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Probabilistic Simulation of Long Term Behavior in Polymer Matrix Composites

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Abstract

A methodology to compute cumulative probability distribution functions (CDF) of fatigue life for different ratios, r of applied stress to the laminate strength based on first ply failure criteria has been developed and demonstrated. Degradation effects due to long term environmental exposure and mechanical cyclic loads are considered in the simulation process. A unified time-stress dependent multi-factor interaction equation model developed at NASA Lewis Research Center has been used to account for the degradation/aging of material properties due to cyclic loads. Fast probability integration method is used to perform probabilistic simulation of uncertainties. Sensitivity of fatigue life reliability to uncertainties in the primitive random variables are computed and their significance in the reliability based design for maximum life is discussed. The results show that the graphite/epoxy $(0/\pm45/90)^\circ$ laminate with ply thickness 0.125 in. has 500,000 cycles life for applied stress to laminate strength ratio of 0.6 and a reliability of 0.999. Also, the fatigue life reliability has been found to be most sensitive to the ply thickness and matrix tensile strength. Tighter quality controls must therefore be enforced

on ply thickness and matrix strength in order to achieve high reliability of the structure.

Introduction

Assured long term behavior with a specified reliability is a prime criteria for High Speed Civil Transport (HSCT) engine structures/components. HSCT engine components are required to have a reliable life of at least 18000 hours. High temperature polymer matrix composites are being considered as prime candidates for some of the propulsion structure components in the HSCT. Therefore, it is of considerable importance to develop methodologies to predict long term behavior of high temperature polymer matrix composite materials (PMCs).

The complexity of predicting composite behavior is compounded by multiple scales (micro, macro, and laminate), fabrication process induced variations, inherent uncertainties occurring in the constituent properties, and an aggressive loading environment. Successful utilization of polymer matrix composites in aerospace structures hinges largely upon the ability to predict and assure their long term behavior. Current practices depends largely on conducting long term testing of materials and components. However, it is time consuming and expensive to conduct a large number of long term tests to capture the effects of all the design variables and loads. Furthermore, accounting for the above discussed uncertainties in the experiments is monetarily infeasible proposition. Therefore in light of these reasons, realization of the full potential of composite materials is more likely to be achieved with the real innovative computational approaches capable of handling these aspects in an integrated manner. Also such computational procedures could aid in identifying critical experiments to be performed as well as reduce the number of tests to be conducted to a manageable number.

Traditional computational approaches are deterministic in nature which do not account for uncertainties associated with composite structures and materials. The focus of ongoing research at the NASA Lewis Research Center has been to develop advanced integrated computational methods and related computer codes to perform a complete probabilistic assessment of composite structures. These methods account for uncertainties in all the constituent properties, fabrication process variables and loads to predict probabilistic micro, ply, laminate and structural responses. These methods have already been implemented in the Integrated Probabilistic Assessment of Composite Structures (IPACS)[1] computer code. More details of IPACS code are given in the next section.

This paper deals with developing a probabilistic computational simulation methodology to predict reliability based long term behavior in polymer matrix composites and implementing it in the ICAN[2] computer code. A unified time, stress and load dependent multi-factor interaction equation (MFIE) model developed at the NASA Lewis Research Center[4] has been used to simulate the long term behavior of polymer matrix composites.

To illustrate the developed methodology a graphite fiber and epoxy matrix composite system is considered. The methodology can also be applied to other types of PMCs as well. The cumulative probability distribution functions (CDF) for the fatigue life cycles of a $(0/\pm45/90)_s$

graphite/epoxy laminate are computed for different applied stress to laminate strength based on first ply failure criteria (hereinafter referred to as laminate strength) ratios. First ply failure criteria assumes that the laminate has failed when any stress component in a ply exceeds its respective allowable. Using these CDFs a fatigue life cycle curve for a reliability of 0.999 is obtained in order to demonstrate how this methodology can be used to aid the designer. Also, the sensitivity of fatigue life to the primitive random variables are computed for a reliability of 0.999.

Computational Simulation

Prediction of long term behavior of PMCs using fundamental governing field equations in the time variable facilitates tracking the uncertainties in random variables throughout the load history. The approach used herein incorporates the micro mechanics theory, time dependent multi-factor interaction equation model, and fast probability integration techniques[5] to account for the physical process of manufacturing a composite material, mechanics governing the individual constituent behavior, their interactions, and constituent property degradation due to aggressive load effects. The following sections describe the developed/modified computer codes and procedures used in the computational simulation of probabilistic long term behavior in PMCs.

Integrated Composite Analyzer: The Integrated Composite Analyzer, ICAN computationally simulates the material behavior of polymer matrix composites from fiber/matrix constituents to the laminate scale including fabrication effects (Figure 1). ICAN uses advanced composite micro-mechanics and laminate analysis based on linear elastic theory to compute constituent, ply, and laminate scale properties and stresses required for global structural analysis (Left hand side of Figure 1). ICAN has an updatable resident database containing room temperature properties of commonly available fibers and matrices. The user needs to input just the names of fibers and matrices used in the laminate which reduces the time required to input the data and eliminates errors. ICAN also decomposes the global structural response to laminate, ply and constituent response levels which helps the user evaluate failure (right hand side of figure 1). Details of the ICAN computer code are given in reference 6. ICAN also performs the failure analysis based on different failure criteria such as first ply failure criteria, fiber break criteria, etc. Failures analysis uses modified distortion energy method or Hoffman's criteria.

For long term behavior prediction the ICAN computer code was modified to implement the time dependent multi-factor interaction equation and perform sensitivity evaluation for primitive random variables. The MFIE model evaluates the degradation magnitude of the constituent material properties at every time step which in turn is used for micro-mechanics and laminate analysis at each step. Sensitivity evaluations of response variables to the random variables at every time step are also performed to compute the respective scatter in response variables. Also, at every time step a failure analysis based on first ply failure and fiber break criteria is performed to determine whether the laminate can take any further load. Failure analysis determines the possible failure modes and maximum load capacity in the respective failure mode. The analysis stops when the laminate is incapable of carrying any more load. In addition, a data base is created to store results required to perform probabilistic life analysis and develop reliability based life assessment of PMCs. Thus, the current version of ICAN is capable of performing probabilistic time variable analysis to incorporate material degradation due to thermal and mechanical cyclic loads.

Time Dependent Multi-Factor Interaction Equation (TMFIE) Model:

It is known that predicting the behavior of composite materials is a difficult task. Accounting for all the physical effects and how they affect the material properties in the time domain is even more complex. Over the years, research in developing a unified law describing the material behavior driven by primitive variables has been an on going activity at NASA Lewis Research Center. The result of this research is the development of a unified multi-factor interaction equation model[7].

Concepts used in the above referred publication have been expanded to include time dependent degradation effect on material behavior due to environmental, fabrication and load effects[4]. A generic form of the above equation is given by:

$$\frac{M_p}{M_{po}} = \prod_{i=1}^{N} \left[\frac{V_F - V}{V_F - V_o}\right]_i^a \qquad (1)$$

where

M	-	material
V	-	primitive variable for a material or load e.g. temperature, stress, mechanical cycles, etc.
N	-	number of effects
S_f	-	Final strength i.e. strength of material before cyclic load is applied
σ_M	-	Mechanical cyclic stress
σ_T	-	Thermal cyclic stress

superscripts

a	-	material exponent for a given variable

subscripts

p	-	material property
i	-	variable effect i
F	-	condition at the final stage
o	-	condition at the reference stage

Each term in parenthesis accounts for a specific physical effect. Any number of effects can be included in one single equation as seen by the nature of the equation. The exponents are determined from the available experimental data or estimated from the anticipated material behavior due to a particular primitive variable. Each primitive variable and the exponent in the above equation can be random with a statistical distribution. The insufficiency of a set of experimental data can be taken into account by means of uncertainties in the exponent.

An important part of the above model is the fact that only one equation can include all the effects with any non-linearity in the material behavior and follow the physics of behavior. It can describe all the interacting effects of different variables (thermal, metallurgical, mechanical, chemical and load). Since variables used are at a primitive level, it simulates the in situ degradation in material properties due to applied cyclic and environmental effects. The specific form of the equation used in this paper to account for time dependent degradation is:

$$\frac{M_p}{M_{po}} = \left(\frac{T_{gw}-T}{T_{gw}-T_o}\right)^{0.5} \left(1-\frac{\sigma}{S_f}\right)^m \left(1-\frac{\sigma t}{S_f t_f}\right)^n \left(1-\frac{\sigma_M N_M}{S_f N_{fM}}\right)^p \left(1-\frac{\sigma_T N_T}{S_f N_{fT}}\right)^q \qquad (2)$$

where:

P - Property, T- temperature, S - strength, σ - stress, N - number of cycles, t - time

subscripts:

gw - wet glass transition temperature, o - reference condition, f - final condition, M - mechanical load, T - thermal cyclic load.

Note that in this paper thermal cyclic loads have not been considered. Only the time effect of mechanical cyclic loads have been considered.

Probabilistic Simulation:

An advanced first order second moment fast probability integration technique[5] is used to compute cumulative probability distribution function of the fatigue life. Results of the random variable perturbations are used to compute the CDF of responses. Fast probability integration (FPI) technique is very efficient compared to Monte-Carlo simulation technique. FPI does not generate random samples nut uses the numerical integration technique to compute the joint probability and probability of failures. It transforms the physical random variable space system into unit normal space to perform probability integration easily and more accurately. On the other hand Monte-carlo method generates a large number of random samples to compute probability of response. Thus, Monte-Carlo method requires many runs to evaluate to response whereas FPI needs selective runs to generate the response surface. Hence, the FPI is computationaly economical than Monte-Carlo. The sensitivity of responses are also computed by FPI. Sensitivity information help improve the design and quality.

Simulation Cases, Results and Discussion

Demonstration examples include only mechanical cyclic load, as mentioned before. However, the methodology is generic in nature and accounts for all types of loads including thermal fatigue. A $(0/\pm45/90)_s$ laminate made of graphite fibers and epoxy matrix is subjected to uniaxial tensile cyclic load as shown in figure 2. The load shown in Figure 2 is sinusoidal. However, the shape does not matter because exponent value in MFIE accounts for the shape. Each ply has a thickness of 0.005 in (each direction has 25 plies thus the total thickness in each direction is 0.125 in). The mean values of all the fiber and matrix properties are given in Table 1. Initially, the laminate was subjected to a mean static load and failure analysis was performed to evaluate static laminate strength. It was found to be 63.0 ksi based on first ply failure criteria. Since the aim of this paper is to develop fatigue life cycles, the laminate was subjected to different applied stress to strength ratios, r. In the following, the computation of a deterministic (mean value) and a 0.999 reliability based fatigue life cycle curves are described.

For the deterministic case typical life cycle curves for all the plies were computed. However for the sake of brevity, only the results for a 90° ply are reported since 90° ply failed first among all the plies. Figure 3 shows the life cycle curve for 90° ply when longitudinal stress exceeds corresponding strength whereas figure 4 shows life cycle curves when transverse stress exceeds transverse strength. It is seen from these figures that the failure is dominated by the longitudinal stress in 90° ply. Also, it is obvious from these figures that the fatigue life reduces at a rapid rate for failure against longitudinal stress in 90° ply as compared to that against transverse stress. Since 90° ply failed first and the longitudinal stresses dominate the fatigue life, Figure 3 represents the fatigue life curve for the entire laminate.

Probabilistic fatigue life cycle evaluation was performed in two stages to reduce the computational time. The first stage involved performing the preliminary analysis to determine significant variables governing the life. Preliminary probabilistic analysis considers all the primitive random variables related to fiber and matrix properties, lay up angle, ply thickness, fiber volume ratio and void volume ratio. The scatter (range of variation) for these variables was assumed to be 5 % and normally distributed as listed in Table 1. The preliminary analysis involved a few sensitivity evaluations and a probabilistic analysis for extreme applied stress to first ply strength load. Based on the preliminary analysis results (not reported here for brevity) it was found that longitudinal fiber modulus, matrix modulus, matrix tensile strength, fiber volume ratio and the ply thickness controlled the fatigue life of the laminate.

As discussed above a final probabilistic fatigue life analysis was performed for a mean applied maximum stress to mean first ply strength ratio, *r* of 0.6, 0.7, 0.8 and 0.9. Cumulative probability distribution function (CDF) curves for each r value were obtained. Figures 5a through 8a depict CDF for *r* values of 0.6, 0.7, 0.8 and 0.9 respectively. Also, the corresponding sensitivity of random variables are plotted respectively in Figures 5b through 8b.

Computations show that the scatter in fatigue life for *r* values of 0.6, 0.7, 0.8 and 0.9 was 14.84 %, 15.23%, 15.46% and 15.87 % respectively. Thus as the *r* value increases, the amount of scatter in the fatigue life increases at a very low rate. Also, for a probability of failure equal to 0.001 (meaning that a reliability of 0.999 OR only 1 out of 1000 laminates has chance of failing), the fatigue life for *r* values of 0.6, 0.7, 0.8 and 0.9 are 51 %, 35.6 %, 25.65 % and 18.86 % of the endurance limit (N_{mf}, defined as the maximum stress level at which material can resist up to 1,000,000 cycles of load) respectively. Figures 5b through 8b also show the sensitivity of fatigue life to the random variables for *r* values of 0.6, 0.7, 0.8 and 0.9 respectively for a reliability of 0.999. These figures show that the uncertainty in the fatigue life is more sensitive to the scatter in the thickness followed by matrix tensile strength, fiber volume ratio, longitudinal fiber modulus and matrix modulus at all *r* values. Therefore, it can be said that the increase in *r* value decreases the fatigue life but the scatter remains the same for all practical purposes.

An important inference from figures 5 through 8 would be that for all *r* values, if the design reliability requirement is higher than 0.999, the scatter in the ply thickness, matrix tensile strength and fiber volume ratio must be reduced by using a tighter quality control. The usefulness of the obtained CDF curves is demonstrated by developing a reliability based fatigue life design curves. Figure 9 shows a fatigue life curve for a reliability of 0.999. The designer can obtain fatigue life for a reliability of 0.999 directly by knowing the magnitude of the applied stress to first ply strength ratio or for a given number of cycles the maximum load on the laminate can be obtained.

Conclusion

A methodology to compute the probabilistic fatigue life of polymer matrix composites has been developed, implemented in the in-house computer code ICAN and demonstrated by examples. The methodology incorporates the ICAN computer code, a generic time dependent multi factor interaction equation model and fast probability integration technique. Also, the developed methodology has been implemented in ICAN computer code to facilitate its integration with IPACS computer code to perform probabilistic composite structural analysis. Cumulative distribution functions of fatigue life cycles for a $(0/\pm45/90)_s$ Graphite/Epoxy laminate under uniaxial tensile load were computed. Sensitivities of fatigue life with 0.999 reliability were plotted. A fatigue life curve for 0.999 reliability was generated. It was observed that for a reliability of 0.999 the life of the laminate subjected to 60 % of its first ply strength in tension is half the "endurance limit". The fatigue life at 0.999 reliability was most sensitive to thickness and matrix tensile strength since tension failure in 90° ply controls the life. Also, the average scatter in fatigue at all applied stress to strength ratios was about 14.5 %. The reliability based fatigue curves are useful in determining allowable load on the structure or assessing the life of a given component. Sensitivity information provides guidelines to the designer to improve the reliability of the structure's fatigue life.

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Table 1 Uncertainties in the Primitive Variables for Graphite/Epoxy Composites

Uncertain Variable	Mean Value	Scatter	Distribution Type
Fiber:			
Normal Modulus, E_{f11}	31 mpsi	5 %	Normal
Normal Modulus, E_{f22}	2 mpsi	5 %	Normal
Poisson's ratio, ν_{12}	0.20	5 %	Normal
Poisson's ratio, ν_{23}	0.25	5 %	Normal
Shear Modulus, G_{f12}	2.0 mpsi	5 %	Normal
Shear Modulus, G_{f23}	1.0 mpsi	5 %	Normal
Tensile strength, S_{fT}	400 ksi	5 %	Normal
Compressive strength, S_{fC}	400 ksi	5 %	Normal
Matrix:			
Normal Modulus, E_m	0.5 mpsi	5 %	Normal
Poisson's ratio, ν_m	0.35	5 %	Normal
Tensile strength, S_{mT}	15.0 ksi	5 %	Normal
Compressive strength, S_{mC}	35.0 ksi	5 %	Normal
Shear strength, S_{mS}	13.0 ksi	5 %	Normal
Fabrication variables:			
Fiber volume ratio	60 %	5 %	Normal
Void volume ratio	2 %	5 %	Normal
Ply Thickness	0.125	5 %	Normal
Ply misalignment	0	1°	Normal

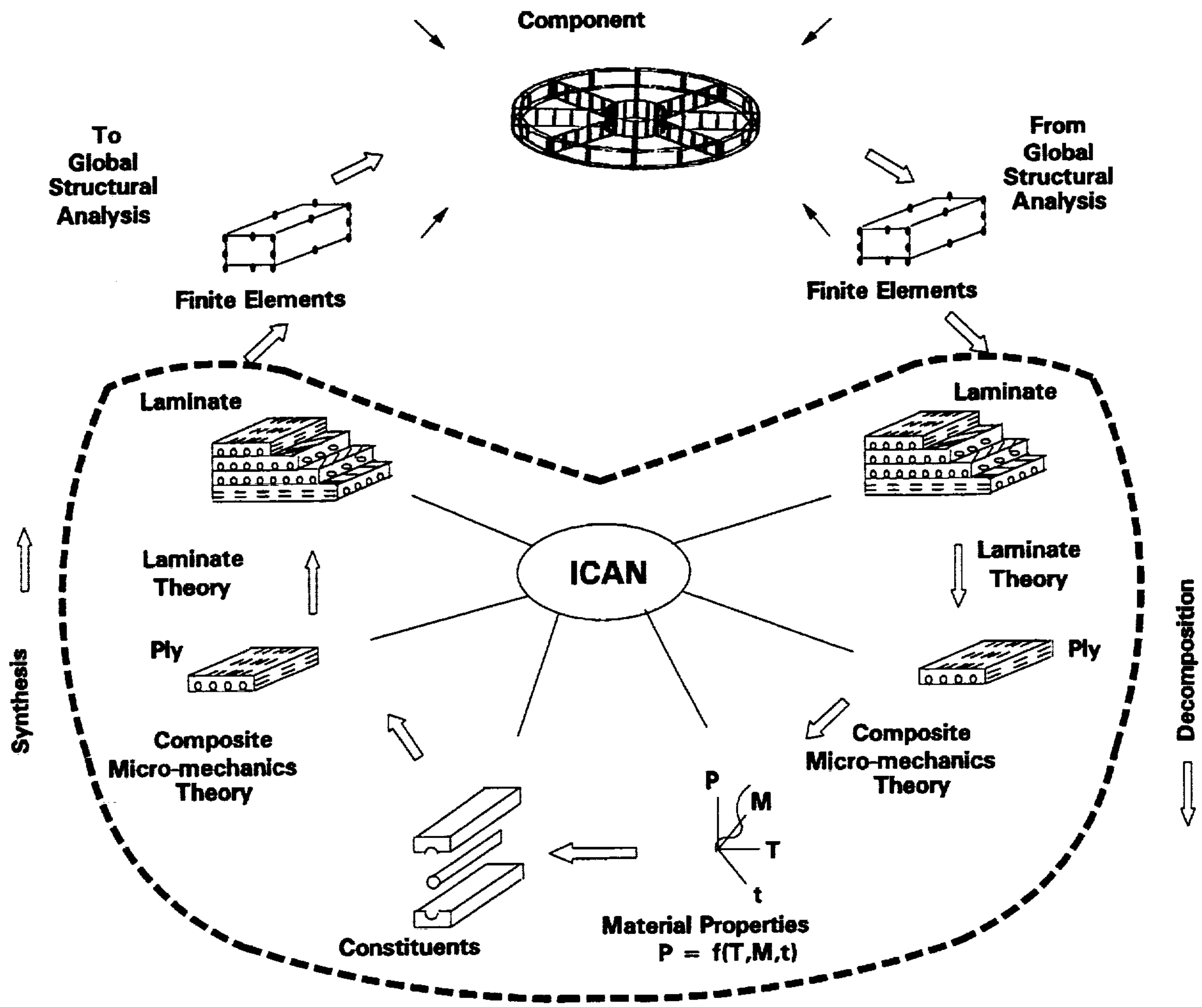

Figure 1 Schematic of ICAN Computer code

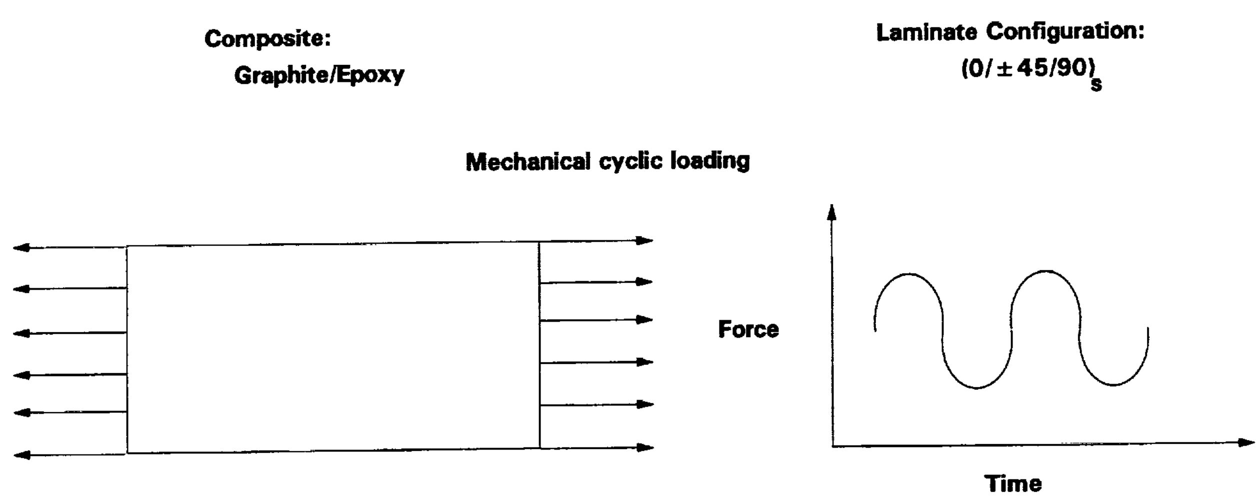

Figure 2 Description of mechanical cyclic load

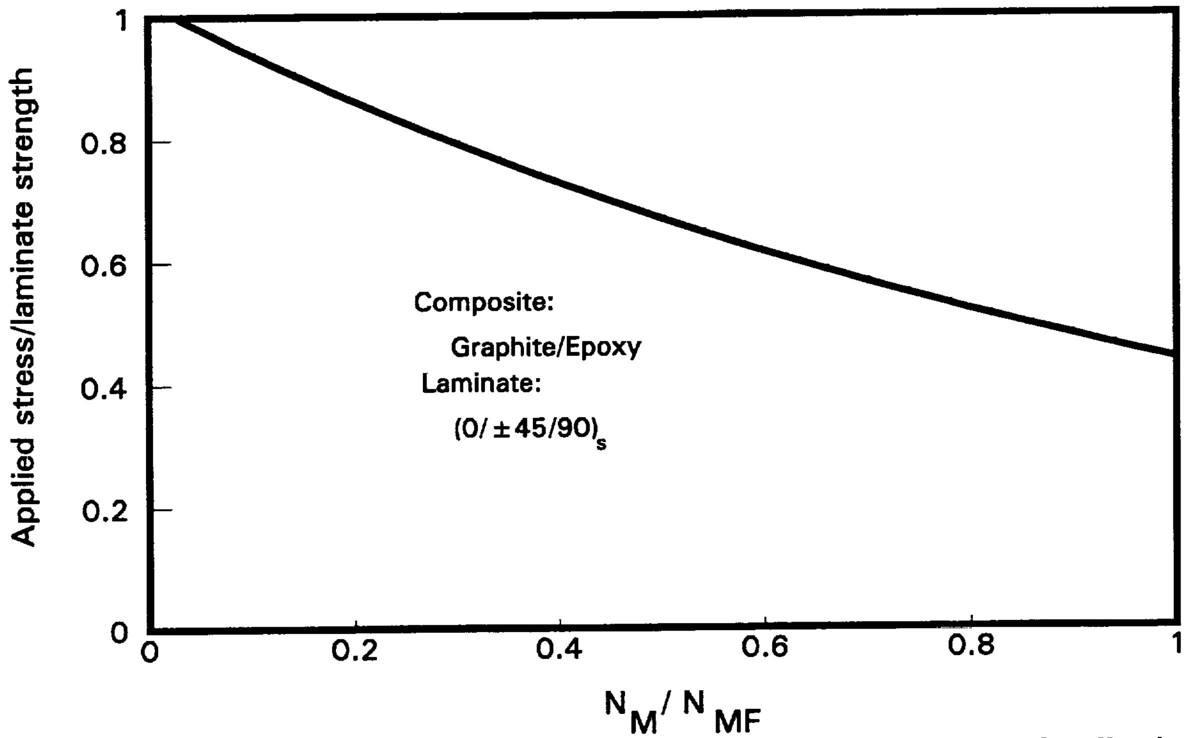

Figure 3 Fatigue life cycle curve for failure in longitudinal direction of ply 5

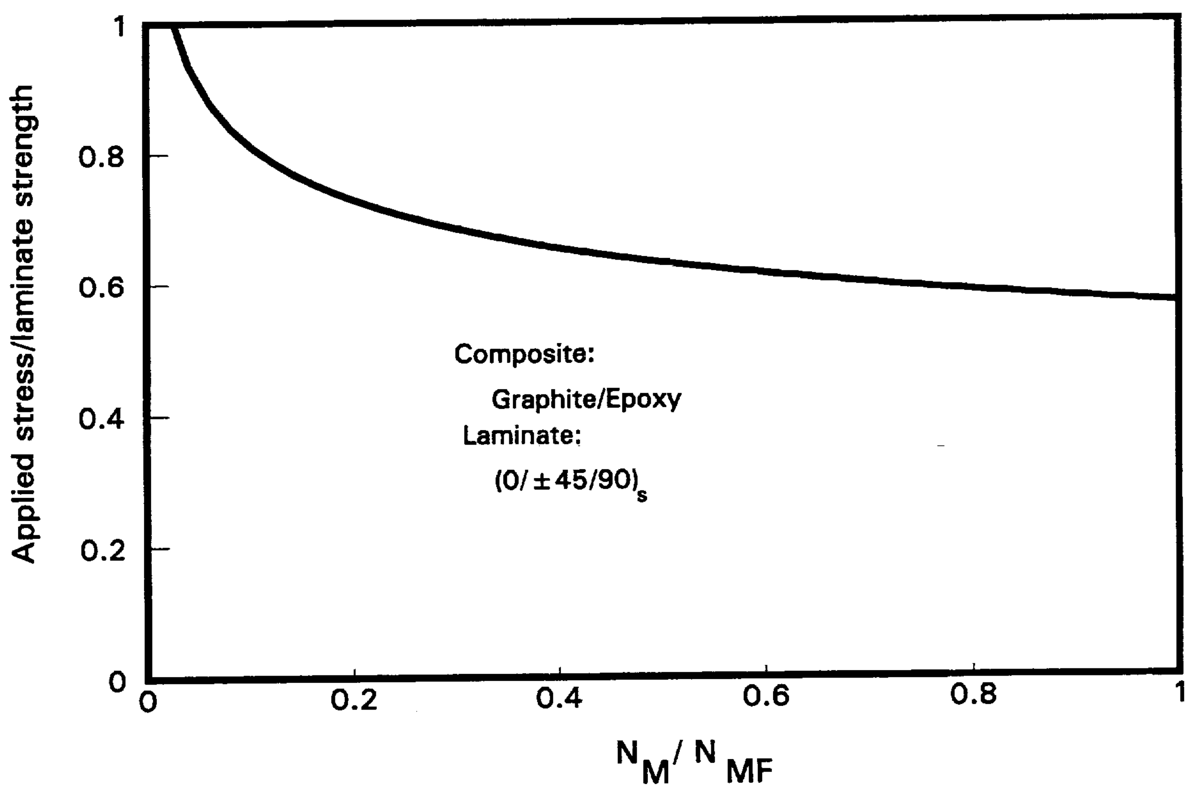

Figure 4 Fatigue life cycle curve for failure in transverse direction of ply 5

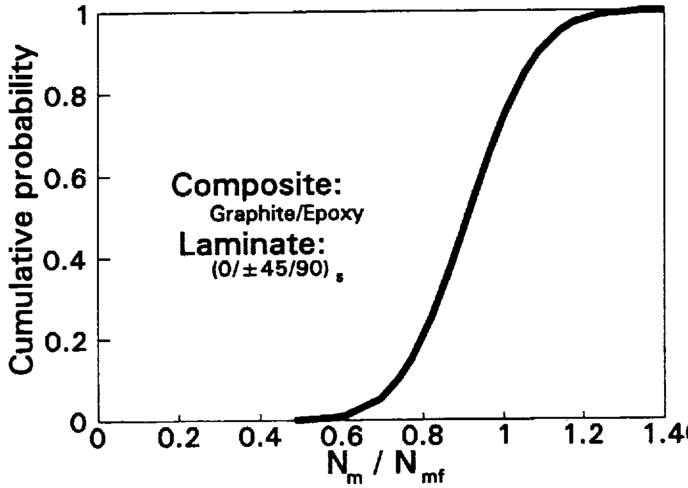

Figure 5a Cumulative probability distribution function of fatigue life applied stress/laminate strength = 0.6

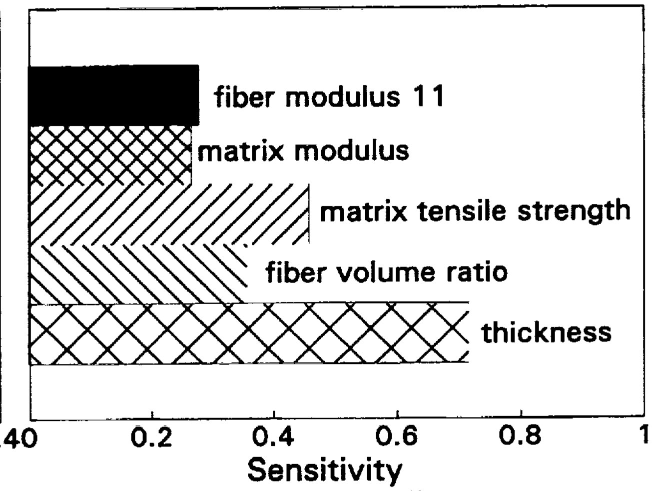

Figure 5b Sensitivity of fatigue life to primitive variables at a reliability of 0.999

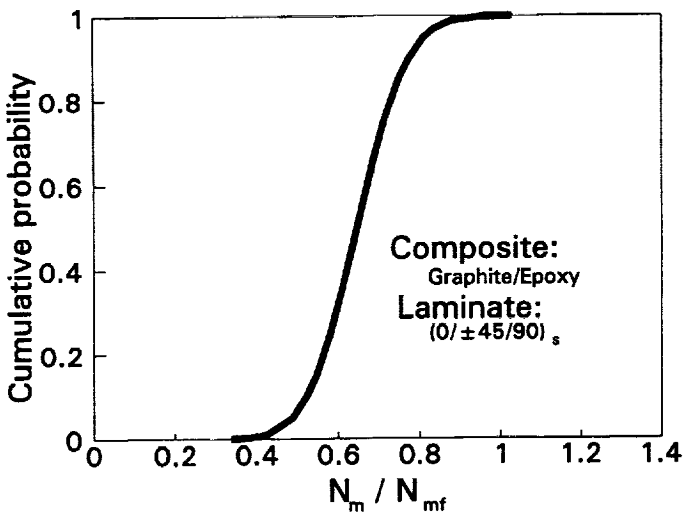

Figure 6a Cumulative probability distribution function of fatigue life applied stress/laminate strength = 0.7

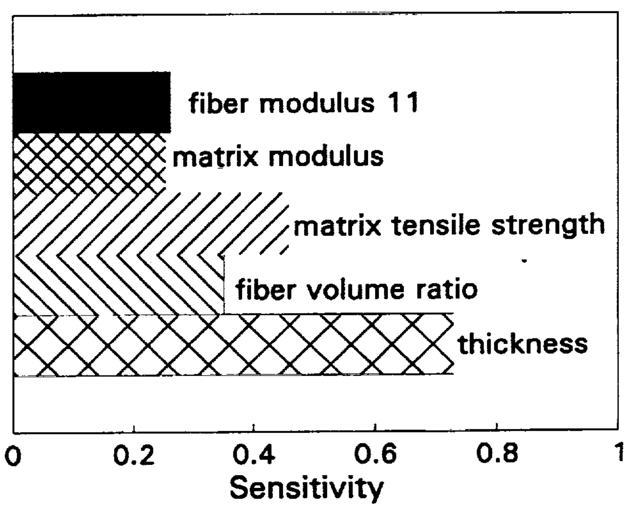

Figure 6b Sensitivity of fatigue life to primitive variables at a reliability of 0.999

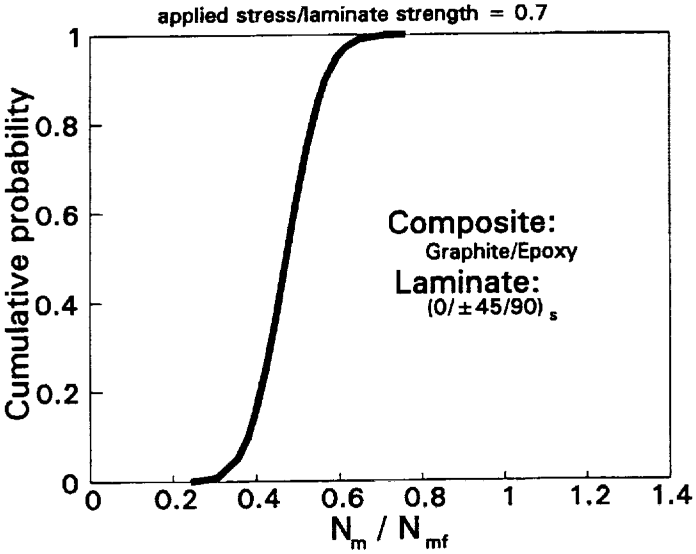

Figure 7a Cumulative probability distribution function of fatigue life applied stress/laminate strength = 0.8

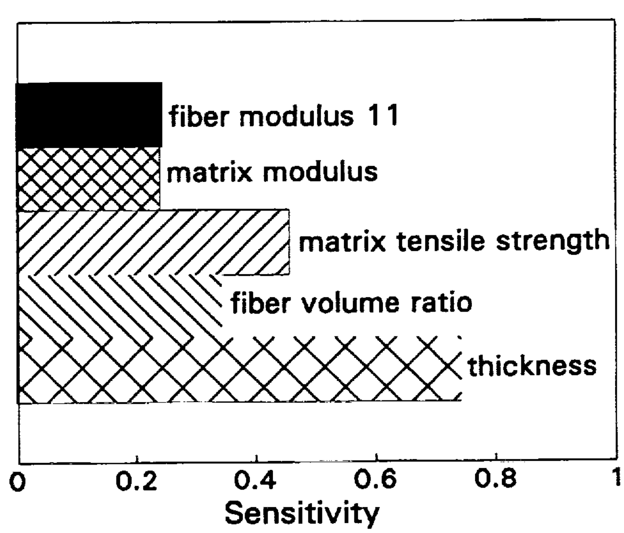

Figure 7b Sensitivity of fatigue life to primitive variables at a reliability of 0.999

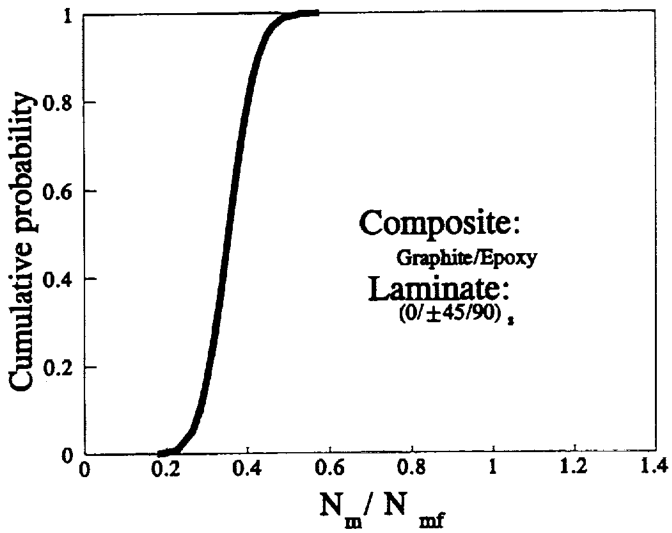

Figure 8a Cumulative probability distribution function of fatigue life applied stress/laminate strength = 0.9

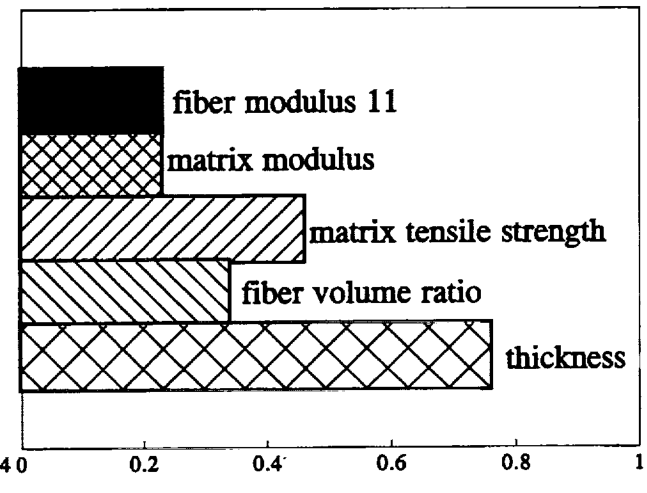

Figure 8b Sensitivity of fatigue life to primitive variables at a reliability of 0.999

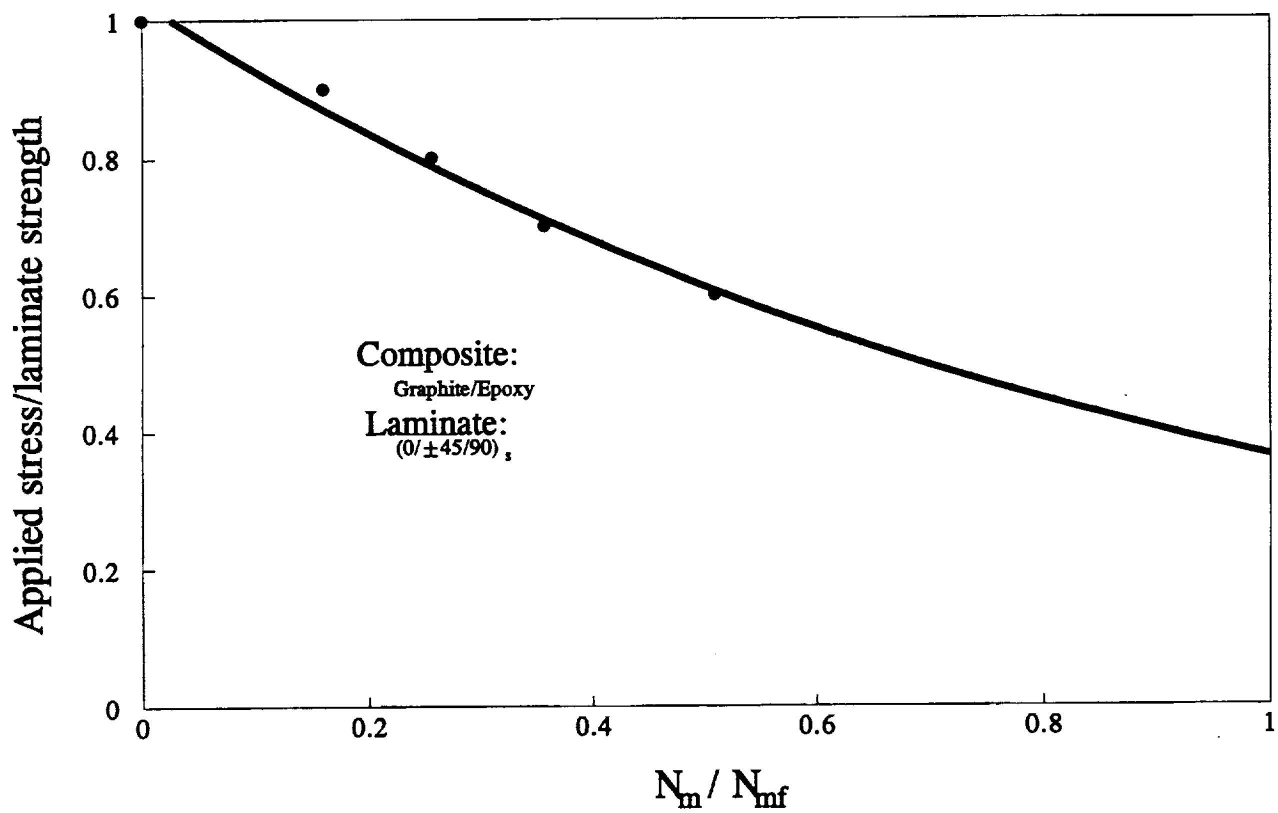

Figure 9 Life cycle curve for Graphite/Epoxy $(0/\pm45/90)_s$ composite for a reliability of 0.999

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13. ABSTRACT ***(Maximum 200 words)***

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