

NASA Contractor Report 195454

1N-02  
44401  
24P

# Enhanced Analysis and Users Manual for Radial-Inflow Turbine Conceptual Design Code RTD

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March 1995

Prepared for  
Lewis Research Center  
Under Grant NAG3-1165



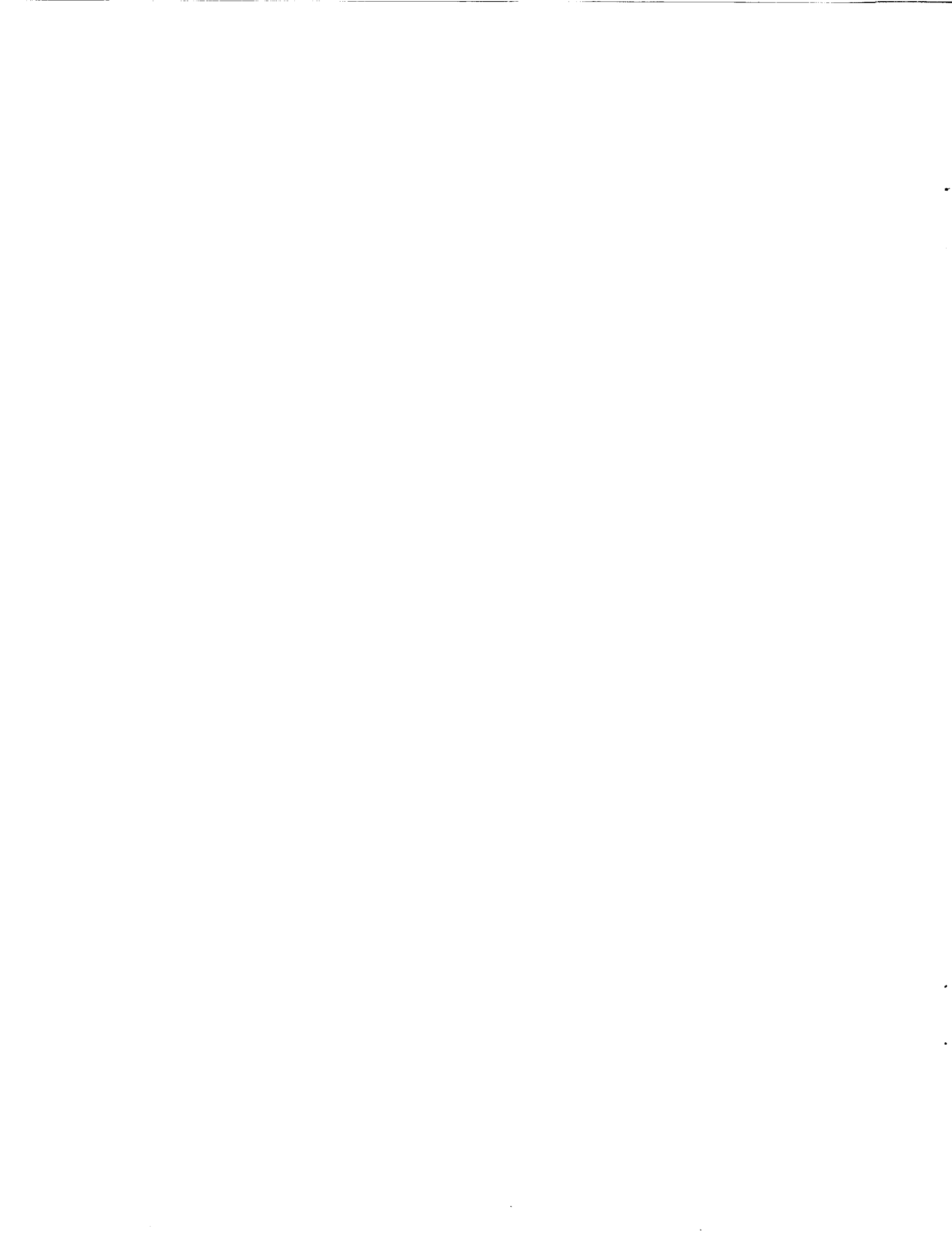
National Aeronautics and  
Space Administration

(NASA-CR-195454) ENHANCED ANALYSIS  
AND USERS MANUAL FOR RADIAL-INFLOW  
TURBINE CONCEPTUAL DESIGN CODE RTD  
Final Contractor Report (Toledo  
Univ.) 24 p

N95-23462

Unclass

G3/02 004401



# Enhanced Analysis and Users Manual for Radial-Inflow Turbine Conceptual Design Code RTD

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## SUMMARY

This report presents modifications made to a radial-inflow turbine conceptual design code, herein named RTD. The analysis is based on meanline flow through the stator and into the rotor, and on equal span-fraction sectors at the rotor exit. Input design variables include flow, power, and rotative speed. The output presents rotor tip diameter, flowpath dimensions, diagram velocities and angles, and efficiencies. Design parameters that can be varied include stator-exit angle, rotor-exit tip and hub radius ratios, and rotor swirl distribution.

Modifications were made to enhance the design-code capabilities consistent with those of a companion off-design code. For pivoting vanes, a stator-endwall clearance flow was modeled by a vaneless-space flow through the clearance height. The rotor-inlet slip model and the rotor-blade-number calculation were changed to allow rotor-blade-inlet angles other than radial, thus permitting the use of swept rotor blades. Also for the rotor, splitter blades can now be used between the full blades.

Added to the loss model were stator and rotor trailing-edge blockage losses and a vaneless-space friction loss. The disk-friction model and the rotor-exit clearance loss correlation were modified. The Reynolds number dependency and level of the stator- and rotor-passage losses were then calibrated based on experimental turbine performance data from several radial-inflow turbines. The selected model performed very well in predicting the efficiency variation over a more than tenfold range of Reynolds number. The predicted efficiencies for three turbines had a maximum deviation of about 1 point as compared to measured values.

This report also serves as an updated users manual for the RTD code. Program input and output are described, and sample cases are included for illustration.

## INTRODUCTION

Performing preliminary studies for power or propulsion systems requires the capability to rapidly produce conceptual designs of the components in order to determine geometry, performance, and weight. The typical turbomachine "design" code enables a study of the interrelationship of the number of stages, the flowpath dimensions, the gas velocities and flow angles, and the resultant variation in efficiency. A computer code capable of performing

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this function for a radial-inflow turbine was presented in reference 1.

A recent evaluation of the aforementioned code, herein named RTD, resulted in the identification of desirable modifications. To make the modeling compatible with the off-design code of reference 2, capabilities were required to accommodate pivoting stators, swept (i.e., non-radial) rotor blades, and splitter blades. Also, the components of the loss model would have to be expanded to include stator and rotor trailing-edge losses and a vaneless-space loss. The disk-friction and the rotor-exit clearance loss models needed adjustment. Since the loss model of reference 1 was calibrated using only a single experimental point, the use of a more extensive database was highly desirable.

This report presents the analytical modeling associated with the above modifications. Also presented is the calibration of the loss model using data from several turbines to define the Reynolds number variation and the level of the efficiency. The report also serves as an updated users manual for the RTD code. Program input and output are described and sample cases are included for illustration.

## SYMBOLS

A	area, $\text{ft}^{2\kappa Z}$
$D_{\text{eq}}$	equivalent diameter, ft
$f$	Fanning friction factor
$g$	gravitational constant, $(\text{lbm})(\text{ft})/(\text{lbf})(\text{sec}^2)$
$\Delta h'$	turbine specific work, Btu/lbm
J	energy dimension constant, $(\text{ft})(\text{lb})/\text{Btu}$
$K_{\text{bf}}$	backface radius correction factor
$K_{\text{Re}}$	Reynolds number correlation coefficient
$K_{\text{rr}}$	radius-ratio correction factor
L	loss, Btu/lbm
l	surface length, ft
$l$	vaneless-space flow length, ft
n	number of blades
$\Delta p'$	absolute total pressure drop, psf
$\Delta p''$	relative total pressure drop, psf
q	blade-to-blade distance, ft
Re	Reynolds number
r	radius, ft
t	trailing-edge thickness, ft
U	blade speed, ft/sec
V	absolute velocity, ft/sec
W	relative velocity, ft/sec
w	mass flow rate, lbm/sec
$\alpha$	absolute flow angle, deg

$\beta$	relative flow angle, deg
$\gamma$	meridional flow angle, deg
$\epsilon_{lim}$	limiting radius ratio for correction factor
$\theta$	momentum thickness, ft
$\mu$	viscosity, lbm/(ft)(sec)
$\rho$	density, lbm/ft <sup>3</sup>

#### Subscripts:

av	average
b	blading
bf	backface
cl	clearance
h	hub
m	mean
opt	optimum
ref	reference
r	rotor or radial component
s	stator
t	tip
te	trailing edge
tot	total
u	tangential component
VD	velocity diagram
vs	vaneless space
0	stator inlet
1	stator exit - outside trailing edge
1a	rotor inlet
1i	stator exit - inside trailing edge
2	rotor exit - outside trailing edge
2i	rotor exit - inside trailing edge

## ANALYTICAL MODELING

Enhancements were made to an existing radial-inflow turbine design code (ref. 1) to improve its accuracy and to make the modeling compatible with that of a companion off-design code (ref. 2). The analytical models, where applicable, are those used in reference 2; in some instances, however, changes were made due to shortcomings in the original modeling. These changes also have been made to the off-design code so that the two codes are compatible. Presented in this section are the analytical models for the stator-endwall clearance flow, the rotor-inlet slip, the rotor-inlet blade number, the splitter blades, the rotor clearance loss, the disk-friction loss, the stator and rotor trailing-edge losses, and the vaneless-space loss.

## Stator-Endwall Clearance Flow

The stator-endwall clearance flow is modeled exactly as in reference 2. At the stator exit, the clearance flow has the same total temperature and static pressure as the passage flow. By assuming the same loss in total pressure, the clearance and passage flows have the same velocities, static temperatures, and densities. The flow angle and flow rate for the clearance flow are then determined from conservation of tangential momentum and continuity, respectively, in the clearance space.

$$V_{u,1i,cl} = r_0 V_{u,0} / r_1 \quad (1)$$

$$\alpha_{1i,cl} = \sin^{-1}(V_{u,1i,cl} / V_{1i}) \quad (2)$$

$$w_{1i,cl} = \rho_{1i} V_{1i} A_{1i} (h_{cl} / h_s)_{1i} \cos \alpha_{1i,cl} \quad (3)$$

This clearance flow is then mixed with the passage flow at the stator exit, with the after-mixed velocity and flow angle obtained from conservation of both angular momentum and mass.

## Rotor-Inlet Slip

The tangential component of velocity for optimum incidence at rotor inlet is obtained from the Wiesner slip factor correlation (ref.3) as adapted to a turbine in reference 2.

$$V_{u,1a,opt} = K_{rr} U_{1a} [1 - \sqrt{(\cos \beta_{b,1a}) / n_{r,1a}{}^7}] / (1 - \tan \beta_{b,1a} / \tan \alpha_{1a}) \quad (4)$$

The radius-ratio correction factor  $K_{rr}$ , which is applied only if  $r_{2,av} / r_{1a}$  is greater than  $\epsilon_{lim}$ , is defined

$$K_{rr} = 1 - [(r_{2,av} / r_{1a} - \epsilon_{lim}) / (1 - \epsilon_{lim})]^3 \quad (5)$$

where

$$r_{2,av} = (r_{2,h} + r_{2,t}) / 2 \quad (6)$$

and

$$\epsilon_{lim} = 1 / \exp(8.16 \cos \beta_{b,1a} / n_{r,1a}) \quad (7)$$

For radial blades, this correlation yields an optimum tangential velocity only a few percent different from that of the correlation used in reference 1.

## Rotor-Inlet Blade Number

The number of blades required at the rotor inlet is based on maintaining a positive velocity everywhere within the rotor channel. This is estimated by formulating the blade-to-blade velocity gradient equation at the rotor inlet and then setting the pressure-surface velocity to zero. Starting from a general velocity gradient equation such as equation (5-70) of reference 4, and applying it to a constant-radius blade-to-blade distance at the inlet to a radial-inflow turbine rotor (i.e., at station 1a), yields

$$dW / dq = -\cos \beta (W_u / r + 2 U / r - dW_u / dr) \quad (8)$$

where  $W$  is the local value of relative velocity along blade-to-blade distance  $q$ . Assuming a linear variation of  $W$  along  $q$  for one passage, setting  $W=0$  at  $q=0$  (i.e., pressure surface), expressing blade-to-blade distance as  $2\pi r/n$ , and using velocity-diagram relationships yields

$$n_{r,1a} = \pi \cos^2 \beta_{1a} [2 \tan \alpha_{1a} - \tan \beta_{1a} - (r / V_r)_{1a} (dW_u / dr)_{1a}] \quad (9)$$

The derivative on the RHS of equation (9) is approximated by using an equation from reference 5 to estimate the radius at which the flow follows the blade

$$r_b = r_{1a} \exp(-1.42 \pi / n_{r,1a}) \quad (10)$$

and assuming that  $W_u$  varies linearly with  $r$  between  $r_{1a}$  and  $r_b$ . Using this approximation in equation (9) finally yields

$$n_{r,1a} = \pi \cos^2 \beta_{1a} \{2 \tan \alpha_{1a} - \tan \beta_{1a} [1 + 1/(1 - \exp(-1.42 \pi / n_{r,1a}))]\} \quad (11)$$

The details of the complete derivation of equation (11) are presented in the Appendix.

This calculation is for the total number of blades (full plus splitter) at the rotor inlet. It can be overridden by input if desired.

## Splitter Blades

The option is now available to directly specify (by input) the use of full blades only for the rotor, the use of splitter blades between the full blades, or a default selection to be made by the program. The default is to use full blades only when the number of blades obtained from equation (11) is less than 16 and to use splitter blades for a larger number.

When splitter blades are used, some minor modifications are required for determining the rotor loss coefficient given by equation (7) of reference 1. The blade spacing is based on the number of full blades. The blade length and blade area terms are arbitrarily increased by 50 percent to account for the splitters.

## Rotor Clearance Loss

The rotor-shroud clearance loss is based on the experimental results of reference 6, wherein a radial-inflow turbine was tested over wide ranges of rotor-inlet axial clearances and rotor-exit radial clearances. The sensitivity of efficiency loss to the normalized clearance (i.e., the ratio of clearance to passage height) was almost an order of magnitude less for the axial clearance than for the radial clearance; thus, axial clearance loss was not included in the loss model.

In reference 1, the efficiency loss per percent of normalized clearance was taken as a constant value of one, which was the average slope over the full range of test clearances (which extended to seven percent). In reality, the data of reference 6 shows that the slope is greatest at zero clearance and decreases with increasing clearance. A parabolic curve fit of the experimental performance yielded

$$L_{cl} / \Delta h'_{VD,av} = - 1.9522 (h_{cl} / h_{r,2}) + 11.207 (h_{cl} / h_{r,2})^2 \quad ( 12 )$$

For a one percent clearance-to-passage ratio, the clearance loss is about two percent, which is double that of the previous model.

## Disk-Friction Loss

The disk-friction loss in reference 1 was for a rotor having a backface extending to the tip radius. Some rotors have a cut-back or scalloped backface, which would reduce the disk friction. To account for this, the ratio (or an effective ratio) of backface radius to tip radius was added to the loss calculation as a correction.

$$L_{df} = 0.02125 K_{bf} \rho_{1a} U_{1a}^3 r_{1a}^2 / [g J w (\rho U r / \mu)_{1a}^{-2}] \quad ( 13 )$$

where the backface radius correction is

$$K_{bf} = (r_{bf} / r_{1a})^{4.6} \quad ( 14 )$$

## Trailing-Edge Losses

To make the loss model compatible with that of the off-design code of reference 2, stator and rotor trailing-edge blockage losses were added.

Stator.- In reference 2, the trailing-edge blockage loss is dependent upon the reduction in radial component of velocity as the flow suddenly expands from inside to outside the trailing



edge. This gives satisfactory results when there is no stator clearance flow, but the mixing of clearance flow with the passage flow changes the radial component of velocity, which then is no longer dependent just on the blockage. Therefore, the model was changed so as to use the physical blockage itself to determine the trailing-edge loss.

$$\Delta p'_{te,s} = \rho_{1i} V_{1i}^2 / (2 g) [n_s t_s / (2 \pi r_1 \cos \alpha_{1i})]^2 \quad (13)$$

Rotor.- The rotor trailing-edge loss in reference 2 was computed as a loss in absolute total pressure proportional to the absolute kinetic energy at the rotor exit. Since this is a cascade loss, the computation was changed to a loss in relative total pressure proportional to the relative kinetic energy at the rotor exit. Also for consistency with the stator, the loss was made dependent on the physical blockage rather than on the reduction of axial component of velocity.

$$\Delta p''_{te,r} = \rho_{2i} W_{2i}^2 / (2 g) [n_{r,2i} t_{r,m} / (2 \pi r_{2,m} \cos \beta_{2i,m})]^2 \quad (14)$$

### Vaneless-Space Loss

The vaneless-space loss is modeled as a friction loss due to the endwalls. The pressure loss is obtained as in reference 2.

$$\Delta p'_{vs} = 2 f / \rho_1 V_1^2 / (g D_{eq}) \quad (15)$$

A correction has been made to the calculation of equivalent diameter, which has been changed to

$$D_{eq} = 2 h_s \quad (16)$$

## CALIBRATION OF LOSS MODEL

The variation of efficiency with Reynolds number and the level of efficiency were defined by using experimental performance data to calibrate the stator and rotor passage loss models. The design characteristics of the turbines used for loss model calibration are presented in table I. The first four of these turbines were for space-power system application while the last two were for small airbreathing engines.

### Reynolds Number Dependency

Performance measurements for three radial-inflow turbines, which are scaled versions of the same design having 6.02-, 4.59-, and 3.50-inch rotor-tip diameters, over ranges of Reynolds number are presented in references 7-11. The three turbines had the same design velocity diagrams, with variations in design pressure and rotative speed accounting for the different sizes. The Reynolds number variation for each turbine was achieved by testing over a range of inlet pressures. These turbines are used only to determine Reynolds number

dependency because they are off-the-shelf industrial turbocharger hardware and, therefore, do not represent aerospace design technology. Plotted in figure 1 against Reynolds number are points representing efficiency at design speed and design pressure ratio for the three turbines. These points were obtained from smoothed curves through experimental points of efficiency plotted against pressure ratio.

Within the experimental variability noted in figure 1, the Reynolds number dependency for the three turbines can be considered the same. Using the type of function

$$\text{Loss} \propto K_{Re} + (1 - K_{Re}) Re^{-2} \quad (17)$$

for the stator and rotor passage losses as was used in references 7 and 9 for overall loss, a value of  $K_{Re}=0.3$  produces the desired variation of efficiency with Reynolds number, as shown by the curves in figure 2. The level of turbine efficiency is determined by the stator and rotor reference loss coefficients, which for the purposes of figure 2 were arbitrarily set to match the efficiencies of the 6.02- and 3.50-inch diameter turbines. The efficiencies for the 4.59-inch diameter turbine were lower than those for the other two turbines by about two points, about half of which is accounted for by the loss model due to a larger rotor-exit clearance.

#### Efficiency Level

The geometries and experimental performances for three other radial-inflow turbines at their design Reynolds numbers (see table I for design characteristics) are presented in references 12-15. These turbines were specifically designed for their intended applications using aerospace design technology and, therefore, should provide the best achievable performance.

The 5.92- and 14.46-inch diameter turbines were designed for "off-design" conditions with loadings higher than optimum and, therefore, efficiencies less than maximum. Since this computer code produces optimum designs, the measured "design-point" efficiencies need correction for proper comparison. For the 5.92-inch diameter turbine (ref. 13), where extensive data is presented, the maximum efficiency on the design speed line is used. In the case of the 14.46-inch diameter turbine (refs. 14 and 15), where the data is not as definitive, the incidence loss determined by the analysis in reference 16 is added to the measured design-point efficiency. In both cases, the efficiency is 1.5 points higher than the measured design-point value.

The stator and rotor reference loss coefficients (used in eqn. (7) of ref. 1) selected to best fit the data are

$$(\theta_{tot} / l)_{ref,s} = 0.0023119 \quad (18)$$

and

$$(\theta_{\text{tot}} / l)_{\text{ref,r}} = 0.0066267$$

( 19 )

The ratio of values is somewhat arbitrary, reflecting the higher loading and diffusion in the rotor. Detailed measurements and/or analysis are required for better definition.

Using these values of reference loss coefficients, the computed efficiencies are compared to the measured values in figure 3 for the three turbines. The maximum deviation is about one point. The 5.92-inch diameter turbine was purposely designed with very thick stator and rotor trailing edges, to simulate cooled blading, as well as a relatively long vaneless space, and these features are reflected in the lower efficiency of this turbine.

## DESCRIPTION OF INPUT AND OUTPUT

This section presents a complete description of the input and output for code RTD. The input and corresponding output for two sample cases are included for illustration. There are several error messages indicating the nonexistence of a solution satisfying the specified input requirements. These messages are self explanatory and, usually, reflect input error.

### Input

The program input, which is read on unit 05, consists of a title line and one NAMELIST dataset. The title, which is printed as a heading on the output, can contain up to 79 characters located anywhere in columns 2 through 80 on the title line. A title line, even if it is left blank, must be the first line for each case. The data are input in data records having the NAMELIST name INPUT. The variables that compose INPUT are defined below along with units and default values. Either SI units or U.S. customary units may be used with this program.

A sample input file containing two cases is shown in table II. Each case is headed by a title line. The first case is for the turbine of reference 12, which was tested with argon. This turbine was designed for optimum rotor incidence. The second case, which is an air turbine, is for an optimum design corresponding to the design requirements of reference 13.

ALPHA0	stator-inlet flow angle from radial direction, deg (input only when NSTAR = 0 or 1)
ALPHA11	stator-exit blade angle from radial direction, deg
BETAB	rotor blade inlet angle, deg (back sweep is positive - default=0.0)
CDT2	ratio of clearance to rotor-exit tip diameter (default=0.0)
CFU1A	multiplier for optimum tangential velocity at rotor inlet (default=1.0)

**CLFR1**      ratio of stator clearance to stator passage height (default=0.0)  
**CR**            rotor loss-coefficient multiplier (default=1.0)  
**CS**            stator loss-coefficient multiplier (default=1.0)  
**EBARR**       rotor loss coefficient (input only to override the internal computation)  
**EBARS**       stator loss coefficient (input only to override the internal computation)  
**GAM**          specific heat ratio  
**INRBL**       number of blades at rotor inlet (default=-1)  
                  = -1 - computed internally  
                  > 0 - number of blades  
**INSTV**       number of stator vanes (default=-1)  
                  =-1 - computed from input or default SIGSIN  
                  > 0 - number of vanes  
**IOFF**         switch for writing off-design code input file (default=0)  
                  = 0 - no  
                  = 1 - yes  
**ISP**          switch for splitter blades (default=-1)  
                  = -1 - decision by internal rule  
                  = 0 - all rotor blades are full blades  
                  = 1 - rotor blades are half full blades and half splitter blades  
**IU**            units indicator (default=2)  
                  = 1 - SI units  
                  = 2 - U.S. customary units  
**K**             number of radial sectors at rotor exit  
**MU**          gas viscosity, (N)(sec)/m<sup>2</sup>; lb/(sec)(ft)  
**N**             rotative speed, rad/sec; rev/min

NSTAR	stator geometry indicator = 0 - cambered vane with input diameter ratio = 1 - cambered vane with input aspect ratio = 2 - uncambered vane
POW	shaft power, kW; hp
PTIN	inlet total pressure, N/cm <sup>2</sup> ; lb/in. <sup>2</sup>
R	gas constant, J/(kg)(K); (ft)(lbf)/(lbm)(°R)
RBFR1A	rotor backface- to rotor-blade-tip-radius ratio (default=1.0)
RH2RT2	rotor-exit-hub- to tip-radius ratio
RT2R1A	rotor-exit-tip- to rotor-inlet-radius ratio
RV1AAV	ratio of rotor-inlet angular momentum to average change in angular momentum (default=1.0)
RV2I2M(I) I = 1, K	ratio of i <sup>th</sup> sector to mean-sector rotor-exit angular momentum (default=1.0 for all I)
R0R1A	stator-inlet- to rotor-inlet-radius ratio
R1R1A	stator-exit- to rotor-inlet-radius ratio
SIGSIN	stator solidity based on exit spacing (can be omitted when INSTV is input - (default=1.2)
STAR	stator aspect ratio (input only for NSTAR = 1 or 2)
TOH1	ratio of stator trailing-edge thickness to stator height (default=0.05)
TOH2M	ratio of rotor mean-section trailing-edge thickness to rotor-exit passage height (default=0.04)
TTIN	inlet total temperature, K; °R
W	mass flow rate, kg/sec; lb/sec

## Output

The program output consists of title headings, the input values, and the computed results. Table III presents the output that corresponds to the sample input shown in table II. The top line of output is a program identification title that is printed with the first case of each data file. The second line is the case title from the input title line. The next three lines show the units used for the different variables.

The heading **\*INPUT\*** is followed by the input values used for this case. Identification of the items is self-explanatory. The zero value shown for stator aspect ratio indicates that this was not an input for this case. Values of 1.0000 are shown for the stator and rotor loss coefficients to indicate that the actual values are computed using the internal loss model.

The heading **\*OUTPUT\*** is followed by the computed results. Temperatures, pressures, flow angles, velocities, and velocity ratios, along with diameter, are shown for each calculation station, which are stator inlet, stator exit inside trailing edge, stator exit outside trailing edge, rotor inlet, rotor exit inside trailing edge, and rotor exit outside trailing edge. At the rotor exit, the computed values are shown at the mean diameter of each sector as well as for the hub and the tip. Additional output for the rotor-exit sectors include flow rate, specific work, and total and static efficiencies. Also shown are the computed loss coefficients for the stator and rotor, the stator height, number of stator vanes, and number of rotor blades, both full and splitter.

Under the heading **\*OVERALL PERFORMANCE\*** are the turbine total-to-total and total-to-static pressure ratios, diagram specific work, and both diagram and net total and static efficiencies. Also shown are the individual loss components as fractions of the turbine ideal work, the specific speed, the blade-jet speed ratio, and the work factor.

The first case uses U.S. customary units while the second case uses SI units. Since the 4.97-inch diameter turbine of reference 12 was an optimum design, the rotor tip diameter computed for the first case should be close to the actual value; and it is, differing only by 2.5 percent (5.093 inches). For the second case, the computed optimum diameter (17.13 cm.) was 14 percent higher than the diameter (15.04 cm.) of the reference 13 turbine, which had been designed for a higher than optimum loading.

## SUMMARY OF RESULTS

This report presents modifications made to a radial-inflow turbine conceptual design code, herein named RTD. The analysis is based on meanline flow through the stator and into the rotor, and on equal span-fraction sectors at the rotor exit. Input design variables include flow, power, and rotative speed. The output presents rotor-tip diameter, flowpath dimensions, diagram velocities and angles, and efficiencies. Design parameters that can be varied include stator-exit angle, rotor-exit tip and hub radius ratios, and rotor swirl distribution.

Modifications were made to enhance the design-code capabilities consistent with those of

a companion off-design code. To accommodate pivoting vanes, stator-endwall clearance effects were modeled by a vaneless-space flow through the clearance height, and this clearance flow was then mixed with the passage flow just beyond the stator trailing edge. The rotor-inlet slip model and the rotor-blade-number calculation were changed to allow rotor-blade-inlet angles other than radial, thus permitting the use of swept rotor blades. For radial blades, the computed slip and blade number were very close to those from the previous models. In addition, splitter blades between the full blades can now be optionally specified.

Added to the loss model were stator and rotor trailing-edge blockage losses and a vaneless-space loss. The trailing-edge losses are reductions in kinetic energy that depend on the fraction of flow area that is blocked. The vaneless-space loss is computed as a friction loss for flow between the parallel endwalls. A change was made to the disk-friction loss to account for backfaces that do not fully extend to the rotor tip. The rotor-clearance loss was changed to reflect the non-linear variation of loss with clearance height.

The variation of turbine efficiency with Reynolds number and the level of efficiency were defined by calibrating the stator and rotor passage losses using experimental performance data from six radial-inflow turbines. The selected model performed very well in predicting the turbine efficiency variation for three industrial turbines over a more than tenfold range of Reynolds number. The efficiencies predicted for three aerospace turbines had a maximum deviation of one point as compared to the measured values.

This report also serves as an updated users manual for the RTD code. Program input and output are described, and sample cases are included for illustration.

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## Appendix

### DERIVATION OF ROTOR-INLET BLADE NUMBER EQUATION

The velocity gradient along an arbitrary curve  $q$  can be expressed (ref. 4)

$$dW / dq = a (dr/dq) + b (dx/dq) + c (d\theta/dq)$$

where

$$a = W \cos \gamma \cos^2 \beta / r_c - W \sin^2 \beta / r + \sin \gamma \cos \beta (dW_m/dm) - 2 \omega \sin \beta$$

$$b = - W \sin \gamma \cos^2 \beta / r_c + \cos \gamma \cos \beta (dW_m/dm)$$

$$c = W \sin \gamma \sin \beta \cos \beta + r \cos \beta [(dW_u/dm) + 2 \omega \sin \gamma]$$

Since our interest is in the blade-to-blade distance (constant  $r$  and constant  $x$ ),

$$dr/dq = dx/dq = 0$$

and the velocity gradient equation reduces to

$$dW / dq = c (d\theta/dq) = [W \sin \gamma \sin \beta \cos \beta + r \cos \beta [(dW_u/dm) + 2 \omega \sin \gamma]] d\theta/dq$$

With the flow being radially inward ( $\gamma = -90^\circ$  and  $\sin \gamma = -1$ ), and since  $dq = r d\theta$ ,  $W_u = W \sin \beta$ , and  $U = r\omega$ , the velocity gradient equation is further reduced to

$$dW / dq = -\cos \beta (W_u / r - dW_u / dr + 2 U / r)$$

which is equation (8) in the body of the report.

As the blade number selection criterion, let  $W=0$  at the pressure surface. Assuming a linear variation of  $W$  along  $q$ ,

$$dW / dq = \Delta W / \Delta q = 2 W / \Delta q = -2 W / (r \Delta\theta)$$

The minus sign is a result of the positive  $q$  direction from pressure to suction surface being in the negative  $\theta$  direction. With this assumption, the velocity gradient equation then becomes

$$W = r \Delta\theta \cos \beta (W_u / r - dW_u / dr + 2 U / r) / 2$$

Substituting  $W = V_r / \cos \beta$ ,  $\Delta\theta = 2\pi/n$ , and  $U = V_u - W_u$  into the above equation then yields

$$Vr = \pi \cos^2 \beta (2 V_u - W_u - r dW_u / dr) / n$$

Using the velocity-diagram relationships  $\tan \alpha = V_u / V_r$  and  $\tan \beta = W_u / V_r$ , subscripting for the rotor inlet station, and rearranging gives

$$n_{r,1a} = \pi \cos^2 \beta_{1a} [2 \tan \alpha_{1a} - \tan \beta_{1a} - (r / V_r)_{1a} (dW_u / dr)_{1a}]$$

This is equation (9) in the body of the report.

The derivative  $dW_u/dr$  is approximated by using an equation from reference 5 to estimate the radius at which the flow follows the blade.

$$r_b = r_{1a} \exp(-1.42 \pi / n_{r,1a})$$

Assuming that  $W_u$  varies linearly between  $r_{1a}$  and  $r_b$  and that  $\beta=0$  at  $r_b$ , we can write

$$dW_u / dr = (W_u - 0) / [(r_{1a} - r_{1a} \exp(-1.42 \pi / n_{r,1a}))] = W_u / [r_{1a} [1 - \exp(-1.42 \pi / n_{r,1a})]]$$

Finally, with this approximation for  $dW_u/dr$ , the equation for number of blades at the rotor inlet becomes

$$n_{r,1a} = \pi \cos^2 \beta_{1a} [2 \tan \alpha_{1a} - \tan \beta_{1a} [1 + 1/(1 - \exp(-1.42 \pi / n_{r,1a}))]]$$

This is equation (11) in the report.

TABLE I  
DESIGN CHARACTERISTICS OF RADIAL-INFLOW  
TURBINES USED FOR LOSS MODEL EVALUATION

Tip diameter, inches	Eq. total pressure ratio	Corrected work Btu/lb	Corrected tip speed, ft/sec	Blade-jet speed ratio	Reynolds number $\times 10^{-3}$	Reference
6.02	1.496	11.9	592	.697	63.7	7, 8
4.59	1.496	11.9	592	.697	83.2	9, 10
3.50	1.496	11.9	592	.697	109.	11
4.97	1.645	14.8	644	.690	76.2	12
5.92	3.028	29.7	813	.609	228.	13
14.46	4.88	41.8	975	~.62	422.	14, 15

TABLE II. - SAMPLE INPUT

Radial Turbine with Argon - TN D-5090

```
&input gam=1.667,alpha0=45.0,alphai=72.50,
rlrla=1.0298,r0rla=1.3392,rt2rla=0.6990,
rh2rt2=0.5245,cdt2=.00193,k=3,nstar=0,r=38.68,iu=2,
ttin=610.,ptin=7.0,n=28359.,w=.2655,pow=5.150,mu=.165e-4,
&end
```

Radial Turbine with Air - TP 1730

```
&input gam=1.4,alpha0=0.0,alphai=73.385,toh1=.1007,
rlrla=1.111,r0rla=1.292,rt2rla=0.6315,toh2m=.1557,
rh2rt2=0.4814,cdt2=.00312,k=3,nstar=0,r=286.8,iu=1,rvlaav=.963,
instv=29,inrbl=12,
ttin=322.2,ptin=13.79,n=3482.7,w=.3054,pow=22.5,mu=.170e-4,
&end
```

TABLE III. - SAMPLE OUTPUT

RADIAL INFLOW TURBINE VELOCITY DIAGRAM DESIGN ANALYSIS

Radial Turbine with Argon - TN D-5090

THIS OUTPUT IS IN THE FOLLOWING UNITS.

TEMPERATURE DEG RANKINE	PRESSURE LB/SQ IN	GAS CONST BTU/LB-R	ROT SPEED REV/MIN	MASS FLOW LB/SEC	POWER HP	VISCOSITY LB/FT-SEC	VELOCITY FT/SEC	SPEC WORK BTU/LB	DIAMETER IN	ANGLE DEGREES
*INPUT*										
INLET TEMP =	610.0000			45.00						
INLET PRESS =	7.0000			72.50						
ROTAT SPEED =	28359.0000			.0000						
MASS FLOW =	.2655									
SHAFT POWER =	5.150									
ROTOR SWEEP =	.00									
*OUTPUT*										
STATOR INLET	6.821	610.00	6.907	558.24	5.534	6.892	45.00	153.57	72.50	567.41
LOSS COEF =	.0543	610.00	6.904	558.71	5.543	6.892	45.00	153.57	72.50	567.41
INSIDE TE	5.245	610.00	6.904	558.71	5.543	6.892	45.00	153.57	72.50	567.41
STATOR EXIT	5.245	610.00	6.904	558.71	5.543	6.892	45.00	153.57	72.50	567.41
ROTOR INLET	5.093	610.00	6.896	555.53	5.458	73.20	582.12	.598	630.23	560.93
LOSS COEF =	.2043	497.08	4.042	488.49	3.869	.00	231.21	.263	231.06	505.67
ROTOR EXIT	2.149	497.08	4.030	489.06	3.869	.00	223.42	.254	265.97	508.46
INSIDE TE	2.714	497.08	4.002	490.44	3.869	.00	203.30	.231	335.80	515.21
	3.278	497.08	3.967	492.14	3.869	.00	175.37	.199	405.62	523.53
	3.560	497.08	3.948	493.11	3.869	.00	157.23	.179	440.53	528.28
ROTOR EXIT	1.867	497.08	4.038	490.49	3.906	.00	202.47	.230	231.06	505.67
OUTSIDE TE	2.149	497.08	4.027	490.70	3.899	.00	199.16	.226	265.97	508.46
	2.714	497.08	3.998	491.54	3.888	.00	185.68	.211	335.80	515.21
	3.278	497.08	3.964	492.83	3.880	.00	162.73	.185	405.62	523.53
	3.560	497.08	3.945	493.62	3.876	.00	146.73	.167	440.53	528.28

CL HT/ROT EX TIP D = .0019

STATOR HGT = .3882, NUMBER OF VANES = 13.0

GAS CONSTANT = 38.6800

SPEC HT RATIO = 1.6670

VISCOSITY = .1650E-04

REL TOTAL PRESS

REL FLOW ANGLE

REL VELO-CITY

REL CRIT VEL RAT

NUMBER OF ROTOR BLADES = 11.0 FULL + 11.0 SPLITTER

\*OVERALL PERFORMANCE\*

TOT-TOT PRESSURE RATIO =	1.7507
TOT-STAT PRESSURE RATIO =	1.8005
DIAG AVG SPECIFIC WORK =	14.0372
DIAG TOTAL EFFICIENCY =	.9206
DIAG STATIC EFFICIENCY =	.8830
NET TOTAL EFFICIENCY =	.8997
NET STATIC EFFICIENCY =	.8630

LOSS/IDEAL T-S DEL H

STATOR TE =	.0232
STATOR T-S DEL H =	.0008
VANELESS =	.0021
ROTOR TE =	.0514
ROTOR T-S DEL H =	.0012
WINDAGE =	.0067
CLEARANCE =	.0133
EXIT KE =	.0416

SPECIFIC SPEED = 74.953

BLADE-JET RAT. = .7066

WORK FACTOR = .8842

TABLE III. - Concluded.

Radial Turbine with Air - TP 1730

THIS OUTPUT IS IN THE FOLLOWING UNITS.

TEMPERATURE KELVIN	PRESSURE N/SQ CM	GAS CONST JLS/KG-K	ROT SPEED RAD/SEC	MASS FLOW KG/SEC	POWER KW	VISCOSITY N-SEC/SQ M	VELOCITY M/SEC	SPEC WORK JLS/GM	DIAMETER CM	ANGLE DEGREES
<p>*INPUT*</p> <p>INLET TEMP = 322.2000            INLET PRESS = 13.7900            ROTAT SPEED = 3482.700            MASS FLOW = .3054            SHAFT POWER = 22.500            ROTOR SWEEP = .00</p> <p>STATOR IN ANGLE = .00            STATOR EX ANGLE = 73.39            STATOR ASPECT RAT = .0000            DIAM RATIOS            STAT IN/ROT IN = 1.2920            STAT EX/ROT IN = 1.1110            ROT EX TP/ROT IN = .6315            ROT EX HUB/TIP = .4814</p>										
<p>*OUTPUT*</p> <p>STATOR INLET            LOSS COEF = .0520            INSIDE TE            STATOR EXIT</p> <p>ROTOR INLET            LOSS COEF = .2734            ROTOR EXIT            INSIDE TE</p> <p>ROTOR EXIT            OUTSIDE TE</p>										
DIA-METER	22.136	322.20	321.42	13.674	.00	39.49	298.35	9.651		
ABS TOTAL TEMP	322.20	322.20	321.42	13.674	.00	39.49	298.35	9.651		
ABS TOTAL PRESS	13.790	13.553	295.62	10.027	73.39	231.02	90.70	5.289	-44.18	174.75
ABS TOTAL TEMP	322.20	322.20	296.14	10.050	75.42	228.74	106.98	5.376	-47.53	160.77
ABS TOTAL PRESS	13.387	13.502	296.14	10.050	75.42	228.74	106.98	5.376	-47.53	160.77
ABS TOTAL TEMP	246.29	246.29	237.99	4.258	-13.93	129.12	253.20	5.289	-44.18	174.75
ABS TOTAL PRESS	4.801	4.773	238.52	4.267	-12.18	124.88	254.80	5.376	-47.53	160.77
ABS TOTAL TEMP	246.29	246.29	239.72	4.276	-10.13	114.82	258.80	5.590	-54.72	195.70
ABS TOTAL PRESS	4.608	4.608	241.16	4.281	-9.29	101.52	263.86	5.865	-62.01	213.47
ABS TOTAL TEMP	246.29	246.29	241.97	4.282	-9.25	93.11	266.78	6.026	-65.68	223.17
ABS TOTAL PRESS	4.555	4.555	244.63	4.551	-32.53	57.81	253.20	5.134	-68.19	131.18
ABS TOTAL TEMP	246.29	246.29	244.13	4.495	-23.61	65.80	254.80	5.221	-65.67	146.34
ABS TOTAL PRESS	4.570	4.570	243.54	4.394	-15.79	74.26	258.80	5.435	-65.90	175.01
ABS TOTAL TEMP	246.29	246.29	243.55	4.314	-12.76	74.14	263.86	5.710	-69.01	201.89
ABS TOTAL PRESS	4.438	4.438	243.78	4.282	-12.16	71.06	266.78	5.871	-71.14	214.91

REL CRIT VEL RAT	REL VELOCITY	REL FLOW ANGLE	REL TOTAL PRESS	REL TOTAL TEMP	REL ABS CRIT VEL RAT	REL BLADE SPEED	REL TOTAL TEMP	REL TOTAL PRESS	REL FLOW ANGLE	REL VELOCITY
.274	85.95	-37.56	9.651	293.44	.777	298.35	293.44	9.651	-37.56	85.95
.600	174.75	-44.18	5.289	253.20	.450	90.70	253.20	5.289	-44.18	174.75
.619	160.77	-47.53	5.376	254.80	.435	106.98	254.80	5.376	-47.53	160.77
.665	195.70	-54.72	5.590	258.80	.400	139.55	258.80	5.590	-54.72	195.70
.718	213.47	-62.01	5.865	263.86	.354	172.12	263.86	5.865	-62.01	213.47
.747	223.17	-65.68	6.026	266.78	.324	188.41	266.78	6.026	-65.68	223.17
.451	131.18	-68.19	5.134	253.20	.201	90.70	253.20	5.134	-68.19	131.18
.501	146.34	-65.67	5.221	254.80	.229	106.98	254.80	5.221	-65.67	146.34
.595	175.01	-65.90	5.435	258.80	.259	139.55	258.80	5.435	-65.90	175.01
.679	201.89	-69.01	5.710	263.86	.258	172.12	263.86	5.710	-69.01	201.89
.719	214.91	-71.14	5.871	266.78	.248	188.41	266.78	5.871	-71.14	214.91

STATOR HGT = .7497, NUMBER OF VANES = 29.0

CL HT/ROT EX TIP D = .0031

GAS CONSTANT = 286.8000  
 SPEC HT RATIO = 1.4000  
 VISCOSITY = .1700E-04

NUMBER OF ROTOR BLADES = 12.0 FULL + .0 SPLITTER

LOSS/IDEAL T-S DEL H  
 STATOR TE = .0162  
 VANELESS = .0078  
 ROTOR TE = .0214  
 WINDAGE = .0095  
 CLEARANCE = .0184  
 EXIT KE = .0290

SPECIFIC SPEED = .476  
 BLADE-JET RAT = .7019  
 WORK FACTOR = .8561

\*OVERALL PERFORMANCE\*

TOT-TOT PRESSURE RATIO = 3.0174  
 TOT-STAT PRESSURE RATIO = 3.1382  
 DIAG AVG SPECIFIC WORK = 76.1977  
 DIAG TOTAL EFFICIENCY = .8676  
 DIAG STATIC EFFICIENCY = .8436  
 NET TOTAL EFFICIENCY = .8388  
 NET STATIC EFFICIENCY = .8157

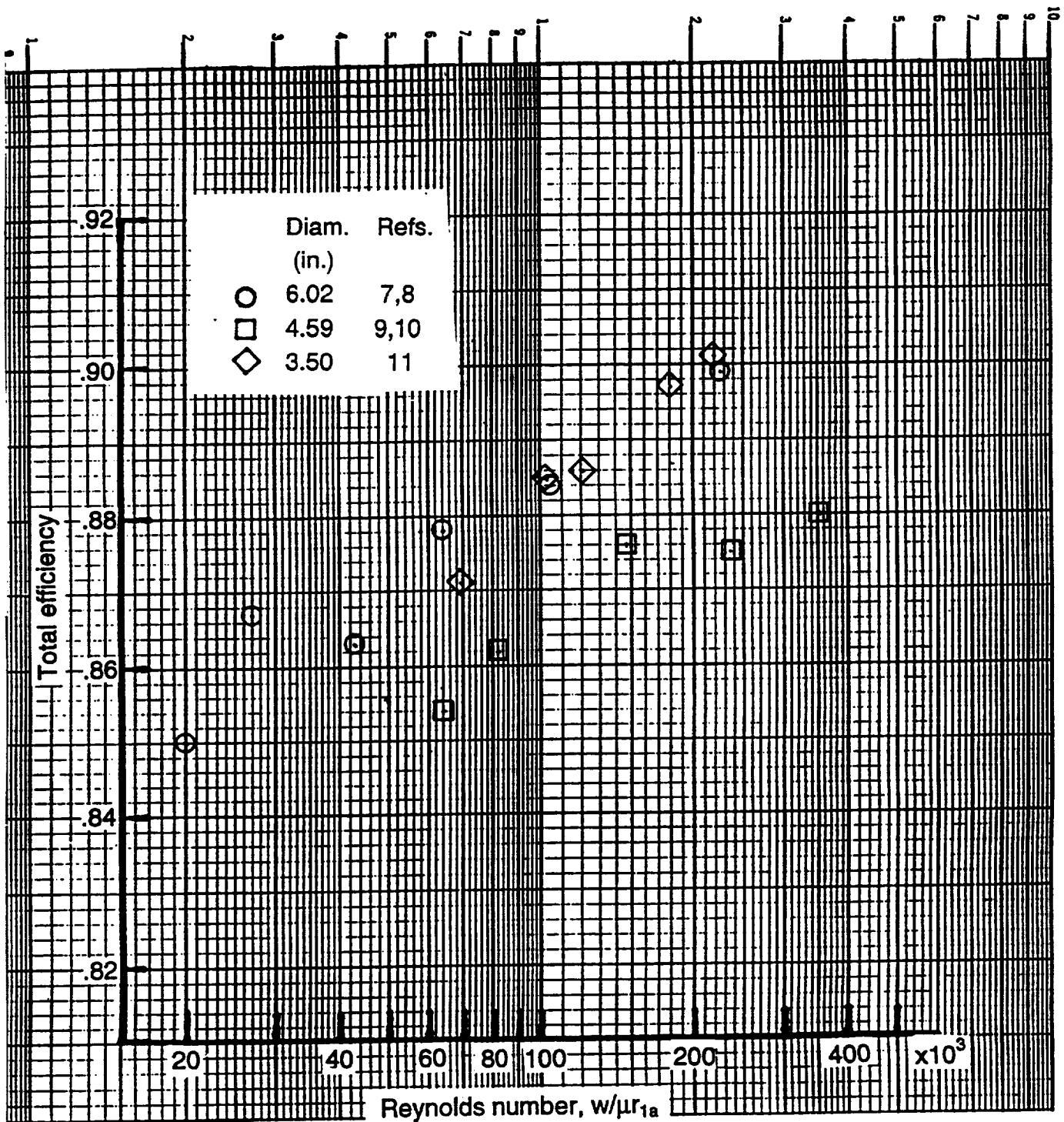


Figure 1 - Measured variation of efficiency with Reynolds number.

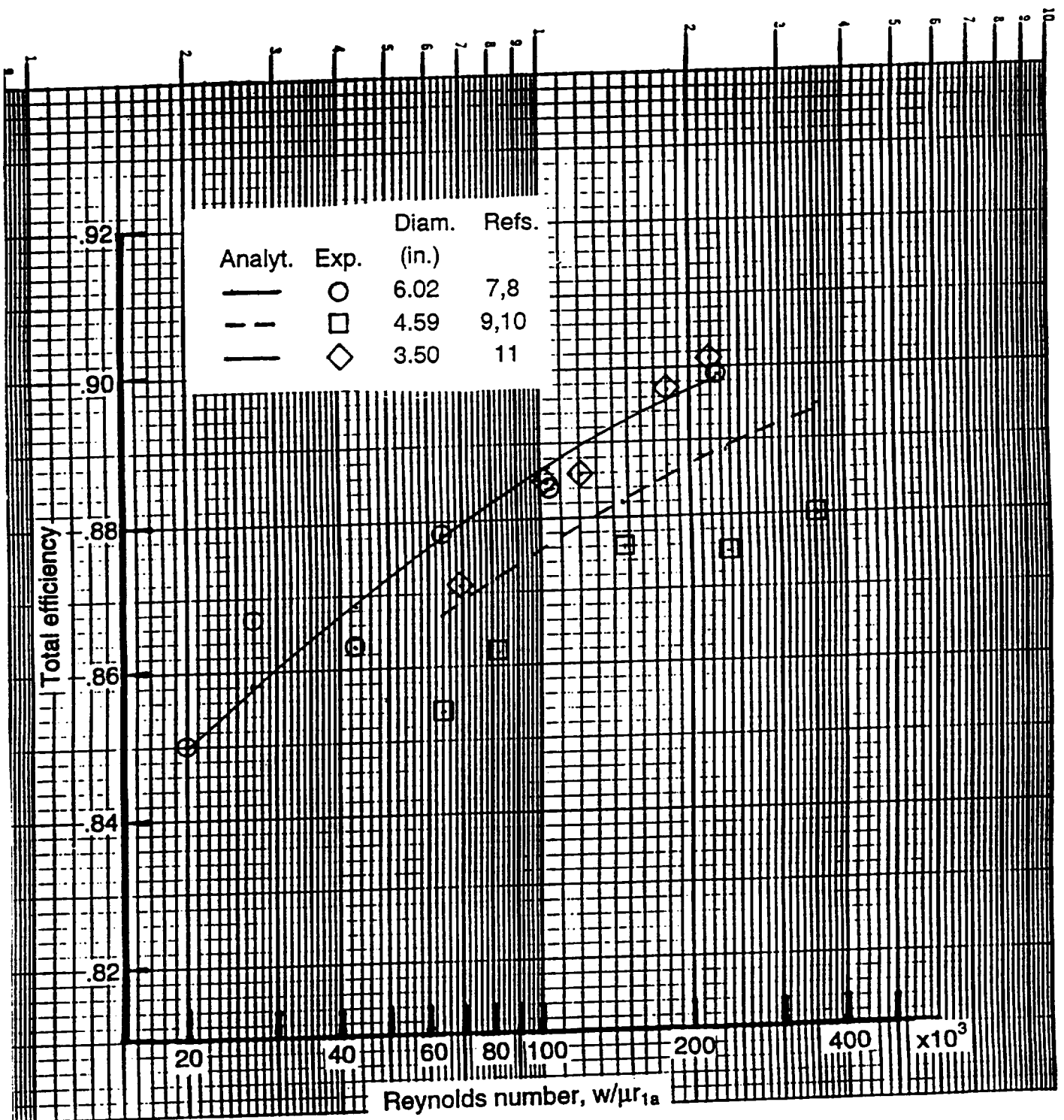


Figure 2 - Comparison of computed and measured variations of efficiency with Reynolds number.

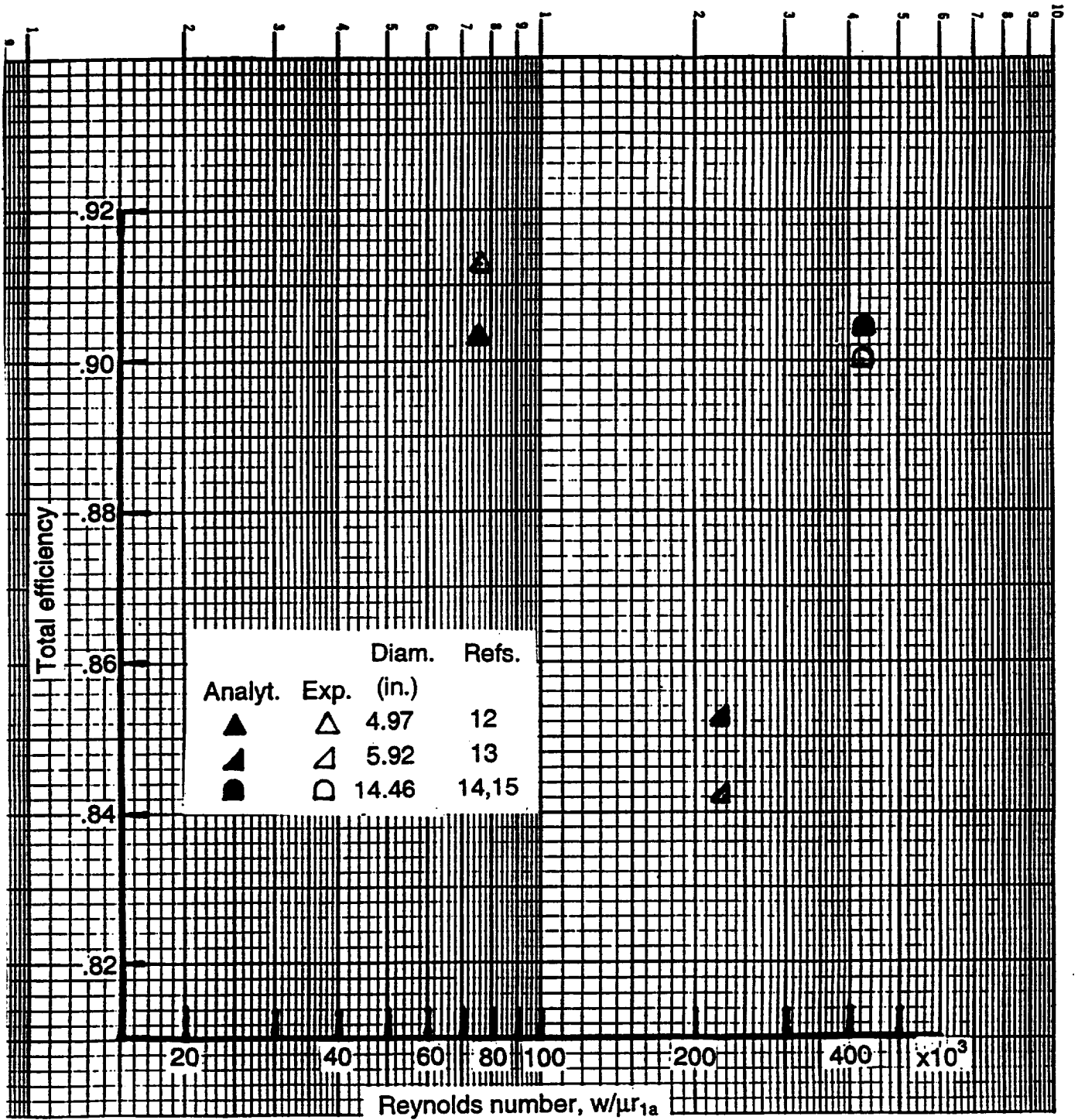


Figure 3 - Comparison of computed and measured efficiencies.



# REPORT DOCUMENTATION PAGE

Form Approved  
OMB No. 0704-0188

Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.

1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE March 1995	3. REPORT TYPE AND DATES COVERED Final Contractor Report	
4. TITLE AND SUBTITLE Enhanced Analysis and Users Manual for Radial-Inflow Turbine Conceptual Design Code RTD		5. FUNDING NUMBERS  WU-505-69-50 G-NAG3-1165	
6. AUTHOR(S)  Arthur J. Glassman		8. PERFORMING ORGANIZATION REPORT NUMBER  E-9538	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)  University of Toledo Toledo, Ohio 43606		10. SPONSORING/MONITORING AGENCY REPORT NUMBER  NASA CR-195454	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)  National Aeronautics and Space Administration Lewis Research Center Cleveland, Ohio 44135-3191		11. SUPPLEMENTARY NOTES Arthur J. Glassman, University of Toledo, Toledo, Ohio 43606 and Resident Research Associate at NASA Lewis Research Center. Project Manager, Thomas M. Lavelle, Aeropropulsion Analysis Office, NASA Lewis Research Center, organization code 2410, (216) 977-7042.	
12a. DISTRIBUTION/AVAILABILITY STATEMENT  Unclassified - Unlimited Subject Category 02  This publication is available from the NASA Center for Aerospace Information, (301) 621-0390.		12b. DISTRIBUTION CODE	
13. ABSTRACT (Maximum 200 words)  Modeling enhancements made to a radial-inflow turbine conceptual design code are documented in this report. A stator-endwall clearance-flow model was added for use with pivoting vanes. The rotor calculations were modified to account for swept blades and splitter blades. Stator and rotor trailing-edge losses and a vaneless-space loss were added to the loss model. Changes were made to the disk-friction and rotor-clearance loss calculations. The loss model was then calibrated based on experimental turbine performance. A complete description of code input and output along with sample cases are included in the report.			
14. SUBJECT TERMS  Turbine analysis; Radial turbines		15. NUMBER OF PAGES 24	16. PRICE CODE A03
17. SECURITY CLASSIFICATION OF REPORT Unclassified	18. SECURITY CLASSIFICATION OF THIS PAGE Unclassified	19. SECURITY CLASSIFICATION OF ABSTRACT Unclassified	20. LIMITATION OF ABSTRACT

