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A CASE STUDY OF VIEW-FACTOR RECTIFICATION PROCEDURES FOR DIFFUSE-GRAY RADIATION ENCLOSURE COMPUTATIONS

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ABSTRACT

The view factors which are used in diffuse-gray radiation enclosure calculations are often computed by approximate numerical integrations. These approximately calculated view factors will usually not satisfy the important physical constraints of reciprocity and closure. In this paper several view-factor rectification algorithms are reviewed and a rectification algorithm based on a least-squares numerical filtering scheme is proposed with both weighted and unweighted classes. A Monte-Carlo investigation is undertaken to study the propagation of view-factor and surface-area uncertainties into the heat transfer results of the diffuse-gray enclosure calculations. It is found that the weighted least-squares algorithm is vastly superior to the other rectification schemes for the reduction of the heat-flux sensitivities to view-factor uncertainties. In a sample problem, which has proven to be very sensitive to uncertainties in view factor, the heat transfer calculations with weighted least-squares rectified view factors are very good with an original view-factor matrix computed to only one-digit accuracy. All of the algorithms had roughly equivalent effects on the reduction in sensitivity to area uncertainty in this case study.

INTRODUCTION

It is general knowledge in the radiation heat transfer literature that the view factors in diffuse-gray radiation enclosure calculations should be computed in such a way that they satisfy the physical constraints of reciprocity and closure. For systems with a large number of surfaces, the only practical way to compute the view factors is by approximate numerical integrations. Monte-Carlo integration is a popular technique which is robust and has the added advantage of providing an estimate of the uncertainty in each calculation. These approximately computed view factors will only in the rarest of coincidences satisfy the reciprocity and closure constraints, and artificial means of enforcement must be adopted.

Most heat transfer textbooks adopt a naive enforcement. Only the view factors above the diagonal in the view-factor matrix are computed. The view factors below the diagonal are computed using reciprocity relationships, and the view factors along the diagonal are computed using closure. This technique is naive because it allows the view factors along the diagonal to be negative. Negative view factors are of course blatant physical impossibilities. Tsuyuki [1] presents a refined form of the naive enforcement which avoids negative view factors. van Leersum [2] presents an iterative approach which enforces closure and reciprocity on an approximate set of view factors and avoids negative instances.

It is often stated in the radiation heat transfer literature (Brewster [3] for example) that reciprocity and closure are required to avoid ill-conditioned matrixes in the linear equation set that results from the diffuse-gray enclosure analysis. Taylor et al. [4, 5] have demonstrated that diffuse-gray radiation enclosure problems can be very sensitive to errors in the view factors even when the coefficient matrixes are very well-conditioned with condition numbers of order 2 and 3. In their work, they found that the simultaneous enforcement of reciprocity and closure using the naive algorithm

described above will greatly reduce this sensitivity. Also, Taylor et al. demonstrated that enforcement of closure and reciprocity reduced the sensitivity of the heat-flux results to uncertainties in the surface areas.

This paper extends the previous work of Taylor et al. by considering more advanced reciprocity and closure enforcement algorithms and comparing the propagation of the view-factor errors and surface-area errors into computed heat-flux results of the diffuse-gray enclosure analysis for the different methods.

Four view-factor enclosure algorithms are discussed and compared

    1)   No enforcement--all view factors independently computed.
    2)   Naive enforcement.
    3)   van Leersum's enforcement.
    4)   Optimal enforcement.

The optimal enforcement algorithm uses a least-squares optimization which finds the minimum root-sum-square charge in the view factors which will simultaneously enforce reciprocity and closure. Nonnegativity conditions can also be included in the optimization algorithms.

The technique used for the comparison is a Monte-Carlo uncertainty analysis of a sample problem which has proven to be hypersensitive to errors in the view factors when reciprocity and closure are not enforced. The results are the distributions in computed surface heat fluxes for assumed uncertainty distributions of the original unrectified view factors and for assumed uncertainty distributions in surface areas.

DIFFUSE-GRAY ENCLOSURE FORMULATION

Radiation exchange between finite diffuse-gray areas which form an enclosure is discussed in almost all general heat transfer textbooks. Excellent detailed discussions can be found in any thermal radiation heat transfer textbook (Brewster [3] and Siegel and Howell [6], for example). The basic restrictions are that each surface have uniform temperature, uniform radiative properties which are diffuse and gray, and uniform radiosity. Boundary conditions for the k-th surface are expressed by specifying either the surface heat flux, q_k, or the surface temperature, t_k. Mixed boundary conditions cause no problem. If all of the surfaces with specified heat flux are considered first as surfaces 1 through M and the surfaces with specified temperatures numbered M + 1 through N, the following set of linear equations can be obtained for the radiosity values [4,5]

$$\left[I - \left(I - D_\epsilon^M\right) D_a^{-1} F^T D_a\right] q_o = b \qquad (1)$$

where D_a is a diagonal matrix with areas as elements, F is the view factor matrix, D_ϵ^M is a diagonal matrix with zeros for elements in rows 1 through M and ϵ_k in rows k = M + 1 to N, b is a vector whose first M elements are q_k (k = 1,2,···,M) and whose last N-M elements are $\epsilon_k \sigma t_k^4$ (k = M+1,···,N), and q_o is the vector of radiosities.

Equation (1) is solved for the radiosities. If the result r is taken to be the vector whose first M elements are $\epsilon_k \sigma t_k^4$ (k = 1,2,···,M) and whose last N-M elements are q_k (k = M+1,···,N), the final equation is

$$r = \left[I - \left(I - D_\epsilon^{*M}\right) D_a^{-1} F^T D_a\right] q_o \qquad (2)$$

where D_ϵ^{*M} is the complement of D_ϵ^M and has ϵ_k for the first M elements and zeros for the last N-M elements.

Usually at this stage of the development, the view-factor reciprocity relationship

$$F^T D_a = D_a F \tag{3}$$

is substituted into equations (1) and (2) to simplify the formulas. However, in this investigation, we are interested in cases where reciprocity is not strictly enforced. In that case, it is more appropriate to work with equations (1) and (2).

VIEW-FACTOR RECTIFICATION

Three view-factor rectification schemes are considered: 1) Naive, 2) Leersum's, and 3) least-squares optimum. For the least-squares optimum three subsets are considered: 1) unweighted without nonnegativity, 2) unweighted with nonnegativity, and 3) weighed with nonnegativity. Each of these procedures is discussed below.

Naive Rectification

For the naive rectification, the view factors above the main diagonal in the view-factor matrix, **F**, are retained and all others are discarded. The upper-triangular matrix containing these remaining view factors is designated as **U** and its transpose as U^T. Equation (3) can then be used to compute the missing view factors below the diagonal. If the lower-triangular matrix containing the view factors below the main diagonal calculated by reciprocity is designated L_N, equation (3) can be written as

$$L_N = D_a^{-1} U^T D_a \tag{4}$$

The rectified view-factor matrix excluding the diagonal is obtained by combining the lower- and upper-triangular matrixes

$$F_N = L_N + U \tag{5}$$

Next the diagonal elements are computed using the closure relation

$$f_{Nii} = 1 - \sum_{\substack{j=1 \\ j \neq i}}^{N} f_{Nij} \tag{6}$$

No attempt is made to ensure nonnegative view factors. The physically impossible negative view factors are naively accepted.

Leersum's Rectification

van Leersum (1989) has published an iterative scheme which can be considered a refinement of

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the naive rectification. His method spreads the closure adjustments over all of the view factors and assures nonnegative view factors. His algorithm is given below.

1) For each row in the **F**, compute a correction factor based on closure

$$d_i = \left(1 - \sum_{k=1}^{N} f_{ik}\right)\Big/ m \tag{7}$$

where m is the number of nonzero view factors in row i.

2) For each nonzero view factor in row i, apply the correction

$$f_{Lik} = f_{ik} + d_i, \quad k = 1, ..., N \tag{8}$$

If any $f_{Lij} < 0$, decrease m by the number of negative values and recalculate d_i bypassing the view factors which made the previous f_{Lij} negative. Repeat this procedure until no negative view factors are obtained.

3) Enforce reciprocity by computing values for column i

$$f_{Lki} = \frac{a_i f_{Lik}}{a_k}, \quad k = 1, ..., N \tag{9}$$

4) Repeat this process in turn for each row.
5) Since the enforcement of reciprocity in 3) disturbs the closure forced in 1) and 2), repeat the entire process iteratively until the values of d_i are arbitrarily small.

The step-by-step enforcement of reciprocity in 3) over wrights all of the original view factors below the main diagonal; therefore, Leersum's procedure only considers the diagonal and upper triangular elements in the original view-factor matrix. Also, it is not clear why zero-valued view factors are considered to be exact and are not allowed to be modified.

Least-Squares Optimum

The least-squares optimization problem can be posed as the quadratic minimization of

$$y = \sum_{i=1}^{N} \sum_{j=1}^{N} w_{ij}(f_{oij} - f_{ij})^2 \tag{10}$$

subject to the closure and reciprocity equality constraints where the f_{ij}'s are the original approximately determined view factors, f_{oij}'s are the corrected view factors, and w_{ij}'s are the weights used when the view factors have unequal uncertainty. The closure and reciprocity constraints are

$$\sum_{j=1}^{N} f_{oij} = 1, \quad i = 1, ..., N \tag{11}$$

$$a_j f_{oji} - a_i f_{oij} = 0 \quad i = 1, N - 1, j = i + 1, N \tag{12}$$

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If nonnegativity is desired, the inequality constraints can be applied

$$f_{oij} \geq 0 \quad i = 1, \ldots, N, \, j = 1, \ldots, N \tag{13}$$

This problem can be readily solved using any number of nonlinear-programming techniques.

However, considerable insight can be gained and a computational formula can be derived if the problem is viewed from a geometric standpoint. First, the view factors are grouped into a column vector instead of a matrix. The view-factor matrix is stacked in row-major form; for example, the 2 × 2 view-factor matrix becomes

$$F = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix} \rightarrow f = \begin{bmatrix} f_{11} \\ f_{12} \\ f_{21} \\ f_{22} \end{bmatrix} \tag{14}$$

Closure and reciprocity are enforced by applying the equality constraints (equations 11 and 12) to form a set of linear equations

$$R \cdot f = c \tag{15}$$

The 2 × 2 system would yield, for example

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & a_1 & -a_2 & 0 \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{12} \\ f_{21} \\ f_{22} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \tag{16}$$

Equation (15) has $N(N - 1)/2$ degrees of freedom. The Naive rectification is obtained by specifying the $N(N - 1)/2$ view factors above the main diagonal and computing the remainder from equation (15). However, it is more desirable to use all of the N^2 view factors.

The solutions of equation (15) can be factored into two orthogonal subspaces--the rowspace and the nullspace. The rowspace component of the solution is computed using the expression (Strang [7])

$$f_{row} = R^T (R R^T)^{-1} c \tag{17}$$

This vector is the particular solution of equation (15) which has the least norm. It is a unique and necessary component of all solutions of equation (15). The other component of the solution ($f - f_{row}$) should lie in the nullspace of the reciprocity and closure matrix R and can be expressed as a linear combination of basis vectors for the nullspace

$$(f - f_{row}) = N_b x \tag{18}$$

where N_b is the matrix whose columns form that basis and x contains the weights of the linear combination. However, if there are errors in the computed view factors, f, equations (18) will not be consistent, and we must resort to the least-squares solution [7]

$$x = \left(N_b^T N_b\right)^{-1} N_b^T (f - f_{row})$$ (19)

The projection of (f - f_{row}) onto the nullspace, then, is the desired set of corrected view factors

$$f_{null} = N_b\left(N_b^T N_b\right)^{-1} N_b^T (f - f_{row})$$ (20)

and the least-squares optimum set of view factors is

$$f_{opt} = f_{row} + f_{null}$$ (21)

When the data are not all equally reliable (usually the case for view factors), weighted least squares should be used for the solution of equations (18) [7]

$$f_{null}^w = N_b\left(N_b^T V^{-1} N_b\right)^{-1} N_b^T V^{-1}(f - f_{row})$$ (22)

where V is the covariance matrix, and

$$f_{opt}^w = f_{row} + f_{null}^w$$ (23)

The view-factor rectifications computed using equations (21) and (23) do not enforce nonnegativity.

The least-squares optimum view-factor rectification obtained through equations (21) and (23) are exactly the same as those which would be obtained by solving the quadric minimization problem in equations (10), (11), and (12) without considering the nonnegativity constraints.

As discussed before the view factors must be nonnegative to be physically realistic; a negative view factor is meaningless. It is our opinion and experience that allowing slightly negative values in the rectified view-factor matrix does not seriously impact the fidelity of the heat transfer results. Certainly, the strict enforcement of reciprocity and closure has had a much stronger impact on our results.

A two-step procedure which is easy to implement and closely approximates the results of the nonlinear-programming solution with the nonnegativity constraints is to apply equations (21) or (23) and to assume that the equality in equations (13) would be enforced on all negative values. These view factors are set to zero and removed from consideration obtaining a reduced order problem, and the process is then repeated with the reduced set of data. This procedure has proven to give exactly the same set of rectified view factors as the nonlinear-programming solution in about 90% of the cases and only slightly different ones in the other 10% of the cases.

The rectification algorithm for the least-squares projection is as follows
1) Construct the closure and reciprocity matrix R.
2) Compute the row space component f_{row} using equation (17).
3) Construct the nullspace matrix N. (This can be constructed using standard routines).
4) Compute the nullspace least-squares projection using equation (20) or equation (22) for the

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weighted case.

5) Compute the optimum rectified view factors using equation (21) or (23). View factors f_{opt} or f_{opt}^w now satisfy reciprocity and closure.

6) If nonnegativity is enforced, search f_{opt} or f_{opt}^w for negative entries, and set these to zero.

7) Remove all zero view factors from consideration. Remove the columns of matrix R corresponding to each diagonal zero element. For each off-diagonal zero reciprocal pair, remove the corresponding columns and reciprocity rows from the matrix R. The process is run a second time starting with step 2 and the reduced set of original view factors.

All of the rectification algorithms presented herein apply reciprocity using the best estimates of the areas in equation (15) as if the areas were known exactly. This is usually not a serious deficiency since the areas can usually be determined with low uncertainty. The authors are currently exploring procedures to properly weight the rectification procedure to account for area variance.

NUMERICAL EXAMPLES

The following problem from the heat transfer text by Incropera and Dewitt [8] is used as a basis of comparison of the different techniques in this paper.

13.62   A room (Figure 1) is represented by the following enclosure, where the ceiling (1) has an emissivity of 0.8 and is maintained at 40°C by embedded electrical heating elements. Heaters are also used to maintain the floor (2) of emissivity 0.9 at 50°C. The right wall (3) of emissivity 0.7 reaches a temperature of 15°C on a cold, winter day. The left wall (4) and end walls (5A, 5B) are very well insulated. To simplify the analysis, treat the two end walls as a single surface (5). Assuming the surfaces are diffuse-gray, find the net radiation heat transfer from each surface.
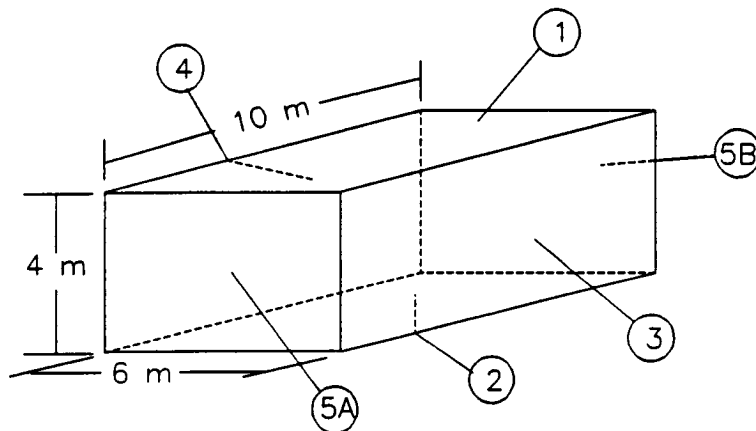

Figure 1. Schematic of a Room for the Example Problem.

This problem was the genesis of our interest in the subject of view-factor sensitivity and rectification. This problem was assigned in the second heat transfer course at Mississippi State University during the Fall 1992 term. Two students, Miguel and Simon, ignored the simplification and worked the problem as a six-sided enclosure. Miguel computed his view factors to four-digit accuracy and Simon to two-digit accuracy; they got radically different answers for the heat fluxes. An analysis of this problem and the cause for this hypersensivity are discussed in a previous publication (Taylor et al. [4]).

The view-factor matrix computed to four-digit accuracy is

$$
F = \begin{bmatrix}
0.0 & 0.394 & 0.1921 & 0.1921 & 0.1109 & 0.1109 \\
0.394 & 0.0 & 0.1921 & 0.1921 & 0.1109 & 0.1109 \\
0.2881 & 0.2881 & 0.0 & 0.196 & 0.1139 & 0.1139 \\
0.2881 & 0.2881 & 0.196 & 0.0 & 0.1139 & 0.1139 \\
0.2774 & 0.2774 & 0.1898 & 0.1898 & 0.0 & 0.066 \\
0.2774 & 0.2774 & 0.1898 & 0.1898 & 0.066 & 0.0
\end{bmatrix}
\tag{24}
$$

Seven different numerical experiments have been performed. In each case, the starting point was the view-factor matrix listed above. Random errors were then introduced by sampling a random-number generator which produced normally distributed values. The difference in each experiment resulted from the way that the variance of these random errors was assigned to the view factors. A thousand view-factor trials were conducted for each case. This was followed by a thousand trials where the areas were varied randomly. In all of the following, covariance terms are assumed to be negligible.

Equal Variance

The first numerical experiment considered the view factors to have equal variance with a view-factor standard deviation of 0.01 and the areas to be fixed. Table 1 gives mean values of the heat flux for several of the rectification schemes. Since the variances were all equal and the covariances were assumed to be zero, the weighted least-squares optimum scheme and the unweighted schemes are identical. The exact solution is computed using the view factors in equation (24) directly. Table 2 shows the standard deviations for the heat flux calculations and the root-mean-square average standard deviation for each treatment.

Table 1. Mean Heat-Flux Values for Equal View-Factor Variance Case.

| Surface | Least-Squares | | | | | | |
	Not Nonneg.	N.L.P. Nonneg.	Nonneg.	Leersum	Naive	No Recti- fication	Exact
1	-3.6273	-3.6030	-3.6036	-3.6231	-3.6570	-3.9179	-3.6891
2	83.9532	83.7224	83.7249	83.7249	83.9429	83.9205	83.8721
3	-120.4888	-120.1792	-120.1820	-120.4798	-120.3951	-120.3969	-120.5353

Table 2. Standard Deviations in Heat Flux for Equal View-Factor Variance Case.

| Surface | Least-Squares | | | | | No Recti- fication |
	Not Nonneg.	N.L.P. Nonneg.	Nonneg.	Leersum	Naive	
1	0.4374	0.3449	0.3447	0.6364	1.0005	10.2714
2	0.5776	0.4394	0.4398	0.8416	1.5774	11.8809
3	0.7272	0.5138	0.5148	0.9571	2.2777	12.3575
rms-avg	0.5927	0.4382	0.4383	0.8225	1.7007	11.5379

From Table 1, all of the rectification schemes seem to have means which are roughly equal to the exact solution. Table 2 shows, however, that there is a large difference in the standard deviations of the calculated heat fluxes. For surface 1, the case of no rectification has a standard deviation which is almost an order of magnitude larger than the rectified values. Figure 2 shows histograms of surface-3 heat-flux distributions for each rectification scheme and for no rectification.

The tables and figure reveal that all of the rectifications are effective for this problem. The nonnegative least-squares procedures are about twice as effective in reducing errors in the heat flux calculations as Leersum's rectification which in turn is about twice as effective as the Naive rectification. Among the least-squares, the nonnegative projection scheme and the nonlinear-programming scheme yield almost identical results as expected, and the least-squares without nonnegativity has very slightly larger errors in heat flux than its nonnegative counter parts.

Next, the view factors were set at the values given in equation (24) and the areas were varied using a standard deviation of 1% for each area. Table 3 gives the standard deviations for the heat-flux calculations and the rms average standard deviation for each treatment.

Table 3.   Standard Deviations in Heat Flux for the Area Variance Case with Equal View-Factor Variance.
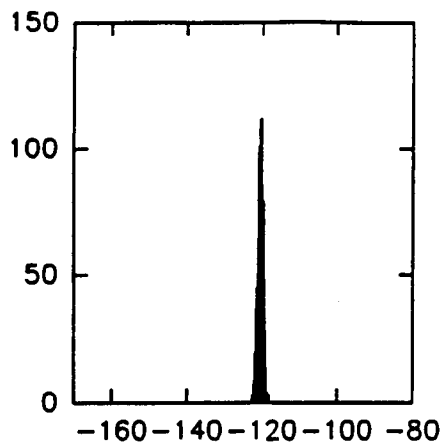
| Surface | Least-Squares | | Leersum | Naive | No Rectification |
	Not Nonneg.	Nonneg.			
1	0.4414	0.5132	0.4601	0.2285	4.641
2	0.5281	0.4678	0.4225	0.4928	4.377
3	0.3211	0.3334	0.4915	0.8307	4.461
rms-avg	0.4385	0.4447	0.4586	0.5730	4.794

The table shows that with no rectification the area uncertainties result in considerable uncertainties in the heat fluxes. However, when the view factors were rectified by enforcing closure and reciprocity these uncertainties in heat flux are reduced by an order of magnitude. All of the algorithms give about the same decrease in the sensitivity to area uncertainty for this case study.
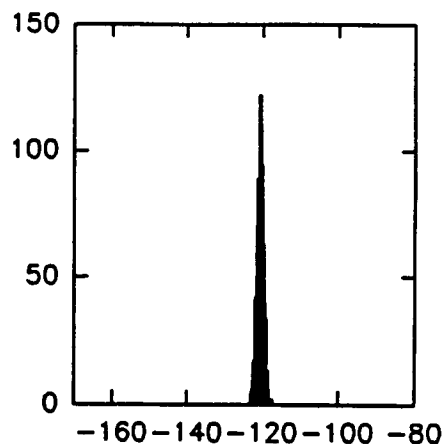
Unequal Variance

Six cases were considered which contained unequal variance: 1) diagonal-dominated, 2) counter-diagonal-dominated, 3) row-dominated, 4) column-dominated, 5) upper-triangle-dominated, and 6) random variances. Depending on the location of the uncertainties in the view-factor matrix, the relative success of the rectification schemes with respect to the sensitivity to view-factor uncertainty is vastly different from that seen for equal variance.
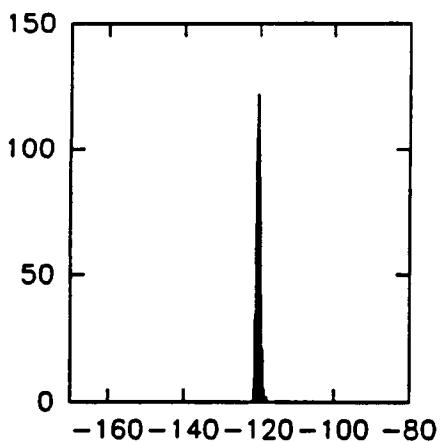
For the diagonal-dominated case-study the view factors along the main diagonal are considered to have standard deviations which are 100 times as large as the off-diagonal view factors. The standard-deviation matrix corresponding to the view-factor matrix is
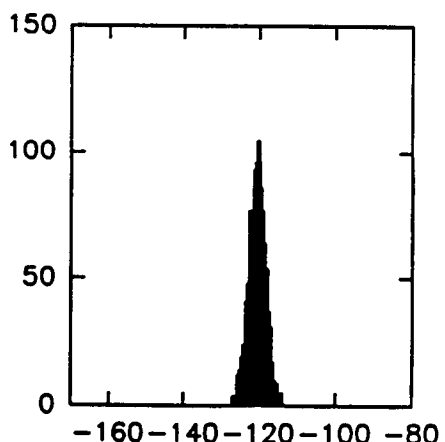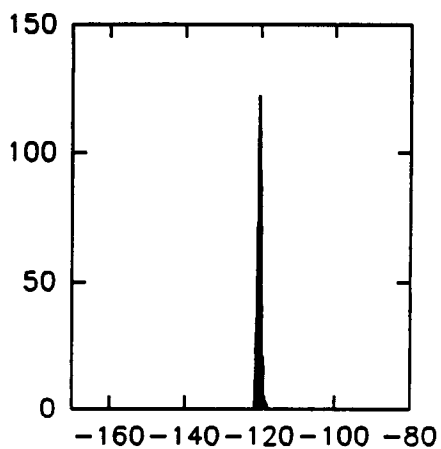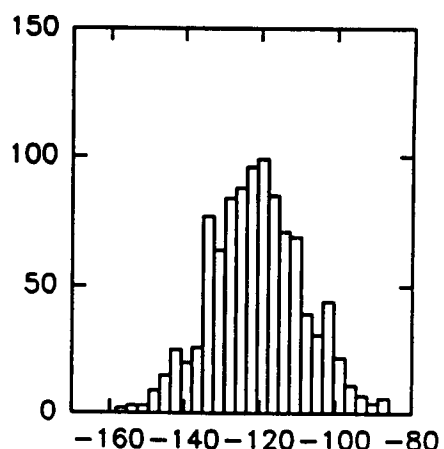
Figure 2. Histogram of Surface-3 Heat Flux [watts/m^2] for Equal Variance Case.

$$S = \begin{bmatrix} \mathbf{0.1000} & 0.0010 & 0.0010 & 0.0010 & 0.0010 & 0.0010 \\ 0.0010 & \mathbf{0.1000} & 0.0010 & 0.0010 & 0.0010 & 0.0010 \\ 0.0010 & 0.0010 & \mathbf{0.1000} & 0.0010 & 0.0010 & 0.0010 \\ 0.0010 & 0.0010 & 0.0010 & \mathbf{0.1000} & 0.0010 & 0.0010 \\ 0.0010 & 0.0010 & 0.0010 & 0.0010 & \mathbf{0.1000} & 0.0010 \\ 0.0010 & 0.0010 & 0.0010 & 0.0010 & 0.0010 & \mathbf{0.1000} \end{bmatrix} \qquad (25)$$

Tables 4 and 5 show the mean and standard deviations of the heat fluxes for 1000 trials where the view factors in equation (24) were perturbed by values from a gaussian random number generator with standard deviations given by equation (25).

Table 4. Mean Heat-Flux Values for the Diagonal-Dominated View-Factor Variance Case.

Surface	Nonnegative Least-Squares		Leersum	Naive	Exact
	Unweighted	Weighted			
1	-3.4722	-3.6263	-3.4719	-3.6264	-3.6891
2	82.1136	83.8987	83.9797	83.9417	83.9417
3	-117.9622	-120.4087	-120.7616	-120.4729	-120.5353

Table 5. Standard Deviations in Heat Flux for the Diagonal-Dominated View-Factor Variance Case.

Surface	Nonnegative Least-Squares		Leersum	Naive
	Unweighted	Weighted		
1	1.7256	0.0468	3.5462	0.1029
2	2.3167	0.0623	5.1518	0.1584
3	3.3123	0.0912	6.4956	0.2284
rms-avg	2.5374	0.0693	5.2061	0.1711

The rectification schemes are the unweighted and weighted nonnegative least-squares projection methods, Leersum's method, and the Naive method. Figure 3 shows histograms for the heat-flux distributions for surface 3.

For this case study, Leersum's rectification is seen to be the least effective at reducing errors in the heat flux. The unweighted nonnegative least-squares projection is about twice as effective as Leersum's scheme, but the weighted nonnegative least-squares projection is an order of magnitude more effective. For this case, the Naive rectification is almost as good as the weighted least-squares projection.

Recall that the Naive rectification scheme lumps all of the corrections into the diagonal elements for closure enforcement while Leersum's scheme evenly distributes the corrections over all of the nonzero values. Therefore, when the view factor variance is mostly along the diagonal, we expect the Naive scheme to perform well and Leersum's to not perform well. When the variances are all equal, Leersum's is expected to perform well, as it did in the previous case study.
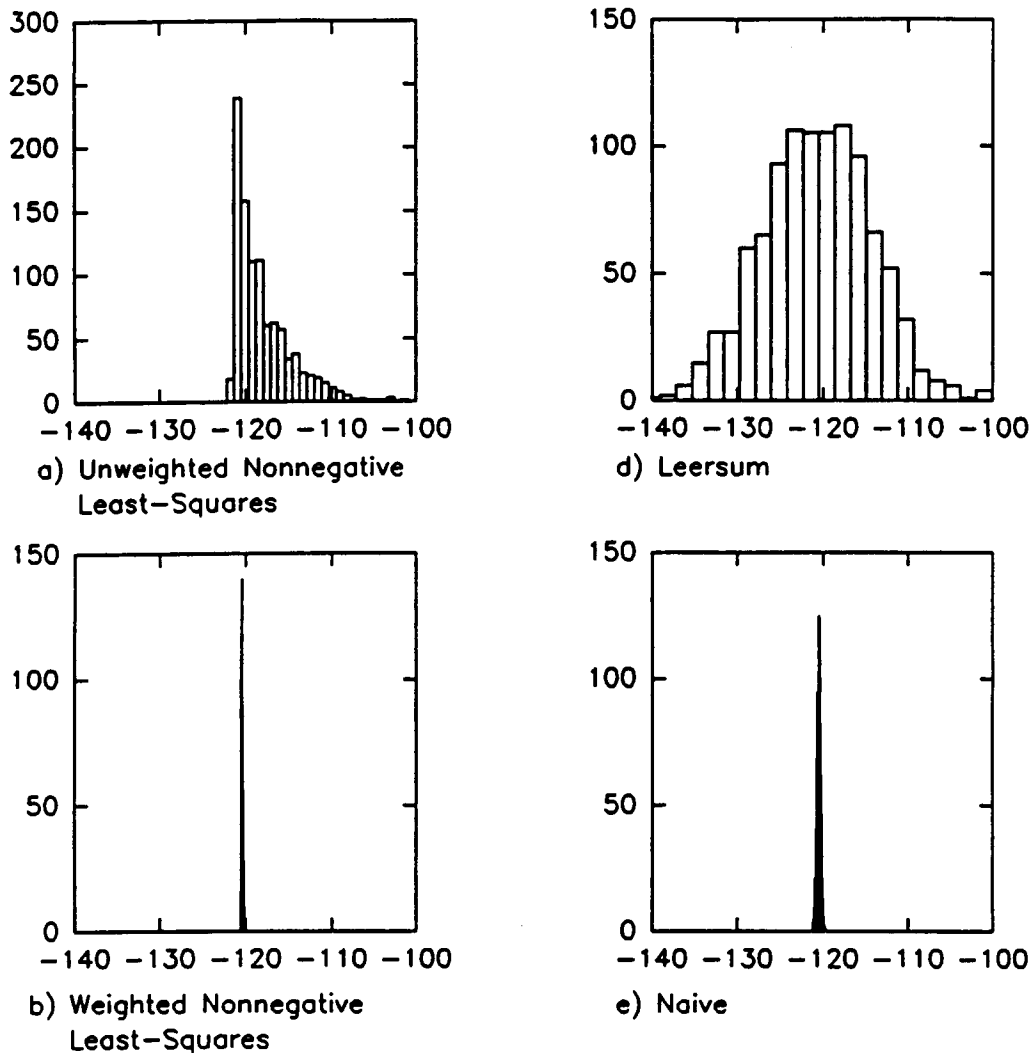
Figure 3. Histograms of Surface-3 Heat Flux [watts/m^2] for the Diagonal-Dominated Case.

Figure 3 shows that there is a considerable skew to the heat flux distributions for the nonnegative least-squares cases. It is believed that this is caused by the nonnegativity constraints. The diagonal elements of the view-factor matrix have nominal values which are zero; therefore, the Monte-Carlo procedure will produce many negative diagonal view factors that are then set to zero.

For the area uncertainties, a random perturbation is added to the view-factor matrix using a gaussian random-number generator with the standard deviations given above. The F-matrix is then frozen and the Monte-Carlo analysis is performed for the area uncertainties using a gaussian random-number generator and area standard deviations equal to 1% of each area. Table 6 shows the standard deviations for the resulting heat-flux calculations.

Table 6. Standard Deviations in Heat Flux for Area Variance with Diagonal-Dominated View-Factor Error.

Surface	Nonnegative Least-Squares		Leersum	Naive
	Unweighted	Weighted		
1	0.4462	0.3718	0.4197	0.2298
2	0.5079	0.5679	0.3978	0.4840
3	0.3160	0.4742	0.5208	0.8534
rms-avg	0.4309	0.4781	0.4493	0.5818

As seen above, all of the rectification schemes have about the same effect on sensitivity to area uncertainty.

The same procedure is followed for the other unequal variance case studies. For all of the cases with regional dominance, the base view-factor standard deviation is 0.001, and the value in the dominate region is 0.1. For the counter-diagonal-dominated case, the larger values of standard deviation are obviously along the counter diagonal. For the row-dominated and column-dominated cases, the larger values are on the second row and second column respectively. For the upper-triangle-dominated case, the six elements in the upper-right corner have the larger values. For the random-variance case, the standard deviations were assigned randomly in the range 0-0.1.

Table 7 shows the mean heat flux values, and Table 8 shows the standard deviations of the heat fluxes for the various 1000 trial Monte-Carlo studies. The tables reveal that the weighted nonnegative

Table 7. Mean Heat-Flux Values for the Other Unequal View-Factor Variance Cases.

Surface	Nonnegative Least-Squares		Leersum	Naive	Exact
	Unweighted	Weighted			
	Counter-Diagonal-Dominated View-Factor Variance				
1	-3.5645	-3.6229	-3.4449	-3.1886	-3.6891
2	82.2329	83.9206	83.0833	83.8364	83.8721
3	-119.5026	-120.4466	-120.4577	-120.4717	-120.5353
	Row-Dominated View-Factor Variance				
1	-2.7005	-3.6096	-2.6789	-1.7829	-3.6891
2	82.0741	83.9014	81.9244	79.4481	83.8721
3	-119.0604	-120.4378	-118.8689	-116.4979	-120.5353
	Column-Dominated View-Factor Variance				
1	-2.6462	-3.6033	-3.7737	-3.5184	-3.6891
2	81.8186	83.8880	83.7971	83.8305	83.8721
3	-118.7586	-120.4271	-120.0350	-120.4682	-120.5353
	Upper-Triangle-Dominated View-Factor Variance				
1	-3.5754	-3.6222	-3.3351	-3.3087	-3.6891
2	83.6157	83.9198	82.6190	80.2599	83.8721
3	-120.0605	-120.4450	-118.9259	-115.4267	-120.5353
	Random View-Factor Variance				
1	-3.2880	-3.2593	-3.3021	-2.0925	-3.6891
2	82.4014	82.7955	82.8567	80.3335	83.8721
3	-118.6701	-119.3044	-119.3318	-117.3613	-120.5353

least-squares projection scheme is vastly superior to the others. Overall its mean heat fluxes most closely agree with the exact values, and with the exception of the random-variance case, its standard deviation is one to four orders of magnitude smaller than those for the other schemes. For the random-variance case, the weighted least-squares scheme gives the best results, but the unweighted least-squares and Leersum's schemes also give good results since the uncertainties are more-or-less evenly distributed.

For some cases, the naive rectification scheme fails completely. Table 9 gives the range of computed heat fluxes for the naive rectification with the upper-triangle-dominated view-factor uncertainties. Clearly, any single heat-flux computation from this set is meaningless.

It should be noted that this is a terribly damaged view-factor matrix. For this case, the 95%-confidence uncertainty in view factor is approximately 0.1, or the view factors are considered to have approximately 1 digit accuracy. This would correspond to very crudely computed view factors. However, properly rectified cases yield very meaningful heat flux computations.

Table 10 gives the rms averaged heat flux standard deviations for the area uncertainty Monte-Carlo analysis. As seen before, all of the rectification schemes seem equally good at reducing the sensitivity of the heat flux calculations to the uncertainties in the areas for this case study.

Table 8. Standard Deviations in Heat Flux for the Other Unequal View-Factor Variance Cases.

Surface	Nonnegative Least-Squares Unweighted	Weighted	Leersum	Naive
Counter-Diagonal-Dominated View-Factor Variance				
1	0.5277	0.0368	1.4061	3.4348
2	0.6197	0.0460	2.0133	5.2123
3	0.8957	0.0563	1.6163	6.2030
rms-avg	0.6988	0.0470	1.6973	5.0810
Row-Dominated View-Factor Variance				
1	1.7794	0.0477	3.6299	5.4372
2	2.7123	0.0630	7.0196	16.1912
3	2.5028	0.0618	5.8428	19.2367
rms-avg	2.3655	0.0579	5.6742	14.8523
Column-Dominated View-Factor Variance				
1	1.9819	0.0550	3.9047	5.1064
2	3.5176	0.0819	4.8266	5.2548
3	3.4125	0.0701	3.9496	0.3175
rms-avg	3.0521	0.0699	4.2482	4.2343
Upper-Triangle-Dominated View-Factor Variance				
1	0.2454	0.0353	2.0937	11.0725
2	0.3273	0.0438	1.9262	22.9427
3	0.4017	0.0535	2.2979	43.0558
rms-avg	0.3310	0.0448	2.1114	28.8835
Random View-Factor Variance				
1	2.6444	2.0461	4.7752	14.7834
2	2.6682	2.0091	6.1736	17.8497
3	2.4439	1.7696	5.3161	15.3694
rms-avg	2.5875	1.9454	5.4521	16.0560

Table 9. Range of Naive Heat-Flux Values for the Upper-Triangle-Dominated Variance Case.

Surface	1	2	3
max	38.9123	106.9031	991.5615
mean	-3.3087	80.2599	-115.4267
min	-281.7724	-389.7757	-205.6421

Table 10. Root-Mean-Square Averaged Standard Deviations in Heat Flux for Area Variance Cases with Other Unequal View-Factor Variance.

View-Factor Variance Case	Nonnegative Least-Squares		Leersum	Naive
	Unweighted	Weighted		
Counter-Diagonal	0.4397	0.4395	0.4605	0.5556
Row-Dominated	0.3916	0.4721	0.3570	0.4075
Column-Dominated	0.4071	0.4461	0.4417	0.5859
Upper-Triangle	0.4386	0.4432	0.4748	0.5556
Random	0.4243	0.4565	0.4355	0.5734

CONCLUSIONS

Several view-factor rectification schemes have been compared. Figure 4 summarizes the rms-averaged standard deviation results for heat flux when view-factor uncertainty is considered. The Naive scheme, where all of the corrections are placed in the diagonal elements of the view-factor matrix, has proven to be erratic and sometimes results in meaningless calculations. Leersum's iterative scheme is also erratic but, on average gives considerably better results than the Naive scheme. Leersum's scheme is most viable when the view factors have equal variance. The unweighted version of the nonnegative least-squares projection scheme is better behaved than either the Naive or Leersum's scheme; however, when the view-factor variance is not equally distributed, the unweighted nonnegative least-squares projection is consistently superior for all cases. In the cases where the variances were not equally distributed the weighted nonnegative least-squares projection gives heat-flux results which were orders of magnitude better than the other schemes.

The Naive scheme is not recommended. If no knowledge on the relative sizes of the view-factor variances is available, either Leersum's scheme or the unweighted nonnegative least-squares projection will take fairly crudely calculated view factors and compute meaningful heat transfer results. The least-squares projection is recommended since the computational tasks are roughly equivalent and it is about twice as effective. If information is available on the relative variance of the view factors (which is always the case for Monte-Carlo integrations), the weighted nonnegative least-squares projection should be used.

The weighted nonnegative least-squares projection can be thought of as a numerical filter for noisy view-factor data. In the examples given here, very good heat transfer calculations were made for cases with very crudely defined view-factor data (roughly 1 digit accuracy). View-factor calculations are the most computationally intensive part of many radiation enclosure problems. There is the possibility of considerable improvement in computational efficiency by combining this excellent filter with relatively crude computations of the view factor values. To properly make such a compromise, sensitivity estimates [5] of the heat transfer calculations would be required.
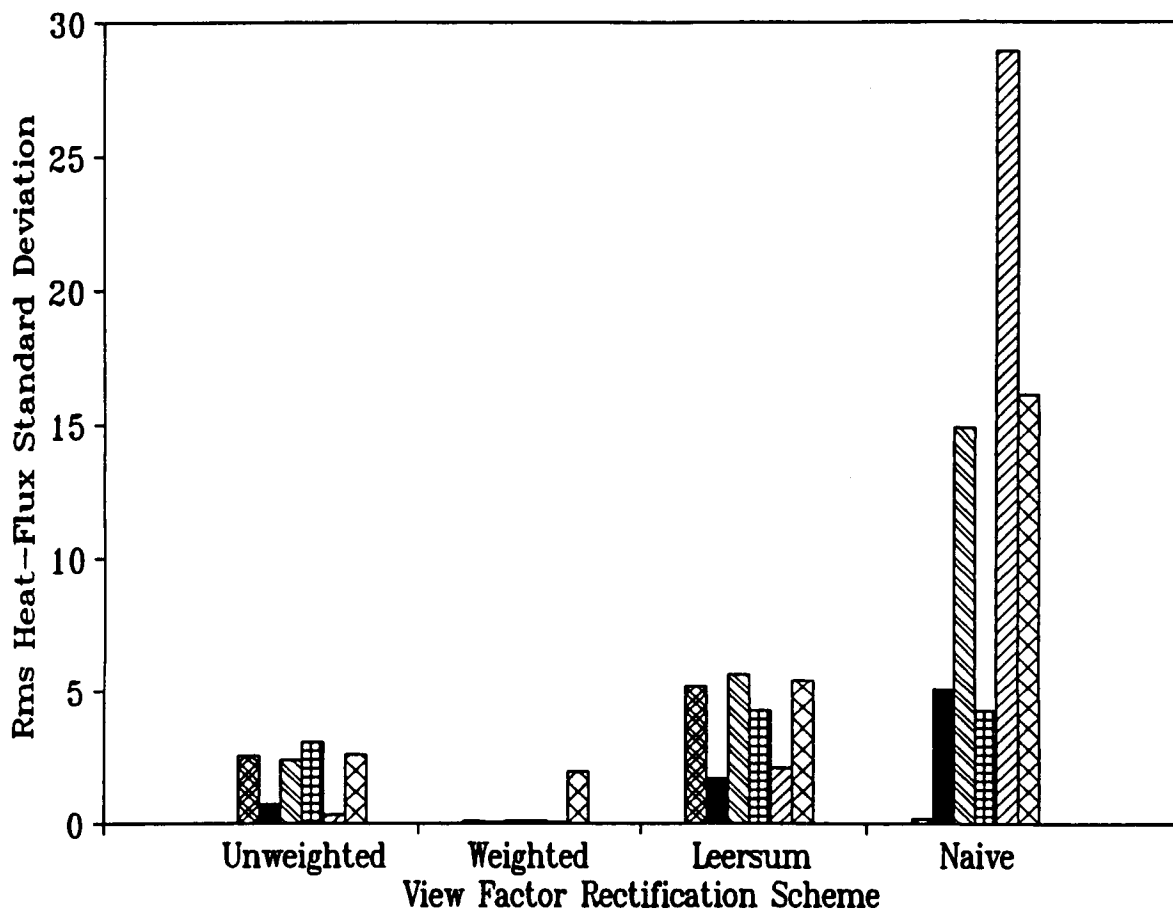
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Figure 4. Summary of Surface-3 Standard Deviation Results for View-Factor Variance.

All of the schemes were roughly equal with regards to the propagation of uncertainties in the surface areas. When the view factors were rectified in this case study, the heat flux uncertainties were roughly an order of magnitude less than the case when no rectification was applied. The proper weighting procedure for the enforcement of reciprocity with uncertain areas is a topic of current research.

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