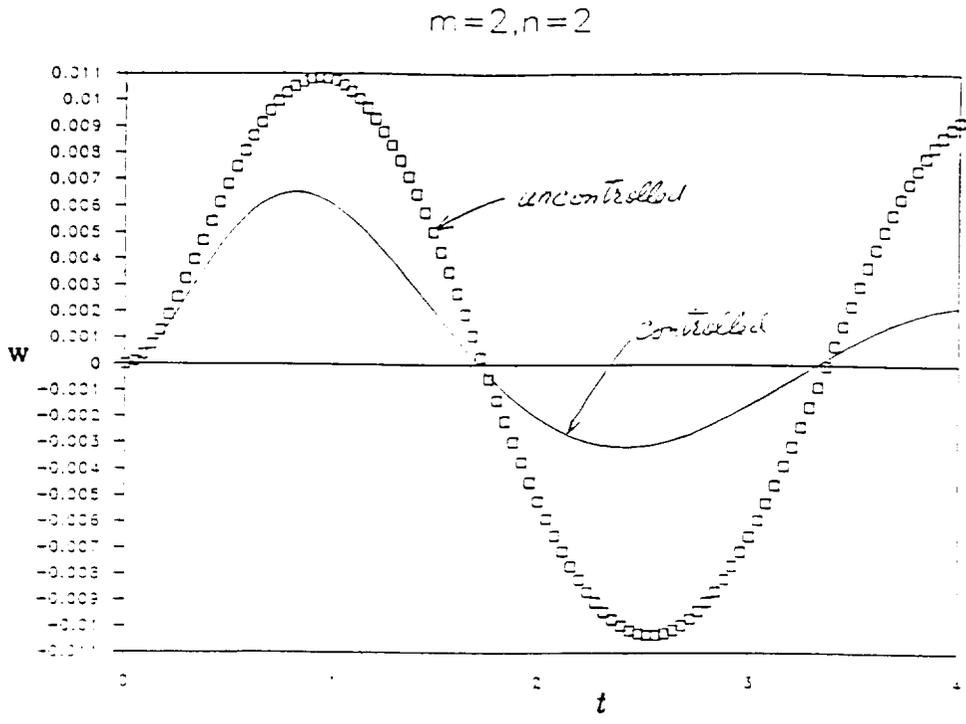


Figure 11: Amplitudes of controlled and uncontrolled vibration at the center of the plate with $m = 2, n = 2$ and $T = 4$.

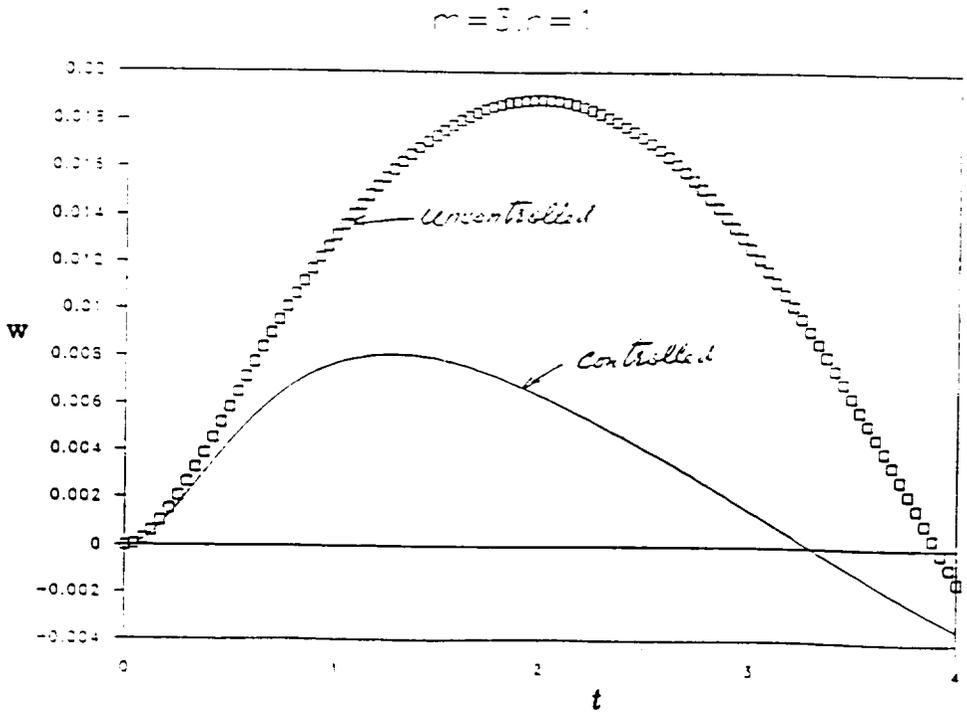


Figure 12: Amplitudes of controlled and uncontrolled vibration at the center of the plate with $m = 3, n = 1$ and $T = 4$.

$$m=1, n=3$$

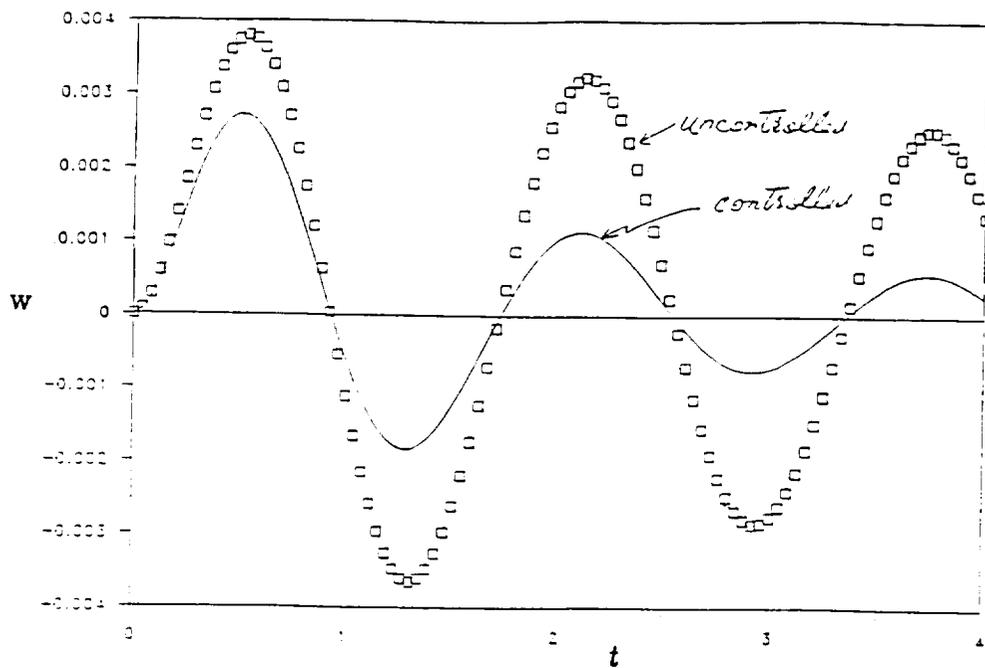


Figure 13: Amplitudes of controlled and uncontrolled vibration at the center of plate with $m = 1, n = 3$ and $T = 4$.

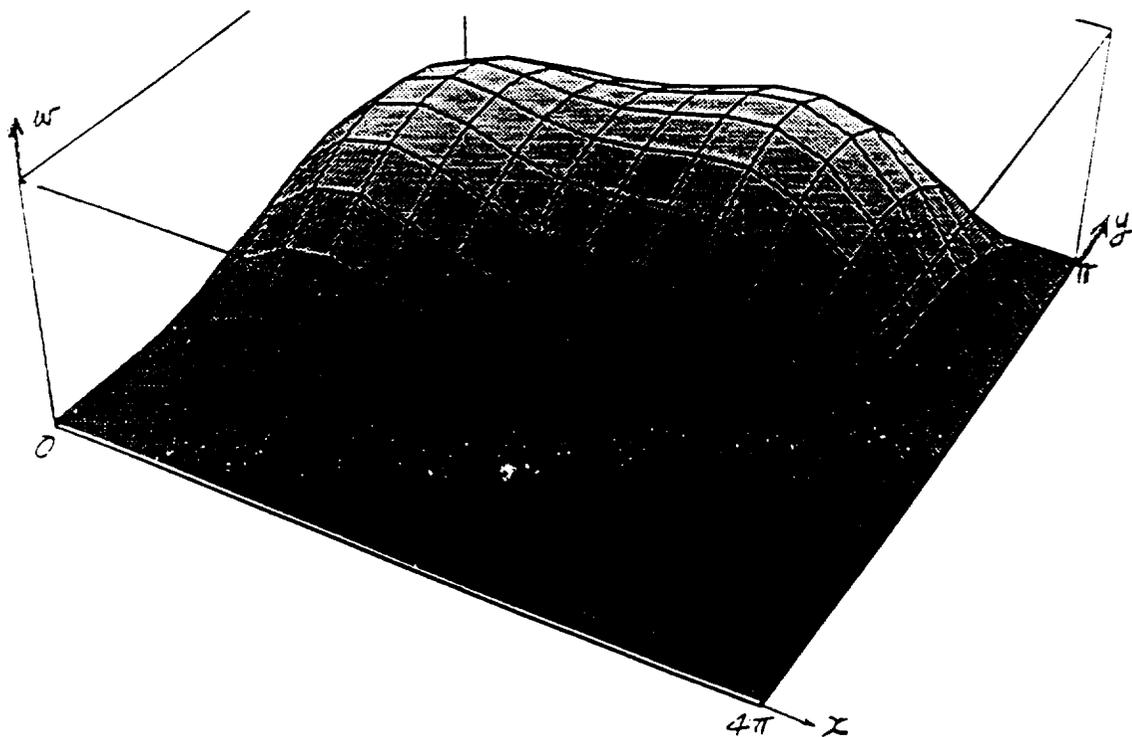


Figure 14: The uncontrolled mode shape with $m = 2, n = 2$ of $T = 4$.

Coefficients I (m=3)

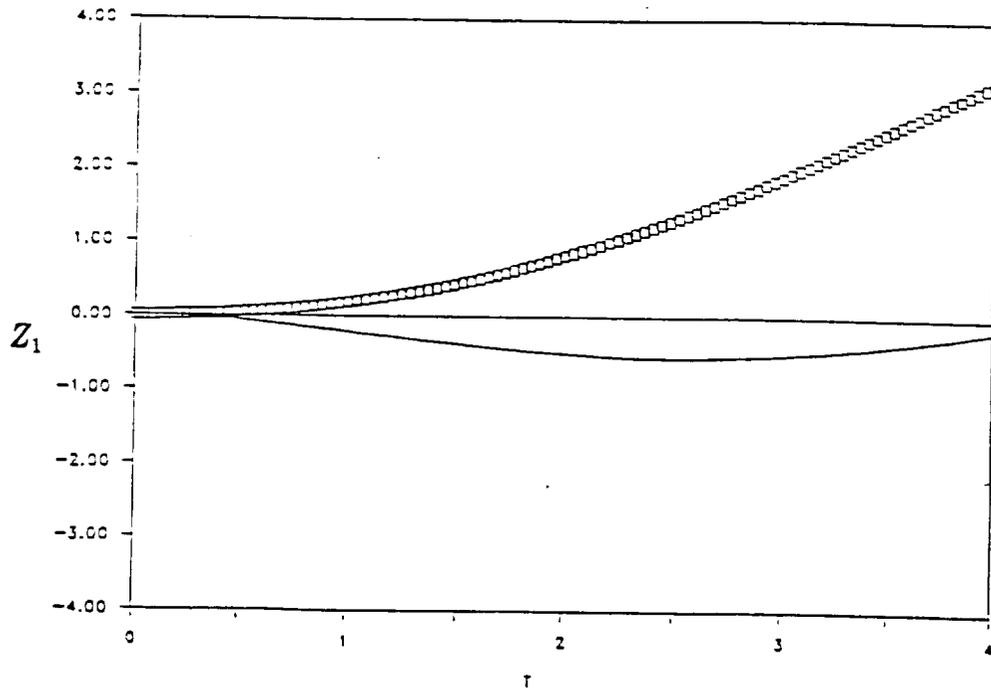


Figure 15: The controlled and uncontrolled amplitudes of the first mode at the center with $m = 3$ and $T = 4$.

Coefficients II (m=3)

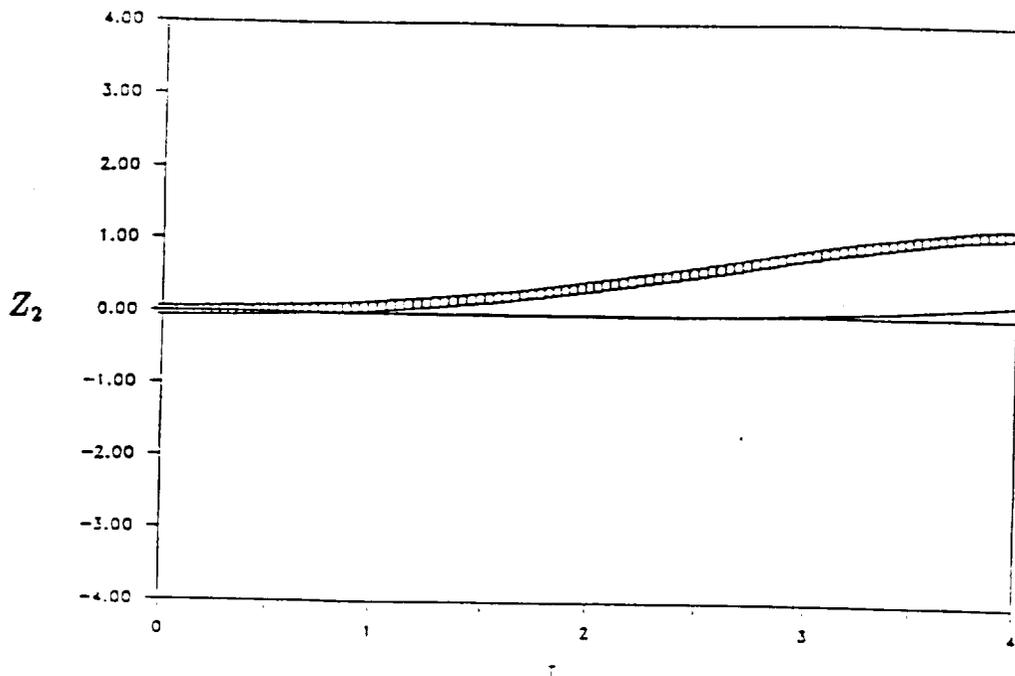


Figure 16: The controlled and uncontrolled amplitudes of the second mode at the center with $m = 3$ and $T = 4$.

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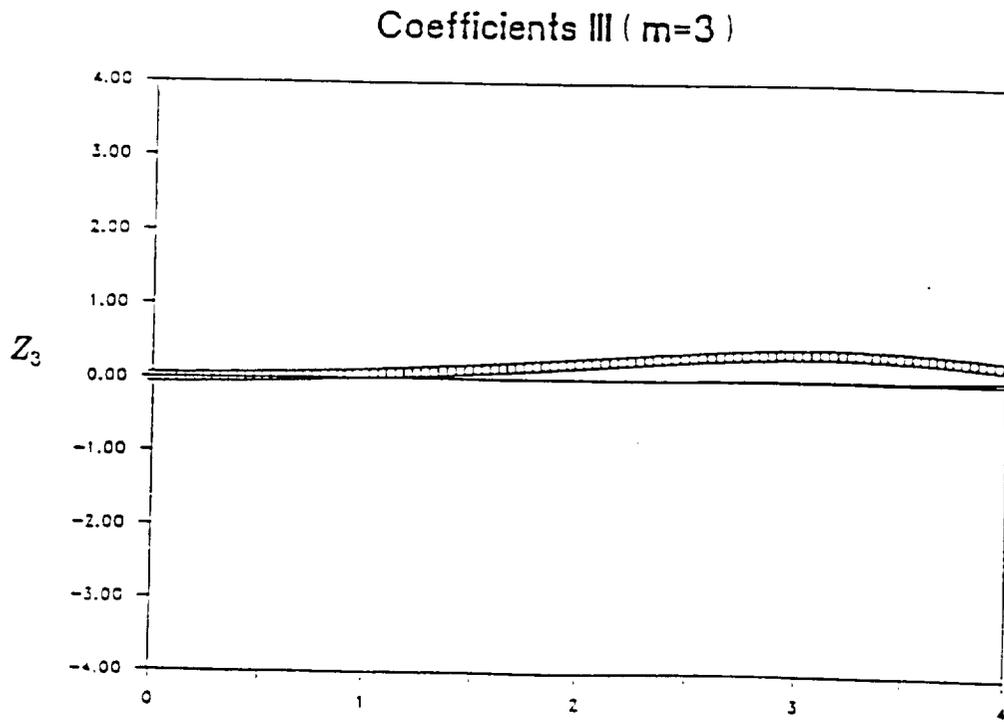


Figure 17: The controlled and uncontrolled amplitudes of the third mode at the center with $m = 3$ and $T = 4$.