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# EFFECT OF HOOP STRESS ON BALL BEARING LIFE PREDICTION

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ABSTRACT

A finite-element analysis (FEA) of a generic, dimensionally normalized inner race of an angular-contact ball bearing was performed under varying conditions of speed and the press (or interference) fit of the inner-race bore on a journal. The FEA results at the ball-race contact were used to derive an equation from which was obtained the radius of an equivalent cylindrical bearing race with the same or similar hoop stress. The radius of the equivalent cylinder was used to obtain a generalized closed-form approximation of the hoop stresses at the ball–inner-race contact in an angular-contact ball bearing. A life analysis was performed on both a 45- and a 120-mm-bore, angular-contact ball bearing. The predicted lives with and without hoop stress were compared with experimental endurance results obtained at 12 000 and 25 000 rpm with the 120-mm-bore ball bearing. A life factor equation based on hoop stress is presented.

SYMBOLS

| | |
|---|---|
| $a$ | semimajor radius of contact ellipse, m (in.) |
| $a_2$ | life factor, materials and processing |
| $a_3$ | life factor, operating conditions |
| $b$ | semiminor radius of contact ellipse, m (in.) |
| $D_b$ | ball diameter, m (in.) |
| $D_i$ | inner-race bore diameter, m (in.) |
| $D_o$ | outer-race outside diameter, m (in.) |
| $D_p$ | pitch diameter, m (in.) |
| $e$ | Weibull slope |
| $g$ | gravitational constant, m/s$^2$ (in./s$^2$) |
| $k$ | function (see Eq. (20)) |
| $L_B$ | bearing life, hr or number of race revolutions |
| $L_b$ | ball life, hr or number of race revolutions |
| $L_i$ | inner-race life, hr or number of race revolutions |

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| | |
|---|---|
| $L_o$ | outer-race life, hr or number of race revolutions |
| $L_{10}$ | 10-Percent life, or life at which 90 percent of bearings or components will survive, hr or number of race revolutions |
| LF | life factor |
| $P$ | bearing thrust load, N (lbf) |
| $p$ | pressure due to press (or interference) fit, Pa (psi) |
| RF | radius factor, $r_i/W$ |
| $r$ | radius or distance from center of race, m (in.) |
| $r_{eff}$ | effective outer radius, $\eta r_i$, m (in.) |
| $r_i$ | inner-race bore radius, m (in.) |
| $r_o$ | outer-race radius of thick-wall cylinder, m (in.) |
| $S_H$ | threshold hoop stress, Pa (psi) |
| $S_{max}$ | maximum Hertz stress, Pa (psi) |
| $S_T$ | principal stress in rolling direction, Pa (psi) |
| $S_{TH}$ | principal stress in rolling direction due to hoop stress, $S_T + \sigma_\theta$, Pa (psi) |
| $W$ | bearing width, m (in.) |
| $z$ | depth to maximum shear stress, m (in.) |
| $\beta$ | contact angle, deg |
| $\delta$ | density, $N/m^3$ ($lbf/in.^3$) |
| $\eta$ | geometry factor |
| $\eta_p$ | geometry factor due to press (or interference) fit |
| $\eta_\omega$ | geometry factor due to speed |
| $\nu$ | Poisson's ratio |
| $\sigma_\theta$ | resultant hoop stress, $\sigma_{\theta,p} + \sigma_{\theta,\omega}$, Pa (psi) |
| $\sigma_{\theta,p}$ | hoop stress due to press (or interference) fit, Pa (psi) |
| $\sigma_{\theta,\omega}$ | hoop stress due to speed, Pa (psi) |
| $\tau_{max}$ | maximum shear stress, Pa (psi) |

$\tau_{max,R}$ resultant maximum shear stress, $\tau_{max} - ½\sigma_\theta$, Pa (psi)

$\omega_i$ inner-race speed, rad/s

## INTRODUCTION

To prevent motion of the inner race around a shaft, designers have been specifying extremely tight interference fits between the inner race and the shaft when it is not practical to provide a keyway or locknut arrangement. The interference fit is usually based on the anticipated growth of the shaft and the bearing under the most severe operating conditions. These conditions sometimes exist only for short periods in the machine's operating cycle. Nevertheless, it is an extremely important design consideration for both safety of operation and maintainability. In recent years some engineers have noticed that bearings with tighter than usual press (or interference) fits may have shorter field lives than anticipated or calculated. The failure mechanism is usually classical rolling-element (subsurface) fatigue. There has been no public documentation of the phenomenon (Ref. 1).

Czyzewski (Ref. 2) first postulated that tensile stresses in a cylindrical race imposed on a lubricated Hertzian contact would affect shear stresses and hence rolling-element fatigue life. He performed an analysis and rolling-element fatigue tests of 45-mm-bore roller bearing inner races subjected to mechanically induced tensile stress. The maximum Hertz stress was approximately 700 MPa (102 ksi). There was a suggestion of an inverse ninth power stress-life relation. Czyzewski found that at a hoop tensile stress of 80 MPa (12 ksi) the failure mode appeared to be a surface fatigue spall accompanied by ring fracture. At lower hoop stresses the failure mode was classical rolling-element fatigue.

Coe and Zaretsky performed an analysis to determine the effects of inner-ring speed and press fit on the life of first the inner ring (Ref. 3) and then an entire roller bearing assembly (Ref. 1). They considered the effects of the resultant hoop and radial stresses on the principal stresses. They determined the maximum shear stresses below the Hertzian contact of a cylindrical roller for various conditions of inner-race speed and load. The results of their analysis indicated that hoop stresses caused by press fit and centrifugal force can reduce bearing life by as much as 90 percent (Ref. 1).

Clark (Ref. 4) suggested the same approach for angular-contact ball bearings. The relations developed by Coe and Zaretsky (Refs. 1 and 3) cannot be used for angular-contact ball bearings in their present form. The primary reason for this is that the inner race has a nonuniform cross section for which a closed-form solution does not exist. Therefore, Zaretsky and August (Ref. 5) used a finite-element analysis (FEA) to determine the resultant hoop stresses under each condition of inner-race speed and press fit. From the FEA they developed a closed-form approximation of the hoop stresses in the ball–inner-race contact of an angular-contact ball bearing.

It was the objective of the analysis reported herein to expand and combine the work of Coe, Zaretsky, and August (Refs. 1, 3, and 5) in order (1) to predict the lives of angular-contact ball bearings under conditions of hoop stress at the ball–inner-race contact caused by press fit and speed and (2) to compare the analysis with existing rolling-element bearing fatigue data.

## BEARING INNER-RACE GEOMETRY AND STRESSES

A schematic of an angular-contact ball bearing drawn without a cage or retainer and having a generic geometry is shown in Fig. 1. The geometry of the inner and outer races is referred to as being "relieved." That is, the bearing can only be thrust loaded in a single direction.

The nominal contact angle $\beta$ changes with thrust load $P$ and inner-race speed $\omega_i$, as illustrated in Fig. 2. As speed increases, the contact angle at the outer race decreases while the contact angle at the inner race increases. The locations at which the ball contacts the inner and outer races therefore change. The contact (Hertz) stress at the inner race decreases and the stress at the outer race increases.

The resultant Hertz stress distributions and principal stresses in the normal and rolling directions are shown in Fig. 3. Both the maximum shear stress and orthogonal shear stress are generally accepted for rolling-element fatigue analysis. The maximum shear stress is affected by both residual and hoop stresses. The orthogonal shear stress is unaffected by these stresses. The maximum shear stress occurs at a depth $z$ below the race surface. The maximum shear stress is

$$\tau_{\max} = \frac{S_N - S_T}{2} \tag{1}$$

Inner-race life is

$$\text{Life} \sim \left(\frac{1}{\tau_{\max}}\right)^{9} \tag{2}$$

If hoop stresses are superimposed on the tangential principal stress $S_T$, as shown in Fig. 4, $S_T$ becomes $S_{TH}$. As a result the maximum shear stress increases and the inner-race life decreases.

For most angular-contact ball bearings all bearing dimensions can be related to the bearing inner-race bore diameter $D_i$ and the bearing width $W$ (Fig. 1). Hence, for the inner race all dimensions can be expressed and normalized as a function of $D_i$ or $r_i$ and $W$.

## BASIC METHODOLOGY

Coe and Zaretsky (Ref. 3) presented a closed-form solution for the hoop stresses and the life of a cylindrical roller bearing race. The method or approach presented herein was to find an effective outer radius $r_{eff}$ (Fig. 5) of an equivalent thick-wall cylinder geometry such that the hoop stress at $r_{eff}$ is approximately equal to the hoop stress of an angular-contact inner race at the ball-race contact (critical location). This was accomplished for both rotation and press fits independently.

The effective outer radius $r_{eff}$ can be expressed in terms of the inner-race bore radius $r_i$ by means of a geometry factor $\eta$. A finite-element stress analysis of the generic bearing for various conditions of speed and press fit was used to determine a geometry factor for each condition whereby

$$r_{eff} = \eta r_i \tag{3}$$

Two geometry factors were determined: $\eta_p$ for the press-fit effect, and $\eta_\omega$ for the speed effect. To further reduce the number of variables, the inner-race bore radius $r_i$ was normalized by dividing by the bearing width $W$ to give the radius factor:

$$\text{RF} = \frac{r_i}{W} \tag{4}$$

## FINITE-ELEMENT MODEL

A finite-element model of a relieved inner bearing race was based on the generic bearing geometry shown in Fig. 1. The dimensions of the generic bearing are also shown in Fig. 1.

The MSC/NASTRAN finite-element code was used for the analysis. Isoparametric, triangular-cross-section ring elements were used to model the bearing race so as to take advantage of the bearing's axisymmetric structure. The global coordinate system was based on cylindrical coordinates. The bearing's plane cross section had been discretized to produce an element mesh similar to that shown in Fig. 6. The model comprised 168 grid points and 204 three-node MSC/NASTRAN CTRIARG elements. Changes to the inner-race bore radius of the bearing model were easily effected by using a geometric offset from the bearing axis defined in the NASTRAN CORD2R card.

Centrifugal loads were applied by using a NASTRAN RFORCE card, which defines a rotation vector acting along the bearing axis. Internal pressure loads were modeled as concentrated line loads applied at the bearing's inner-race, bore-radius grid circles. The boundary conditions imposed on the model allowed only axial and radial translations for all the grid points. Nodal rotations were fixed, as was translation in the circumferential direction.

## ANALYSIS

For purposes of analysis it was assumed that the bearing inner race was equivalent to a thick-wall cylinder with fixed ends. The rationale for this assumption was that shrink fitting the bearing race on a journal results in the bearing race ends being fixed at the bore-journal interface. From Saada (Ref. 6) the hoop stress for a rotating, thick-wall cylinder with fixed ends is

$$\sigma_{\theta,\omega} = \frac{3 - 2\nu}{8(1 - \nu)} \frac{\delta}{g} \omega^2 \left[ r_i^2 + r_o^2 + \left( \frac{r_i r_o}{r} \right)^2 - r^2 \left( \frac{1 + 2\nu}{3 - 2\nu} \right) \right] \tag{5}$$

The outer radius of the cylinder that will effect the same hoop stress as that in the bearing race can be expressed in terms of the inner-race bore radius:

$$r_o = r_{\text{eff}} = \eta_\omega r_i \tag{6}$$

Substituting into Eq. (5) and evaluating at $r_{\text{eff}}$ give

$$\begin{aligned} \sigma_{\theta,\omega}\Big|_{r=r_{\text{eff}}} &= \frac{3 - 2\nu}{8(1 - \nu)} \frac{\delta}{g} \omega^2 \left[ 2r_i^2 + (\eta_\omega r_i)^2 - (\eta_\omega r_i)^2 \left( \frac{1 + 2\nu}{3 - 2\nu} \right) \right] \\ &= \frac{3 - 2\nu}{4(1 - \nu)} r_i^2 \frac{\delta}{g} \omega^2 \left[ 1 + \frac{\eta_\omega^2 (1 - 2\nu)}{3 - 2\nu} \right] \end{aligned} \tag{7}$$

Similarly, from Saada (Ref. 6) hoop stress at radius $r$ in a thick-wall cylinder with fixed ends due to an internal pressure is given as

$$\sigma_{\theta,p} = \frac{p \left[ 1 + \left( \frac{r_{\text{eff}}}{r} \right)^2 \right]}{\left( \frac{r_{\text{eff}}}{r_i} \right)^2 - 1} \tag{8}$$

Defining an effective outer radius of the cylinder, as before, as

$$r_{\text{eff}} = \eta_p r_i \tag{9}$$

and substituting into Eq. (8) give

$$\sigma_{\theta,p} = \frac{2p}{\eta_p^2 - 1} \tag{10}$$

Finite-element analysis was used to solve for $\sigma_{\theta,\omega}$ and $\sigma_{\theta,p}$ for the inner race of an angular-contact ball bearing having both a relieved inner race (Fig. 1) and a split inner race. Differences in values of stress between the relieved inner race and the split inner race were found to be insignificant. Values of $\eta_\omega$ and $\eta_p$ for the relieved inner race were determined from Eqs. (7) and (10), respectively. The values of the dimensionless effective radius $r_{eff}/r_i$ were plotted on the log-log plot of Fig. 7 as a function of the radius factor RF. A best fit of these calculated values resulted in the following relationships for the geometry factors:

$$\eta_\omega = 1.4625(\mathrm{RF})^{-0.1796} \tag{11}$$

$$\eta_p = 1.2638(\mathrm{RF})^{-0.1188} \tag{12}$$

Equations (11) and (12) should be used only where RF is less than 7.5. At higher values of RF, $r_{eff}$ can be assumed to be equivalent to $r_i$. The value of $r_{eff}$ related to press fit will vary depending on the bearing bore size and be less than the radius of the inner-race shoulder. However, for values of $r_{eff}$ related to speed, because of growth and distortion of the inner race, $r_{eff}$ will be greater than the radius of the inner-race shoulder and vary as a function of bore diameter (see Appendix for discussion).

Using Eqs. (11) and (12) in combination with Eqs. (7) and (10), respectively, provides values of $\sigma_{\theta,\omega}$ and $\sigma_{\theta,p}$. The total hoop stress is

$$\sigma_\theta = \sigma_{\theta,\omega} + \sigma_{\theta,p} \tag{13}$$

From Fig. 3 the resultant principal stress in the rolling direction is

$$S_{TH} = S_T + \sigma_\theta \tag{14}$$

The resultant maximum shear stress is

$$\tau_{\max,R} = \frac{S_N - S_{TH}}{2} \tag{15}$$

The life of the inner race is

$$\mathrm{Life} \sim \left(\frac{1}{\tau_{\max,R}}\right)^9 \tag{16}$$

Combining Eqs. (14) to (16) gives

$$\mathrm{Life} \sim \left(\frac{1}{\tau_{\max} - \dfrac{\sigma_\theta}{2}}\right)^9 \tag{17}$$

Because the values of $\tau_{max}$ (from Eq. (1)) will calculate to be negative and those of $\sigma_\theta$ to be positive, the resultant shear stress $\tau_{max,R}$ will always be greater than $\tau_{max}$, decreasing the inner-race life. As an example, a 10-percent increase in $\tau_{max,R}$ from $\tau_{max}$ will result in a 50-percent decrease in inner-race life.

## RESULTS AND DISCUSSION

Zaretsky and August (Ref. 5) showed that for a 45-mm-bore, angular-contact ball bearing inner race the difference between the hoop stress calculated using FEA and the closed-form solution approximation was approximately 4 percent. For their calculations, at inner-race speeds to 25 000 rpm, press-fit pressures to 20.7 MPa (3 ksi), and maximum Hertz stresses to 2.76 GPa (400 ksi), the hoop stress could theoretically reduce inner-race life by as much as 85 percent.

The method of Coe and Zaretsky (Refs. 1 and 3) uses classical Lundberg-Palmgren theory (Refs. 7 and 8) to calculate life. This method does not consider the life of the rolling elements (balls or rollers) separate from the life of the races. Zaretsky (Ref. 9) considered this restriction and was able to separate the respective lives of the rolling elements and races. As a result, changes made to any component that affects the life of the bearing could be evaluated separately from the other components, where

$$\frac{1}{L_B^e} = \frac{1}{L_i^e} + \frac{1}{L_b^e} + \frac{1}{L_o^e} \tag{18}$$

Zaretsky's rules for separating rolling-element life from race life without the presence of hoop stress are as follows:

1. For thrust-loaded bearings, such as angular-contact ball bearings, where the Hertz stress is greater on the inner race than on the outer race, the lives of the rolling elements as a group will be equal to or greater than the inner-race life and less than the outer-race life. Conservatively, let rolling-element life equal inner-race life.
2. For thrust-loaded bearings where the Hertz stress is greater on the outer race than on the inner race, rolling-element life will be equal to or less than outer-race life. Let rolling-element life equal outer-race life.
3. For radially loaded rolling-element bearings, rolling-element life will be equal to or greater than outer-race life. Let rolling-element life equal outer-race life.

By using the simplified equations of Hamrock and Anderson (Ref. 10) to determine the ellipticity ratio of the contact, the maximum shear stress can be calculated. Use of Eq. (13) then permits the addition of hoop stress from race speed and press fit. Once the values of shear stress with and without hoop stress are determined, the life of the inner race with the hoop stress can be calculated by using Zaretsky's rule, and the life of the bearing modified by the presence of hoop stress can be determined.

Bearing life was calculated, with and without the effect of hoop stress, for a generic angular-contact ball bearing (Fig. 1) having a 45-mm bore, a free contact angle of 30°, speeds of 15 000 and 30 000 rpm, and five thrust loads of 1334, 2224, 3114, 4003, and 4893 N (300, 500, 700, 900 and 1100 lbf). The results are shown in Fig. 8 and summarized in Table I.

For the conditions selected, at 15 000 rpm the inner-race life is reduced by approximately 21 to 31 percent. However, when the reduced life of the inner race due to hoop stress was factored into Eq. (18), the overall bearing life was reduced by approximately 11 to 17 percent. In normal bearing operation these differences would not be noticeable. At an operating speed of 30 000 rpm the inner-race life was reduced approximately 37 to 52 percent, but the overall bearing life was reduced by approximately 21 to 22 percent.

Bearing $L_{10}$ life as measured in inner-race revolutions (Fig. 8) increased with speed for this bearing because the elastohydrodynamic film thickness increases with increasing speed. Hence, contrary to popular belief, in this instance, bearing life increased with speed instead of decreasing. The effect of hoop stress would be expected to be negligible in this instance.

Lundberg and Palmgren (Ref. 7) normalized their life prediction equations by using bearing fatigue data obtained with 45-mm-bore ball bearings. It may be reasonably concluded, although not intended by Lundberg and Palmgren (Ref. 7), that the hoop stress effect was incorporated in their material constants to predict bearing life. The conclusion was indirectly addressed in Ref. 9 wherein it was stated that, "... where normally recommended press fits are used under normal machine operating speeds, the effect of hoop stress can be ignored in the life calculations. However, where the bearings are operated at high speeds or at speeds higher than normally recommended by the bearing manufacturers and at higher than recommended press fits, hoop stress effects on bearing life must be considered."

To test this premise, it is necessary to compare predicted results of larger bore ball bearings with actual life results. Although most researchers presenting life data in the open literature have not reported nor probably considered the effect of bore-shaft interference fits and ring growth due to centrifugal effects, the endurance data necessary to conduct an analysis on the effect of hoop stress on large-bore, high-speed bearings were obtained by NASA in the 1970's (Ref. 11).

The Ref. 11 data were obtained with 120-mm-bore, angular-contact ball bearings manufactured from a single heat of vacuum-induction-melted, vacuum-arc-remelted (VIM–VAR) AISI M–50 steel. This was the first reported use of double-vacuum melting process for aircraft-quality bearings. The bearing verification and life factors are given in Table II. Two groups of 30 bearings each were endurance tested at speeds of 12 000 and 15 000 rpm ($1.44\times10^6$ and $3.0\times10^6$ $DN$), respectively, at thrust loads of 22 241 N (5000 lbf) and a temperature of 218 °C (425 °F). Calculated hoop stresses for these bearings were 37.0 and 70.6 MPa (5.4 and 10.2 ksi, respectively). The theoretical bearing lives were reanalyzed by using STLE life factors and the method outlined in Ref. 9 and Eq. (18) that considers the effect of hoop stress. These results are summarized in Table III. The theoretical bearing life results with and without hoop stress for 12 000 and 25 000 rpm are shown in Fig. 9, together with the experimental $L_{10}$ lives at these two respective speeds.

Theoretical life predictions, for the 120-mm-bore, angular-contact ball bearings showed life decreasing with speed. Hence, the centrifugal effects were greater than the effects of EHD film thickness for these bearings. At 12 000 rpm the hoop stress effect on the inner race would theoretically reduce its life approximately 60 to 73 percent. Using Eq. (18), the bearing life is reduced approximately 38 to 40 percent. The results are plotted in Fig. 9. At 25 000 rpm the hoop stress effect on the inner race would theoretically reduce its life approximately 78 to 89 percent. The bearing life would be accordingly reduced by approximately 18 to 41 percent. These results are also plotted in Fig. 9 and compared with the experimental data from Ref. 11.

The experimental life $L_{10}$ obtained at 12 000 rpm exceeded the predicted lives with and without hoop stress by approximately 30 and 80 percent, respectively. The experimental $L_{10}$ life obtained at 25 000 rpm exceeded the predicted lives with and without hoop stress by approximately 41 and 140 percent, respectively.

The shaft on which the 120-mm-bore bearings were fitted was originally designed with conventional press fits normally designated by engineering design practice. At 25 000 rpm the bearing inner ring grew more than the hollow shaft, allowing the bearing ring to spin on the shaft at operating speed. To counter this effect, the interference between the shaft and the bore was increased so that the bearing ring would not rotate at the increased speed. Even so, the major effect of the hoop stress reported in Table III was from centrifugal effects. Quantitatively, using accepted life factors, the hoop stress effect would tend to underpredict the bearing life. Qualitatively, the results are somewhat better. The life of the bearing at 12 000 rpm without hoop stress was 41 percent higher than at 25 000 rpm without hoop stress. Considering the effect of hoop stress, at 12 000 rpm the bearing would be expected to have a 50 percent longer life that at 25 000 rpm. The experimental results show that the 12 000 rpm $L_{10}$ life was 30 percent longer than the 25 000-rpm $L_{10}$ life.

The reason that the quantitative life predictions are conservative may be because hoop stress was factored unintentionally into the original Lundberg-Palmgren material constant. For the 120-mm-bore bearings this would only account for part of the difference. However, experience has shown that most bearing fatigue data are generally repeatable within a range of ±50 percent of a mean value. Additionally, the life factors of Ref. 9 were designed to conservatively predict bearing life. Hence, the predicted values presented herein are reasonably within an acceptable range discussed in Ref. 9. As stated by Zaretsky (Ref. 9), "both the use of life factors and the results obtained therefrom must be subject to engineering judgment and experience."

Clark (Ref. 4) suggested a "threshold tensile stress before life reduction is noted." It can be reasonably assumed that there exists a threshold hoop stress $S_H$ below which hoop stress will not significantly affect bearing life when Lundberg-Palmgren analysis and STLE life factors are used in combination. Using this assumption, a life factor (LF) for hoop stress may be written that is based on Eq. (17) as follows:

$$LF = \left( \frac{\tau_{max} + \frac{S_H}{2}}{\tau_{max} + \frac{\sigma_\theta}{2}} \right)^9 \tag{19}$$

and should only be used where $\sigma_\theta > S_H$.

The threshold hoop stress $S_H$ is the value below which, for a given material, hoop stress would be ineffective in decreasing life. The probable reason for this occurrence would be either compressive residual stress being present in the unrun bearing inner race and/or the inducing of such stress during bearing operation. Unlike Eq. (17), Eq. (19) is written so that the user can ignore the negative sign for $\tau_{max}$ and the positive sign for $\sigma_\theta$ discussed previously for Eq. (17).

To normalize Eq. (19) to the STLE life factors (Ref. 9), values of $S_H$ can be assumed from the current analysis. For air-melted AISI 52100, from Table I, $S_H$ = 37 MPa (5.4 ksi). For VIM–VAR AISI M–50, from Table III, $S_H$ = 242 MPa (35.1 ksi). These values should be subject to further experimental verification or change on the basis of field experience.

## SUMMARY

A finite-element analysis (FEA) of a generic, dimensionally normalized inner race of an angular-contact ball bearing was performed under varying conditions of speed and press (or interference) fit of the inner-race bore on a journal. The FEA results at the ball-race contact were used to derive an equation from which was obtained the radius of an equivalent cylindrical bearing race with the same or similar hoop stress. The radius of the equivalent cylinder was used to obtain a generalized closed-form approximation of the hoop stresses at the ball–inner-race contact in an angular-contact ball bearing. A life analysis was performed on both 45- and 120-mm-bore, angular-contact ball bearings. The predicted lives with and without hoop stress were compared with experimental endurance results obtained at 12 000 and 25 000 rpm with the 120-mm-bore ball bearing. A life factor equation based on hoop stress was presented. The following results were obtained:

1. The experimental 10-percent life ($L_{10}$ life) obtained at 12 000 rpm for the 120-mm-bore, angular-contact ball bearings exceeded the predicted lives with and without hoop stress by approximately 30 and 80 percent, respectively. The experimental $L_{10}$ at 25 000 rpm, exceeded the predicted lives with and without hoop stress by approximately 41 and 140 percent, respectively. These results are not unreasonable considering the conservative nature of the Lundberg-Palmgren analysis and the STLE life factors.

2. For the 45-mm-bore, angular-contact ball bearing the theoretical life was reduced by approximately 11 to 17 percent at 15 000 rpm and by 21 to 22 percent at 30 000 rpm with hoop stress. For this size bearing hoop stress effects would not be expected to be apparent in operation for normal press (or interference) fits.

3. For large-bore ball bearings operating at high speeds qualitative effects of hoop stress would be expected. To account for the differences between the measured and predicted effects of hoop stress, the concept of a threshold hoop stress $S_H$ was introduced. The initial data available suggest that hoop stresses below $S_H$ would not cause a life reduction when the prediction is based on classical Lundberg-Palmgren analysis and STLE life factors.

## APPENDIX—DISCUSSION OF EFFECTIVE OUTER RADIUS

Jones (Ref. 12) suggested that for (ball bearing) calculations involving the force required to accomplish press fitting a ball bearing inner race on a shaft, the equivalent (cylindrical race outer) diameter is chosen at a point 1/3 of the race (groove) depth from the shoulder for full raceways, and 1/2 of the race (groove) depth for partial (relieved) raceways. This diameter has been commonly used as the effective outer diameter of an equivalent cylindrical inner race to determine equivalent hoop stress due to press fit of a ball bearing on a shaft. It is also probable, although unreported, that the same effective diameter has been used in some manner to determine speed effects on hoop stress.

Another method for determining an effective diameter is based on a cylindrical race cross-sectional area equal to that of the ball bearing inner race. That is, when combined with the bore and race width, the effective diameter would yield the same cross-sectional area as the actual race. The authors are unaware of any discussion of this method in the open literature. However, it would most probably be used to determine the effect of speed on hoop stress.

The issue is whether there are differences between these two methods and that proposed in the current paper. Because hoop stress would be a function of the effective outer radius $r_{eff}$, the differences in $r_{eff}$ values may indicate the relative differences of each method in determining hoop stress.

Table IV compares the normalized effective outer radius $r_{eff}$ as a function of the normalized radius of the bore $r_i$. The method of Jones (Ref. 12) and the uniform-cross-section method gave nearly identical results. Further, for the current method, which is based on a finite-element analysis, $r_{eff}$ is different for press fit and speed effects. However, the difference between the $r_{eff}$ values for press fit with the current method and with the Jones and uniform-cross-section methods appears at first to be minimal.

From Eq. (10) the normalized values of the hoop stress due to press fit were calculated for each of the effective radii of Table IV. These values are summarized in Table V. What is immediately apparent from this table is that as the value of $r_i$ increases, there is a significant deviation in calculated values between the current method and the Jones and uniform-cross-section methods. The latter methods underpredicted hoop stress by as much as 61 percent when compared with the finite-element analysis.

For values of hoop stress due to speed effects, Eq. (7) can be written as follows:

$$\sigma_{\theta,\omega} = k\left[1 + \frac{\eta_\omega^2 (1 - 2\nu)}{3 - 2\nu}\right] \tag{20}$$

where

$$k = \frac{3 - 2\nu}{4(1 - \nu)} r_i^2 \frac{\delta}{g} \omega^2$$

Using Eq. (20), the normalized hoop stress was calculated for $r_{eff}$ values from Table IV. In this instance (Table VI) as $r_i$ values increased, the results of the current method and the Jones and uniform-cross-section methods, which had lower values, converged. The maximum difference between these methods from finite-element analysis was approximately 6 percent.

It becomes intuitively obvious that the current method provides both an ease in calculations to determine an appropriate value of $r_{eff}$ and a more accurate answer than state-of-the-art practice.

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TABLE I.—SUMMARY OF OPERATING CONDITIONS AND EFFECT OF HOOP STRESS ON THEORETICAL LIFE OF GENERIC 45-mm-BORE, ANGULAR-CONTACT BALL BEARING (see Fig. 1)

[Ball diameter, 9.5 mm (0.374 in.); number of balls, 12; conformity, 52 percent; contact angle, 30°; race temperature, 121 °C (250 °F); bulk oil temperature, 82 °C (180 °F); lubricant, MIL–L–23699; life factors, $a_2$ (materials and processing) = 1, $a_3$ (operating conditions) as indicated.]

| Thrust load | | Maximum Hertz stress | | | | Life factor $a_3$ (EHD film thickness) | | Theoretical $L_{10}$ life, millions of inner-race revolutions | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| N | lbf | Inner race | | Outer race | | Inner race | Outer race | Inner race | | Outer race | Balls | Bearing | |
| | | GPa | ksi | GPa | ksi | | | Without hoop stress | With hoop stress | | | Without hoop stress | With hoop stress |
| Speed, 15 000 rpm; hoop stress, 37.0 MPa (5.4 ksi) | | | | | | | | | | | | | |
| 1334 | 300 | 1.37 | 199 | 1.20 | 174 | 0.49 | 0.64 | 4475 | 3074 | 19 467 | 4 475 | 2189 | 1807 |
| 2224 | 500 | 1.63 | 237 | 1.39 | 201 | .43 | .58 | 844 | 614 | 4 773 | 874 | 421 | 358 |
| 3114 | 700 | 1.83 | 265 | 1.53 | 222 | .40 | .54 | 290 | 218 | 1 872 | 290 | 146 | 126 |
| 4003 | 900 | 1.98 | 287 | 1.66 | 240 | .38 | .51 | 131 | 101 | 870 | 131 | 66 | 58 |
| 4893 | 1100 | 2.12 | 307 | 1.76 | 255 | .37 | .49 | 71 | 56 | 484 | 71 | 36 | 32 |
| Speed, 30 000 rpm; hoop stress, 70.6 MPa (10.2 ksi) | | | | | | | | | | | | | |
| 1334 | 300 | 1.32 | 192 | 1.40 | 203 | 1.50 | 1.73 | 17 137 | 8260 | 11 567 | 11 567 | 4775 | 3751 |
| 2224 | 500 | 1.58 | 229 | 1.52 | 221 | 1.39 | 1.68 | 5 311 | 2873 | 3 234 | 3 234 | 1368 | 1142 |
| 3114 | 700 | 1.78 | 258 | 1.63 | 237 | 1.32 | 1.61 | 1 129 | 682 | 2 803 | 1 129 | 517 | 402 |
| 4003 | 900 | 1.94 | 281 | 1.74 | 252 | 1.25 | 1.57 | 506 | 305 | 1 628 | 506 | 239 | 185 |
| 4893 | 1100 | 2.08 | 301 | 1.83 | 265 | 1.20 | 1.54 | 267 | 167 | 1 023 | 267 | 129 | 101 |

TABLE II.—BEARING SPECIFICATIONS AND LIFE FACTORS FOR SPLIT-INNER-RACE, 120-mm-BORE, ANGULAR-CONTACT BALL BEARINGS

(a) Bearing geometry

| | |
|---|---|
| Outer diameter, mm | 190.0 |
| Inner diameter (bore), mm | 120.0 |
| Bearing semiwidth, mm | 17.5 |
| Pitch diameter, mm | 155.0 |
| Ball diameter, mm | 20.6375 |
| Number of balls | 15 |
| Contact angle, deg | 24 |
| Outer-race conformity | 0.52 |
| Inner-race conformity | 0.54 |

(b) Life factors

| | |
|---|---|
| $a_2$ (materials and processing): | |
| Bearing steel, AISI M-50 | 2.0 |
| Melting process, VIM–VAR | 6.0 |
| Metal working, forged inner race | 1.2 |
| $a_3$ (operating conditions): | |
| Hoop stresses | (See Table III) |
| Speed | (See Table III) |
| Temperature 218 °C (425 °F) and hardness of Rockwell C61 | 1.13 |
| EHD film thickness | (See Table III) |
| Oil filtration (3 μm) | 1.4 |

TABLE III.—SUMMARY OF THEORETICAL LIFE WITH AND WITHOUT HOOP STRESS AND EXPERIMENTAL LIFE OF SPLIT-INNER-RACE, 120-mm-BORE, ANGULAR-CONTACT BALL BEARING

[Contact angle, 24°; race temperature, 218 °C (425 °F); bulk oil temperature, 191 °C (375 °F); material VIM–VAR AISI M–50 steel; inner race, forged; material hardness, 63; difference between hardness of rolling elements and races, 0; lubricant, MIL–L–23699; life factors, from Table II.]

| Thrust load | | Maximum Hertz stress | | | | Life factor $a_3$ (EHD film thickness) | | Theoretical $L_{10}$ life with life factors $a_2$ and $a_3$, millions of inner-race revolutions | | | | | | Experimental $L_{10}$ life (Ref. 11), billions of inner-race revolutions |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| N | lbf | Inner race | | Outer race | | Inner race | Outer race | Inner race | | Outer race | Balls | Bearing | | |
| | | GPa | ksi | GPa | ksi | | | Without hoop stress | With hoop stress | | | Without hoop stress | With hoop stress | |
| Speed, 12 000 rpm; hoop stress, 144 MPa (20.9 ksi) | | | | | | | | | | | | | | |
| 6 667 | 1500 | 1.42 | 206 | 1.38 | 200 | 2.82 | 2.92 | 204.0 | 55.0 | 178.6 | 170.1 | 67.4 | 36.6 | --- |
| 13 333 | 3000 | 1.79 | 260 | 1.58 | 229 | 2.88 | 2.92 | 27.9 | 9.8 | 72.4 | 23.2 | 11.8 | 6.8 | --- |
| 22 241 | 5000 | 2.13 | 309 | 1.79 | 260 | 2.95 | 2.77 | 5.5 | 2.2 | 19.2 | 4.6 | 2.4 | 1.5 | 2.7 |
| Speed, 16 000 rpm; hoop stress, 167 MPa (24.2 ksi) | | | | | | | | | | | | | | |
| 6 667 | 1500 | 1.40 | 203 | 1.54 | 223 | 2.97 | 2.82 | 252.5 | 55.5 | 77.9 | 77.9 | 36.9 | 25.2 | --- |
| 13 333 | 3000 | 1.77 | 257 | 1.69 | 245 | 2.82 | 2.93 | 28.6 | 8.3 | 28.6 | 23.8 | 9.9 | 5.5 | --- |
| 22 241 | 5000 | 2.10 | 305 | 1.87 | 271 | 2.90 | 2.88 | 6.6 | 2.3 | 13.3 | 5.5 | 3.2 | 1.7 | --- |
| Speed, 20 000 rpm; hoop stress, 196 MPa (28.4 ksi) | | | | | | | | | | | | | | |
| 6 667 | 1500 | 1.39 | 202 | 1.72 | 249 | 3.00 | 3.00 | 1761.0 | 53.4 | 25.1 | 25.1 | 13.3 | 11.2 | --- |
| 13 333 | 3000 | 1.75 | 254 | 1.83 | 265 | 2.83 | 2.85 | 33.0 | 7.9 | 14.9 | 14.9 | 6.7 | 4.2 | --- |
| 22 241 | 5000 | 2.08 | 302 | 1.97 | 286 | 2.87 | 2.94 | 6.7 | 2.0 | 7.3 | 6.7 | 2.4 | 1.3 | --- |
| Speed, 25 000 rpm; hoop stress, 242 MPa (351 ksi) | | | | | | | | | | | | | | |
| 6 667 | 1500 | 1.38 | 200 | 1.96 | 284 | 2.87 | 3.00 | 349.0 | 38.3 | 9.7 | 9.7 | 5.1 | 4.7 | --- |
| 13 333 | 3000 | 1.73 | 251 | 2.03 | 294 | 2.83 | 2.86 | 40.0 | 6.8 | 6.4 | 6.4 | 3.2 | 2.4 | --- |
| 22 241 | 5000 | 2.05 | 297 | 2.13 | 309 | 2.90 | 2.93 | 8.1 | 1.8 | 3.8 | 3.8 | 1.7 | 1.0 | 2.4 |

TABLE IV.—COMPARISON OF VALUES OF EFFECTIVE OUTER RADIUS $r_{eff}$

| $r_i/W$ | Current method | | Jones (Ref. 12) | Uniform cross section |
|---|---|---|---|---|
| | Press-fit effects | Speed effects | | |
| | $r_{eff}/r_i$ | | | |
| 1 | 1.26 | 1.46 | 1.30 | 1.28 |
| 2 | 1.16 | 1.29 | 1.15 | 1.14 |
| 3 | 1.11 | 1.20 | 1.10 | 1.13 |
| 4 | 1.10 | 1.14 | 1.08 | 1.07 |
| 5 | 1.04 | 1.10 | 1.06 | 1.06 |
| 6 | 1.02 | 1.06 | 1.05 | 1.05 |

TABLE V.—COMPARISON OF NORMALIZED HOOP STRESS AT BALL-RACE CONTACT FOR VALUES OF EFFECTIVE OUTER RADIUS $r_{eff}$ DUE TO PRESS FIT (TABLE IV)

| $r_i/W$ | Current method | Jones (Ref. 12) | Uniform cross section |
|---|---|---|---|
| | $\sigma_{\theta,p}/p$ (percent change from current method) | | |
| 1 | 3.40 (—) | 2.90 (−14.7) | 3.13 (−7.9) |
| 2 | 5.79 (—) | 6.20 (−7.1) | 6.68 (−15.4) |
| 3 | 8.62 (—) | 9.52 (+10.5) | 7.22 (−16.2) |
| 4 | 9.52 (—) | 12.02 (+26.3) | 13.80 (+45.0) |
| 5 | 24.51 (—) | 16.18 (−34.0) | 16.18 (−34.0) |
| 6 | 49.5 (—) | 19.51 (−60.6) | 19.51 (−60.6) |

TABLE VI.—COMPARISON OF NORMALIZED HOOP STRESS AT BALL-RACE CONTACT FOR VALUES OF EFFECTIVE OUTER RADIUS $r_{eff}$ DUE TO SPEED EFFECTS (TABLE IV)

| $r_i/W$ | Current method | Jones (Ref. 12) | Uniform cross section |
|---|---|---|---|
| | $\sigma_{\theta,\omega}/k$ (percent change from current method) | | |
| 1 | 1.35 (—) | 1.28 (−5.2) | 1.27 (−5.9) |
| 2 | 1.28 (—) | 1.22 (−4.7) | 1.22 (−4.7) |
| 3 | 1.24 (—) | 1.20 (−3.2) | 1.21 (−2.4) |
| 4 | 1.22 (—) | 1.19 (−2.4) | 1.19 (−2.4) |
| 5 | 1.20 (—) | 1.19 (−0.8) | 1.19 (−0.8) |
| 6 | 1.19 (—) | 1.18 (−0.8) | 1.18 (−0.8) |

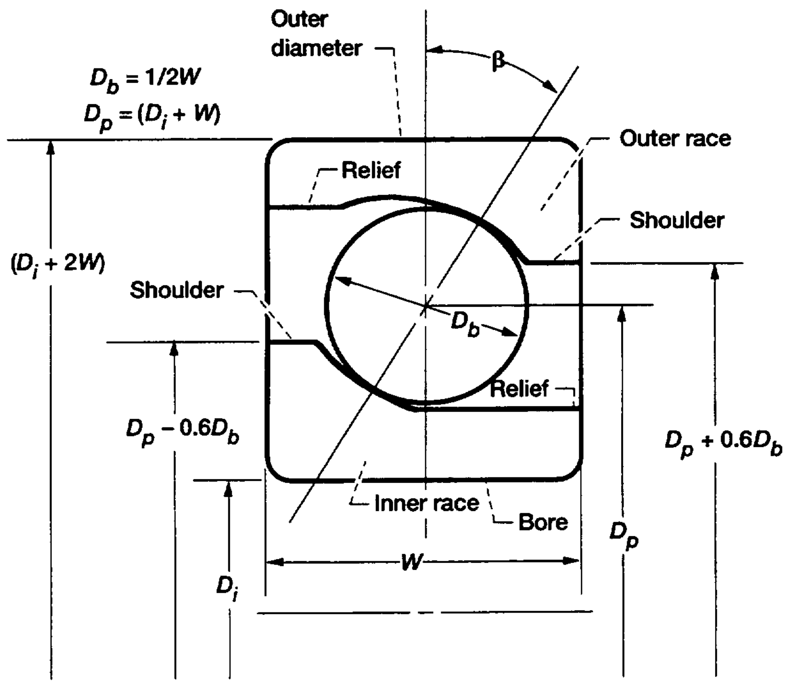


Figure 1.—Generic geometry for angular-contact ball bearing.

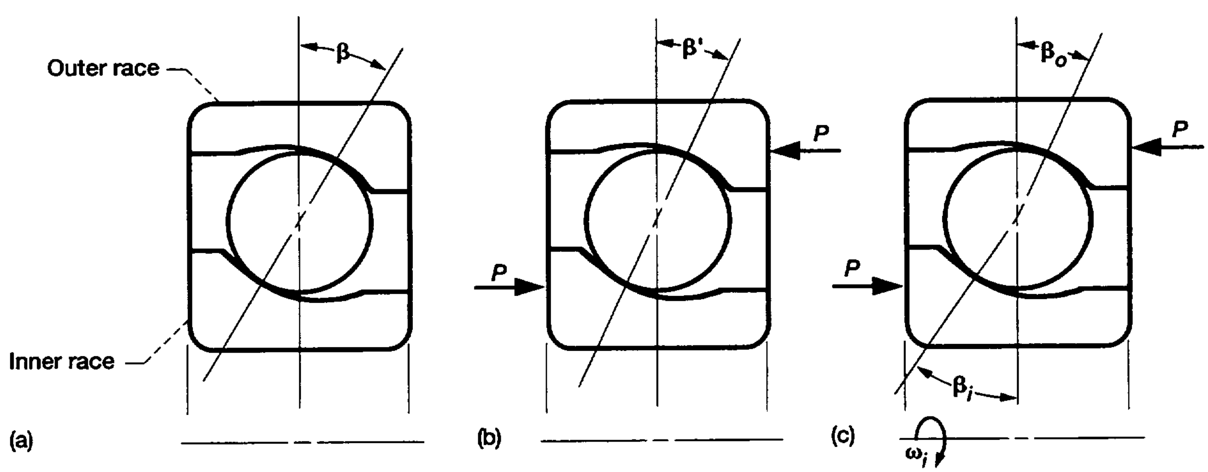


Figure 2.—Changes in contact angle with load and speed. (a) Free-contact angle (no load). (b) Contact angle when under load. (c) Contact angles when under load and at high speed.

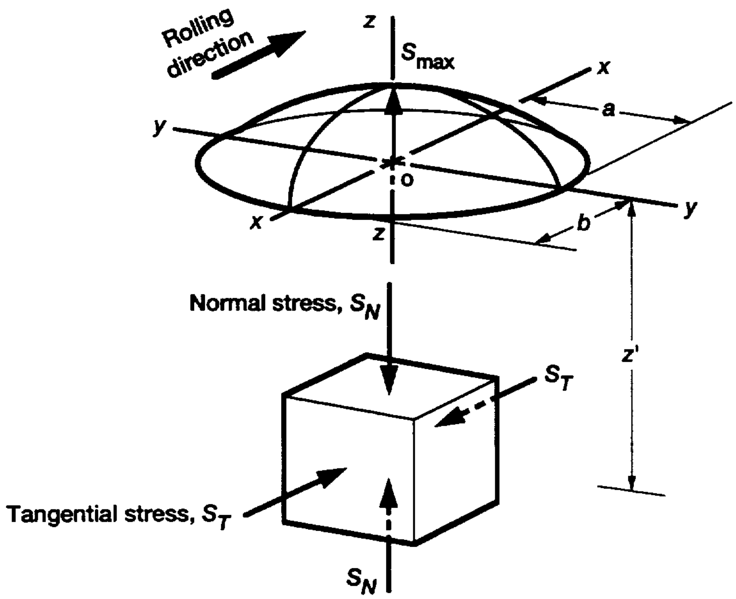


Figure 3.—Surface Hertz (contact) stress distribution and principal stresses below surface in normal and rolling directions.

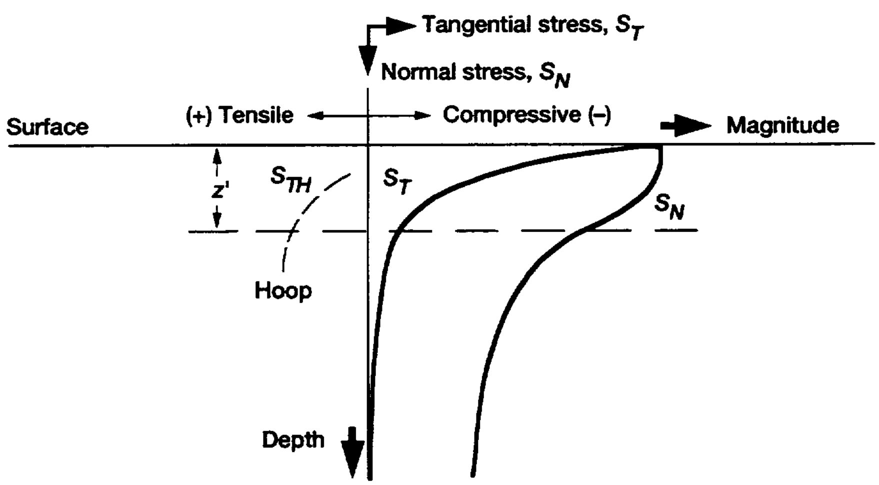


Figure 4.—Resultant effect of hoop stresses on principal stresses in rolling direction.

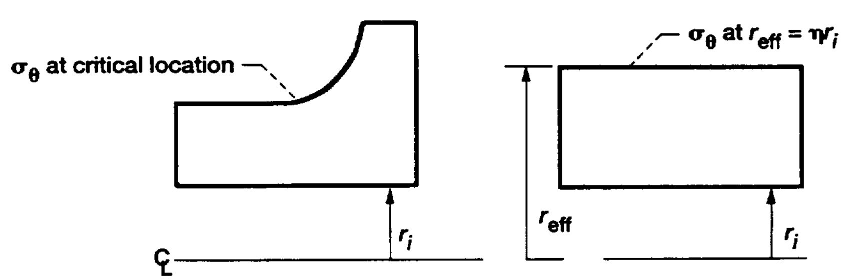


Figure 5.—Equivalent geometries for hoop stresses for angular-contact inner race and thick-wall cylinder.

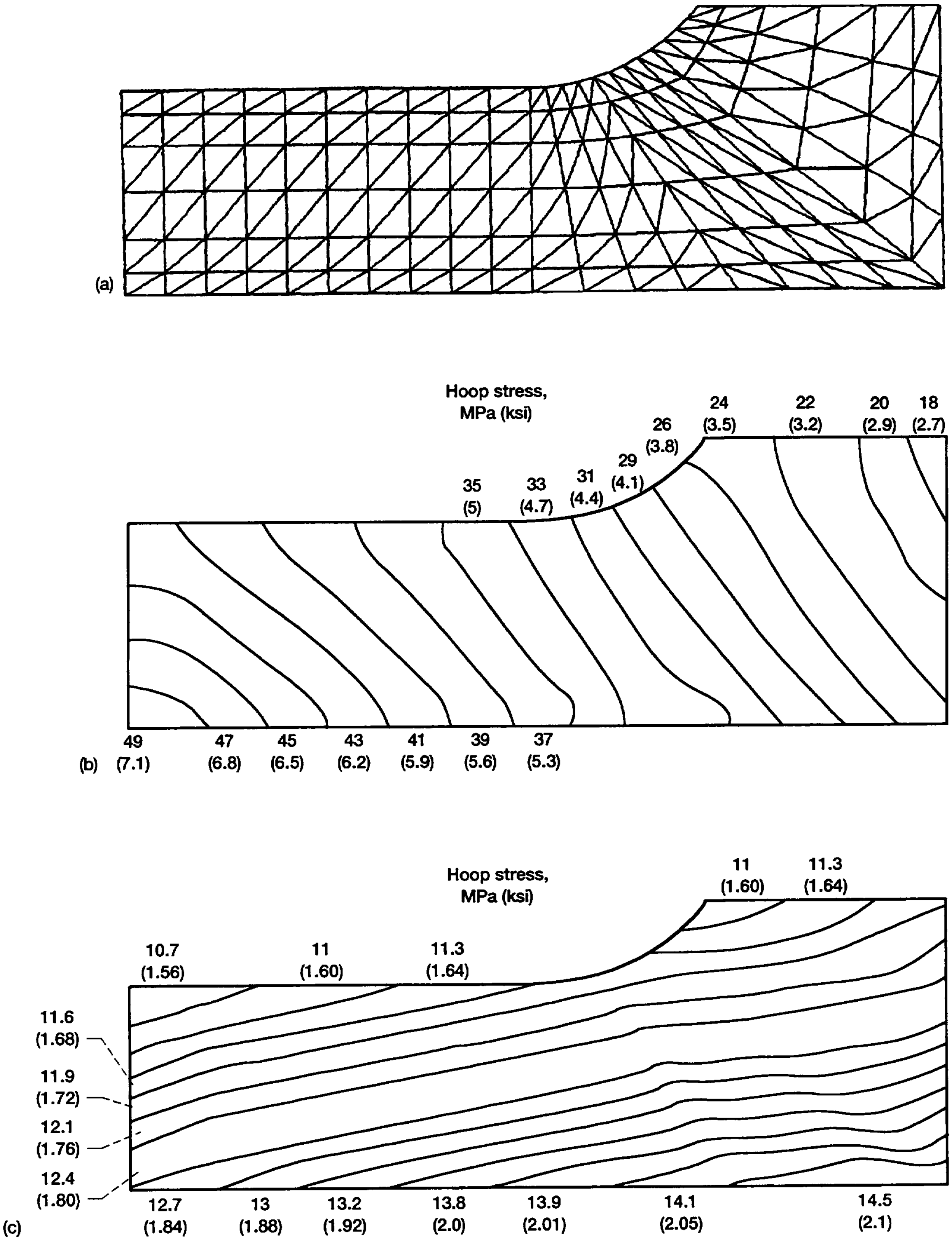


Figure 6.—Finite-element analysis for 45-mm-bore, angular-contact ball bearing inner race. (a) FEA grid. (b) Press fit for 6.9-kPa (1-ksi) bore pressure. (c) Effect of speed (15 000 rpm) without press fit.

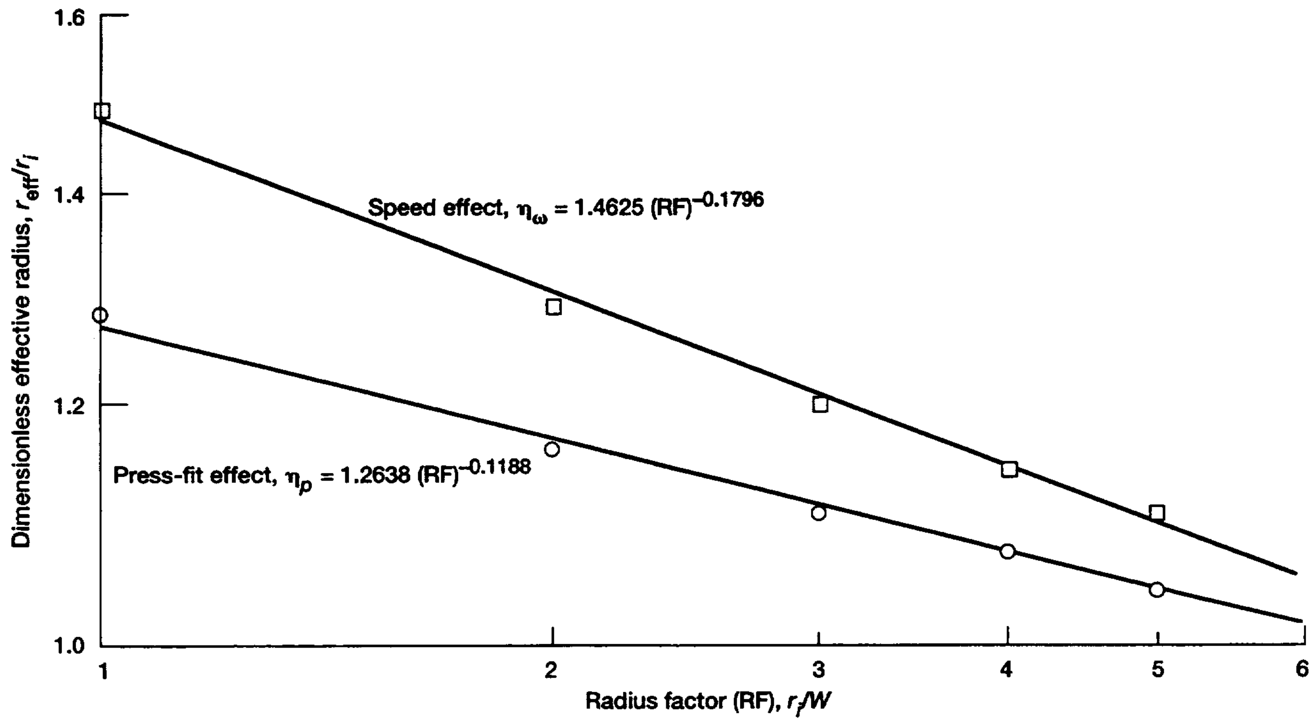


Figure 7.—Effective radius as function of rotational speed and press fit.

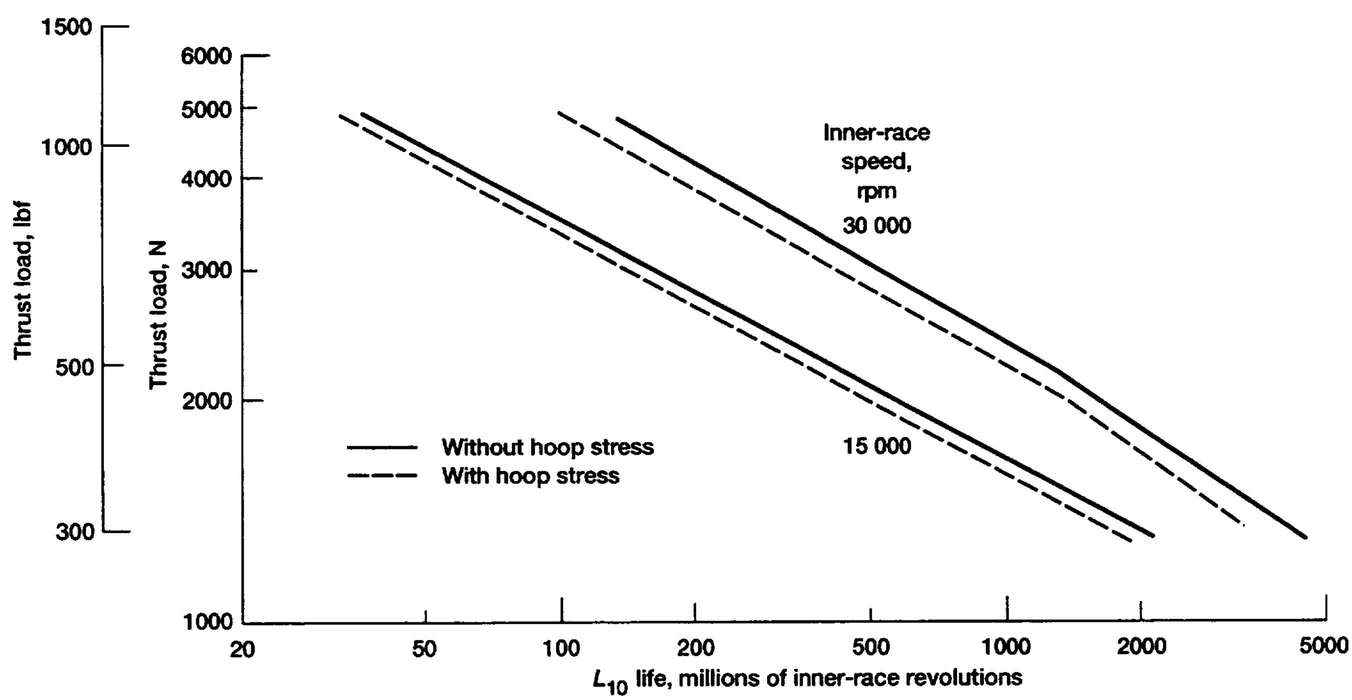


Figure 8.—Theoretical effect of hoop stress on life of 45-mm-bore, angular-contact ball bearing at two speeds. Contact angle, 30°; race temperature, 121 °C (250 °F); bulk oil temperature, 82 °C (180 °F); material, air melt AISI 52100 steel; material hardness, Rc 60; difference between hardness of rolling elements and inner race, 0; lubricant, MIL–L–23699; life factor, $a_3$ (EHD film thickness).

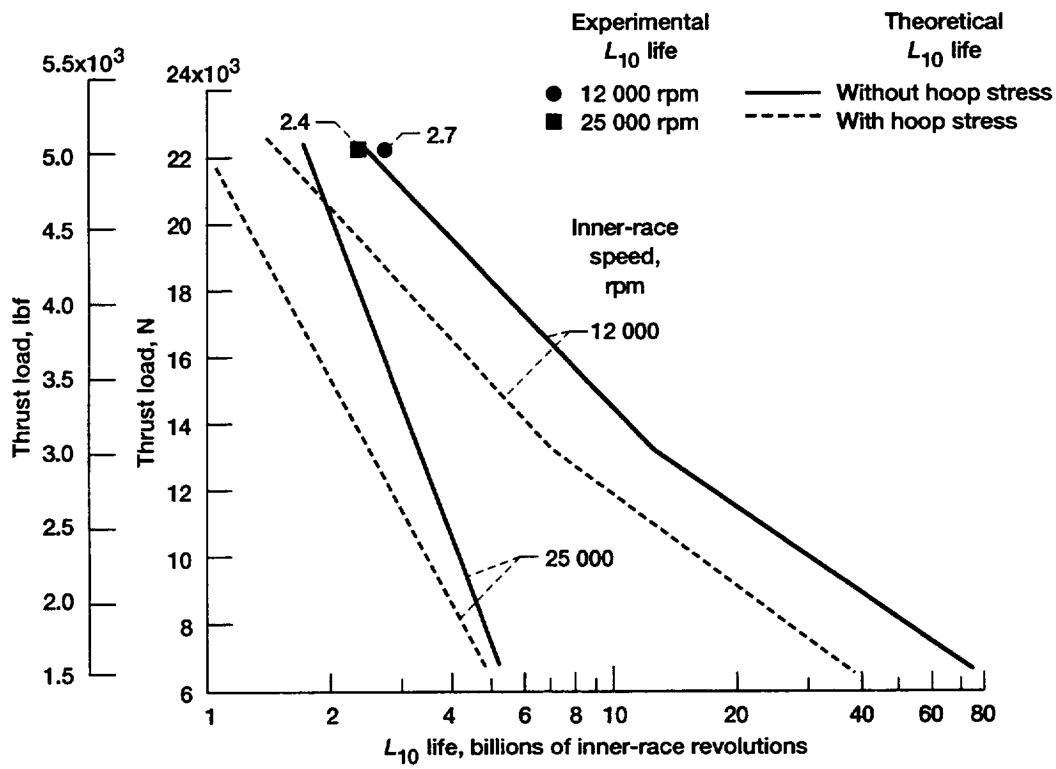


Figure 9.—Comparison of theoretical $L_{10}$ life with and without hoop stress and experimental $L_{10}$ life at a thrust load of 22 241 N (5000 lbf) for 120-mm-bore, angular-contact ball bearings. Contact angle, 24°; race temperature, 218 °C (425 °F); bulk oil temperature, 191 °C (375 °F); material, VIM–VAR AISI M–50 steel; inner race, forged; material hardness, Rc 63; difference between hardness of rolling elements and inner race, 0; lubricant, tetraester MIL–L–23699; life factors, $a_2$ (material and processing), $a_3$ (operating conditions). (Experimental results from Ref. 11.)

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**13. ABSTRACT** ***(Maximum 200 words)***

A finite-element analysis (FEA) of a generic, dimensionally normalized inner race of an angular-contact ball bearing was performed under varying conditions of speed and the press (or interference) fit of the inner-race bore on a journal. The FEA results at the ball-race contact were used to derive an equation from which was obtained the radius of an equivalent cylindrical bearing race with the same or similar hoop stress. The radius of the equivalent cylinder was used to obtain a generalized closed-form approximation of the hoop stresses at the ball–inner-race contact in an angular-contact ball bearing. A life analysis was performed on both a 45- and a 120-mm-bore, angular-contact ball bearing. The predicted lives with and without hoop stress were compared with experimental endurance results obtained at 12 000 and 25 000 rpm with the 120-mm-bore ball bearing. A life factor equation based on hoop stress is presented.

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