APPENDIX G


by

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Dependence of the multiplicities of secondary particles on the impact parameter in collisions of high-energy neon and iron nuclei with photoemulsion nuclei

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Received 8 January 1992
(Revised 6 July 1992)

Abstract: A method is proposed for finding the dependence of mean multiplicities of secondaries on the nucleus-collision impact parameter from the data on the total interaction ensemble. The impact parameter has been shown to completely define the mean characteristics of an individual interaction event. A difference has been found between experimental results and the data calculated in terms of the cascade-evaporation model at impact-parameter values below 3 fm.

1. Introduction

The characteristics of secondaries produced in an interaction between two nuclei are defined as a first approximation by the energy of the projectile nucleus, the masses of the colliding nuclei, and the impact parameter of the collision. As a rule, the energy of the projectile nucleus and the masses of the colliding partners are known, whereas the impact parameter is difficult to determine for a particular interaction. The collision centrality can be estimated from the characteristics of the projectile-nucleus spectator fragments produced by a given collision, just as was done in the experiment \(^1\) by detecting charged secondaries of pseudo-rapidities \(\eta > 1.3\) generated in relativistic oxygen-ion interactions with various targets. In experiments which measure all charged secondaries, however, the study of the dependence of interaction characteristics on impact parameter has been qualitative.
and dealt mainly with two extreme situations, namely, the extremely peripheral and central collisions\(^2\)\(^4\). The present work is aimed at (a) testing the methods of determining the collision impact parameter; (b) finding the dependence of the mean multiplicities of secondaries on the impact parameter; and (c) studying the impact-parameter dependence as a function of the masses and energies of interacting nuclei and of the secondary species.

2. Initial data and method

The analyzed experimental ensembles are composed of 497 inelastic interactions of Fe nuclei (1.8 GeV/nucleon) and 236 interactions of Ne nuclei (3.6 GeV/nucleon) with photoemulsion nuclei\(^7\)\(^9\). The target nucleus (H, CNO or AgBr), the individual charges of all particles, and the polar (\( \theta \)) and azimuthal (\( \psi \)) angles of the latter were found for each of the interactions. To classify interactions according to the type of target nucleus, we used the results of refs.\(^10\)\(^11\) where the characteristic features of collisions on each group of photoemulsion nuclei were studied using nuclear emulsions with different concentrations of light components (of H, C and O nuclei). Energy was determined for secondaries with energies below 400 MeV/nucleon. All singly charged particles were treated as protons, and all doubly charged particles as \( \alpha \)-particles. Following Antonchik et al. and Krasnov et al.\(^4\)\(^5\), all secondaries were classified into “black” particles (b-particles, \( E < 26 \) MeV/nucleon), “grey” particles (g-particles, \( 26 < E \leq 400 \) MeV/nucleon) and “shower” particles (s-particles, \( E > 400 \) MeV/nucleon), except the projectile fragments.

Each of the interactions was characterized by the particle number of a particular species \((n_h, n_g, n_s)\), by the total number of all charged particles \((n_{ch})\) produced in an interaction, and by the total charge of the non-interacting fragments of a projectile nucleus \((Q)\). Targets in the emulsion experiments were identified by criteria developed in Dudkin et al.\(^7\)\(^8\), and did not bias the multiplicity analysis.

The experimental results were compared with the cascade-evaporation model using two calculated ensembles of the interactions on H, CNO, Br, Ag nuclei: the first ensemble was produced by 1.8 GeV/nucleon \(^{56}\)Fe nuclei - 4767 events were calculated with the DCM version of the model\(^12\), and the second ensemble was produced by 3.6 GeV/nucleon \(^{22}\)Ne nuclei - 4976 events with the CEM version\(^13\). In both cases all the events were summed with weights corresponding to the observed cross sections and to the emulsion composition.

We use the experimental integral distributions which define the probability for producing interactions with the number of rays of type \(i\) equalling or exceeding a certain number \(n_i\):

\[
W_i \cdot n_i = \sum_{k_i \cdot n_i} W(k_i \cdot n_i) = W_{(n_i+1)} + W_{(n_i+2)} + \cdots ,
\]

(1)
where \( W_{(n_i)} \) is the experimental estimate of the probability of disintegration with the number of rays \( n_i \) to occur, which equals the ratio of the number of stars with \( n_i \) to the total number of stars in an ensemble. The subscript \( i \) indicates the type of ray (b-, g-, s-particles) or the total charge \( Q \) of non-interacting fragments.

In terms of the geometrical approach to nucleus-nucleus collisions, it was postulated that as the degree of overlap of two nuclei increases (the impact parameter decreases), the multiplicity of secondaries increases in strictly monotonic fashion \(^{14} \). The highest impact parameter \( b_i \) in a nucleus-nucleus interaction which gives rise to disintegration with the number of rays of type \( i \) equalling or exceeding \( n_i \) will, then, be

\[
b_i = \sqrt{\sigma/\pi} \sqrt{W_{(n_i)}}.
\]

(2)

where \( \sigma \) is the total inelastic cross section (cm\(^2\)); \( b \) is the impact parameter (cm); the subscript \( i \) indicates that the impact parameter is estimated from the integral star distribution of the particles of type \( i \). Note that the disintegrations with \( n_i \) particles will correspond to the collisions whose impact parameters belong to the range:

\[
\Delta b_i = \sqrt{\sigma/\pi} \left( \sqrt{W_{(n_i)}} - \sqrt{W_{(n_{i+1})}} \right).
\]

(3)

Having postulated a strictly monotonic decrease in the number of non-interacting protons of a projectile nucleus (\( Q \)) with decreasing impact parameter, after similar reasoning we obtain

\[
b_0 = \sqrt{\sigma/\pi} \sqrt{1 - W_{(Q)}}.
\]

(4)

Thus, using the interaction ensemble data and the above formulae, we may transform the integral distributions \( W_{(n_i)} \) to obtain the dependence of the number of particles of type \( i \) in an individual disintegration on the impact parameter \( n_i(b_i) \), and to find the range of the impact parameters of nucleus-nucleus interactions which give rise to interactions with a definite number \( (n_i) \) of secondaries. Also, the experimental correlations of multiplicities of type \( \langle n_i \rangle(Q) \) and \( \langle n_i \rangle(Q) \) inferred from the same ensemble of events may be used together with the relations between \( n_i \) and \( b_i \) [eqs. (2)-(4)] to obtain the dependence of the mean multiplicity of particles of type \( j \) on impact parameter \( \langle n_i \rangle(b_i) \) and \( \langle n_i \rangle(Q) \). The above-mentioned transformation procedure is shown schematically in fig. 1.

Let us assume a total ensemble of events in which the number of all charged particles and g-particles is recorded for each of the events. The data on the ensemble can then be used to obtain the integral distribution of the number of all charged particles \( (n_{ch}) \) in the event (fig. 1a) and to obtain the correlation dependence \( \langle n_{g-}\rangle(n_{ch}) \) of the mean number of g-particles in the disintegration on \( n_{ch} \) (fig. 1b). Eq. (3) is used to find the range of the impact parameters corresponding to the interactions with a particular \( n_{ch} \) value. For example, the events with \( n_{ch} = 4 \) at \( \langle n_{g-}\rangle = 0.75 \pm 0.09 \) belong to the variation range from \( W_{(n_{ch})} = 0.09 \) to \( W_{(n_{ch})} = 0.25 \) and, according to eq. (3), correspond to the impact parameters ranging from \( 0.3\sqrt{\sigma/\pi} \) to \( 0.5\sqrt{\sigma/\pi} \).
Thus, in terms of our geometrical concepts, the mean g-particle multiplicity for the 0.3√σ/π - 0.5√σ/π interval will be 0.75 ± 0.09.

The scheme shown in fig. 1 illustrates the use of an ensemble of events when only the numbers n_\(n_e\) and n_e were recorded. In the case where additional information has been obtained (i.e. the numbers n_\(n\), Q, etc.), the impact-parameter dependence of mean particle multiplicity may be found by different techniques (from different correlations of \(\langle n_j \rangle\) with other \(n_i\) and Q).

3. Results and discussion

The above formulae and the CEM-calculated ensembles of interactions of 3.6 GeV/nucleon 22Ne nuclei with H nuclei (617 events), CNO nuclei (1629 events),
and Ag, Br nuclei (2730 events) were used to obtain the dependencies of the number of all charged particles \( n_{ch} \) per event and of the mean multiplicity of these particles in a star \( \langle n_{ch} \rangle \) on the impact parameter (fig. 2a). Fig. 2b shows similar dependence of the multiplicity and the mean multiplicity of g-particles in a disintegration. The techniques treated in this work were checked for consistency by using the impact parameter prescribed in the CEM calculations for a particular event and by obtaining the "direct" dependencies of the mean multiplicities \( \langle n_{ch} \rangle \) and \( \langle n_g \rangle \) on the impact parameter. From fig. 2 it is seen that the dependencies on the Monte Carlo sample

![Figure 2](image_url)

Fig. 2. The impact-parameter dependencies of the number and of the mean multiplicity of all charged particles (a) and of g-particles (b) in calculated interactions of 3.6 GeV/nucleon \(^{22}\text{Ne}\) nuclei with hydrogen nuclei (1), with CNO nuclei (2) and with AgBr nuclei (3). The histograms are the dependencies \( n_{ch}(b_i) \) and \( n_g(b_i) \) inferred from the \( n_{ch} \) and \( n_g \) multiplicity distributions. The filled-in circles and triangles are the dependencies \( \langle n_{ch}(b_i) \rangle \) and \( \langle n_g(b_i) \rangle \) inferred from correlations with \( n_{ch} \) and with \( n_g \), respectively. The open circles are the results of the straightforward verification by CEM calculations.
\langle n_{\text{ch}}(b) \rangle \) and \langle n_q(b) \rangle obtained by different techniques, including the direct methods, coincide with each other in remarkable agreement. The agreement is observed for all three reactions which differ substantially in the masses of interacting nuclei, in the variation range of the mean particle multiplicity, and in the variation range of the impact parameter in a disintegration. [The analysis of the dependence of particle number on \( b \) has shown that the functions \( n_i(b) \) have significant dispersion, so only the dependencies of mean multiplicities inferred from correlations should be treated.]

The results obtained indicate that the impact parameter defines unambiguously the mean characteristics of a nucleus-nucleus interaction event. Therefore, given five dependencies of the types \( \langle n_h \rangle, \langle n_q \rangle, \langle n_s \rangle, \langle n_{\text{ch}} \rangle \) and \( \langle Q \rangle \) on impact parameter, one can readily obtain twenty correlation dependencies of the types \( \langle n_i, \rangle \langle n_j \rangle \) which are frequently analyzed in theoretical and experimental studies of nucleus-nucleus interactions.

The experimental dependencies of the mean multiplicities of particles produced during the first (fast) stage of nucleus-nucleus interaction on impact parameter and those calculated in terms of two modifications of the cascade-evaporation model \( ^{12,13} \) are compared in figs. 3 and 4 and table 1.

From the data obtained (fig. 3a, c and table 1) it follows that the mean total charge of the non-interacting fragments of a projectile nucleus (whose velocities are close to the projectile velocity and whose transverse momenta are lower than 230 MeV/c nucleon) depends on the impact parameter and, obviously, on the masses of colliding nuclei. A nearly linear dependence \( \langle Q \rangle(b) \) with a positive coefficient depending on projectile-nucleus mass is obtained in the collisions of relativistic nuclei with light (CNO) and heavy (AgBr) nuclei in most of the impact-parameter ranges. The coefficient proved to be 1.4 charge unit/fm in the case of \(^{22}\text{Ne}, \) and almost 2.5 times as high (3.4 charge unit/fm) in the case of \(^{56}\text{Fe}. \) In central collisions (at low values of \( b \)) the coefficient decreases with decreasing \( b. \) The effect is enhanced by increasing the mass of the colliding nuclei.

Both cascade-evaporation model versions describe qualitatively the trend in the experimental correlation dependencies but show a systematic excess over experimental data for a range of small \( b. \)

The mean multiplicity of s-particles (the produced pions with energy \( E > 60 \text{ MeV}, \) and the interacting protons with energy \( E > 400 \text{ MeV} \) and transverse momentum \( P > 230 \text{ MeV/c} \)) increases with decreasing impact parameter of collision (see fig. 3b, d) in all the interaction types examined. Note the similar absolute values of the impact-parameter dependencies of the s-particle multiplicities for different beams and energies.

The data of table 1 indicate that the multiplicity \( n_i \) increases with target-nucleus mass. In the case of interactions at the impact parameters ranging from 2 to 4 fm, for example, the experimental value of \( \langle n_i \rangle \) is \( 8.9 \pm 1.2 \) for the \(^{22}\text{Ne} + \text{CNO} \) collisions and \( \langle n_i \rangle = 30 \pm 4 \) for the \(^{22}\text{Ne} + \text{AgBr} \) collisions.
The examination of the impact-parameter dependencies of the mean multiplicities of g-particles (the 26-400 MeV knocked-out from target nuclei), \( \langle n_g \rangle (b) \) (fig. 4a, c), has shown that the form of \( \langle n_g \rangle (b) \) is substantially affected by the target-nucleus mass. In the case of interactions with CNO nuclei, \( \langle n_g \rangle \) proves in practice to be nearly independent of impact parameter at \( b = 0-3 \) fm; afterwards, the multiplicity decreases slightly with increasing \( b \). The variation of the projectile-nucleus mass by a factor of more than 2 (when going from \(^{22}\)Ne to \(^{56}\)Fe) does not result in any pronounced changes of the character or numerical values of the given correlation.

In the case of collisions with heavy nuclei, on the contrary, the target-nucleus mass affects substantially the g-particle multiplicity. In this case, a nearly linear
dependence $\langle n_g \rangle(b)$ is observed at a high negative coefficient amounting to 4 g-particles/fm.

The comparison between the experimental and model-calculated dependencies $\langle n_g \rangle(b)$ and $\langle n_s \rangle(b)$ has shown that the models can quantitatively represent the peripheral collisions only. At low values of impact parameters ($b < 5$ fm for $^{22}$Ne + AgBr and $b < 6$ fm for $^{56}$Fe + AgBr collisions), the calculation results are systematically in excess of the experimental data.

For the purposes of further analysis, it is expedient to present the s- and g-particle multiplicities per single interacting proton of a projectile nucleus:

$$k_s = \frac{\langle n_s \rangle}{Z - \langle Q \rangle}, \quad k_g = \frac{\langle n_g \rangle}{Z - \langle Q \rangle},$$

where $Z$ is the charge of the projectile nucleus.
Mean values of g- and s-particle multiplicities and of Q as functions of impact parameter in 56Fe and 22Ne collisions with light and heavy photoemulsion nuclei. Shown in parentheses are the cascade-evaporation model calculation results.

<table>
<thead>
<tr>
<th>Reaction type</th>
<th>Type of secondary</th>
<th>Range of impact parameter (fm)</th>
<th>0-2</th>
<th>2-4</th>
<th>4-6</th>
<th>6-8</th>
<th>8-10</th>
<th>10-12</th>
<th>0-12</th>
</tr>
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<tbody>
<tr>
<td>Fe ± AgBr</td>
<td>\langle n_g \rangle</td>
<td></td>
<td>34±3</td>
<td>28±2</td>
<td>24±1</td>
<td>14.1±0.8</td>
<td>3.3±0.2</td>
<td>1.6±0.2</td>
<td>12.9±1.1</td>
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<tr>
<td></td>
<td>\langle n_s \rangle</td>
<td></td>
<td>33±3</td>
<td>33±3</td>
<td>24±2</td>
<td>13±1</td>
<td>3.5±0.3</td>
<td>1.9±0.2</td>
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<tr>
<td></td>
<td>\langle Q \rangle</td>
<td></td>
<td>3.2±0.4</td>
<td>5.1±0.8</td>
<td>9.4±1.1</td>
<td>15.8±0.9</td>
<td>23.0±0.3</td>
<td>25.1±0.2</td>
<td>16.1±0.8</td>
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<td>Fe ± CNO</td>
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<td>\langle n_s \rangle</td>
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<td>\langle Q \rangle</td>
<td></td>
<td>10.1±1.4</td>
<td>17.8±1.1</td>
<td>21.8±0.6</td>
<td>24.0±0.5</td>
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<tr>
<td>Ne ± AgBr</td>
<td>\langle n_g \rangle</td>
<td></td>
<td>33±4</td>
<td>22±4</td>
<td>14±3</td>
<td>6.5±0.8</td>
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<tr>
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<td>\langle n_s \rangle</td>
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<td>35±4</td>
<td>30±4</td>
<td>18±3</td>
<td>9.5±0.9</td>
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<tr>
<td></td>
<td>\langle Q \rangle</td>
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<tr>
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<td>8.9±1.2</td>
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<tr>
<td></td>
<td>\langle Q \rangle</td>
<td></td>
<td>3.3±0.5</td>
<td>6.4±0.8</td>
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<td>6.8±0.3</td>
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</tr>
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</table>

The resultant impact-parameter dependencies of the multiplicities in the case of the interactions of 22Ne and 56Fe nuclei with heavy photoemulsion nuclei (fig. 4b, d) exhibit very different trends. The specific multiplicities decrease systematically with increasing impact parameter in the case of 22Ne+AgBr interactions, whereas any noticeable correlation between k_s, k_p and b is absent in the case of 56Fe+AgBr interactions.

The difference in the absolute values of k, for 22Ne and 56Fe projectiles is readily explained by a greater fraction of charged pions, \langle n_p \rangle = k - 1, per single interacting proton of a projectile nucleus when changing from 1.8 GeV/nucleon (56Fe) interactions to the 3.6 GeV/nucleon (22Ne) interactions. Such an explanation is corroborated by the similar variations of k_p in the interactions with light (CNO) nuclei (see table 1).

The \( k_p \) value defines, first of all, the ratio of the masses of interacting nuclei. If the target-nucleus mass exceeds the projectile mass, the specific g-particle multiplicity
will exceed unity, and vice versa (see table 1). Any increase in the collision energy enhances the cascading because the latter involves both the produced particles and the singly interacting nucleons. The increase of $k_s$ and $k_t$ with decreasing impact parameter in the case of $^{22}$Ne + AgBr, and the absence of any definite dependence in the case of $^{56}$Fe + AgBr, seem to us to indicate a substantial effect of the produced particles on the cascading, especially in the central and nearly central interactions. The cascade-type models represent quite adequately the experimental dependencies $k_s(b)$ and $k_t(b)$, thereby indicating that the nucleus-nucleus interaction can be described in terms of these models. At the same time, the quantitative differences between the calculated and experimental data rise systematically with an increasing interacting nucleon number in an individual collision. For example, the model can well describe the $^{22}$Ne collisions with light photoemulsion nuclei [except the range $b = 0$–2 fm; the interval of differences increases up to $b = 0$–4 fm in the case of $^{22}$Ne + AgBr collisions and reaches 0–6 fm in the case of $^{56}$Fe + AgBr interactions].

4. Conclusions

The main results of the present work can be summarized as follows:

(1) The impact parameter of a collision has been shown to unambiguously define the mean multiplicities of secondaries in the interaction;

(2) The techniques for finding the impact-parameter dependence of mean particle multiplicity from the data on the total ensemble of interactions have been proposed and tested;

(3) The projectile-nucleus energy, the masses of the colliding nuclei, and the species of the examined secondaries have been found to substantially affect the impact-parameter dependencies of the mean and specific multiplicities of the secondaries.

(4) The experimental results have been compared with the data calculated in terms of the cascade-evaporation model. The particular impact-parameter values at which the theoretical data differ from the experimental results have been found.

E.V. Benton would like to acknowledge partial support by NASA under grant number NAG9-235.

References

4) V.A. Antonchik et al., JINR Preprint, Dubna, P1-83-577 (1983)
5) S.A. Krasnov et al., JINR Rapid Communications, Dubna, 16-86 (1986) 11
13) V.S. Barashenkov et al., JINR Preprint, Dubna, E2-83-117 (1983)