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A GENERALIZED EULERIAN-LAGRANGIAN ANALYSIS, WITH APPLICATION TO LIQUID FLOWS WITH VAPOR BUBBLES†

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ABSTRACT

Under a NASA MSFC SBIR Phase II effort an analysis has been developed for liquid flows with vapor bubbles such as those in liquid rocket engine components. The analysis is based on a combined Eulerian-Lagrangian technique, in which Eulerian conservation equations are solved for the liquid phase, while Lagrangian equations of motion are integrated in computational coordinates for the vapor phase. The novel aspect of the Lagrangian analysis developed under this effort is that it combines features of the so-called particle distribution approach with those of the so-called particle trajectory approach and can, in fact, be considered as a generalization of both of those traditional methods. The result of this generalization is a reduction in CPU time and memory requirements. Particle time step (stability) limitations have been eliminated by semi-implicit integration of the particle equations of motion (and, for certain applications, the particle temperature equation), although practical limitations remain in effect for reasons of accuracy. The analysis has been applied to the simulation of cavitating flow through a single-bladed section of a labyrinth seal. Models for the simulation of bubble formation and growth have been included, as well as models for bubble drag and heat transfer. The results indicate that bubble formation is more or less "explosive": for a given flow field, the number density of bubble nucleation sites is very sensitive to the vapor properties and the surface tension. The bubble motion, on the other hand, is much less sensitive to these properties, but is affected strongly by the local pressure gradients in the flow field. In situations where either the material properties or the flow field are not known with sufficient accuracy, parametric studies can be carried out rapidly to assess the effect of the important variables. Future work will include application of the analysis to cavitation in inducer flow fields.

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OBJECTIVE:

DEVELOPMENT OF AN ANALYSIS FOR LIQUID FLOWS WITH VAPOR BUBBLES (SUCH AS THOSE IN BEARINGS, SEALS, AND PUMPS) FOR USE IN DESIGN OF LIQUID ROCKET ENGINE COMPONENTS

APPROACH:

USE A COMBINED EULERIAN-LAGRANGIAN ANALYSIS

- CONTINUOUS (LIQUID) PHASE TREATED BY SOLVING EULERIAN CONSERVATION EQS. (N-S EQS.) WITH DISCRETE PHASE SOURCE TERMS
- DISCRETE (VAPOR BUBBLE) PHASE TREATED BY INTEGRATING LAGRANGIAN EQUATIONS OF MOTION IN COMPUTATIONAL COORDINATES

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LAGRANGIAN ANALYSIS - METHODOLOGY

- GENERALIZATION OF PARTICLE TRAJECTORY AND PARTICLE DISTRIBUTION MODELS

  ⇒ REDUCED CPU TIME AND STORAGE REQUIREMENTS

- STABLE INTEGRATION OF LAGRANGIAN EQUATIONS OF MOTION IN COMPUTATIONAL COORDINATES

  ⇒ NO PARTICLE TIME STEP LIMITATION OTHER THAN FOR ACCURACY

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LAGRANGIAN ANALYSIS - FEATURES

- LAGRANGIAN DISCRETE PHASE ANALYSIS INTERFACES WITH EULERIAN CONTINUOUS PHASE ANALYSIS VIA ONE SINGLE SUBROUTINE
- GENERALIZED ARRAY ADDRESSING AND INDEX / VARIABLE NUMBERING
  - ⇒ LAGRANGIAN ANALYSIS CAN BE HOOKED UP TO A VARIETY OF EULERIAN CODES ("FLOW SOLVERS")
- CODE IS MODULAR W.R.T. PHYSICAL MODELS

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APPLICATION TO VAPOR BUBBLES

- MODELS NEEDED FOR BUBBLE FORMATION, GROWTH / COLLAPSE AND MOTION
- CRITICAL PHYSICAL PARAMETERS:
  - SURFACE TENSION
  - VAPOR PRESSURE - TEMPERATURE RELATION (CLAUSIUS-CLAPEYRON EQUATION)

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BUBBLE FORMATION

- NUCLEATION DUE TO LOCAL P DROP;
  USE HOMOGENEOUS NUCLEATION MODEL
  (KATZ AND BLANDER)

$$J\left(\frac{\text{NUCLEI}}{M^3 S}\right) = n\left(\frac{2\sigma}{\pi M}\right)^{1/2} \exp\left[\frac{-16\pi\sigma^3}{3\kappa T (P_e - P_L)^2}\right]$$

J : NUMBER OF NUCLEATION SITES PER UNIT VOLUME AND TIME

P_e : EQUILIBRIUM VAPOR PRESSURE

P_L : PRESSURE IN LIQUID

T : TEMPERATURE OF THE LIQUID

σ : SURFACE TENSION

k : BOLTZMANN CONSTANT

M : MOLECULAR WEIGHT

n : MOLECULAR NUMBER DENSITY IN LIQUID

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BUBBLE GROWTH

- AFTER THE INITIAL GROWTH PERIOD (WHICH IS INERTIA CONTROLLED) THE PRESSURE INSIDE THE BUBBLE IS DETERMINED BY THE YOUNG-LAPLACE EQUATION

$$P_e - P_L = 2\sigma / R$$

- THE BUBBLE TEMPERATURE IS ASSUMED TO BE EQUAL TO THE SATURATION TEMPERATURE AT THE PRESSURE P_e

- BUBBLE GROWTH IS DETERMINED BY THE ENERGY EQUATION

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ENERGY EQUATION FOR BUBBLE

$$q = m\, c_p \dot{T}_p + \dot{m}\, h_{fg}$$

c_p : VAPOR-PHASE SPECIFIC HEAT

h_{fg} : LATENT HEAT OF EVAPORATION

$\dot{m}$: BUBBLE MASS GROWTH RATE ("EVAPORATION RATE")

$\dot{T}_p$: RATE OF CHANGE OF BUBBLE TEMPERATURE

$$\dot{T}_p = \left(\frac{dT}{dp}\right)_{sat} \vec{U} \cdot \nabla p$$

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HEAT TRANSFER RATE TO BUBBLE

$$q = Nu\, \pi\, D\, \kappa \left(T - T_p\right)$$

Nu : NUSSELT NUMBER

D : BUBBLE DIAMETER

κ : THERMAL CONDUCTIVITY OF THE LIQUID

T : LIQUID TEMPERATURE

T_p : BUBBLE TEMPERATURE

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BUBBLE EQUATION OF MOTION

$$\vec{F} = m\left(1+\frac{\Delta_A}{2}\frac{\rho}{\rho_p}\right)\frac{d^2\vec{X}}{dt^2}$$

ρ : LIQUID DENSITY

ρ_p : VAPOR DENSITY

Δ_A : ADDED MASS COEFFICIENT (=1 FOR STOKES FLOW)

m : BUBBLE MASS

$\vec{X}$: BUBBLE POSITION VECTOR

$\vec{F}$: FORCE ON BUBBLE
- DRAG FORCE
- BUOYANCY
- PRESSURE GRADIENT

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DRAG COEFFICIENT

- FOR BUBBLES

$$C_D\left(=\frac{2F_d}{\pi\rho a^2 u^2}\right) = 14.9 / Re^{0.78} \quad Re > 2$$

$$= 16 / Re \quad Re < 2$$

GOOD FIT TO NUMERICAL SOLUTION
COMPARES WELL WITH EXPERIMENTS, WIDELY USED

- FOR SOLID SPHERES

$$C_D = \frac{24}{Re}\left[1 + 0.15\, Re^{0.687}\right] \quad Re < 1000$$

$$= 0.438, \quad Re > 1000$$

- NOTE: FOR SAME a AND u, DRAG ON A BUBBLE IS SMALLER (FACTOR 6 FOR Re = 900)

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COMPUTATIONAL TEST CASE

SINGLE BLADE LABYRINTH SEAL

APPROXIMATES ONE BLADE OF THE LOX PREBURNER BOOST PUMP IMPELLER SEAL IN THE SSME

FLOW CONDITIONS:

INLET PRESSURE	: 3.67 MPa (532 psi)
INLET TEMPERATURE	: 130 K (234 °R)
DENSITY	: 1088 kg/m^3 (167.9lb/ft^3)
PRESSURE DROP	: 1.47 MPa (213 psi)

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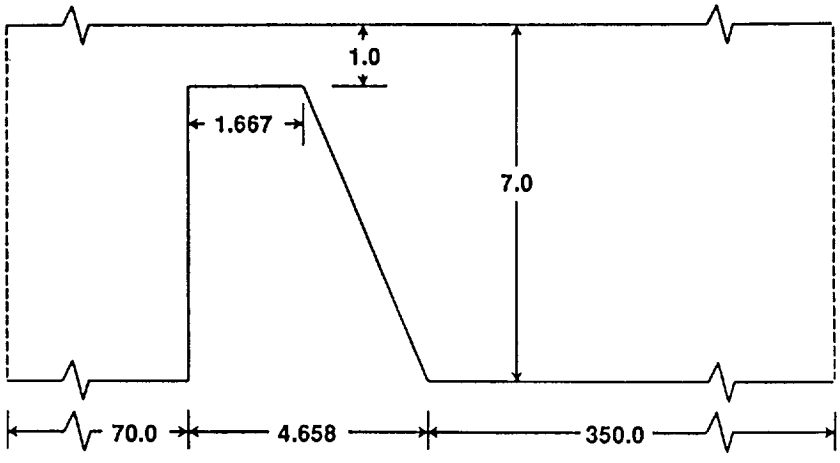
GEOMETRY



SEAL GAP: 1.524 x 10^{-4} m (0.006 inch)

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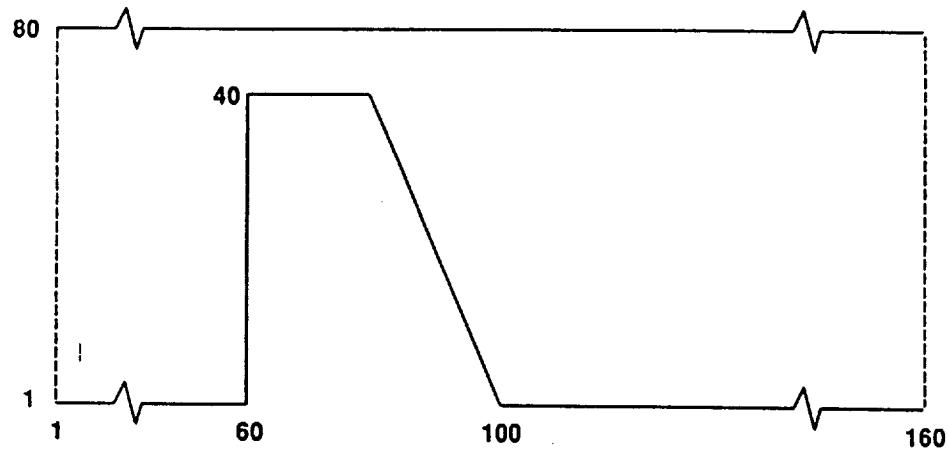
COMPUTATIONAL GRID



80 x 160 GRID

NEAR-WALL RESOLUTION IN THE GAP = 1.0×10^{-4} (= 0.015 μm)

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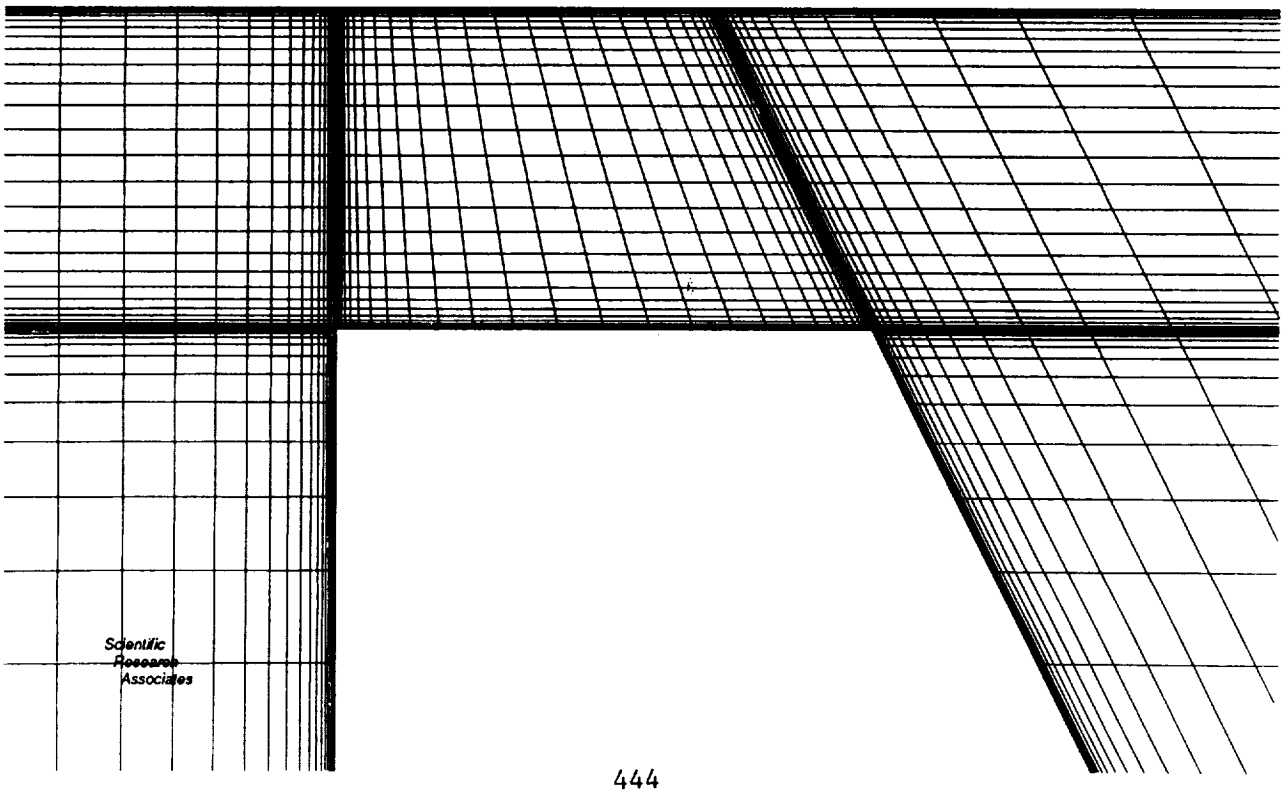
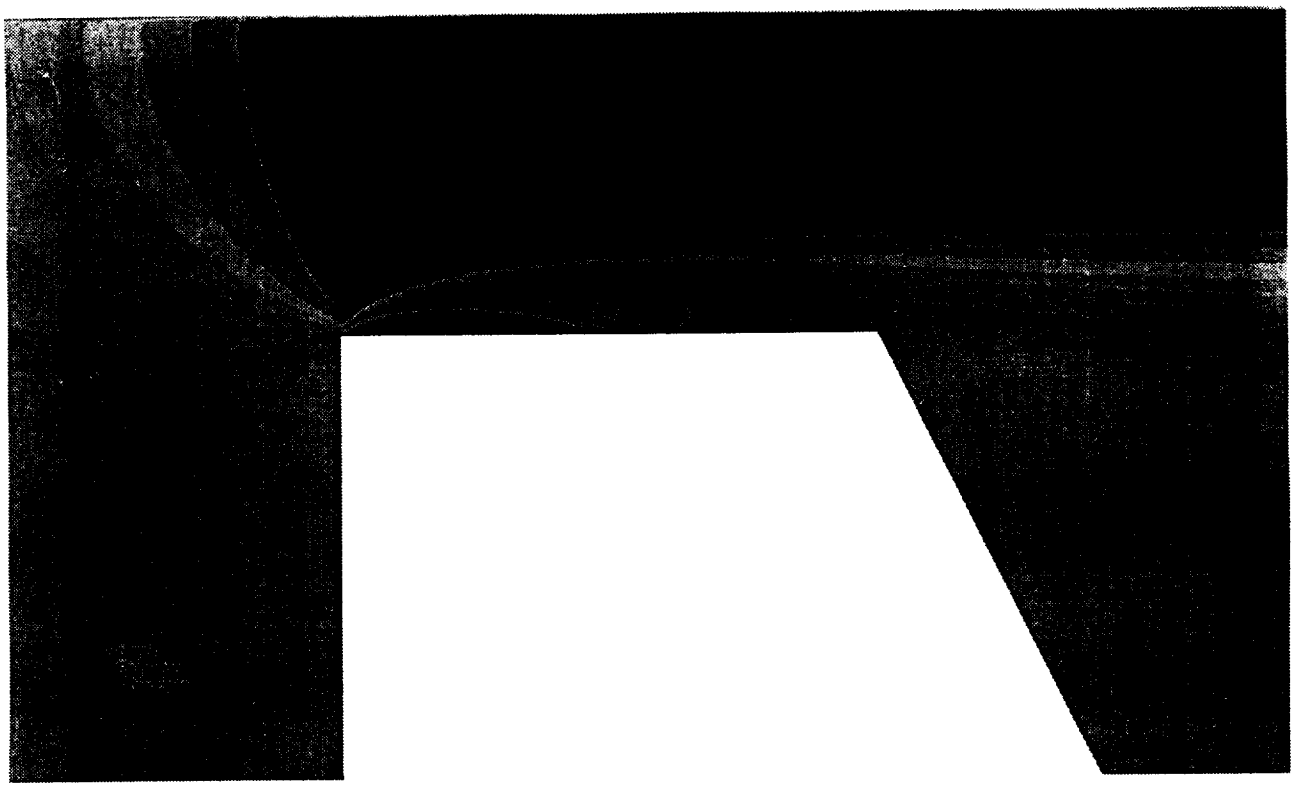
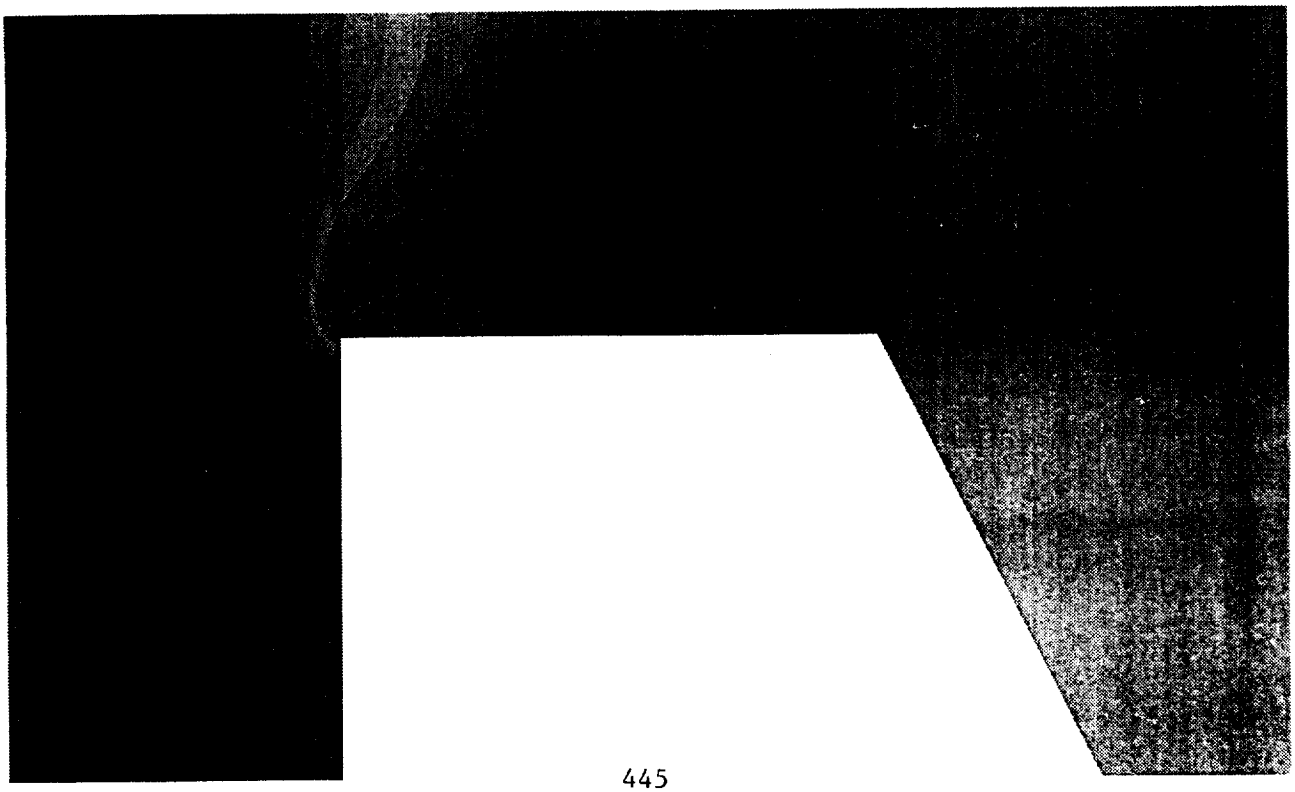
COMPUTATIONAL GRID

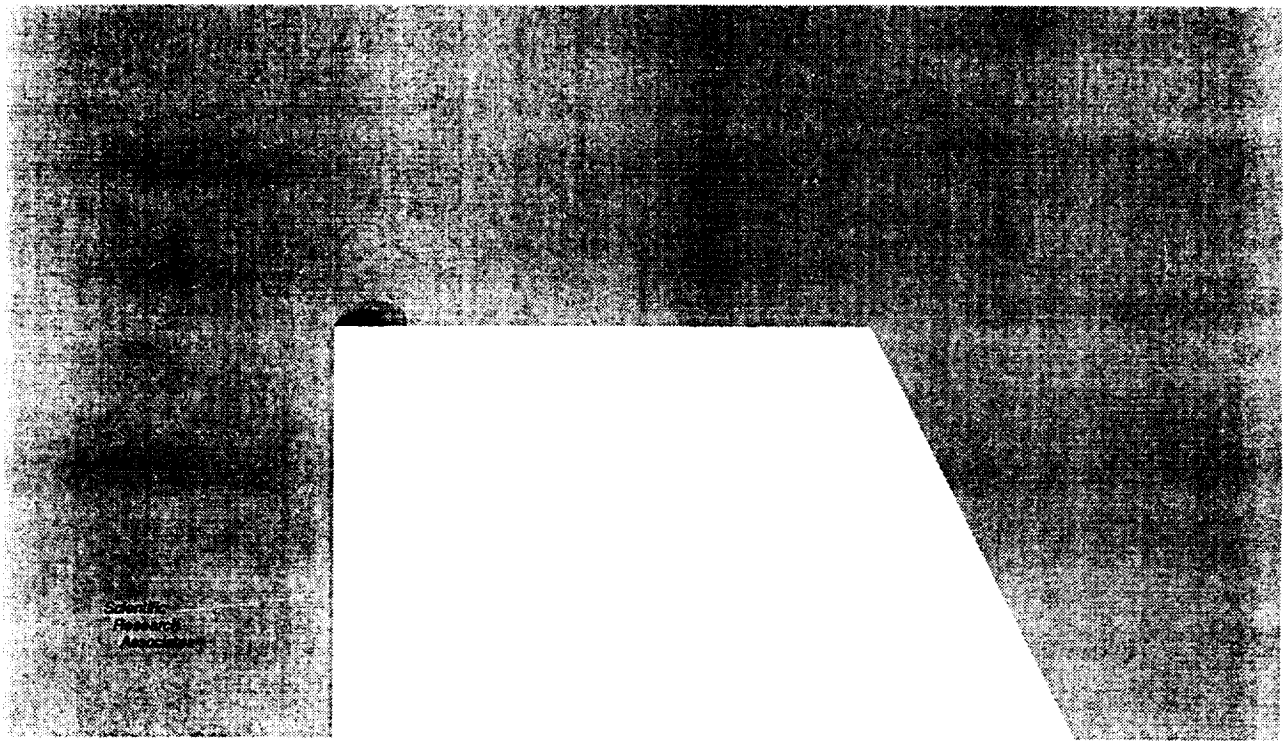


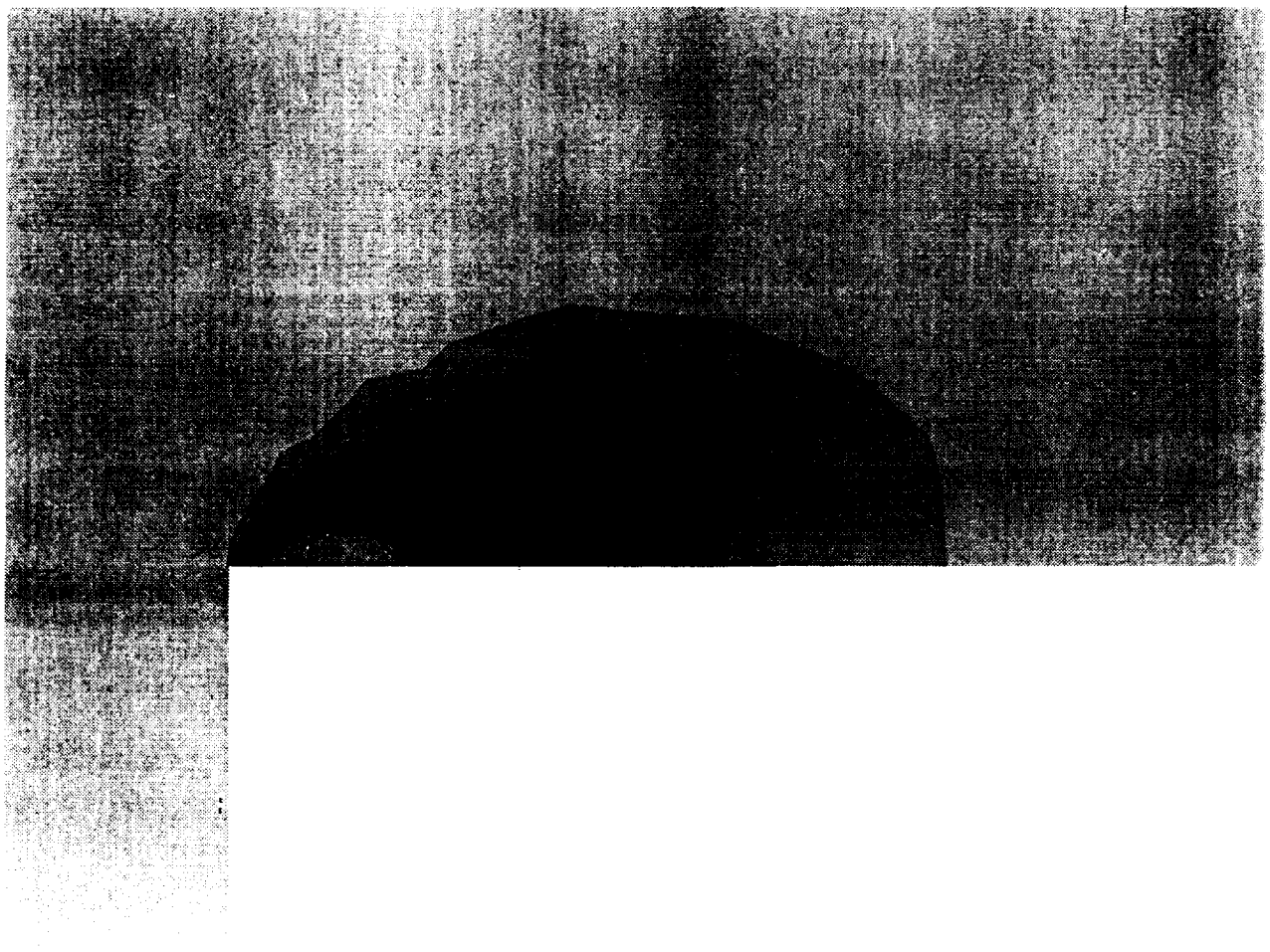
AXIAL VELOCITY

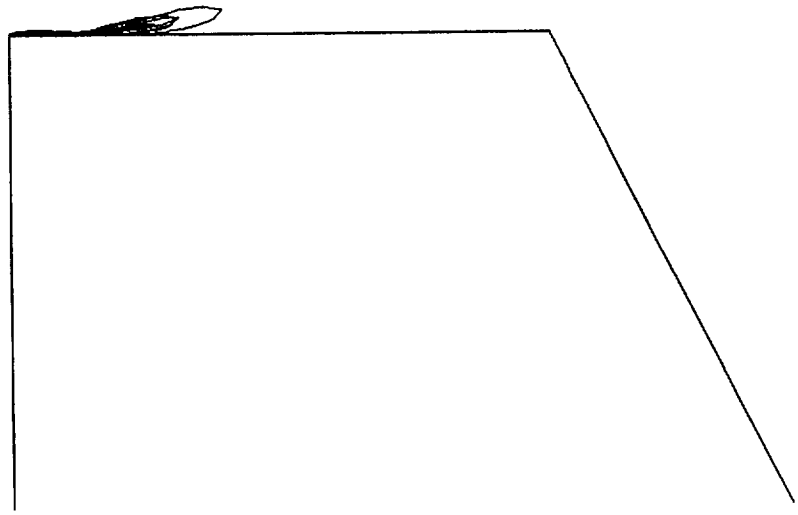


PRESSURE









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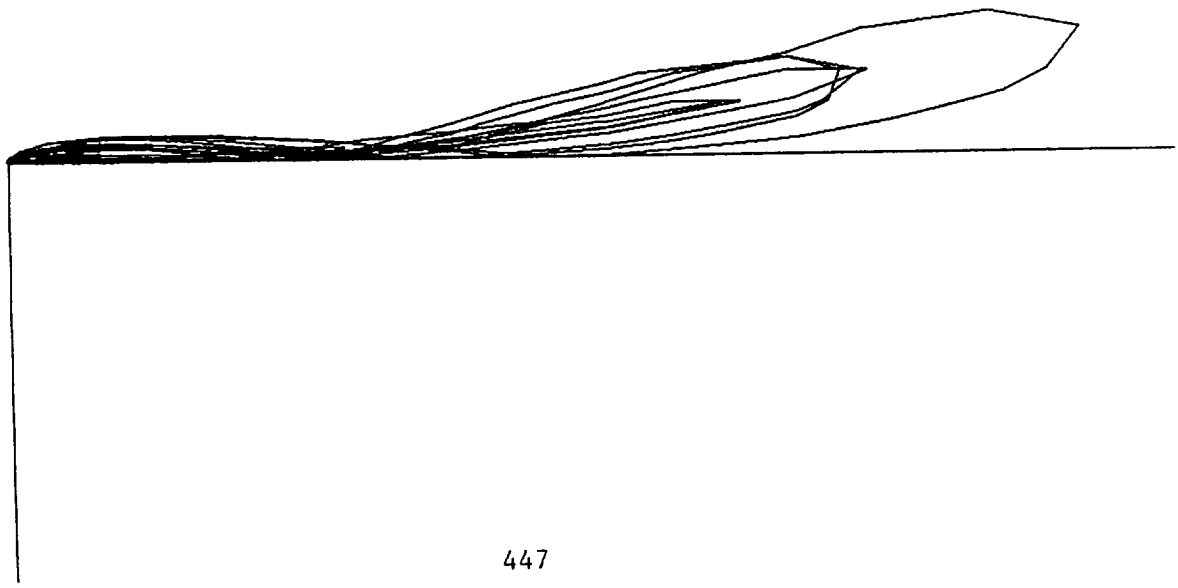

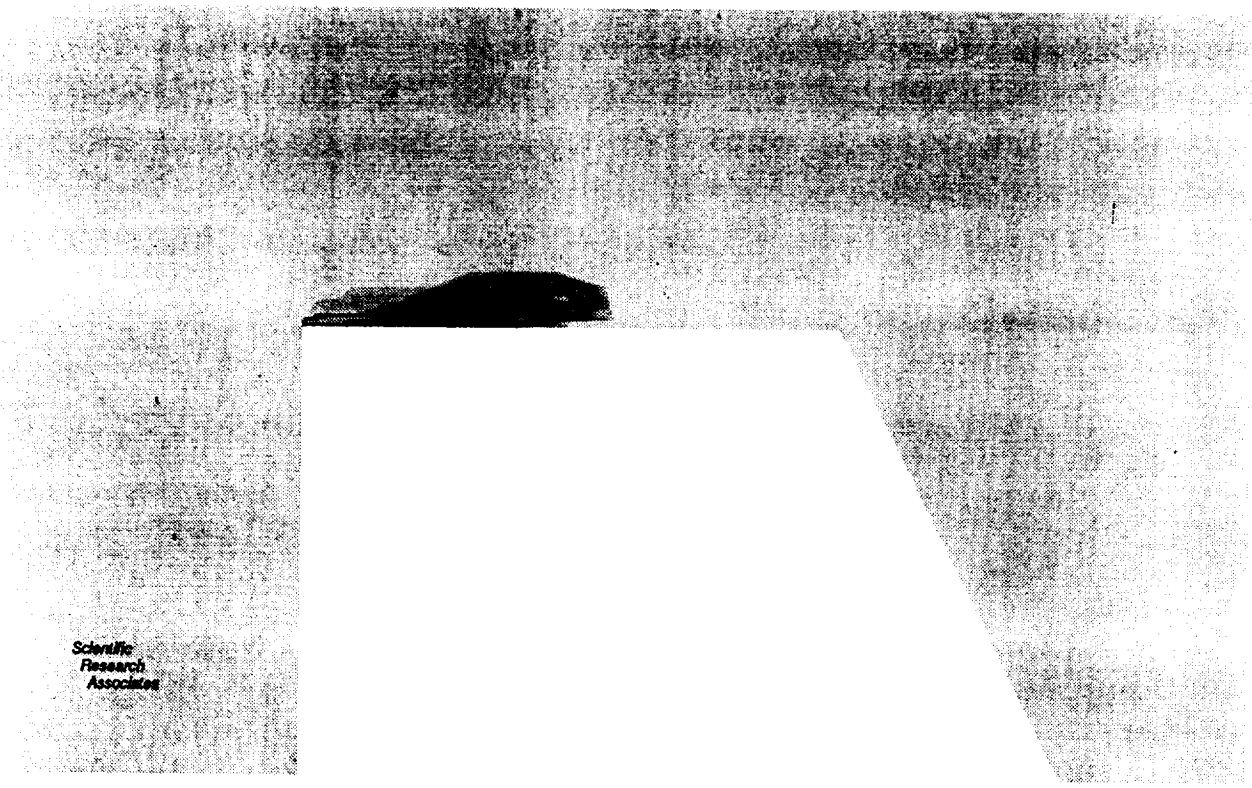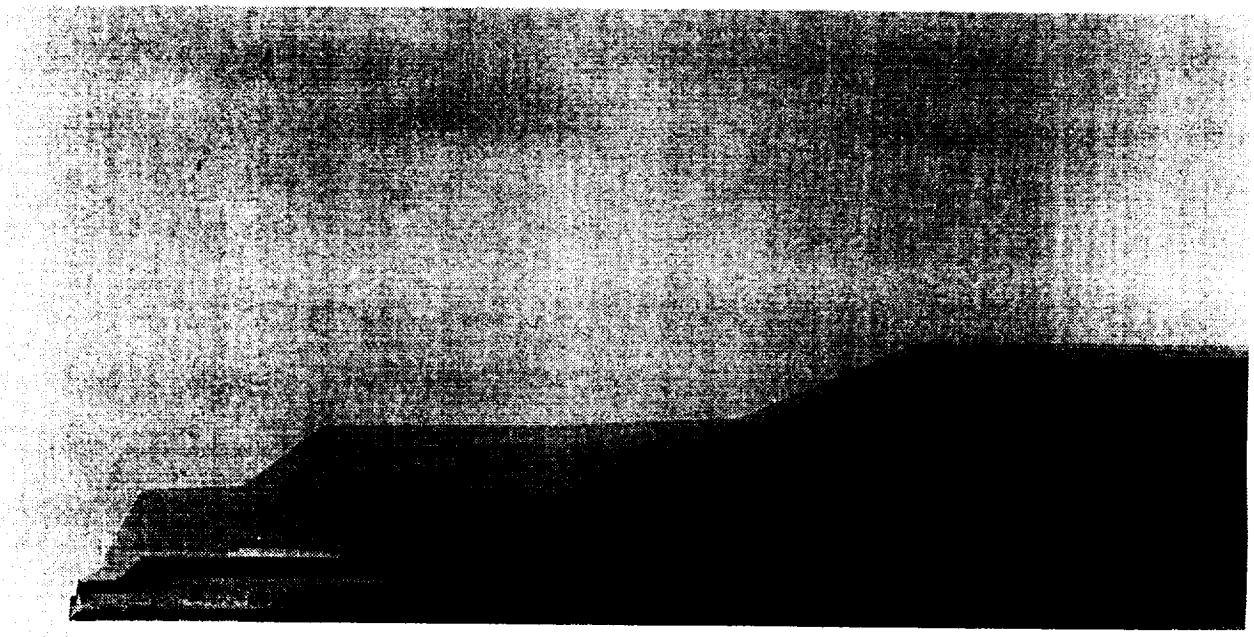
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