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NAG 2-570
Final Report

The Calculation of the Mass Moment of Inertia of a Fluid in a Rotating Rectangular Tank

SUMMARY

This analysis calculated the mass moment of inertia of a non-viscous fluid in a slowly rotating rectangular tank. Given the dimensions of the tank in the x, y, and z coordinates, the axis of rotation, the percentage of the tank occupied by the fluid, and angle of rotation, an algorithm was written that could calculate the mass moment of inertia of the fluid. While not included in this paper, the change in the mass moment of inertia of the fluid could then be used to calculate the force exerted by the fluid on the container wall.

ANALYSIS

Depending on the dimensions of the tank, the amount of fluid in the tank, and the angle of rotation, the resulting shape of the fluid can be broken down into simple geometries. Since the fluid was assumed to be incompressible, non-viscous, and the rotational velocity small, the shape of the fluid was represented by rectangular and triangular prisms. The most simple fluid shapes were represented by a single rectangular or a single triangular prism, while the most complex geometry was represented by two rectangular and one triangular prism (See Fig. 1 and 2). The mass moment of inertia ("Iz" in this coordinate system) of each prism was first calculated relative to its centroid. Then the parallel axis theorem was used to calculate the Iz of the entire volume at either the fluid's center of mass or the container's center of rotation.

The mass moment of inertia of a rectangular prism with respect to its centroid and the axis shown was calculated by the following equation:[1]

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$$\overline{I_z} = \frac{m(a^2+b^2)}{12}$$

where:

I_z = Moment of inertia with respect to the z-axis.
a = Length of rectangular prism in x-axis.
b = Length of rectangular prism in y-axis.
m = Mass of prism.

The mass moment of inertia of a triangular prism with respect to its centroid and the axis shown was calculated by the following equations:[2]

$$\overline{I_z} = \frac{m(a^2+b^2)}{18}$$

The parallel axis theorem:[1]

$$I_z = \overline{I_z} + m(\overline{x^2} + \overline{y^2})$$

where:

x = Distance in x-direction from centroidal to arbitrary axis.
y = Distance in y-direction from centroidal to arbitrary axis.

For comparison, the effective moment of inertia of fluid was calculated by the following formula:[3]

$$\frac{I_{Fy}}{I_{Sy}} = 1 - \frac{4r_1^2}{1+r_1^2} + 2.510\left(\tanh\frac{\pi}{2r_1} + 0.0045\right)\left(\frac{r_1^3}{1+r_1^2}\right)$$

where:

I_{Fz} = Effective moment of inertia of fuel about z-axis.
I_{Sz} = Moment of inertia of solidified fuel about z-axis.
= Iz when theta equals zero.
r1 = b/a = tank aspect ratio in xy-plane.

RESULTS

As the partially filled rectangular tank shown in figures 1 or 2 rotates about its origin, the Iz of the fluid will change. Program MomentOfInertia (See Appendix B) calculated the Iz of the fluid relative to the centroid of the fluid. In tables one through three, three different tank dimensions (all with unit depth) are shown: 1 x 2, 1 x 4, and 1 x 8, respectively. With each tank dimension, the mass moment of inertia for three different fluid volumes were tabulated. Only zero through 90 degrees were calculated since the mass moment of inertia for 90 through 180 degrees are mirror image of the shown data. The data then repeats every 180 degrees. This data is also shown graphically in figures 3 through 6.

As expected, the Iz of tanks with aspect ratio of 1/2 did not change significantly relative to tank rotation at any fluid level. This was due to the proximity of the aspect ratio to unity. With smaller (or greater) aspect ratios, the change in Iz increased significantly. For 50% volume, there was a 112% increase in Iz/rho for a/b = 1/2; while for a/b = 1/8, the change was 278%. Decreasing fluid volume also increased the change in Iz as the tank rotated. For a/b = 1/8, and the fluid volume was 80 percent, the change in Iz was 54%. For 20 percent fluid volume and the same aspect ratio, the change in Iz was 1,700%.

Program Moment1 (See Appendix B) calculated the Iz of the fluid relative to the center of rotation, which in this case was the origin. The calculations were very similar to the previous, except that the center of rotation was used as the axis instead of the C.G. of the fluid. Tables 3 through 6 shown the values calculated for 3 different aspect ratios and 3 different fluid volumes for each aspect ratio. These values are shown graphically in figures 7 thru 9.

As compared to the Iz relative to the C.G. of the fluid, the Iz relative to the center of rotation was greater, given the same aspect ratio and fluid volume. The greatest increase was the 20 percent fluid volume and aspect ratio = 1/2. The smaller percentage filled containers had greater increases than the higher percentage filled containers due to the greater change in the distance between the C.G. of the fluid and the center of rotation. For example, while the Iz of zero degrees, aspect ratio = 1/5, and 80 percent fluid volume changed 17% when the axis changed from C.G. of fluid to center of rotation, the Iz for the same angle and aspect ratio, but only 20% fluid volume, changed 1,873%.

The change in the mass moment of inertia also increased with decreasing aspect ratio and decreasing fluid volume, however, the percent change was not as dramatic. The change in Iz for aspect ratio = 1/2 and 20% fluid volume relative to the C.G. was 248%, while the change in Iz relative to center of rotation was reduced to 48%.

The Iz of a tank with decreasing fluid level was also calculated by altering program MomentCG. The fluid level started at 100% at 0 degrees and decreased to 0% after 360 degrees. The results for three different aspect ratios are plotted in Figures 7 thru 9. This data demonstrated that even with an aspect ratio = 1/2, the change in Iz can be significant when the fluid level decreased. With even smaller aspect ratios, such as wing fuel tanks, the change in Iz was even greater. The "jumps" in the value of Iz corresponds to the a large change in the geometry of the fluid.

For comparison, the effective moment of inertia for various tank aspect ratios were also calculated. The effective moment of inertia is the moment of inertia of an equivalent mechanical system. However, the effective moment applies to small angular displacements only, which was quite different from the case that was analyzed here. These values are plotted in Figures 10 thru 12.

REFERENCES

1. Beer, F.P., and Johnston E.R., Vector Mechanics for Engineers, Dynamics, McGraw-Hill Book Co., New York, 1977.

2. Higdon and Stiles, Engineering Mechanics, Prentice-Hall, New Jersey, 1968.

3. Graham, E.W., and Rodriguez, A.M., "The characteristics of Fuel Motion Which Affect Airplane Dynamics", Journal of Applied Mechanics, September 1952.

APPENDIX A

TABLES AND FIGURES

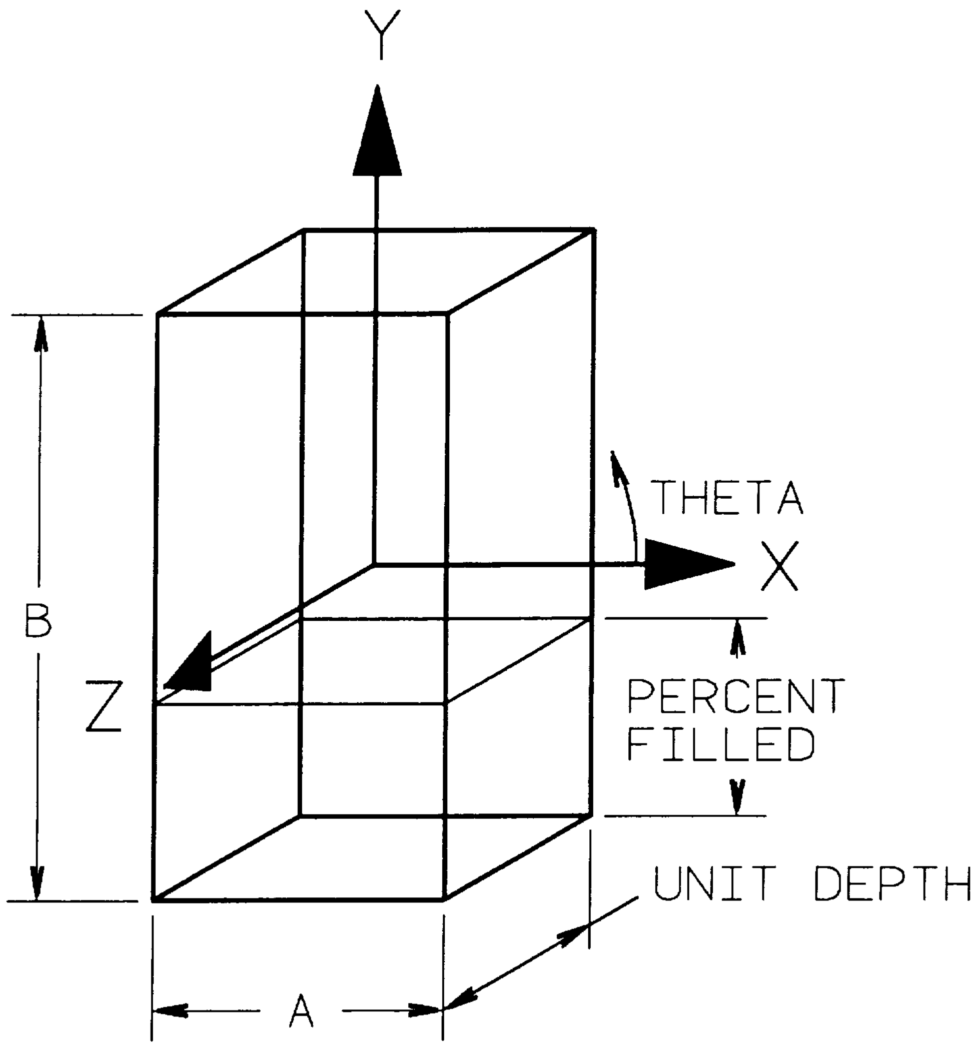


FIGURE 1: TANK CONFIGURATION

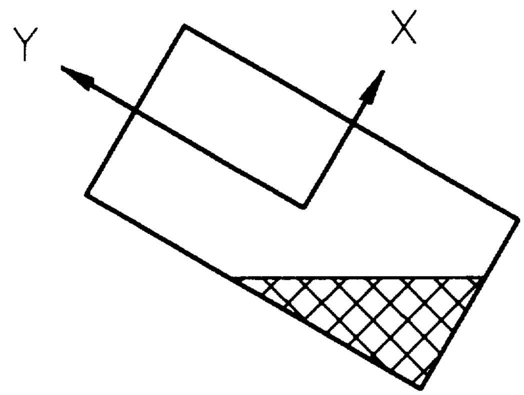


FIGURE 2: MOST SIMPLE GEOMETRY.

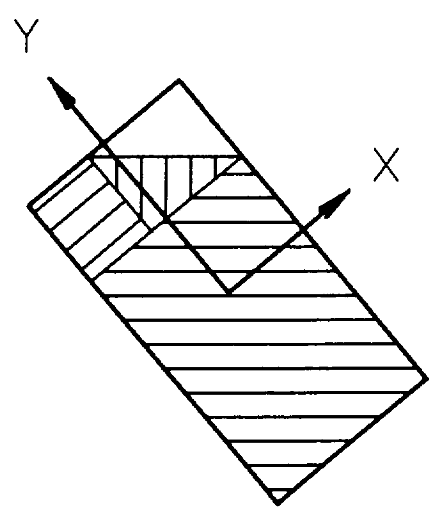


FIGURE 3: MOST COMPLEX GEOMETRY.

Aspect Ratio = 1/2

Theta	Izz/rho 20% Full	Izz/rho 50% Full	Izz/rho 80% Full
0.0	.0387	.1667	.4747
10.0	.0386	.1677	.4766
20.0	.0385	.1712	.4829
30.0	.0380	.1780	.4953
40.0	.0361	.1903	.5178
50.0	.0361	.2125	.5440
60.0	.0411	.2552	.5703
70.0	.0553	.3154	.5989
80.0	.1019	.3454	.6183
90.0	.1347	.3542	.6187
% Change	248	112	30

Table 1: Iz relative to fluid center of gravity for tank dimensions: a = 1.00, b = 2.00, and c = 1.00.

Aspect Ratio = 1/5

Theta	Izz/rho 20% Full	Izz/rho 50% Full	Izz/rho 80% Full
0.0	.1093	.8333	2.9973
10.0	.1101	.8358	3.0014
20.0	.1126	.8439	3.0147
30.0	.1173	.8599	3.0410
40.0	.1256	.8891	3.0894
50.0	.1400	.9450	3.1825
60.0	.1642	1.0651	3.3819
70.0	.2213	1.3867	3.6922
80.0	.4158	2.3042	4.1547
90.0	1.0693	2.7083	4.4373
% Change	878	225	48

Table 2: Iz relative to fluid center of gravity for tank dimensions: a = 1.00, b = 4.00, and c = 1.00.

Aspect Ratio = 1/8

Theta	Izz/rho 20% Full	Izz/rho 50% Full	Izz/rho 80% Full
0.0	.4747	5.6667	22.378
10.0	.4766	5.6718	22.387
20.0	.4829	5.6885	22.467
30.0	.4953	5.7216	22.467
40.0	.5180	5.7826	22.566
50.0	.5610	5.9000	22.755
60.0	.6519	6.1581	23.173
70.0	.8833	6.8869	24.368
80.0	1.6633	10.522	28.232
90.0	8.5387	21.417	34.475
% Change	1699	278	54

Table 3: Iz relative to fluid center of gravity for tank dimensions: a = 1.00, b = 8.00, and c = 1.00.

Aspect Ratio = 1/2

Theta	Izz/rho 20% Full	Izz/rho 50% Full	Izz/rho 80% Full
0.0	.2947	.4167	.5387
10.0	.2931	.4167	.5390
20.0	.2880	.4167	.5453
30.0	.2780	.4167	.5553
40.0	.2596	.4167	.5737
50.0	.2387	.4167	.5946
60.0	.2187	.4167	.6147
70.0	.1986	.4167	.6347
80.0	.1924	.4167	.6409
90.0	.1987	.4167	.6347
% Change	34	0	18

Table 4: Iz relative to center of rotation for tank dimensions: a = 1.00, b = 2.00, and c = 1.00.

Aspect Ratio = 1/5

	Iz/rho		
Theta	20% Full	50% Full	80% Full
0.0	2.1573	2.8333	3.5093
10.0	2.1542	2.8333	3.5124
20.0	2.1441	2.8333	3.5226
30.0	2.1240	2.8333	3.5427
40.0	2.0869	2.8333	3.5797
50.0	2.0153	2.8333	3.6514
60.0	1.8067	2.8333	3.8060
70.0	1.6238	2.8333	4.0428
80.0	1.2927	2.8333	4.3740
90.0	1.1973	2.8333	4.4693
% Change	44	0	27

Table 5: Iz relative to center of rotation for tank dimensions: a = 1.00, b = 4.00, and c = 1.00.

Aspect Ratio = 1/8

	Iz/rho		
Theta	20% Full	50% Full	80% Full
0.0	16.859	21.667	26.475
10.0	16.852	21.667	26.481
20.0	16.832	21.667	26.501
30.0	16.792	21.667	26.541
40.0	16.718	21.667	26.616
50.0	16.575	21.667	26.759
60.0	16.259	21.667	27.075
70.0	15.349	21.667	27.984
80.0	12.413	21.667	30.920
90.0	8.7947	21.667	34.539
% Change	48	0	31

Table 6: Iz relative to center of rotation for tank dimensions: a = 1.00, b = 8.00, and c = 1.00.

Figure 1: Iz/rho Relative to C.G. of Fluid for 1 x 2 Tank

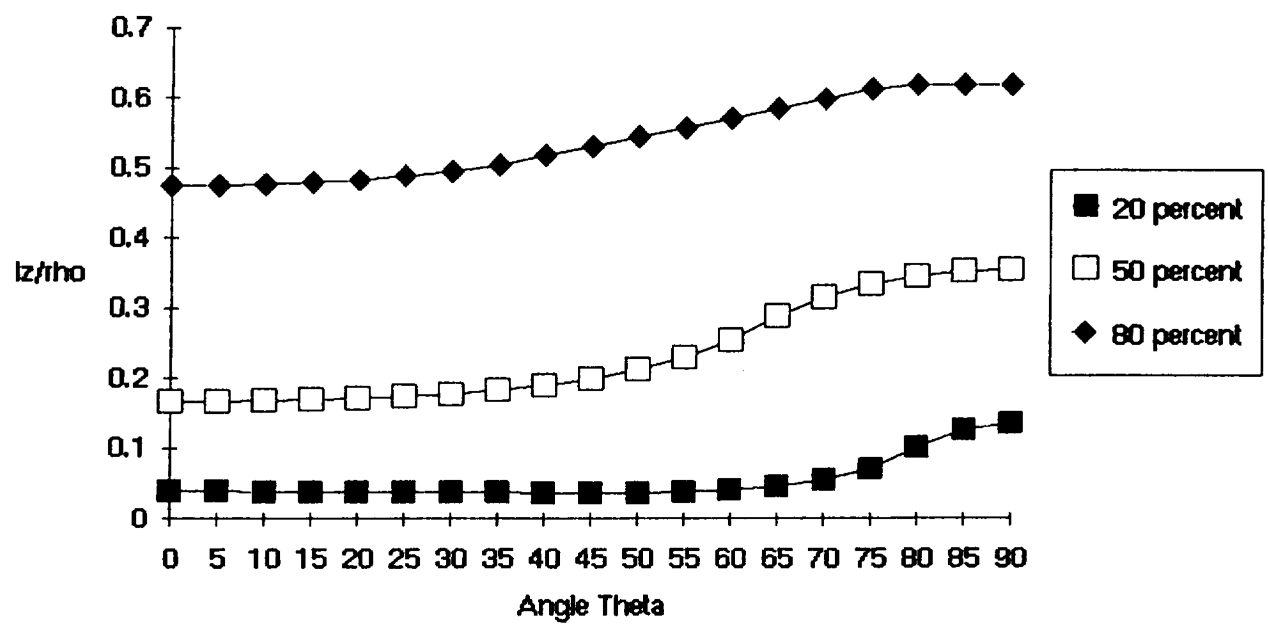


Figure 2: Iz/rho Relative to C.G. of Fluid for 1 x 4 Tank.

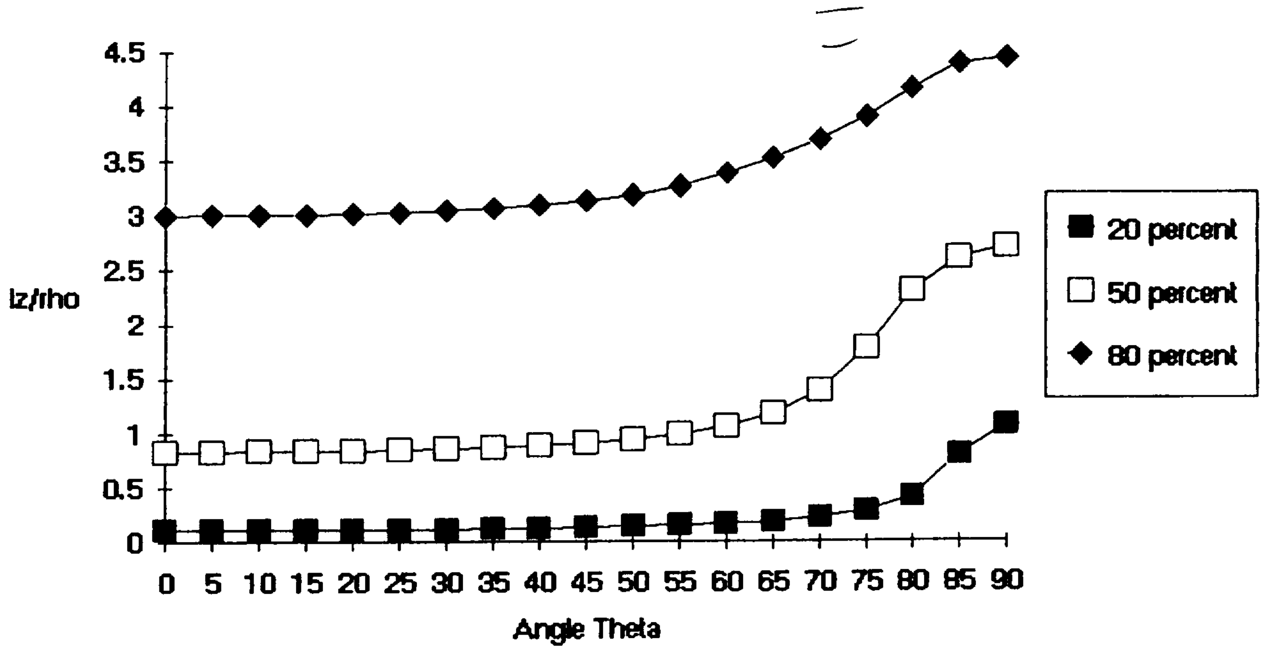


Figure 3: Iz/rho Relative to C.G. of Fluid for 1 x 8 Tank

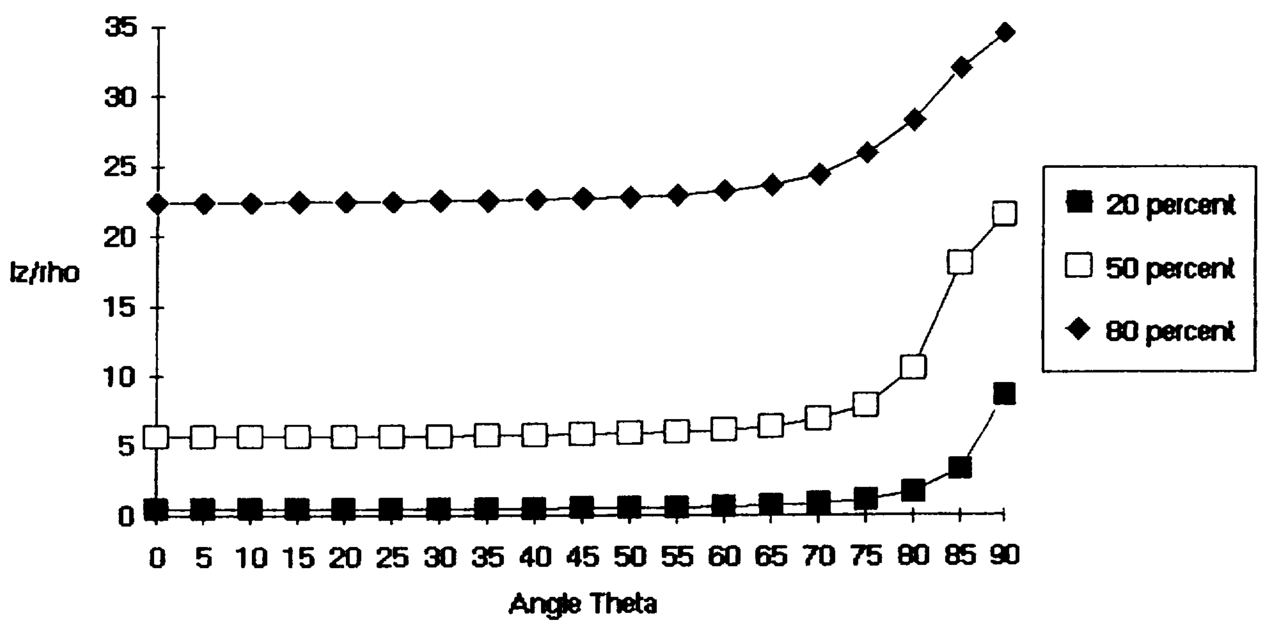


Figure 4: Iz/rho Relative to Center of Rotation for 1x2 Tank

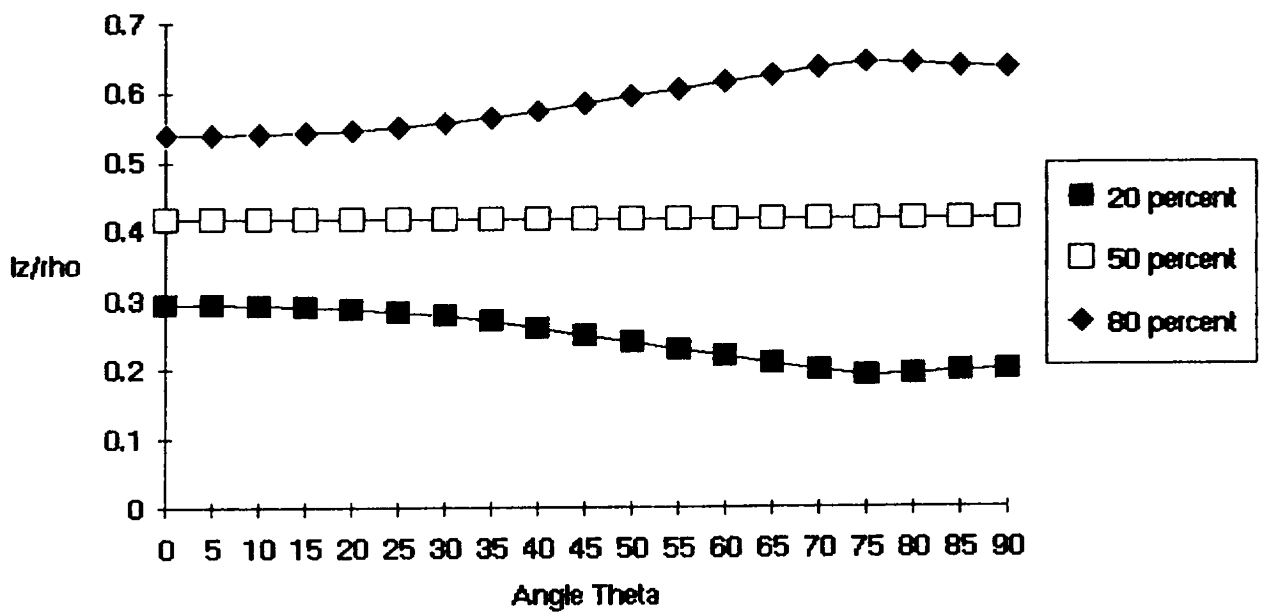


Figure 5: Iz/rho Relative to Center of Rotation for 1 x 4 Tank

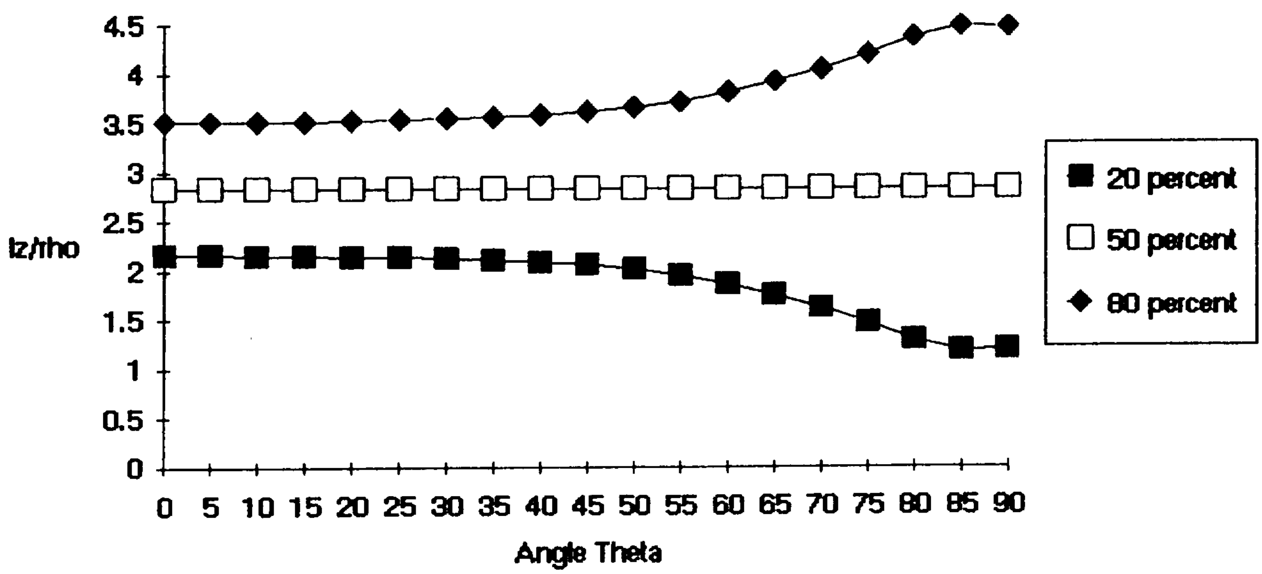


Figure 6: Iz/rho Relative to Center of Rotation for 1 x 8 Tank

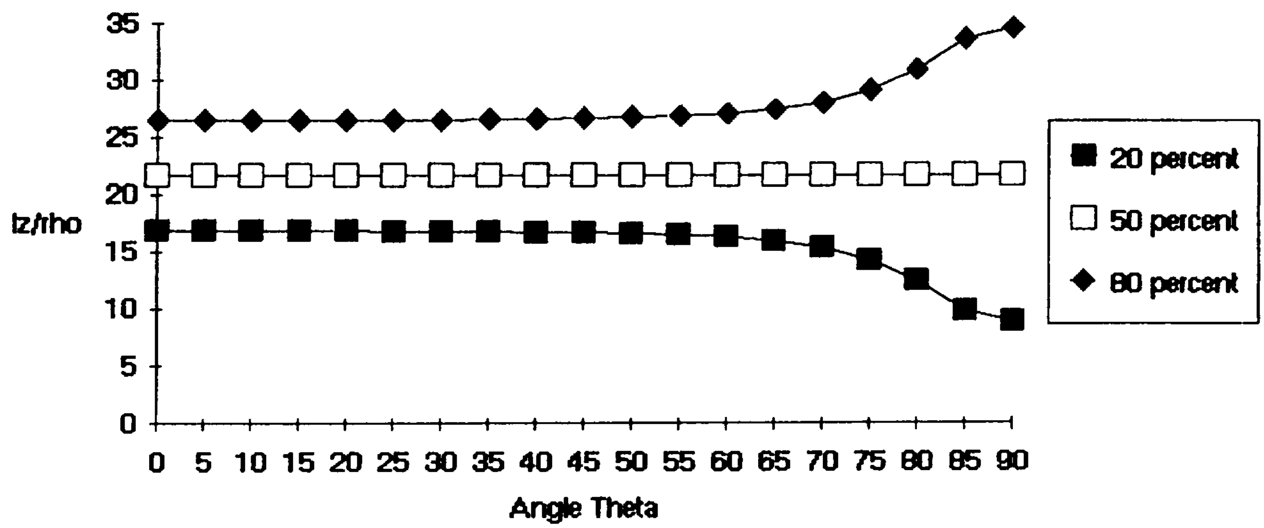


Figure 7: Iz/rho of 1 x 2 Tank with Decreasing Fluid

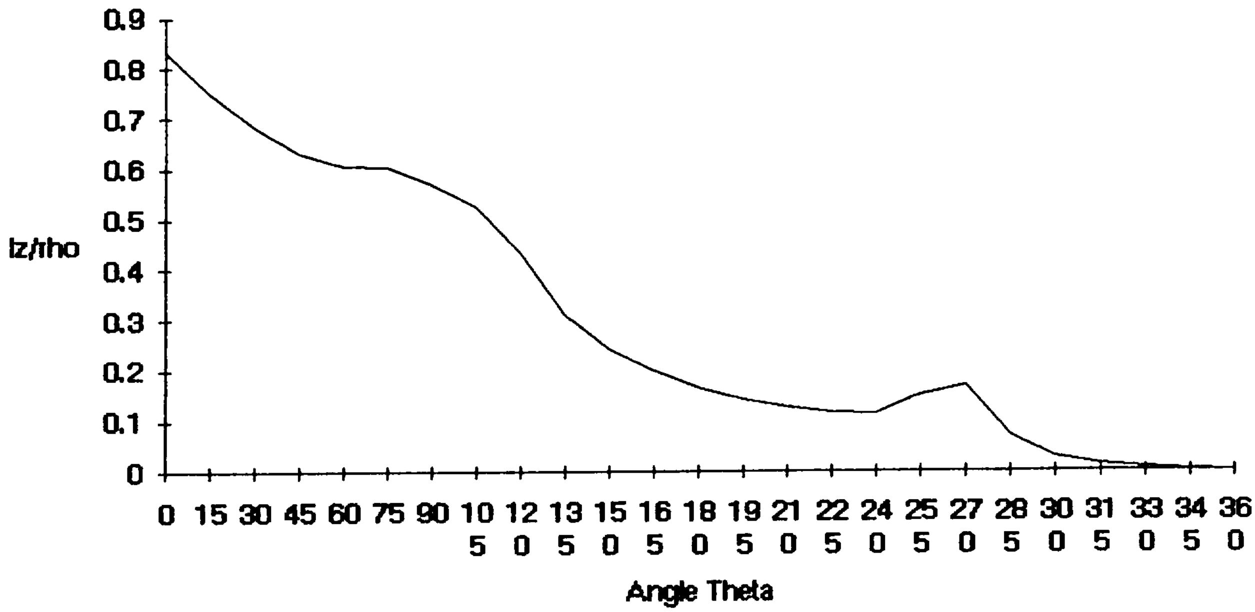


Figure 8: Iz/rho of 1 x 4 Tank with Decreasing Fluid

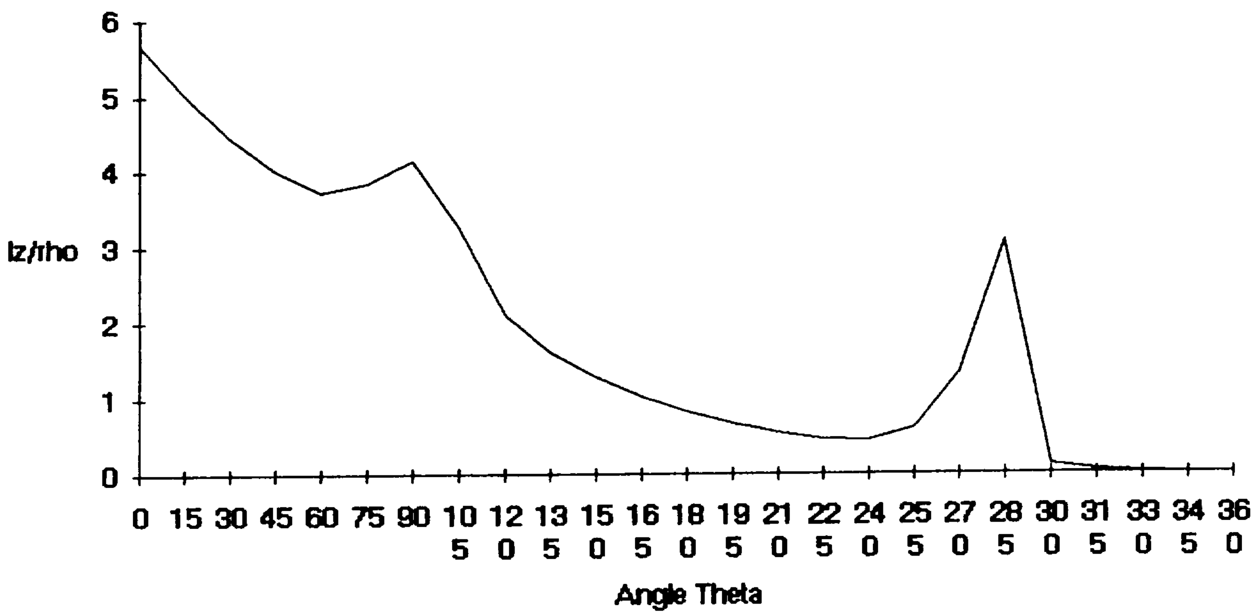


Figure 9: Iz/rho of 1 x 8 Tank with Decreasing Fluid

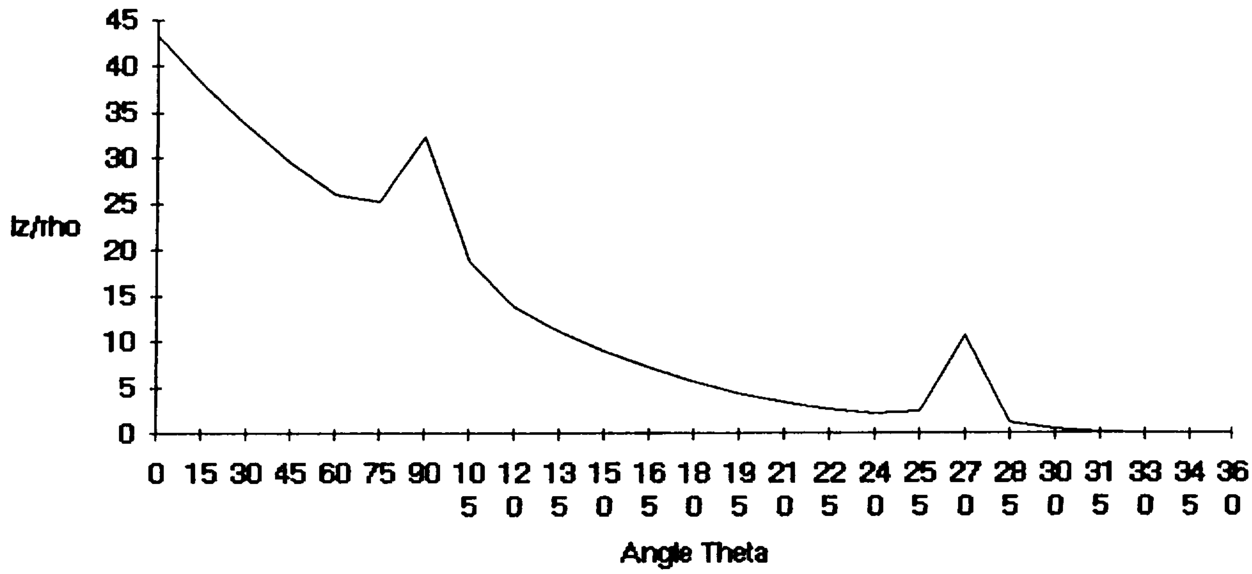


Fig 10: Ify vs. Tank Aspect Ratio, r

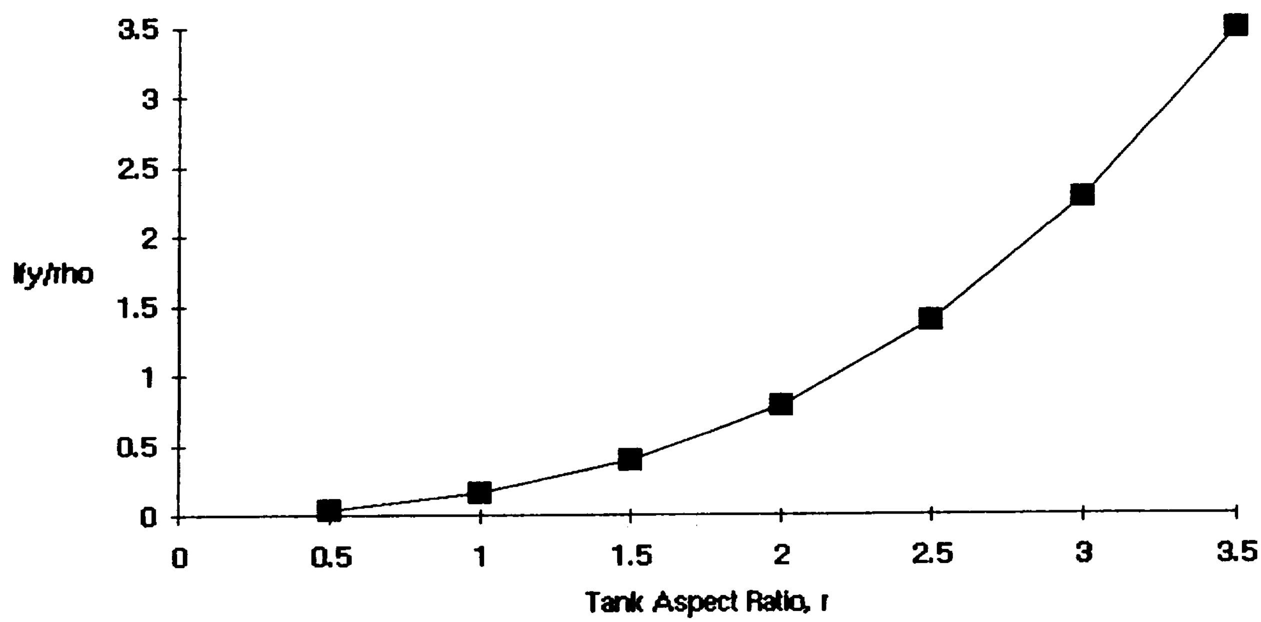


Fig 11: Isy/Iy vs. Tank Aspect Ratio, r

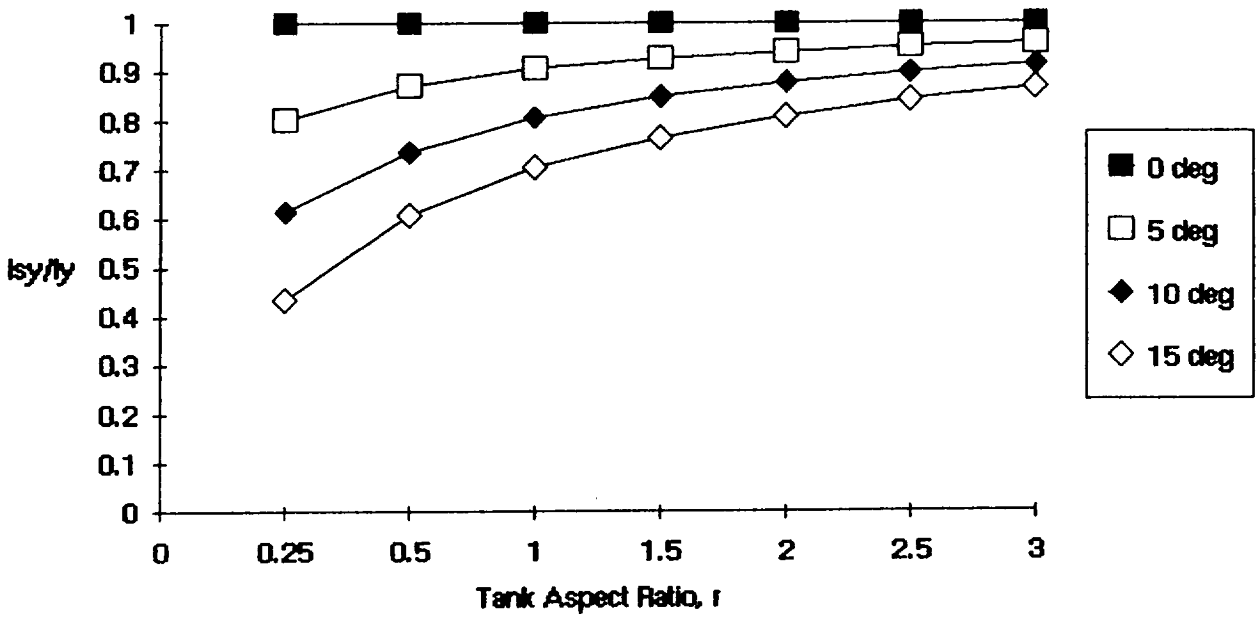
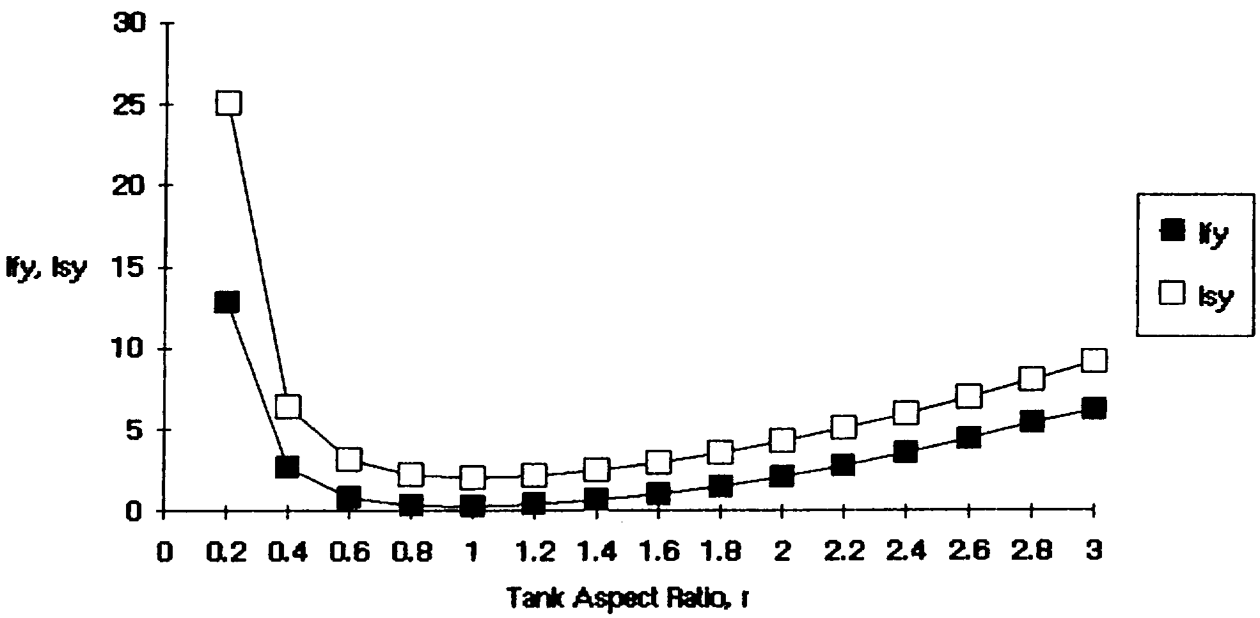


Fig. 13: Ify and Isy vs. Tank Aspect Ratio,r



APPENDIX B

PROGRAM LISTINGS

```
rogram MomentOfInertia;
This program calculates the mass moment of inertia of the fluid in a tank  }
relative the to the c.g. of the fluid.  The tank is dimensioned as 'a' in  }
the x-dir, 'b' in the y-dir(up), and 'c' in the z-dir(into the page).  The }
center of the tank is also the center of rotation and corresponds to the   }
origin of the coordinates.  The tank rotates counterclockwise and this     }
angle is defined as theta.  This version was last modified on 2/5/90.      }
ses
   PasPrinter;
onst
   pi = 3.141593;
   rho = 1.0;
ype
   Number = array [1..90] of real;
ar
   outFile      :text;  {file pointer for output}
   outFileName  :string[15];
   output1:Number;
   output2:Number;
   printer:char;
   a,              {length of tank}
   b,              {height of tank}
   c,              {width of tank}
   angle,          {the angle of rotation in degrees}
   area,           {total area of fluid in xy plane}
   increment,      {Increment by which angle increases}
   aTriangle,      {Max area of triangular shape}
   bTriangle,      {Max area of triangle and rectangle combined}
   percent,        {Fluid level as a percent of height "b"}
   mass,           {Mass of fluid}
   moment,         {Mass moment of inertia of entire shape}
   beta,           {2 - theta}
   tangentBeta,    {tan(beta)}
   tangentTheta,   {tan(theta)}
   theta,          {Angle of rotation CCW in radians}
   zero:real;
   form:integer;   {defines the geometry}

*************************************************************************)

rocedure CounterRotate(var  ax, ay, ax1, ay1:real);
This procedure converts local Cord. to global Cord. given angle theta.}
egin
   ax := (ax1 * cos(theta))  - (ay1 * sin(theta));
   ay := (ax1 * sin(theta))  + (ay1 * cos(theta));
nd;


unction IBarTri(var  mTri,ta,tb:real):real;
This function will calculate the mass moment of inertia of a triangular    }
prism given its mass, base, and height.                                    }
egin
   IBarTri := (mTri/18) * (sqr(ta) + sqr(tb));
nd;


unction IBarRect(var  mRect,ra,rb:real):real;
This function will calculate the mass moment of inertia of a rectangular   }
prism given its mass, length, and height.                                  }
egin
   IBarRect := (mRect/12) * (sqr(ra) + sqr(rb));
nd;


ocedure Centroid(var  xBar,yBar,
                      a1,x1,y1,
                      a2,x2,y2,
                      a3,x3,y3:real);
```

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```
This procedure will calculate the x and y coord of the centroid of the      }
fluid.                                                                       }
ar
   xArea,
   yArea:real;
egin
   xArea  := (a1 * x1) + (a2 * x2) + (a3 * x3);
   yArea  := (a1 * y1) + (a2 * y2) + (a3 * y3);
   xBar  := xArea/area;
   yBar  := yArea/area;
nd;



unction  WhatType:integer;
*This  function  determines  what  shape  the  fluid  is  in.  Type  1 = triangle;  *)
*type  2 = triangle  and  rectangle;  and  type  3 = one  triangle  and  two        *)
*rectangles.                                                                        *)
ar
   aArea,  alpha,  bArea  :real;
egin
   aTriangle  := 0.5 * sqr(a)  * tangentTheta;
   aArea  := aTriangle  + (a * (b - (a * tangentTheta)));
   alpha  := arctan(b/a);
   if tangentTheta  <> 0.0  then
   begin
       bTriangle  := abs((0.5  * sqr(b))/tangentTheta);
       bArea  := bTriangle  + abs((b  * (a - (b/tangentTheta))));
   end;
   if (angle  = 0.0)  then        {It's  a rectangle.}
       WhatType  := 4
   else  if (angle  = 90.0)  then  {It's  still  a rectangle.}
       WhatType  := 5
   else  if theta  <= alpha  then
   begin
       if (aTriangle  > area)  then
       WhatType  := 1
       else  if (aArea  > area)  and  (bArea  > area)  then
       WhatType  := 2
       else  WhatType  := 3;
   end
   else
   begin
       if (bTriangle  > area)  then
       WhatType  := 1
       else  if (bArea  > area)  then
       WhatType  := 6
       else  WhatType  := 7;
   end;
d;


ocedure  TypeOne;
This procedure  will  calculate  the mass moment  of inertia  of a triangular     }
rism.                                                                              }
r
   a2, b2, h, j, cx, cy:real;
gin
   h := sqrt((2  * area)/tangentTheta)    * tangentTheta;
   j := sqrt((2  * area)/tangentTheta);
   a2  := (j/3)  - (a/2);
   b2  := (h/3)  - (b/2);
   CounterRotate(cx,   cy, a2, b2);
   moment  := IBarTri(mass,h,j);
d;


ocedure  TypeTwo;
```

```
{This Procedure will calculate the mass moment of inertia of a volume which}
{can be broken down into one triangular and one rectangular prism.        }
var
    areaTri,
    areaRect1,
    h,           {Height of Rect}
    massTri,     {Mass of triangular prism}
    massRect,    {Mass of Rect. prism}
    momentTri,   {Moment of inertia of triangular prism rel. to origin}
    momentRect,{Moment of inertia of Rect. prism rel. to origin}
    j, gx, gy,
    x,y,
    xCen,
    yCen,
    x2, y2,
    x3, y3,
    x4, y4,
    x5, y5:real;
begin
    h := (area - atriangle)/a;
    j := a * tangentTheta;
    areaTri := atriangle;
    areaRect1 := area - aTriangle;
    massTri := areaTri * c * rho;
    massRect := areaRect1 * c * rho;
    gx := a/3 - a/2;
    gy := h + j/3 - b/2;
    CounterRotate(x2,y2,gx,gy);
    x := 0;
    y := h/2 - b/2;
    CounterRotate(x3,y3,x,y);
    Centroid(xCen,yCen,areaTri,x2,y2,areaRect1,x3,y3,zero,zero,zero);
    x4 := xCen - x2;
    y4 := yCen - y2;
    x5 := xCen - x3;
    y5 := yCen - y3;
    momentTri := IBarTri(massTri,a,j) + (massTri * (sqr(x4) + sqr(y4)));
    momentRect := IBarRect(massRect,a,h) + (massRect * (sqr(x5) + sqr(y5)));
    moment := momentTri + momentRect;
end;


procedure TypeThree;
{This Procedure will calculate the mass moment of inertia of a volume which}
{must be broken down into one triangular and two rectangular prisms.       }
var
    h, j, k, m,
    areaTri,
    areaRect1,
    areaRect2,
    massTri,     {mass of triangle}
    massRect1,   {mass of rectangle below triangle}
    massRect2,   {mass of rectangle next to triangle}
    momentTri,
    momentRect1,
    momentRect2,
    tri,         {the triangular area not filled with fluid}
    xCen, yCen,  {centroidal coord of the fluid}
    x1, y1,
    x2, y2,
    x3, y3,
    x4, y4,
    x5, y5,
    x6, y6,
    x7, y7,
    x8, y8,
    x9, y9,
    moment1, moment2:real;
```

```
egin
   tri  :=  (a * b) - area;
   j := sqrt((2  * tri)/tangentTheta);
   h := j * tangentTheta;
   k := a - j;
   m := b - h;
   areaTri  := 0.5 * h * j;
   areaRect1  := a * m;
   areaRect2  := h * k;
   massTri  := areaTri  * c * rho;
   massRect1  := areaRect1  * c * rho;
   massRect2  := areaRect2  * c * rho;
   x2 := k + j/3 - a/2;
   y2 := b/2 - (2 * h/3);
   CounterRotate(x1,y1,x2,y2);
   x5 := 0;
   y5 := m/2 - b/2;
   CounterRotate(x6,y6,x5,y5);
   x4 := k/2 - a/2;
   y4 := b/2 - h/2;
   CounterRotate(x3,y3,x4,y4);
   Centroid(xCen,yCen,areaTri,x1,y1,areaRect1,x6,y6,areaRect2,x3,y3);
   x7 := xCen  - x1;
   y7 := yCen  - y1;
   x8 := xCen  - x6;
   y8 := yCen  - y6;
   x9 := xCen  - x3;
   y9 := yCen  - y3;
   momentTri   := IBarTri(massTri,h,j)+(massTri     * (sqr(x7   + sqr(y7)));
   momentRect1   := IBarRect(massRect1,a,m)+(massRect1      * (sqr(x8)   + sqr(y8)));
   momentRect2   := IBarRect(massRect2,h,k)+(massRect2      * (sqr(x9)   + sqr(y9)));
   moment  := momentTri   + momentRect1   + momentRect2;
nd;


rocedure  TypeFour;
ar
   c4:real;
egin
   c4 := percent  * b;
   moment  := IBarRect(mass,a,c4);
nd;


rocedure  TypeFive;
ar
   c5:real;
egin
   c5 := percent  * a;
   moment  := IBarRect(mass,c5,b);
nd;


rocedure  TypeSix;
This Procedure  will calculate  the mass moment  of inertia  of a volume  which}
can  be broken  down  into one triangular  and one rectangular  prism.          }
ar
   area2,         {Area  of Rect}
   areaTri,
   areaRect1,
   h,             {Height  of Rect}
   massTri,       {Mass  of triangular  prism}
   massRect,      {Mass  of Rect.  prism}
   momentTri,     {Moment  of inertia  of triangular  prism rel. to origin}
   momentRect,    {Moment  of inertia  of Rect. prism rel. to origin}
   j, gx, gy,
   tri,           {area  of tank  not filled with  fluid}
   x, y,
```

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```
    xCen, yCen,
    x1, y1,
    x2, y2,
    x3, y3,
    x4, y4:real;
egin
    areaTri  := 0.5 * sqr(b)  * tangentBeta;
    areaRect1  := area  - areaTri;
    h := (area  - areaTri)/b;
    j := b * tangentBeta;
    massTri  := areaTri  * c * rho;
    massRect  := areaRect1  * c * rho;
    gx := h + j/3 - a/2;
    gy := b/3 - b/2;
    CounterRotate(x2,y2,gx,gy);
    x := (h/2)  - (a/2);
    y := 0.0;
    CounterRotate(x1,y1,x,y);
    Centroid(xCen,yCen,AreaTri,x2,y2,AreaRect1,x1,y1,zero,zero,zero);
    x3 := xCen  - x2;
    y3 := yCen  - y2;
    x4 := xCen  - x1;
    y4 := yCen  - y1;
    momentTri  := IBarTri(massTri,b,j)    + (massTri   * (sqr(x3)  + sqr(y3)));
    momentRect  := iBarRect(massRect,b,h)    + (massRect  * (sqr(x4)  + sqr(y4)));
    moment  := momentTri  + momentRect;
nd;


rocedure  TypeSeven;
This Procedure  will  calculate  the mass moment  of inertia  of a volume  which}
must  be broken  down  into one triangular  and two rectangular  prisms.        }
ar
   h, j, k, m,
   areaTri,
   areaRect1,
   areaRect2,
   massTri,      {mass  of  triangle}
   massRect1,    {mass  of rectangle  below  triangle}
   massRect2,    {mass  of rectangle  next  to triangle}
   momentTri,
   momentRect1,
   momentRect2,
   tri,          {the  triangular  area  not  filled  with  fluid}
   xCen, yCen,
   ax1, ay1,
   ax2, ay2,
   ax3, ay3,
   x1, x2, x3, x4, x5, x6,
   y1, y2, y3, y4, y5, y6,
   moment1,  moment2:real;
egin
   tri :=  (a * b) - area;
   j := sqrt((2  * tri)/tangentBeta);
   h := j * tangentBeta;
   k := b - j;
   m := a - h;
   areaTri  := 0.5 * h * j;
   areaRect1  := b * m;
   areaRect2  := h * k;
   massTri  := areaTri  * c * rho;
   massRect1  := areaRect1  * c * rho;
   massRect2  := areaRect2  * c * rho;
   x2 := a/2 - (2 * h/3);
   y2 := b/2 - (2 * j/3);
   CounterRotate(x1,y1,x2,y2);
   x5 := m/2 - a/2;
   y5 := 0;
```

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```
   CounterRotate x5,y5,x5,y5);
   x4 := a/2 - t/2;
   y4 := k/2 - c/2;
   CounterRotate x3,y3,x4,y4);
   Centroid(xCen,yCen,areaTri,x1,y1,areaRect1,x6,y6,areaRect2,x3,y3);
   ax1 := xCen - x1;
   ay1 := yCen - y1;
   ax2 := xCen - x6;
   ay2 := yCen - y6;
   ax3 := xCen - x3;
   ay3 := yCen - y3;
   momentTri  := IBarTri(massTri,h,j)+(massTri     * (sqr(ax1)  + sqr(ay1)));
   momentRect1  := IBarRect(massRect1,b,m)+(massRect1     * (sqr(ax2)+sqr(ay2)));
   momentRect2  := IBarRect(massRect2,h,K)+(massRect2     * (sqr(ax3)+sqr(ay3)));
   moment  := momentTri  + momertRect1  + momentRect2;
nd;


*************************************************************************)
egin  (*START  MAIN  PROGRAM*)
   ClearScreen;
   zero := 0;
   writeln ('Enter  dimensions  for rectangular  tank:');
   readln (a,b,c);
   writeln ('Enter  water level as a percentage  of "b".');
   readln (percent);
   while ((percent  >= 1) or (percent  <= 0)) do
   begin
       writeln('0.0  < Water Level < 1.0.  Try again.');
       readln(percent);
   end;
   writeln('Enter  angle  theta  increment.');
   readln(increment);
   writeln('Enter  output  file  name.');
   readln(outFileName);
   angle := 0.0;
   area := a * b * percent;
   mass := a * percent  * b * c;
   rewrite(outFile,outFileName);
   writeln(outFile,'A    = ',a:8:4);
   writeln(outFile,'B    = ',b:8:4);
   writeln(outFile,'C    = ',c:8:4);
   writeln(outFile,'Percent    full  = ',percent:8:4);
   writeln(outFile,'Type      Theta     Moment');
   while (angle  <= 90.0)  do
   begin
       moment  := 0.0;
       beta  := ((90  - angle)/360)*(2   * pi);
       theta  := (angle/360)*(2   * pi);
       tangentBeta   := sin(beta)/cos(beta);
       tangentTheta   := sin(theta)/cos(theta);
       form  := WhatType;
       case  (form)  of
            1:TypeOne;
            2:TypeTwo;
            3:TypeThree;
            4:TypeFour;
            5:TypeFive;
            6:TypeSix;
            7:TypeSeven;
       end;
       writeln(outFile,form,    '     ',angle:6:2,  '     ',moment:8:4);
       angle  := angle  + increment;
   end;
   close(outFile);
d.  (*END  MAIN  PROGRAM*)
```

```
Program Moment1;
{This program calculates the mass moment of inertia of the fluid in a tank  }
{relative the to the center of the tank. The tank is dimensioned as 'a' in }
{the x-dir, 'b' in the y-dir(up), and 'c' in the z-dir(into the page). The }
{center of the tank is also the center of rotation and corresponds to the   }
{origin of the coordinates. The tank rotates counterclockwise and this      }
{angle is defined as theta. This version was lase modified on 2/4/90.       }
Uses
   PasPrinter;
Const
   pi = 3.141593;
   rho = 1.0;
Type
   Number = array [1..90] of real;
Var
   outFile      :text; {file pointer for output}
   outFileName  :string[15];
   output1:Number;
   output2:Number;
   printer:char;
   a,               {length of tank}
   b,               {height of tank}
   c,               {width of tank}
   area,
   angle,           {the angle of rotation in degrees}
   increment,       {Increment by which angle increases}
   aTriangle,       {Max area of triangluar shape}
   bTriangle,       {Max area of triangle and rectangle combined}
   percent,         {Fluid level as a percent of height "b"}
   mass,            {Mass of fluid}
   moment,          {Mass moment of inertia of entire shape}
   beta,            {2 - theta}
   tangentBeta,     {tan(beta)}
   tangentTheta,    {tan(theta)}
   theta:real;      {Angle of rotation CCW in radians}
   form:integer;    {defines the geometry}

{**********************************************************************}

Procedure CounterRotate(var  ax, ay, ax1, ay1:real);
{This procedure converts local Cord. to global Cord. given angle theta.}
Begin
   ax := (ax1 * cos(theta))  - (ay1 * sin(theta));
   ay := (ax1 * sin(theta))  + (ay1 * cos(theta));
End;


Function IBarTri(var  mTri,ta,tb:real):real;
{This function will calculate the mass moment of inertia of a triangular    }
{prism given its mass, base, and height.                                     }
Begin
   IBarTri  := (mTri/18)  * (sqr(ta)  + sqr(tb));
End;


Function IBarRect(var  mRect,ra,rb:real):real;
{This function will calculate the mass moment of inertia of a rectangular   }
{prism given its mass, length, and height.                                   }
Begin
   IBarRect  := (mRect/12)  * (sqr(ra)  + sqr(rb));
End;


Function WhatType:integer;
(*This function determines what shape the fluid is in. Type 1 = triangle;  *)
(*type 2 = triangle and rectangle; and type 3 = one triangle and two       *)
(*rectangles                                                                *)
```

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```
ar
    aArea, alpha, bArea :real;
egin
    aTriangle := 0.5 * sqr(a) * tangentTheta;
    aArea := aTriangle + (a * (b - (a * tangentTheta)));
    alpha := arctan(b/a);
    if tangentTheta <> 0.0 then
    begin
        bTriangle := abs((0.5 * sqr(b))/tangentTheta);
        bArea := bTriangle + abs((b * (a - (b/tangentTheta))));
    end;
    if (angle = 0.0) then        (It's a rectangle.)
        WhatType := 4
    else if (angle = 90.0) then (It's still a rectangle.)
        WhatType := 5
    else if theta <= alpha then
    begin
        if (aTriangle > area) then
        WhatType := 1
        else if (aArea > area) and (bArea > area) then
        WhatType := 2
        else WhatType := 3;
    end
    else
    begin
        if (bTriangle > area) then
        WhatType := 1
        else if (bArea > area) then
        WhatType := 6
        else WhatType := 7;
    end;
nd;


rocedure TypeOne;
This procedure will calculate the mass moment of inertia of a triangular   }
prism.                                                                      }
ar
    a2, b2, h, j, cx, cy:real;
egin
    h := sqrt((2 * area)/tangentTheta) * tangentTheta;
    j := sqrt((2 * area)/tangentTheta);
    a2 := (j/3) - (a/2);
    b2 := (h/3) - (b/2);
    CounterRotate(cx, cy, a2, b2);
    moment := IBarTri(mass,h,j) + (mass *(sqr(cx) + sqr(cy)));
nd;


rocedure TypeTwo;
This Procedure will calculate the mass moment of inertia of a volume which}
can be broken down into one triangular and one rectangular prism.         }
ar
    area2,       (Area of Rect}
    h,           (Height of Rect}
    massTri,     (Mass of triangular prism}
    massRect,    (Mass of Rect. prism}
    momentTri,   (Moment of inertia of triangular prism rel. to origin}
    momentRect,(Moment of inertia of Rect. prism rel. to origin}
    j, gx, gy, xbar, ybar,
    x, y, x3, y3:real;
egin
    h := (area - atriangle)/a;
    j := a * tangentTheta;
    massTri := atriangle * c * rho;
    massRect := (area - atriangle) * c * rho;
    gx := a/3 - a/2;
    gy := h + j/3 - b/2;
```

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```
    CounterRotate(xbar,ybar,gx,gy);
    momentTri  := IBarTri(massTri,a,j)   + (massTri  * (sqr(xbar)  + sqr(ybar)));
    x := 0;
    y := h/2 - b/2;
    CounterRotate(x3,y3,x,y);
    momentRect  := IBarRect(massRect,a,h)   + (massRect  * (sqr(x3)  + sqr(y3)));
    moment  := momentTri  + momentRect;
end;


Procedure  TypeThree;
{This Procedure will calculate the mass moment of inertia of a volume which}
{must be broken down into one triangular  and two rectangular  prisms.       }
var
    h, j, k, m,
    massTri,      {mass of triangle}
    massRect1,    {mass of rectangle  below triangle}
    massRect2,    {mass of rectangle  next to triangle}
    momentTri,
    momentRect1,
    momentRect2,
    tri,          {the triangular  area not filled with fluid}
    x1, x2, x3, x4, x5, x6,
    y1, y2, y3, y4, y5, y6,
    moment1,  moment2:real;
begin
    tri :=  (a * b) - area;
    j := sqrt((2  * tri)/tangentTheta);
    h := j * tangentTheta;
    k := a - j;
    m := b - h;
    massTri  := 0.5 * h * j * c * rho;
    massRect1  := a * m * c * rho;
    massRect2  := h * k * c * rho;
    x2 := k + j/3 - a/2;
    y2 := b/2 - (2 * h/3);
    CounterRotate(x1,y1,x2,y2);
    momentTri  := IBarTri(massTri,h,j)   + (massTri  * (sqr(x1)  + sqr(y1)));
    x5 := 0;
    y5 := m/2 - b/2;
    CounterRotate(x6,y6,x5,y5);
    momentRect1  := IBarRect(massRect1,a,m)+(massRect1     * (sqr(x6)  + sqr(y6)));
    x4 := k/2 - a/2;
    y4 := b/2 - h/2;
    CounterRotate(x3,y3,x4,y4);
    momentRect2  := IBarRect(massRect2,h,k)+(massRect2     * (sqr(x3)  + sqr(y3)));
    moment  := momentTri  + momentRect1  + momentRect2;
end;


Procedure  TypeFour;
var
    c4:real;
begin
    c4 := percent  * b;
    moment  := IBarRect(mass,a,c4)   + (mass  * sqr((b/2)  * (percent  - 1)));
end;


Procedure  TypeFive;
var
    c5:real;
begin
    c5 := percent  * a;
    moment  := IBarRect(mass,c5,b)   + (mass  * (sqr((a/2)  * (percent  - 1))));
end;
```

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```
Procedure  TypeSix;
{This Procedure  will calculate  the mass moment  of inertia  of a volume  which}
{can  be broken  down  into one  triangular  and one rectangular  prism.         }
var
    area2,         {Area  of Rect}
    areaTriangle,
    h,             {Height  of Rect}
    massTri,       {Mass  of triangular  prism}
    massRect,      {Mass  of Rect.  prism}
    momentTri,     {Moment  of inertia  of triangular  prism rel. to origin}
    momentRect,    {Moment  of inertia  of Rect. prism  rel. to origin}
    j, gx, gy, xbar, ybar,
    x, y, x3, y3:real;
begin
    areaTriangle  := 0.5 * sqr(b)  * tangentBeta;
    h := (area  - areaTriangle)/b;
    j := b * tangentBeta;
    massTri  := areaTriangle   * c * rho;
    massRect  := (area  - areaTriangle)   * c * rho;
    gx  := h + j/3 - a/2;
    gy  := b/3 - b/2;
    CounterRotate(xbar,ybar,gx,gy);
    momentTri   := IBarTri(massTri,b,j)    + (massTri   * (sqr(xbar)   + sqr(ybar)));
    x := (h/2)  - (a/2);
    y := 0.0;
    CounterRotate(x3,y3,x,y);
    momentRect   := iBarRect(massRect,b,h)    + (massRect   * (sqr(x3)   + sqr(y3)));
    moment  := momentTri   + momentRect;
end;


Procedure  TypeSeven;
{This  Procedure  will  calculate  the mass  moment  of inertia  of a volume  which}
{must  be broken  down  into one  triangular  and two rectangular  prisms.        }
var
    h, j, k, m,
    massTri,       {mass  of  triangle}
    massRect1,     {mass  of rectangle  below  triangle}
    massRect2,     {mass  of rectangle  next  to triangle}
    momentTri,
    momentRect1,
    momentRect2,
    tri,           {the  triangular  area  not  filled  with  fluid}
    x1, x2, x3, x4, x5, x6,
    y1, y2, y3, y4, y5, y6,
    moment1,  moment2:real;
begin
    tri  :=  (a * b) - area;
    j := sqrt((2  * tri)/tangentBeta);
    h := j * tangentBeta;
    k := b - j;
    m := a - h;
    massTri  := 0.5 * h * j * c * rho;
    massRect1  := b * m * c * rho;
    massRect2  := h * k * c * rho;
    x2 := a/2 - (2 * h/3);
    y2 := b/2 - (2 * j/3);
    CounterRotate(x1,y1,x2,y2);
    momentTri   := IBarTri(massTri,h,j)+(massTri     * (sqr(x1)   + sqr(y1)));
    x5 := m/2 - a/2;
    y5 := 0;
    CounterRotate(x6,y6,x5,y5);
    momentRect1   := IBarRect(massRect1,b,m)+(massRect1      * (sqr(x6)   + sqr(y6)));
    x4 := a/2 - h/2;
    y4 := k/2 - b/2;
    CounterRotate(x3,y3,x4,y4);
    momentRect2   := IBarRect(massRect2,h,k)+(massRect2      * (sqr(x3)   + sqr(y3)));
    moment  := momentTri   + momentRect1   + momentRect2;
```

```
end;


(****************************************************************************)
Begin (*START MAIN PROGRAM*)
    ClearScreen;
    writeln ('Enter dimensions for rectangular tank:');
    readln (a,b,c);
    writeln ('Enter water level as a percentage of "b".');
    readln (percent);
    while ((percent >= 1) or (percent <= 0)) do
    begin
        writeln('0.0 < Water Level < 1.0.  Try again.');
        readln(percent);
    end;
    writeln('Enter angle theta increment.');
    readln(increment);
    writeln('Enter output file name.');
    readln(outFileName);
    angle := 0.0;
    area := a * b * percent;
    mass := a * percent * b * c;
    rewrite(outFile,outFileName);
    writeln(outFile,'A   = ',a:8:4);
    writeln(outFile,'B   = ',b:8:4);
    writeln(outFile,'C   = ',c:8:4);
    writeln(outFile,'Percent   full = ',percent:8:4);
    writeln(outFile,'Type      Theta     Moment');
    while (angle <= 90.0) do
    begin
        moment := 0.0;
        beta := ((90 - angle)/360)*(2 * pi);
        theta := (angle/360)*(2 * pi);
        tangentBeta := sin(beta)/cos(beta);
        tangentTheta := sin(theta)/cos(theta);
        form := WhatType;
        case (form) of
            1:TypeOne;
            2:TypeTwo;
            3:TypeThree;
            4:TypeFour;
            5:TypeFive;
            6:TypeSix;
            7:TypeSeven;
        end;
        writeln(outFile,form,   '     ',angle:6:2,  '     ',moment:8:4);
        angle := angle + increment;
    end;
    close(outFile);
End. (*END MAIN PROGRAM*)
```

APPENDIX B

PROGRAM LISTINGS

```
rogram MomentOfInertia;
This program calculates the mass moment of inertia of the fluid in a tank   }
relative the to the c.g. of the fluid.  The tank is dimensioned as 'a' in   }
the x-dir, 'b' in the y-dir(up), and 'c' in the z-dir(into the page).  The }
center of the tank is also the center of rotation and corresponds to the   }
origin of the coordinates.  The tank rotates counterclockwise and this      }
angle is defined as theta.  This version was last modified on 2/5/90.      }
ses
   PasPrinter;
onst
   pi = 3.141593;
   rho = 1.0;
ype
   Number = array [1..90] of real;
ar
   outFile      :text; {file pointer for output}
   outFileName  :string[15];
   output1:Number;
   output2:Number;
   printer:char;
   a,               {length of tank}
   b,               {height of tank}
   c,               {width of tank}
   angle,           {the angle of rotation in degrees}
   area,            {total area of fluid in xy plane}
   increment,       {increment by which angle increases}
   aTriangle,       {Max area of triangular shape}
   bTriangle,       {Max area of triangle and rectangle combined}
   percent,         {Fluid level as a percent of height "b"}
   mass,            {Mass of fluid}
   moment,          {Mass moment of inertia of entire shape}
   beta,            {2 - theta}
   tangentBeta,     {tan(beta)}
   tangentTheta,    {tan(theta)}
   theta,           {Angle of rotation CCW in radians}
   zero:real;
   form:integer;    {defines the geometry}

*****************************************************************************}

rocedure CounterRotate(var  ax, ay, ax1, ay1:real);
This procedure converts local Cord. to global Cord. given angle theta.}
egin
   ax := (ax1 * cos(theta)) - (ay1 * sin(theta));
   ay := (ax1 * sin(theta)) + (ay1 * cos(theta));
nd;


unction IBarTri(var  mTri,ta,tb:real):real;
This function will calculate the mass moment of inertia of a triangular    }
prism given its mass, base, and height.                                    }
egin
   IBarTri := (mTri/18) * (sqr(ta) + sqr(tb));
nd;


unction IBarRect(var  mRect,ra,rb:real):real;
This function will calculate the mass moment of inertia of a rectangular   }
prism given its mass, length, and height.                                  }
egin
   IBarRect := (mRect/12) * (sqr(ra) + sqr(rb));
nd;


rocedure Centroid(var  xBar,yBar,
                      a1,x1,y1,
                      a2,x2,y2,
                      a3,x3,y3:real);
```

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```
{This procedure will calculate the x and y coord of the centroid of the     }
{fluid.                                                                      }
ar
    xArea,
    yArea:real;
egin
    xArea  := (a1 * x1) + (a2 * x2) + (a3 * x3);
    yArea  := (a1 * y1) + (a2 * y2) + (a3 * y3);
    xBar  := xArea/area;
    yBar  := yArea/area;
end;



Function  WhatType:integer;
(*This  function  determines  what shape the fluid is in.  Type 1 = triangle;  *)
(*type  2 = triangle  and rectangle;  and type 3 = one triangle  and two        *)
(*rectangles.                                                                   *)
ar
    aArea,  alpha,  bArea  :real;
begin
    aTriangle   := 0.5 * sqr(a)  * tangentTheta;
    aArea  := aTriangle  + (a * (b - (a * tangentTheta)));
    alpha  := arctan(b/a);
    if tangentTheta   <> 0.0  then
    begin
        bTriangle   := abs((0.5  * sqr(b))/tangentTheta);
        bArea  := bTriangle  + abs((b  * (a - (b/tangentTheta ))));
    end;
    if (angle  = 0.0)  then          {It's  a rectangle.}
        WhatType  := 4
    else if (angle  = 90.0)  then {It's still  a rectangle.}
        WhatType  := 5
    else if theta  <= alpha  then
    begin
        if (aTriangle   > area)  then
        WhatType  := 1
        else if (aArea  > area)  and  (bArea  > area)  then
        WhatType  := 2
        else WhatType  := 3;
    end
    else
    begin
        if (bTriangle   > area)  then
        WhatType  := 1
        else if (bArea  > area)  then
        WhatType  := 6
        else WhatType  := 7;
    end;
end;


Procedure  TypeOne;
{This procedure  will calculate  the mass moment  of inertia  of a triangular   }
{prism.                                                                          }
ar
    a2, b2, h, j, cx, cy:real;
egin
    h := sqrt((2  * area)/tangentTheta)    * tangentTheta;
    j := sqrt((2  * area)/tangentTheta);
    a2 := (j/3)  - (a/2);
    b2 := (h/3)  - (b/2);
    CounterRotate(cx,    cy, a2, b2);
    moment  := IBarTri(mass,h,j);
nd;


rocedure  TypeTwo;
```

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```
This Procedure will calculate the mass moment of inertia of a volume which}
can be broken down into one triangular and one rectangular prism.        }
ar
   areaTri,
   areaRect1,
   h,          {Height of Rect}
   massTri,    {Mass of triangular prism}
   massRect,   {Mass of Rect. prism}
   momentTri,  {Moment of inertia of triangular prism rel. to origin}
   momentRect,{Moment of inertia of Rect. prism rel. to origin}
   j, gx, gy,
   x,y,
   xCen,
   yCen,
   x2, y2,
   x3, y3,
   x4, y4,
   x5, y5:real;
egin
   h := (area - atriangle)/a;
   j := a * tangentTheta;
   areaTri := atriangle;
   areaRect1 := area - aTriangle;
   massTri := areaTri * c * rho;
   massRect := areaRect1 * c * rho;
   gx := a/3 - a/2;
   gy := h - j/3 - b/2;
   CounterRotate(x2,y2,gx,gy);
   x := 0;
   y := h/2 - b/2;
   CounterRotate(x3,y3,x,y);
   Centroid(xCen,yCen,areaTri,x2,y2,areaRect1,x3,y3,zero,zero,zero);
   x4 := xCen - x2;
   y4 := yCen - y2;
   x5 := xCen - x3;
   y5 := yCen - y3;
   momentTri := IBarTri(massTri,a,j) + (massTri * (sqr(x4) + sqr(y4)));
   momentRect := IBarRect(massRect,a,h) + (massRect * (sqr(x5) + sqr(y5)));
   moment := momentTri + momentRect;
nd;


rocedure TypeThree;
This Procedure will calculate the mass moment of inertia of a volume which}
must be broken down into one triangular and two rectangular prisms.       }
ar
   h, j, k, m,
   areaTri,
   areaRect1,
   areaRect2,
   massTri,      {mass of triangle}
   massRect1,    {mass of rectangle below triangle}
   massRect2,    {mass of rectangle next to triangle}
   momentTri,
   momentRect1,
   momentRect2,
   tri,          {the triangular area not filled with fluid}
   xCen, yCen,   {centroidal coord of the fluid}
   x1, y1,
   x2, y2,
   x3, y3,
   x4, y4,
   x5, y5,
   x6, y6,
   x7, y7,
   x8, y8,
   x9, y9,
   moment1, moment2:real;
```

```
egin
   tri := (a * b) - area;
   j := sqrt((2 * tri)/tangentTheta);
   h := j * tangentTheta;
   k := a - j;
   m := c - h;
   areaTri := 0.5 * h * j;
   areaRect1 := a * m;
   areaRect2 := h * k;
   massTri := areaTri * c * rho;
   massRect1 := areaRect1 * c * rho;
   massRect2 := areaRect2 * c * rho;
   x2 := k + j/3 - a/2;
   y2 := b/2 - (2 * h/3);
   CounterRotate(x1,y1,x2,y2);
   x5 := 0;
   y5 := m/2 - b/2;
   CounterRotate(x6,y6,x5,y5);
   x4 := k/2 - a/2;
   y4 := b/2 - h/2;
   CounterRotate(x3,y3,x4,y4);
   Centroid(xCen,yCen,areaTri,x1,y1,areaRect1,x6,y6,areaRect2,x3,y3);
   x7 := xCen - x1;
   y7 := yCen - y1;
   x8 := xCen - x6;
   y8 := yCen - y6;
   x9 := xCen - x3;
   y9 := yCen - y3;
   momentTri := IBarTri(massTri,h,j)+(massTri * (sqr(x7) + sqr(y7)));
   momentRect1 := IBarRect(massRect1,a,m)+(massRect1 * (sqr(x8) + sqr(y8)));
   momentRect2 := IBarRect(massRect2,h,k)+(massRect2 * (sqr(x9) + sqr(y9)));
   moment := momentTri + momentRect1 + momentRect2;
nd;


rocedure TypeFour;
ar
   c4:real;
egin
   c4 := percent * b;
   moment := IBarRect(mass,a,c4);
nd;


rocedure TypeFive;
ar
   c5:real;
egin
   c5 := percent * a;
   moment := IBarRect(mass,c5,b);
nd;


rocedure TypeSix;
This Procedure will calculate the mass moment of inertia of a volume which}
can be broken down into one triangular and one rectangular prism.         }
ar
   area2,         {Area of Rect}
   areaTri,
   areaRect1,
   h,             {Height of Rect}
   massTri,       {Mass of triangular prism}
   massRect,      {Mass of Rect. prism}
   momentTri,     {Moment of inertia of triangular prism rel. to origin}
   momentRect,    {Moment of inertia of Rect. prism rel. to origin}
   j, gx, gy,
   tri,           {area of tank not filled with fluid}
   x, y,
```

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```
    xCen, yCen,
    x1, y1,
    x2, y2,
    x3, y3,
    x4, y4:real;
egin
    areaTri  := 0.5 * sqr(b)  * tangentBeta;
    areaRect1  := area  - areaTri;
    h := (area  - areaTri)/b;
    j := b * tangentBeta;
    massTri  := areaTri  * c * rho;
    massRect  := areaRect1  * c * rho;
    gx := h + j/3 - a/2;
    gy := b/3 - b/2;
    CounterRotate(x2,y2,gx,gy);
    x := (h/2) - (a/2);
    y := 0.0;
    CounterRotate(x1,y1,x,y);
    Centroid(xCen,yCen,AreaTri,x2,y2,areaRect1,x1,y1,zero,zero,zero);
    x3 := xCen  - x2;
    y3 := yCen  - y2;
    x4 := xCen  - x1;
    y4 := yCen  - y1;
    momentTri  := IBarTri(massTri,b,j)   + (massTri  * (sqr(x3)  + sqr(y3)));
    momentRect  := IBarRect(massRect,b,h)   + (massRect  * (sqr(x4)  + sqr(y4)));
    moment  := momentTri  + momentRect;
nd;


rocedure  TypeSeven;
This Procedure  will  calculate  the mass moment  of inertia  of a volume  which}
must  be broken  down  into one triangular  and two rectangular  prisms.        }
ar
    h, j, k, m,
    areaTri,
    areaRect1,
    areaRect2,
    massTri,       {mass  of  triangle}
    massRect1,     {mass  of rectangle  below  triangle}
    massRect2,     {mass  of rectangle  next  to triangle}
    momentTri,
    momentRect1,
    momentRect2,
    tri,           {the  triangular  area  not  filled  with  fluid}
    xCen,  yCen,
    ax1,  ay1,
    ax2,  ay2,
    ax3,  ay3,
    x1,  x2,  x3,  x4,  x5,  x6,
    y1,  y2,  y3,  y4,  y5,  y6,
    moment1,  moment2:real;
egin
    tri :=  (a * b) - area;
    j := sqrt((2  * tri)/tangentBeta);
    h := j * tangentBeta;
    k := b - j;
    m := a - h;
    areaTri  := 0.5 * h * j;
    areaRect1  := b * m;
    areaRect2  := h * k;
    massTri  := areaTri  * c * rho;
    massRect1  := areaRect1  * c * rho;
    massRect2  := areaRect2  * c * rho;
    x2 := a/2 - (2 * h/3);
    y2 := b/2 - (2 * j/3);
    CounterRotate(x1,y1,x2,y2);
    x5 := m/2 - a/2;
    y5 := 0;
```

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```
    CounterRotate(xb,yb,xE,yE);
    x4 := a/2 - h/2;
    y4 := k/2 - b/2;
    CounterRotate(x3,y3,x4,y4);
    Centroid(xCen,yCen,areaTri,x1,y1,areaRect1,x2,y2,areaRect2,x3,y3);
    ax1  := xCen  - x1;
    ay1  := yCen  - y1;
    ax2  := xCen  - x2;
    ay2  := yCen  - y2;
    ax3  := xCen  - x3;
    ay3  := yCen  - y3;
    momentTri   := IBarTri(massTri,h,j)+(massTri      * (sqr(ax1)   + sqr(ay1)));
    momentRect1   := IBarRect(massRect1,c,m)+(massRect1      * (sqr(ax2)+sqr(ay2)));
    momentRect2   := IBarRect(massRect2,h,k)+(massRect2      * (sqr(ax3)+sqr(ay3)));
    moment  := momentTri   + momentRect1   + momentRect2;
nd;


*************************************************************************)
egin  (*START  MAIN  PROGRAM*)
    ClearScreen;
    zero  := 0;
    writeln  ('Enter  dimensions  for  rectangular   tank:');
    readln  (a,b,c);
    writeln   ('Enter  water  level  as  a  percentage   of  "b".');
    readln  (percent);
    while  ((percent   >= 1)  or  (percent   <= 0))  do
    begin
        writeln('0.1   < Water  Level   < 1.0.   Try  again.');
        readln(percent);
    end;
    writeln('Enter   angle  theta  increment.');
    readln(increment);
    writeln('Enter   output  file  name.');
    readln(outFileName);
    angle  := 0.0;
    area  := a  *  b  * percent;
    mass  := a  * percent   *  b  *  c;
    rewrite(outFile,outFileName);
    writeln(outFile,'A    = ',a:8:4);
    writeln(outFile,'B    = ',b:8:4);
    writeln(outFile,'C    = ',c:8:4);
    writeln(outFile,'Percent     full  = ',percent:8:4);
    writeln(outFile,'Type       Theta      Moment');
    while  (angle  <= 90.0)  do
    begin
        moment  := 0.0;
        beta  := ((90  - angle)/360)*(2   * pi);
        theta  := (angle/360)*(2   * pi);
        tangentBeta   := sin(beta)/cos(beta);
        tangentTheta   := sin(theta)/cos(theta);
        form  := WhatType;
        case  (form)  of
            1:TypeOne;
            2:TypeTwo;
            3:TypeThree;
            4:TypeFour;
            5:TypeFive;
            6:TypeSix;
            7:TypeSeven;
        end;
        writeln(outFile,form,     '      ',angle:6:2,   '      ',moment:8:4);
        angle  := angle  + increment;
    end;
    close(outFile);
nd.  (*END  MAIN  PROGRAM*)
```

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```
Program Moment1;
{This program calculates  the mass moment  of inertia  of the fluid  in a tank  }
{relative  the to the center  of the tank.  The tank is dimensioned  as 'a' in }
{the x-dir, 'b' in the y-dir(up),  and 'c' in the z-dir(into  the page).  The }
{center  of the tank  is also the center  of rotation  and corresponds  to the  }
{origin  of the coordinates.   The tank rotates counterclockwise   and this     }
{angle  is defined  as theta.  This version  was lase modified  on 2/4/90.      }
Uses
    PasPrinter;
Const
    pi = 3.141593;
    rho = 1.0;
Type
    Number  = array [1..90]  of real;
Var
    outFile      :text; {file pointer  for output}
    outFileName   :string[15];
    output1:Number;
    output2:Number;
    printer:char;
    a,              {length  of tank}
    b,              {height  of tank}
    c,              {width  of tank}
    area,
    angle,          {the  angle  of rotation  in degrees}
    increment,      {Increment  by which  angle  increases}
    aTriangle,      {Max  area  of triangluar  shape}
    bTriangle,      {Max  area  of triangle  and rectangle  combined}
    percent,        {Fluid  level  as a percent  of height  "b"}
    mass,           {Mass  of fluid}
    moment,         {Mass  moment  of inertia  of entire  shape}
    beta,           {2  - theta}
    tangentBeta,    {tan(beta)}
    tangentTheta,   {tan(theta)}
    theta:real;     {Angle  of rotation  CCW  in radians}
    form:integer;   {defines  the geometry}

{**********************************************************************}

Procedure  CounterRotate(var   ax, ay, ax1, ay1:real);
{This procedure  converts  local  Cord.  to global  Cord.  given  angle  theta.}
begin
    ax  := (ax1  * cos(theta))  - (ay1  * sin(theta));
    ay  := (ax1  * sin(theta))  + (ay1  * cos(theta));
end;


Function  IBarTri(var  mTri,ta,tb:real):real;
{This  function  will  calculate  the mass moment  of inertia  of a triangular    }
{prism  given  its mass,  base,  and height.                                     }
begin
    IBarTri  := (mTri/18)  * (sqr(ta)  + sqr(tb));
end;


Function  IBarRect(var  mRect,ra,rb:real):real;
{This  function  will  calculate  the mass moment  of inertia  of a rectangular   }
{prism  given  its mass,  length,  and height.                                   }
begin
    IBarRect  := (mRect/12)  * (sqr(ra)  + sqr(rb));
end;



Function  WhatType:integer;
(*This  function  determines  what shape  the fluid  is in.  Type 1 = triangle;  *)
(*type  2 = triangle  and rectangle;  and type 3 = one triangle  and two        *)
(*rectangles                                                                    *)
```

```
ar
    aArea, alpha, bArea :real;
egin
    aTriangle := 0.5 * sqr(a) * tangentTheta;
    aArea := aTriangle + (a * (b - (a * tangentTheta)));
    alpha := arctan(b/a);
    if tangentTheta <> 0.0 then
    begin
        bTriangle := abs((0.5 * sqr(b))/tangentTheta);
        bArea := bTriangle + abs((b * (a - (b/tangentTheta))));
    end;
    if (angle = 0.0) then        {It's a rectangle.}
        WhatType := 4
    else if (angle = 90.0) then {It's still a rectangle.}
        WhatType := 5
    else if theta <= alpha then
    begin
        if (aTriangle > area) then
        WhatType := 1
        else if (aArea > area) and (bArea > area) then
        WhatType := 2
        else WhatType := 3;
    end
    else
    begin
        if (bTriangle > area) then
        WhatType := 1
        else if (bArea > area) then
        WhatType := 6
        else WhatType := 7;
    end;
nd;


rocedure TypeOne;
This procedure will calculate the mass moment of inertia of a triangular     }
prism.                                                                        }
ar
    a2, b2, h, j, cx, cy:real;
egin
    h := sqrt((2 * area)/tangentTheta) * tangentTheta;
    j := sqrt((2 * area)/tangentTheta);
    a2 := (j/3) - (a/2);
    b2 := (h/3) - (b/2);
    CounterRotate(cx, cy, a2, b2);
    moment := IBarTri(mass,h,j) + (mass *(sqr(cx) + sqr(cy)));
nd;


rocedure TypeTwo;
This Procedure will calculate the mass moment of inertia of a volume which}
can be broken down into one triangular and one rectangular prism.       }
ar
    area2,      {Area of Rect}
    h,          {Height of Rect}
    massTri,    {Mass of triangular prism}
    massRect,   {Mass of Rect. prism}
    momentTri,  {Moment of inertia of triangular prism rel. to origin}
    momentRect,{Moment of inertia of Rect. prism rel. to origin}
    j, gx, gy, xbar, ybar,
    x, y, x3, y3:real;
egin
    h := (area - atriangle)/a;
    j := a * tangentTheta;
    massTri := atriangle * c * rho;
    massRect := (area - atriangle) * c * rho;
    gx := a/3 - a/2;
    gy := h + j/3 - b/2;
```

```
    CounterRotate(xbar,ybar,gx,gy);
    momentTri  := IBarTri(massTri,a,j)   + (massTri  * (sqr(xbar)  + sqr(ybar)));
    x := 0;
    y := h/2 - b/2;
    CounterRotate(x3,y3,x,y);
    momentRect  := IBarRect(massRect,a,h)   + (massRect  * (sqr(x3)  + sqr(y3)));
    moment  := momentTri  + momentRect;
end;


Procedure  TypeThree;
{This Procedure  will calculate  the mass moment  of inertia  of a volume  which}
{must  be broken  down  into one triangular  and two rectangular  prisms.       }
var
    h, j, k, m,
    massTri,      {mass  of  triangle}
    massRect1,    {mass  of rectangle  below  triangle}
    massRect2,    {mass  of rectangle  next  to triangle}
    momentTri,
    momentRect1,
    momentRect2,
    tri,          {the  triangular  area  not filled  with  fluid}
    x1, x2, x3, x4, x5, x6,
    y1, y2, y3, y4, y5, y6,
    moment1,  moment2:real;
begin
    tri :=  (a * b) - area;
    j := sqrt((2  * tri)/tangentTheta);
    h := j * tangentTheta;
    k := a - j;
    m := b - h;
    massTri  := 0.5 * h * j * c * rho;
    massRect1  := a * m * c * rho;
    massRect2  := h * k * c * rho;
    x2 := k + j/3 - a/2;
    y2 := b/2 - (2 * h/3);
    CounterRotate(x1,y1,x2,y2);
    momentTri  := IBarTri(massTri,h,j)   + (massTri  * (sqr(x1)  + sqr(y1)));
    x5 := 0;
    y5 := m/2 - b/2;
    CounterRotate(x6,y6,x5,y5);
    momentRect1  := IBarRect(massRect1,a,m)+(massRect1     * (sqr(x6)  + sqr(y6)));
    x4 := k/2 - a/2;
    y4 := b/2 - h/2;
    CounterRotate(x3,y3,x4,y4);
    momentRect2  := IBarRect(massRect2,h,k)+(massRect2     * (sqr(x3)  + sqr(y3)));
    moment  := momentTri  + momentRect1  + momentRect2;
end;


Procedure  TypeFour;
var
    c4:real;
begin
    c4 := percent  * b;
    moment  := IBarRect(mass,a,c4)   + (mass  * sqr((b/2)  * (percent  - 1)));
end;


Procedure  TypeFive;
var
    c5:real;
begin
    c5 := percent  * a;
    moment  := IBarRect(mass,c5,b)   + (mass  * (sqr((a/2)  * (percent  - 1))));
end;
```

```
Procedure TypeSix;
{This Procedure will calculate the mass moment of inertia of a volume which}
{can be broken down into one triangular and one rectangular prism.         }
var
    area2,          {Area of Rect}
    areaTriangle,
    h,              {Height of Rect}
    massTri,        {Mass of triangular prism}
    massRect,       {Mass of Rect. prism}
    momentTri,      {Moment of inertia of triangular prism rel. to origin}
    momentRect,     {Moment of inertia of Rect. prism rel. to origin}
    j, gx, gy, xbar, ybar,
    x, y, x3, y3:real;
begin
    areaTriangle := 0.5 * sqr(b) * tangentBeta;
    h := (area - areaTriangle)/b;
    j := b * tangentBeta;
    massTri := areaTriangle * c * rho;
    massRect := (area - areaTriangle) * c * rho;
    gx := h + j/3 - a/2;
    gy := b/3 - b/2;
    CounterRotate(xbar,ybar,gx,gy);
    momentTri := IBarTri(massTri,b,j) + (massTri * (sqr(xbar) + sqr(ybar)));
    x := (h/2) - (a/2);
    y := 0.0;
    CounterRotate(x3,y3,x,y);
    momentRect := iBarRect(massRect,b,h) + (massRect * (sqr(x3) + sqr(y3)));
    moment := momentTri + momentRect;
end;


Procedure TypeSeven;
{This Procedure will calculate the mass moment of inertia of a volume which}
{must be broken down into one triangular and two rectangular prisms.       }
var
    h, j, k, m,
    massTri,        {mass of triangle}
    massRect1,      {mass of rectangle below triangle}
    massRect2,      {mass of rectangle next to triangle}
    momentTri,
    momentRect1,
    momentRect2,
    tri,            {the triangular area not filled with fluid}
    x1, x2, x3, x4, x5, x6,
    y1, y2, y3, y4, y5, y6,
    moment1, moment2:real;
begin
    tri := (a * b) - area;
    j := sqrt((2 * tri)/tangentBeta);
    h := j * tangentBeta;
    k := b - j;
    m := a - h;
    massTri := 0.5 * h * j * c * rho;
    massRect1 := b * m * c * rho;
    massRect2 := h * k * c * rho;
    x2 := a/2 - (2 * h/3);
    y2 := b/2 - (2 * j/3);
    CounterRotate(x1,y1,x2,y2);
    momentTri := IBarTri(massTri,h,j)+(massTri * (sqr(x1) + sqr(y1)));
    x5 := m/2 - a/2;
    y5 := 0;
    CounterRotate(x6,y6,x5,y5);
    momentRect1 := IBarRect(massRect1,b,m)+(massRect1 * (sqr(x6) + sqr(y6)));
    x4 := a/2 - h/2;
    y4 := k/2 - b/2;
    CounterRotate(x3,y3,x4,y4);
    momentRect2 := IBarRect(massRect2,h,k)+(massRect2 * (sqr(x3) + sqr(y3)));
    moment := momentTri + momentRect1 + momentRect2;
```

```
end;


{****************************************************************************}
Begin (*START MAIN PROGRAM*)
    ClearScreen;
    writeln ('Enter dimensions for rectangular tank:');
    readln (a,b,c);
    writeln ('Enter water level as a percentage of "b".');
    readln (percent);
    while ((percent >= 1) or (percent <= 0)) do
    begin
        writeln('0.0 < Water Level < 1.0. Try again.');
        readln(percent);
    end;
    writeln('Enter angle theta increment.');
    readln(increment);
    writeln('Enter output file name.');
    readln(outFileName);
    angle := 0.0;
    area := a * b * percent;
    mass := a * percent * b * c;
    rewrite(outFile,outFileName);
    writeln(outFile,'A    = ',a:8:4);
    writeln(outFile,'B    = ',b:8:4);
    writeln(outFile,'C    = ',c:8:4);
    writeln(outFile,'Percent    full = ',percent:8:4);
    writeln(outFile,'Type      Theta     Moment');
    while (angle <= 90.0) do
    begin
        moment := 0.0;
        beta := ((90 - angle)/360)*(2 * pi);
        theta := (angle/360)*(2 * pi);
        tangentBeta := sin(beta)/cos(beta);
        tangentTheta := sin(theta)/cos(theta);
        form := WhatType;
        case (form) of
            1:TypeOne;
            2:TypeTwo;
            3:TypeThree;
            4:TypeFour;
            5:TypeFive;
            6:TypeSix;
            7:TypeSeven;
        end;
        writeln(outFile,form,  '     ',angle:6:2,  '     ',moment:8:4);
        angle := angle + increment;
    end;
    close(outFile);
End. (*END MAIN PROGRAM*)
```