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Estimation of spectral emissivity in the thermal infrared

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1. INTRODUCTION

A number of algorithms are available in the literature that attempt to remove most of the effects of temperature from thermal multispectral data where the final goal is to extract emissivity differences. Early approaches include adjacent spectral band ratioing, broad band radiance normalization and the use of one band where emissivities are generally high (e.g., 11 to 12 μm) to determine the temperature (Salisbury, 1992). More recent work (Salisbury, 1992) has produced two techniques that use data averaging to extract temperature to leave a quantity related to emissivity changes. These two techniques (Thermal Log Residuals and Alpha Residuals) have been investigated and compared and appear to provide reasonable results.

The analysis presented in this paper develops a thermal IR multispectral temperature/emissivity estimation procedure based on formal estimation theory, Gaussian statistics, and a stochastic radiance signal model including the effects of both temperature and emissivity. The importance of this work is that this is an optimal estimation procedure which will provide minimum variance estimates of temperature and emissivity changes directly.

Section 2 discusses optimal linear spectral emissivity estimation and Section 3 is a summary.

2. OPTIMAL LINEAR SPECTRAL EMISSIVITY ESTIMATION

A stochastic model for the spectral radiance in the thermal IR using the Planck equation ignoring reflection and atmospheric contributions is:

$$L(\lambda,\overline{T}) = \frac{c_1}{\pi\lambda^5}\,\frac{1}{e^{c_2/\lambda\overline{T}}-1}\left\{\overline{\epsilon}(\lambda) + \Delta\epsilon(\lambda) + \frac{c_2\,\overline{\epsilon}(\lambda)\,\Delta T}{\lambda\overline{T}^2}\right\} \qquad (1)$$

c_1 First Radiation Constant
c_2 Second Radiation Constant
$\overline{T}$ Average background temperature in region
$\overline{\epsilon}(\lambda)$ Average background emissivity in region
λ Wavelength
ΔT Change in background temperature (a zero mean random variable)
$\Delta\epsilon(\lambda)$ Change in background emissivity (a zero mean random process)

The term in braces in Equation 1 is the apparent emissivity, $\varepsilon^{app}(\lambda)$. The model assumes samples are independent and identically distributed spatially. From Equation 1 it is clear that changes in temperature affect all wavelengths uniformly. Thus ΔT can be considered as a random process of rank one with respect to wavelength. In a generalized sense it is also considered to be narrowband since all of the variation induced by temperature will fall along a single direction with a properly chosen expansion basis for the spectral radiance. The covariance of the apparent emissivity can be computed assuming that temperature and emissivity are independent as :

$$K_{\epsilon app}(\lambda,\eta) = K_{\epsilon}(\lambda,\eta) + \frac{C_2^2\,\overline{\epsilon}(\lambda)\,\overline{\epsilon}(\eta)}{\lambda\eta\overline{T}^4}\sigma^2_{\Delta T} \tag{2}$$

The second term on the right hand side of Equation 2 is (to within the temperature variance) known. Moreover in an absolute sense the second term more often is the dominant one (thermal IR phenomenology tells us this, especially in the daytime) since it is the one dependent on temperature. The first term on the right hand side of the equation is more difficult to specify. In order to describe it for a certain class of material an ensemble of measurements is needed for that class. Since only a very limited number of measurements exist for spectral emissivity in the thermal IR, a description of the first term for an arbitrary class is not available. In this development it will be assumed that the first term is stationary white. That is:

$$K_{\epsilon}(\lambda,\eta) = \frac{N_{\epsilon}}{2}\delta(\lambda-\eta) \tag{3}$$

This assumption is very appealing for large regions because large regions usually contain many different classes of materials and it is reasonable to assume the second order statistics of the ensemble are white.

2.1 Cancellation of Narrow Band Interference (Temperature)

What follows here is a procedure for separating temperature and emissivity. What motivates the procedure is the observation that emissivity and temperature cannot be separated from a single multispectral measurement but a statistically based procedure can provide separation with acceptable error. Again writing Equation 1 but including sensor noise $w(\lambda)$:

$$L(\lambda,\overline{T}) = L^{BB}(\lambda,\overline{T})\left\{\overline{\epsilon}(\lambda) + \Delta\epsilon(\lambda) + \frac{C_2\overline{\epsilon}(\lambda)\,\Delta T}{\lambda\overline{T}^2}\right\} + w(\lambda) \tag{4}$$

Since the means are assumed to be known or well estimated from the data, the mean radiance can be subtracted off along with a division by the blackbody function to obtain the following equation for observed variation in apparent emissivity:

$$\Delta\epsilon^{app}(\lambda) = \Delta\epsilon(\lambda) + \frac{C_2\overline{\epsilon}(\lambda)\,\Delta T}{\lambda\overline{T}^2} + w'(\lambda) \tag{5}$$

The quantity $\Delta\varepsilon(\lambda)$ contains the relative emissivity behavior and it is this component that needs to be estimated. To estimate a realization of the random process $\Delta\varepsilon(\lambda)$, the covariance of the apparent emissivity is needed. The covariance of Equation 5 can be written as

$$K_{\epsilon app}(\lambda,\eta) = K_{\epsilon}(\lambda,\eta) + \frac{c_2^2\,\overline{\epsilon}(\lambda)\,\overline{\epsilon}(\eta)}{\lambda\eta\overline{T}^4}\sigma^2_{\Delta T} + \frac{N_o}{2}\,\delta(\lambda-\eta) \tag{6}$$

Here $N_o/2$ is the strength of the white sensor noise. It is recognized that the sensor noise term will not be stationary white due to the division by the Planck function. This will be ignored because modern sensors are generally clutter limited not noise limited, so a stationary assumption should introduce very little error. The second and third term of Equation 6 will be considered as known. Since it is assumed that the emissivity covariance is stationary white, the form of the apparent emissivity covariance is:

$$K_{\epsilon app}(\lambda,\eta) = \frac{c_2^2\,\overline{\epsilon}(\lambda)\,\overline{\epsilon}(\eta)}{\lambda\eta\overline{T}^4}\sigma^2_{\Delta T} + \frac{N_o+N_\epsilon}{2}\,\delta(\lambda-\eta) \tag{7}$$

The estimation filter is required to be linear but not necessarily shift invariant with respect to wavelength. This implies that not only stationary but nonstationary random processes can be estimated as well. In fact the stochastic description of the temperature behavior clearly indicates that it is nonwhite and nonstationary due to its non Toeplitz form:

$$K_{\Delta T}(\lambda,\eta) = \frac{c_2^2\,\overline{\epsilon}(\lambda)\,\overline{\epsilon}(\eta)}{\lambda\eta\,\overline{T}^4}\,\sigma^2_{\Delta T} \tag{8}$$

The optimal linear filter for estimating the temperature driven part of $\epsilon^{app}(\lambda)$, $h_o(\lambda,\eta)$, is obtained from the following integral equation (Van Trees, 1968,1971):

$$\frac{(N_o+N_\epsilon)}{2}\,h_o(\lambda,\eta) + \int_{\lambda_i}^{\lambda_f} h_o(\lambda,x)\,K_{\Delta T}(x,\eta)\,dx = K_{\Delta T}(\lambda,\eta) \quad \lambda_i<\eta,\ \lambda<\lambda_f \tag{9}$$

Equations of this form are fundamental to all linear signal processing. A series solution to this equation is obtained by using the eigenfunctions and eigenvalues of $K_{\Delta T}(\lambda,\eta)$. Since the temperature driven term is rank one there is only one eigenfunction and eigenvalue and the optimal filter can be written as

$$h_o(\lambda,\eta) = \frac{\dfrac{c_2^2\sigma^2_{\Delta T}}{\overline{T}^4}}{\dfrac{(N_o+N_\epsilon)}{2} + \dfrac{c_2^2\sigma^2_{\Delta T}}{\overline{T}^4}\displaystyle\int_{\lambda_i}^{\lambda_f}\frac{\overline{\epsilon}^2(x)}{x^2}\,dx}\;\frac{\overline{\epsilon}(\lambda)}{\lambda}\;\frac{\overline{\epsilon}(\eta)}{\eta} \tag{10}$$

This filter, when applied to the stochastic portion of the signal, will produce an estimate of the correlated component driven by temperature. The filter projects the observables onto the temperature direction defined by $\overline{\epsilon}(\lambda)\,/\,\lambda$. To produce an estimate of the stochastic portion of the emissivity, $\Delta\epsilon(\lambda)$, the weighted projection is subtracted from the observations. The weighting is important because if all the energy were to be taken out in the temperature direction the "space" spanned by the emissivity would be dimensionally too small providing poorer estimates. To make the emissivity estimation equation simple in form the following are defined:

$$\Phi_1(\lambda) = \frac{\overline{\epsilon}(\lambda)}{\lambda}\left[\int_{\lambda_i}^{\lambda_f}\frac{\overline{\epsilon}^2(x)}{x^2}\right]^{-1/2} \tag{11}$$

$$\zeta_1 = \frac{C_2^2\,\sigma_{\Delta T}^2}{\overline{T}^4}\int_{\lambda_i}^{\lambda_f}\frac{\overline{\epsilon}^2(x)}{x^2} \tag{12}$$

$$\sigma^2 = \frac{N_0 + N_\epsilon}{2} \tag{13}$$

The optimal estimate of emissivity becomes

$$\Delta\hat{\epsilon}(\lambda) = \Delta\epsilon^{app}(\lambda) - \frac{\zeta_1}{\sigma^2 + \zeta_1}\left\{\int_{\lambda_i}^{\lambda_f}\Delta\epsilon^{app}(\eta)\;\Phi_1(\eta)\;d\eta\right\}\Phi_1(\lambda) \tag{14}$$

Equation 14 is the best linear minimum mean square estimate of the spectral emissivity which is the spectral apparent emissivity less the contribution from temperature variation.

3. SUMMARY

Temperature variations cause variations in the multispectral data that are highly correlated between spectral bands. Although many phenomenology based techniques have been developed to remove the variations due to temperature from the data, it has been shown in this paper that a formal development of the optimal estimator includes a temperature projection filter to remove the correlated variations due temperature leaving a direct estimate of emissivity changes.

4. REFERENCES

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