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Multi-Spacecraft Coherent Doppler and Ranging for Interplanetary Navigation

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Abstract

Future plans for planetary exploration currently include using multiple spacecraft to simultaneously explore one planet. This never before encountered situation, places new demands on tracking systems used to support navigation. One possible solution to the problem of heavy ground resource conflicts is the use of multi-spacecraft coherent radio metric data, also known as, bent-pipe data. Analysis of the information content of these data types show that the information content of multi-spacecraft Doppler is dependent only on the frequency of the final downlink leg and is independent of the frequencies used on other legs. Numerical analysis shows that coherent bent-pipe data can provide significantly better capability to estimate the location of a lander on the surface of Mars, than can direct lander to Earth radio metric data. However, this is complicated by difficulties in separating the effect of a lander position error from that of an orbiter position error for single passes of data.

Introduction

As plans are being made to send multiple spacecraft simultaneously to the same planet, it has become apparent that this places a considerable burden on the ability of Earth based tracking resources to provide the levels and types of support traditionally provided. In the past NASA's Deep Space Network (DSN) has been able to meet the needs of spacecraft whose visibility periods overlapped by using extra resources or by negotiated compromises in scheduling. This was in part achievable because overlapping visibility periods were generally a transient phenomena, which orbital motion would correct. However, with the development of the *Mars Surveyor* program, it is planned that the DSN will have to support multiple spacecraft in orbit around or landed on the surface of Mars. During certain phases of this program, it is envisioned that four or more spacecraft (some combination of landers and orbiters) may simultaneously be in operation This will require the development of new techniques and operational methods, including in the of navigation.

Traditionally operational deep space navigation has been performed by using coherent 2 way Doppler and ranging between an Earth station and the spacecraft. In this mode of operation an uplink signal is sent from the Earth to a spacecraft, where the frequency of the received signal is used by the spacecraft to control the frequency of the signal transmitted back to Earth. Additionally, a ranging signal (or signals) can be modulated on the uplink, demodulated by the spacecraft receiver and remodulated onto the downlink, allowing for the measure of the round-trip light time to the spacecraft. These tracking data types were provided in passes which typically lasted from four to eight hours. The total amount of coverage varied from three passes/per week to continuous coverage. It can be seen that it would be difficult to provide this level of support to two or more spacecraft which have a 100% visibility overlap without committing large amounts of DSN ground resources for years at a time.

Alternative tracking methods do exist, such as receiving a noncoherent downlink with a multichannel receiver. This however, places a great reliance on the stability of the spacecraft oscillator. Analysis (Ref. 1) indicates that reasonable accuracies can be met with such a noncoherent system, but that these accuracies are not equal to a coherent system. A second option is to track one spacecraft in the traditional manner, and to have that spacecraft receive, process, and telemeter to the Earth noncoherent signals sent by other spacecraft which are nearby. Such a system has significant advantages in that the radio system for the secondary spacecraft can be much smaller in that it is not necessary to provide a link to the Earth. However this system is highly dependent on the stability of oscillators on both spacecraft and on the accuracy of the Doppler extraction and telmetering system on the relay. Analysis performed to support the never exercised MBR relay, between Russian Mars landers and the *Mars Observer* spacecraft (Ref. 2.) indicates that by far the limiting error source for that system was the stability of the lander oscillator. However a system midway between the current coherent tracking process and the telemetered system could be developed. This system would utilize a coherent radio link between the Earth station and both spacecraft. This "bent-pipe" data would not have any dependence on spacecraft oscillators, would not require a Doppler extraction/telemetry system, and would not require the support of simultaneous uplinks from the Earth.

Bent-Pipe Tracking

A bent-pipe tracking scheme is illustrated in Figure 1. In this case a radio signal of frequency, f_{tE}, is broadcast from an Earth station. This signal is received by the first spacecraft (SC_1), with the shifted frequency, f_{r1a}, and then is coherently rebroadcast with frequency, f_{t1a}, to the second spacecraft (SC_2), where the received frequency is f_{r2}. SC_2 then coherently rebroadcasts the data with frequency, f_{t2}, to SC_1, where it is received with the frequency, f_{r1b}, and coherently broadcast with the frequency, f_{t1b}. Finally the signal is received at the Earth station with a measured receipt frequency of f_{rE}. The length and rate of change of the length of the four radio links are designated respectively, ρ_1, $\dot{\rho}_1$, ρ_2, $\dot{\rho}_2$, ρ_3, $\dot{\rho}_3$, and ρ_4, $\dot{\rho}_4$.

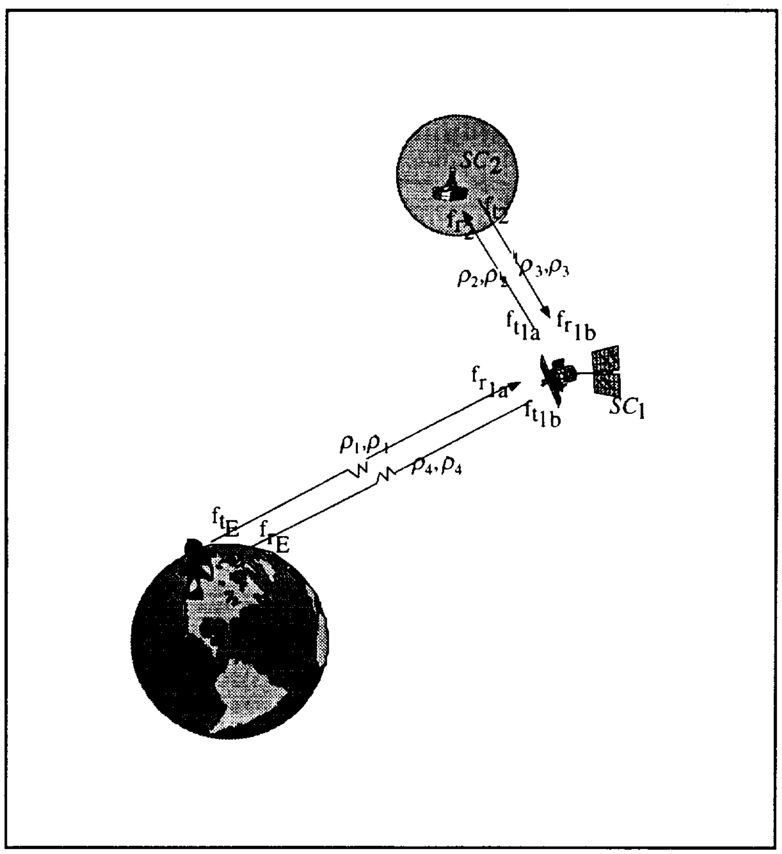

Figure 1: Two spacecraft bent pipe tracking

Observables

By convention, the Doppler radio metric observable, O, is defined as:

$$O = f_{tE}\, r_1\, r_2\, r_3 - f_{rE} \qquad \text{<Eq. 1>}$$

where:

$r_1 \equiv$ the frequency turn around ratio between leg 1 & leg 2 $\left(\frac{f_{t_{1a}}}{f_{r_{1a}}}\right)$

$r_2 \equiv$ the frequency turn around ratio between leg 2 & leg 3 $\left(\frac{f_{t_2}}{f_{r_2}}\right)$

$r_3 \equiv$ the frequency turn around ratio between leg 3 & leg 4 $\left(\frac{f_{t_{1b}}}{f_{r_{1b}}}\right)$

However, working backward from the received signal, f_{r_E}:

$$f_{r_E} = \left(\frac{1 - \frac{\dot{\rho}_4}{c}}{\sqrt{1 - \left(\frac{\dot{\rho}_4}{c}\right)^2}}\right) f_{t_{1b}} + \varepsilon_4 \qquad \text{<Eq. 2>}$$

where:

$\dot{\rho}_4 \equiv$ the rate of change of the length of the final radio link
$\varepsilon_4 \equiv$ noise and other effects (including transmission media) on the final downlink leg
$c \equiv$ the speed of light

In the interest of streamlining the notation, a function is introduced to replace the first factor on the right hand side of Eq.. 2, the Doppler shift multiplier.

$$d(x) = \left(\frac{1 - \frac{x}{c}}{\sqrt{1 - \left(\frac{x}{c}\right)^2}}\right) \qquad \text{<Eq. 3>}$$

thereby reducing Eq.. 2 to:

$$f_{r_E} = d(\dot{\rho}_4)\, f_{t_{1b}} + \varepsilon_4 \qquad \text{<Eq. 4>}$$

Given the definition of the turn around ratio, it is possible to redefine $f_{t_{1b}}$:

$$f_{t_{1b}} = r_3\, f_{r_{1b}} \qquad \text{<Eq. 5>}$$

However,

$$f_{r_{1b}} = d(\dot{\rho}_3)\, f_{t_2} + \varepsilon_3 \qquad \text{<Eq. 6>}$$

where:

$\dot{\rho}_3 \equiv$ the rate of change of the length of the second intermediate leg
$\varepsilon_3 \equiv$ noise and other effects (including transmission media) on the second intermediate leg

recursively substituting Eq.. 6 into Eq.. 5 and that result into Eq.. 4:

$$f_{r_E} = d(\dot{\rho}_4) r_3 \left(d(\dot{\rho}_3) f_{t_2} + \varepsilon_3\right) + \varepsilon_4 \qquad \text{<Eq. 7>}$$

which expands to

$$f_{r_E} = d(\dot\rho_4)\, d(\dot\rho_3)\, r_3\, f_{t_2} + d(\dot\rho_4)\, r_3\, \varepsilon_3 + \varepsilon_4 \qquad \text{<Eq. 7a>}$$

repeating the steps of Eq.. 5 and Eq.. 6 on the transmission leg from SC_1 to SC_2:

$$f_{t_2} = r_2\, f_{r_2} \qquad \text{<Eq. 8>}$$

$$f_{r_2} = d(\dot\rho_2)\, f_{t_{1a}} + \varepsilon_2 \qquad \text{<Eq. 9>}$$

recursively substituting as before

$$f_{r_E} = d(\dot\rho_4)\, d(\dot\rho_3)\, d(\dot\rho_2)\, r_3\, r_2\, f_{t_{1a}} + d(\dot\rho_4)\, d(\dot\rho_3)\, r_3\, r_2\, \varepsilon_2 + d(\dot\rho_4)\, r_3\, \varepsilon_3 + \varepsilon_4 \qquad \text{<Eq. 10>}$$

continuing to the Earth to SC_1 leg:

$$f_{t_{1a}} = r_1\, f_{r_{1a}} \qquad \text{<Eq. 11>}$$

$$f_{r_{1a}} = d(\dot\rho_1)\, f_{t_E} + \varepsilon_1 \qquad \text{<Eq. 12>}$$

and as before

$$\begin{aligned} f_{r_E} = {} & d(\dot\rho_4)\, d(\dot\rho_3)\, d(\dot\rho_2)\, d(\dot\rho_1)\, r_3\, r_2\, r_1\, f_{t_E} + d(\dot\rho_4)\, d(\dot\rho_3)\, d(\dot\rho_2)\, r_3\, r_2\, r_1\, \varepsilon_1 \\ & + d(\dot\rho_4)\, d(\dot\rho_3)\, r_3\, r_2\, \varepsilon_2 + d(\dot\rho_4)\, r_3\, \varepsilon_3 + \varepsilon_4 \end{aligned} \qquad \text{<Eq. 13>}$$

Assuming for the purposes of this data content analysis that the final four terms in Eq.. 13 are very small in comparison to the first term and can be dropped, Eq.. 13 simplifies to

$$f_{r_E} \approx d(\dot\rho_4)\, d(\dot\rho_3)\, d(\dot\rho_2)\, d(\dot\rho_1)\, r_3\, r_2\, r_1\, f_{t_E} \qquad \text{<Eq. 14>}$$

substituting back in the function introduced in Eq.. 3:

$$f_{r_E} \approx \left(\frac{1 - \frac{\dot\rho_4}{c}}{\sqrt{1 - \left(\frac{\dot\rho_4}{c}\right)^2}}\right)\left(\frac{1 - \frac{\dot\rho_3}{c}}{\sqrt{1 - \left(\frac{\dot\rho_3}{c}\right)^2}}\right)\left(\frac{1 - \frac{\dot\rho_2}{c}}{\sqrt{1 - \left(\frac{\dot\rho_2}{c}\right)^2}}\right)\left(\frac{1 - \frac{\dot\rho_1}{c}}{\sqrt{1 - \left(\frac{\dot\rho_1}{c}\right)^2}}\right) r_3\, r_2\, r_1\, f_{t_E} \qquad \text{<Eq. 15>}$$

this then becomes

$$f_{r_E} \approx \frac{\left(1 - \frac{\dot\rho_4}{c} - \frac{\dot\rho_3}{c} + \frac{\dot\rho_3\,\dot\rho_4}{c^2}\right)\left(1 - \frac{\dot\rho_2}{c} - \frac{\dot\rho_1}{c} + \frac{\dot\rho_1\,\dot\rho_2}{c^2}\right)}{\sqrt{\left(1 - \left(\frac{\dot\rho_4}{c}\right)^2 - \left(\frac{\dot\rho_3}{c}\right)^2 + \left(\frac{\dot\rho_3\dot\rho_4}{c^2}\right)^2\right)\left(1 - \left(\frac{\dot\rho_4}{c}\right)^2 - \left(\frac{\dot\rho_3}{c}\right)^2 + \left(\frac{\dot\rho_3\dot\rho_4}{c^2}\right)^2\right)}}\, r_1\, r_2\, r_3\, f_{t_E} \qquad \text{<Eq. 15a>}$$

since all of the $\dot{\rho}$ terms are much smaller than c, the numerator of Eq.. 15a can be approximated as 1 (this has the effect of ignoring the relativistic correction to the Doppler shift) and the numerator expanded.

$$f_{r_E} \approx \left(\begin{array}{c} 1 - \frac{\dot\rho_2}{c} - \frac{\dot\rho_1}{c} + \frac{\dot\rho_2 \dot\rho_1}{c^2} - \frac{\dot\rho_4}{c} + \frac{\dot\rho_4 \dot\rho_2}{c^2} + \frac{\dot\rho_4 \dot\rho_1}{c^2} - \frac{\dot\rho_4 \dot\rho_2 \dot\rho_1}{c^3} \\ - \frac{\dot\rho_3}{c} + \frac{\dot\rho_3 \dot\rho_2}{c^2} + \frac{\dot\rho_3 \dot\rho_1}{c^2} - \frac{\dot\rho_3 \dot\rho_2 \dot\rho_1}{c^3} + \frac{\dot\rho_4 \dot\rho_3}{c^2} - \frac{\dot\rho_2 \dot\rho_3 \dot\rho_4}{c^3} \\ - \frac{\dot\rho_1 \dot\rho_3 \dot\rho_4}{c^3} + \frac{\dot\rho_1 \dot\rho_2 \dot\rho_3 \dot\rho_4}{c^4} \end{array} \right) r_1 r_2 r_3 f_{t_E} \qquad \text{<Eq. 16>}$$

Ignoring all second order effects, Eq.. 16 can be further approximated and simplified to

$$f_{r_E} \approx \left(1 - \frac{\dot\rho_1}{c} - \frac{\dot\rho_2}{c} - \frac{\dot\rho_3}{c} - \frac{\dot\rho_4}{c}\right) r_1 r_2 r_3 f_{t_E} \qquad \text{<Eq. 17>}$$

substituting Eq.. 17 into Eq.. 1:

$$O \approx f_{t_E} r_1 r_2 r_3 - \left(1 - \frac{\dot\rho_1}{c} - \frac{\dot\rho_2}{c} - \frac{\dot\rho_3}{c} - \frac{\dot\rho_4}{c}\right) r_1 r_2 r_3 f_{t_E} \qquad \text{<Eq. 18>}$$

and finally

$$O \approx \left(\frac{\dot\rho_1}{c} + \frac{\dot\rho_2}{c} + \frac{\dot\rho_3}{c} + \frac{\dot\rho_4}{c}\right) r_1 r_2 r_3 f_{t_E} \qquad \text{<Eq. 19>}$$

A bent-pipe range observable, R, is defined as:

$$R = \frac{\rho_1}{c} + \frac{\rho_2}{c} + \frac{\rho_3}{c} + \frac{\rho_4}{c} - n(A) \qquad \text{<Eq. 20>}$$

where:

$\rho_1 \cdots \rho_4 \equiv$ the path length of each radio link
$n \equiv$ unknown integer multiplier
$A \equiv$ range modulus (a function of ground hardware configuration)

Given the definitions of the range and Doppler observables from Eq.'s 20 and 18, the sensitivity of the observable to any parameter z can be readily calculated.

$$\frac{\partial O}{\partial z} \approx \left(\frac{\partial(\dot\rho_1)}{\partial z} + \frac{\partial(\dot\rho_2)}{\partial z} + \frac{\partial(\dot\rho_3)}{\partial z} + \frac{\partial(\dot\rho_4)}{\partial z}\right) \frac{r_1 r_2 r_3}{c} f_{t_E} \qquad \text{<Eq. 21>}$$

$$\frac{\partial R}{\partial z} \approx \frac{\left(\frac{\partial(\rho_1)}{\partial z} + \frac{\partial(\rho_2)}{\partial z} + \frac{\partial(\rho_3)}{\partial z} + \frac{\partial(\rho_4)}{\partial z}\right)}{c} \qquad \text{<Eq. 22>}$$

A conclusion that can be readily drawn from Eq.'s 21 and 22 that is not intuitive is that the information content in a coherent radio metric data point is (to first order) only dependent on the frequency on the final downlink leg. For example, for a radio link where the first leg is X band, the second is S-band, the third is S-band, and the final leg is X-band, typical values for the three turn around ratios and transmit frequency are $r_1 = \frac{221}{749} = 0.2951$, $r_2 = \frac{240}{221} = 1.086$, $r_3 = \frac{880}{240} = 3.666$, and $f_{t_E} = 7.2$ GHz (Ref. 3), while for an S-band/UHF/UHF/X-band link the corresponding values would be $r_1 = \frac{25}{132} = 0.1894$, $r_2 = \frac{749}{737} = 1.016$, $r_3 = \frac{13840}{669} = 20.69$, and $f_{t_E} = 2.1$ GHz. In both of these examples, the term, $r_1 r_2 r_3 f_{t_E}$, is equal to 8.4 GHz. This result is more than simply an interesting detail. Since to first order (and ignoring transmission media effects), the data content does not depend on the frequency of the inter-spacecraft links, the choice of frequencies and transponders for the this link can be made without regard to navigation issues. This can be a significant cost savings issue. Additionally from an operational perspective, it is not possible (without extra information) for the operator of the ground tracking system to know what frequency is being used on the spacecraft/spacecraft link. However all that is required is that the product of the three turn around ratios be known.

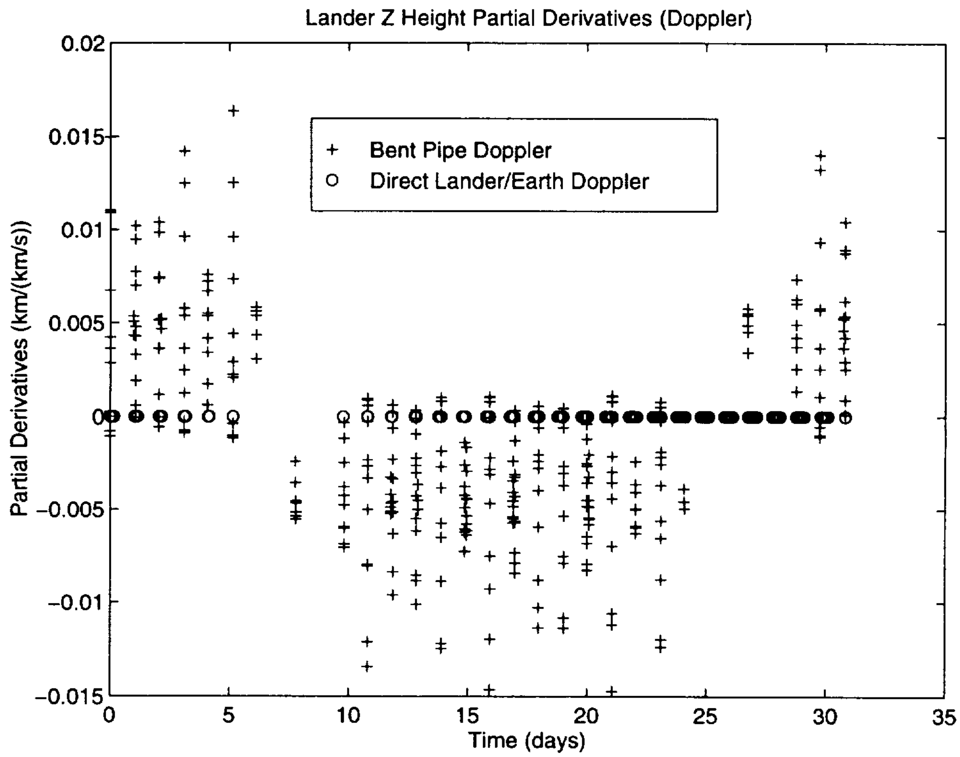

Figure 2: Comparison of bent pipe Doppler and traditional Doppler lander Z-height sensitivity

Numerical Analysis

Equations 19 and 22 give the partial derivatives of a Doppler and range observation with respect to an arbitrary parameter z. From these it is possible to calculate the approximate sensitivity of a number of observations to parameters of interest. For the purpose of this initial study, the case of an orbiter about Mars and a lander on the surface is examined. The orbiter in question, is in a near polar orbit with a semi-major axis of 3775 km. This is the approximate orbit planned for *Mars Observer* and currently planned for *Mars Global Surveyor* (Ref. 4). A lander is located at approximately 30° North latitude. One quantity of strong interest is the ability to determine the location of the lander on the surface. It has been known for some time that Earth based tracking of landers on Mars has difficulty in determining the Z - height component of the position vector in a cylindrical coordinate frame. Figures 2 and 3 clearly show that bent-pipe Doppler and range data exhibit a sensitivity to this parameter that is more than an order of magnitude larger than that for direct lander/Earth tracking. It should be noted that given the low polar orbiter chosen for this case, the sensitivity to this parameter in the bent pipe data is much greater than it would be for a high equatorial orbiter.

The bent pipe Doppler data also exhibit much larger sensitivity to lander spin axis knowledge and longitude knowledge than the traditional Earth based lander Doppler. Figure 4 clearly indicates that the partials for spin axis and longitude are approximately 10 to 20 times larger than the corresponding partials for the conventional Doppler.

Unfortunately, this enhanced sensitivity does little good, if it is not possible to separate the lander position from other parameters. Detailed covariance analysis of a similar problem (Ref. 2), indicates that tracking arcs on the order of a week to a month are required to completely separate the knowledge of the orbiter position and the lander position. Single passes are extremely poor in the ability to separate the two spacecraft. The reason for this is clearly indicated in Figure

5. The partial derivatives of lander position and orbiter epoch state are given over a single pass (the first pass in the data arc). The similarity in structure between the two sets of partials, especially the orbiter Cartesian x, and the lander z-height location, make it very nearly impossible to separate the position estimates for the two spacecraft given a short data arc.

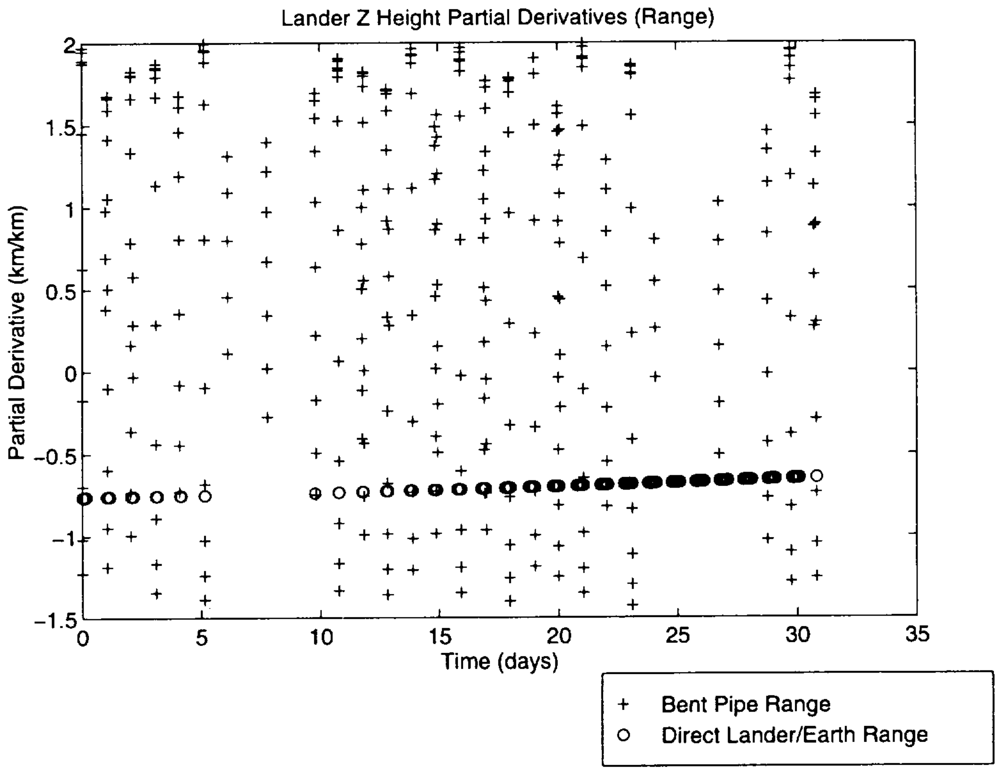

Figure 3: Comparison of bent pipe range and traditional range lander Z-height sensitivity

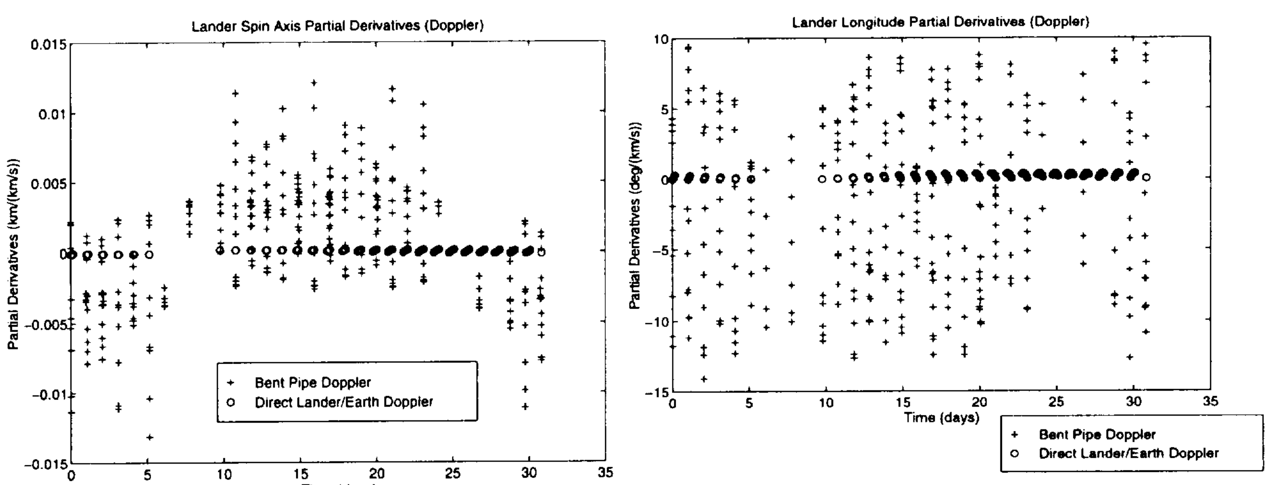

Figure 4: Comparison of bent pipe & traditional Doppler sensitivities to lander location spin axis & longitude

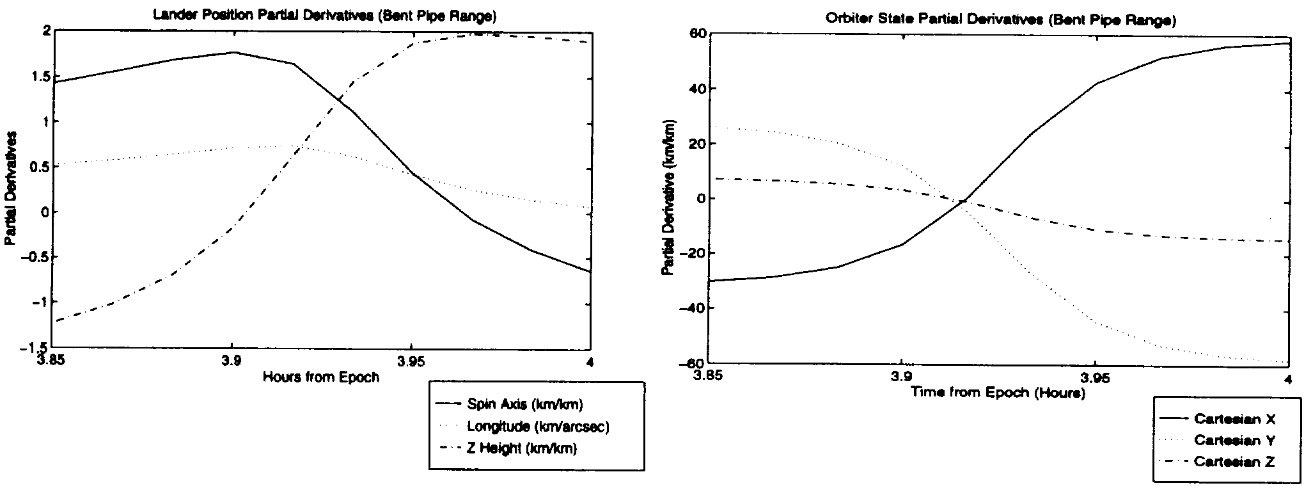

Figure 5: Comparison Orbiter state and lander location partials for a single pass of bent pipe Doppler

Operational Considerations

It is apparent that there is a high probability that coherent bent pipe Doppler and ranging data will provide sufficient information to allow the navigation of an orbiter and a lander at Mars. However, a brief examination of the operational feasibility of such a system from a ground and spacecraft hardware position is in order. Given current analog phase-lock-loop (PLL) receivers used by interplanetary spacecraft, the procedure for acquiring a coherent bent pipe link would be to, first, sweep the uplink to the first spacecraft, slow enough and wide enough to ensure lock. Then repeat the sweep to attempt to acquire lock of the second spacecraft. This second sweep will have to be slow enough that the first spacecraft does not drop lock. Then a sweep of the signal to reacquire the downlink signal at the first spacecraft may be needed. Finally the signal is received on the ground and a coherent link is established. This process would place considerable overhead on the tracking bandwidth of the spacecraft receivers, the width of the total tracking loop, and the amount of time required to acquire a signal. Given that for the geometry identified in this short study, the longest pass of bent pipe Doppler data acquired is 12 minutes long, it seems improbable that a link could be set up in this time. However if the orbiter were in a somewhat higher orbit, and directly controlled receivers used, it should be possible to set up a link. However the need for a controlled receiver could possibly offset the cost savings accrued due to the lack of a required direct to Earth link.

Once a coherent link is set up, if the frequency shifts too much or too quickly radio lock may be lost. Given that the radio signal to be received by the spacecraft will have the Doppler shifts of multiple legs it is of some concern that the total shift would be too great to maintain lock. Figure 6 shows the range rates for the lander and the orbiter for both bent pipe and traditional tracking methods. It can be seen that the motion of Mars relative to the Earth station is the dominant error source and the summation of the two signals would result in less than 40% increase in maximum Doppler shift over that from conventional Doppler. Thus it is unlikely that this alone could preclude the acquisition of bent pipe Doppler data.

Conclusion

Coherent bent pipe Doppler and ranging data can provide useful information for the navigation of multiple spacecraft at a given target which is independent of the frequency used on the inter-spacecraft link. However, the operational complexities involved in acquiring a link would most likely require the use of a controlled receiver, rather than the analog PLL receivers currently used for the majority of deep space missions and would preclude the acquisition of data during extremely short visibility periods. Consequently, this data type would not be useable for the support of a lander and a low mapping orbiter of the *Mars Observer* or *Mars Global Surveyor* type. However, for some types of missions such a system could significantly decrease the resource conflicts inherent in supporting multiple spacecraft at a single source.

More study is needed of the detailed requirements on the spacecraft telecommunications system of acquiring a coherent bent pipe link. Additionally, the ability to separate the position knowledge of a lander and an orbiter or of two orbiters needs to be more fully investigated than was possible in the scope of the study. Finally, other data types such as two-way coherent telemetered Doppler between an orbiter and a lander should be investigated. This data has similar information content, and fewer separability problems, but may have additional theoretical and implementation obstacles.

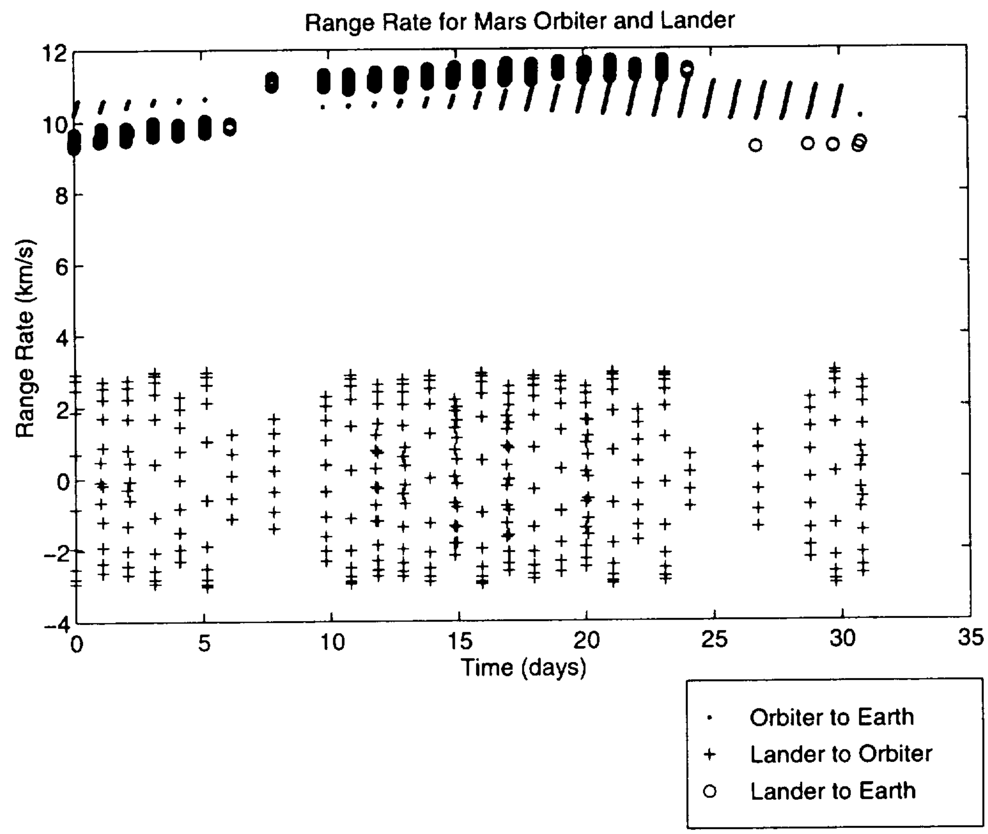

Figure 6: Comparison of range rates for various combinations of lander and orbiter observations

Acknowledgments

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