N95- 28425

EFFECT OF THE FIBER-MATRIX INTERPHASE ON THE TRANSVERSE TENSILE STRENGTH OF THE UNIDIRECTIONAL COMPOSITE MATERIAL

S5-24
57373
P-17

H. C. Tsai and A. M. Arocho
Naval Air Development Center
Warminster, PA

ABSTRACT

A simple one-dimensional fiber-matrix interphase model has been developed and analytical results obtained correlated well with available experimental data. It was found that by including the interphase between the fiber and matrix in the model, much better local stress results were obtained than with the model without the interphase. A more sophisticated two-dimensional micromechanical model, which includes the interphase properties was also developed. Both one-dimensional and two-dimensional models were used to study the effect of the interphase properties on the local stresses at the fiber, interphase and matrix. From this study, it was found that interphase modulus and thickness have significant influence on the transverse tensile strength and mode of failure in fiber reinforced composites.

INTRODUCTION

Unidirectional fiber reinforced composites have a very low transverse tensile strength. This strength, in general, is much lower than the strength of the pure matrix and limits the performance of the composite system. The transverse tensile strength of a composite is dependent upon the fiber-matrix interfacial bonding strength, matrix strength, transverse fiber strength and stiffness ratio between fiber and matrix, etc. Material defects such as voids in the matrix, broken fibers, microcracks and fiber-matrix disbonds will degrade the transverse tensile strength of composite materials. Conventional analysis methods (Refs. 1-3), which treat the fiber and matrix as two phase, homogeneous and isotropic materials, provide ways to predict the transverse tensile strength of the composite, but the correlation between these analytical results and test results is not satisfactory.

Recently, several researchers (Refs. 4, 5) have suggested that the volume of matrix material immediately surrounding the fiber is significantly different from the bulk matrix. This volume of material is commonly referred to as the interphase. It is believed that the interphase, though small in thickness, has significant effect on the strength and fracture toughness of the composite. To verify this concept, these authors (Ref. 6) have developed micromechanical models which treat the fiber reinforced composite as a three-phase material; namely, fiber, interphase and bulk matrix. Through the analysis of the microdebonding problem, it has been shown that the model including an interphase between fiber and matrix provides a much better prediction of debonding loads than the model without the interphase.

The purpose of this study is to determine the effect of the fiber-matrix interphase on the transverse tensile strength of the composite. Since the transverse tensile strength is very low, there is a potential for engineering material improvements based upon a better understanding of the transverse failure mechanics, so that higher composite transverse tensile strength can be obtained.

In the following discussions, composites without material defects are considered.

METHOD OF ANALYSIS

The composite material is assumed to consist of a square array of unidirectionally elastic circular filaments in an infinite elastic matrix as shown in Figure 1. This simple packing geometry has been known to give satisfactory results (Ref. 7). The filaments are assumed to be perfectly bonded to the matrix. By assuming a square packing arrangement, a repeating unit can be isolated, as indicated by the solid lines a-a-a-a shown in figure 1. For a composite subjected to a remotely applied average tensile stress in the x direction, σ_T, the conditions of symmetry require that boundary lines of this repeating unit in the deformed composite remain parallel to the corresponding lines of the undeformed material, i.e., as shown in Figure 1, line a′- a′ is parallel to the corresponding boundary line a - a. This implies that the gross strains over the length 2L in directions x and z remain constant.

$$\frac{1}{2L}\int_{-L}^{L} \epsilon_x \, dx = \epsilon_T = \text{constant (at any z)} \tag{1}$$

$$\frac{1}{2L}\int_{-L}^{L} \epsilon_z \, dz = \epsilon_z = \text{constant (at any x)} \tag{2}$$

Also, at any section across the element 2L the conditions of force equilibrium require that

$$\sigma_T = \frac{1}{2L}\int_{-L}^{L} \sigma_x \, dz \tag{3}$$

$$0 = \frac{1}{2L}\int_{-L}^{L} \sigma_z \, dx \tag{4}$$

Having established the assumptions of square packing, gross strain and stress conditions, the physical problem can be formulated as follows:

ONE-DIMENSIONAL PREDICTION METHODOLOGY

The repeating unit to be modeled which includes the interphase is shown in Figure 2. The basic concept for the one-dimensional model is shown in Figure 3. The material response to the external transverse tensile stress, σ_T, can be approximated by the response of a one-dimensional series spring system.

In the region consisting of fiber, interphase and matrix, there will be three distinct stiffnesses corresponding to each constituent. In the region consisting of interphase and matrix, there will be two distinct stiffnesses corresponding to interphase and matrix. At the region $Z \geq R + t_i$, there will be only one stiffness, namely, stiffness of matrix. From the basic concept, we can conclude that

$$\begin{aligned} &\sigma_f = \sigma_i = \sigma_m = \sigma_L && 0 \leq Z \leq R \\ &\sigma_m = \sigma_i = \sigma_L && R \leq Z \leq R + t_i \end{aligned} \tag{5}$$

where σ_f, σ_i, σ_m = stress in the fiber, interphase and matrix respectively

σ_L = local stress

Based on this concept, the local stress at three regions can be derived as follows:

(1) $0 \leq Z \leq R$

This region includes three phases; namely, fiber, matrix and interphase. The total displacement, as shown in figures 2 and 3, at $x = L$, can be written as

$$U = \epsilon_f \cdot \overline{AB} + \epsilon_i \cdot \overline{BC} + \epsilon_m \cdot \overline{CD} \tag{6}$$

where

ϵ_f = strain in the fiber

ϵ_i = strain in the interphase

ϵ_m = strain in the matrix

Divide both sides of equation (6) by L. We have

$$\epsilon_T = \epsilon_f k_f + \epsilon_i k_i + \epsilon_m (1 - k_f - k_i) \tag{7}$$

From Reference 8, we have

$$k_f = \left(\frac{4V_f}{\pi}\right)^{1/2} \left\{ 1 - \left(\frac{Z}{R}\right)^2 \right\}^{1/2}$$

$$k_i = \left(\frac{4V_f}{\pi}\right)^{1/2} \left\{ \left[\left(1 + \frac{t_i}{R}\right)^2 - \left(\frac{Z}{R}\right)^2\right]^{1/2} - \left[1 - \left(\frac{Z}{R}\right)^2\right] \right\}^{1/2}$$

$$k_m = 1 - k_f - k_i$$

$$V_f = \text{volume fraction of the fiber} \tag{8}$$

From the stress-strain relationship, we have

$$\epsilon_f = \frac{\sigma_f}{E_f}$$

$$\epsilon_i = \frac{\sigma_i}{E_i} \tag{9}$$

$$\epsilon_m = \frac{\sigma_m}{E_m}$$

Substituting equation (9) into (7) and making use of equation (5), the local stress, σ_L, in this region can be expressed as follows

$$\sigma_L = \epsilon_T \frac{1}{\dfrac{k_f}{E_f} + \dfrac{k_i}{E_i} + \dfrac{(1 - k_f - k_i)}{E_m}} \tag{10}$$

(2) $R \leq Z \leq R + t_i$

This region includes matrix and interphase. The total displacement, as shown in figure 2, contributed from these constituents can be written as follows:

$$U = \epsilon_i \cdot \overline{EF} + \epsilon_m \cdot \overline{FG} \tag{11}$$

Divide both sides of equation (11) by L, we have

$$\epsilon_T = \epsilon_i \, k_i + \epsilon_m \, (1 - k_i) \tag{12}$$

where

$$k_i = \left(\frac{4V_f}{\pi}\right)^{\frac{1}{2}} \left\{ \left(1 + \frac{t_i}{R}\right)^2 - \left(\frac{Z}{R}\right)^2 \right\}^{\frac{1}{2}} \tag{13}$$

By this same token, the local stress, σ_L, in this region can be written as follows

$$\sigma_L = \epsilon_T \frac{1}{\left\{\dfrac{k_i}{E_i} + \dfrac{(1 - k_i)}{E_m}\right\}} \tag{14}$$

(3) $R + t_i \leq Z \leq 1$

This region consists of matrix material only. By the same token, the local stress, σ_L, in the region can be expressed as

$$\sigma_L = \epsilon_T \quad \frac{1}{\left(\frac{1}{E_m}\right)} \tag{15}$$

If ε_T is known, then the local stress, σ_L, can be obtained from equations (10), (14) and (15) for the corresponding region.

In the case where only the applied transverse tensile stress, σ_T, is known, then equation (3) will be used to obtain the local stress.

From reference 8, the local stress, σ_L, can be expressed as follows:

$$\sigma_L = \sigma_T \left(\frac{E_m}{E_T}\right) \frac{1}{[1 - k_f\,(1 - E_m/E_f) - k_i\,(1 - E_m/E_i)]} \qquad 0 \le Z \le R \tag{16}$$

$$\sigma_L = \sigma_T \left(\frac{E_m}{E_T}\right) \frac{1}{[1 - k_i\,(1 - E_m/E_i)]} \qquad R \le Z \le R + t_i \tag{17}$$

$$\sigma_L = \sigma_T \left(\frac{E_m}{E_T}\right) \qquad R + t_i \le Z \le L \tag{18}$$

where

$$\frac{E_T}{E_m} = \frac{1}{L}\left\{\int_0^R \frac{dz}{[1 - k_i\,(1 - E_m/E_i) - k_f\,(1 - E_m/E_f)]} + \int_R^{R + t_i} \frac{dz}{[1 - (1 - E_m/E_i)\,k_i]} + L - (R + t_i)\right\} \tag{19}$$

Equation (19) can be solved through numerical integration

Define

$$\text{S.C.F.} = \text{stress concentration factor} = \frac{\sigma_L}{\sigma_T} \tag{20}$$

Making use of equations (16) through (20), we can obtain S.C.F. at any location in the composite material.

TWO DIMENSIONAL MODEL

The basic concept, which was orginated from reference 2, will be adopted in this paper as shown in Figure 4. The solution procedure can be divided into two parts. One is to solve the two-dimensional plane strain problem with $U = \Delta U$ at $x \pm L$, and $V = 0$, at $Z = \pm L$; the other is to solve the two-dimensional plane strain problem with $V = \Delta V$ at $Z = \pm L$, $U = 0.0$ at $x = \pm L$. The final solution will be the linear combination of these two solutions so that equations (3) and (4) will be met. The detailed derivation is in Reference 8.

Only one quadrant of the circular fiber cross section and the surrounding interphase and matrix material needs be analyzed due to symmetry. The finite element model used in this study is shown in Figure 5. The mesh refinement at the fiber-matrix interfacial region is evident. The 'ABAQUS' computer code was used to solve this two-dimensional plane strain problem.

VALIDATION OF ANALYTICAL METHODS

In this analysis, it is assumed that an individual point failure immediately causes failure of the whole material. We recognize that, depending upon the failure locations and the properties of the constituents, this assumption may not be true. Yet the presence of an individual point failure suggests failure of the whole material and provides a conservative prediction. A maximum normal stress failure criterion will be used to determine the failure of the individual point in the material. According to this criterion, fracture is assumed to have occurred if any one of the three principal stresses at this individual point reach the ultimate strength of the corresponding constituent.

VALIDATION OF ONE-DIMENSIONAL MODEL

The comparison between the theoretical and experimental transverse stress distribution is shown in Figure 6. The test-theory correlation for the maximum stress concentration is shown in Figure 7.

The test data (Ref. 1) were obtained from transverse tensile loading tests on plates containing aluminum inclusions imbedded in epoxy resin. As shown in figure 6, both one-dimensional theories, with and without interphase, provide accurate predictions as compared with test data. The model including interphase predicts better results than the model without interphase. Matching analytical results to test data requires that the interphase modulus be seven times smaller than matrix modulus and the interphase thickness be about 1.5 percent of the radius of the fiber.

Figure 7 shows the maximum transverse tensile stress, which occurs at $Z = 0.0$ as a function of the volume fraction of the fiber. Both one-dimensional theories provide very good predictions. The interphase modulus and thickness are assumed the same for all fiber volume fractions. As shown in Figure 7, the inclusion of the interphase in the model does not seem to provide better results than the model withoutinterphase for $V_F = 0.65$. This is because as V_f increases the plane strain condition becomes more evident. The one-dimensional model does not include the effect of the plane strain condition. In the next section, it can be seen that the two-dimensional model including the interphase provides much better results than the two-dimensional model without interphase.

Another example to verify the accuracy of the one-dimensional theory is shown in Figure 8. The test data were obtained from Reference 9. The correlation between analytical results and test results is reasonably good. Since most test data is above the curve with $E_m/E_i = 1.0$, this may suggest that the interphase is a soft interphase.

VALIDATION OF TWO-DIMENSIONAL MODEL

The same composite as shown in Figures 6 and 7 is modelled, with the same interphase modulus and thickness as used in the one-dimensional model. The comparison between the theoretical and experimental transverse stress distribution is shown in Figures 9 and 10. The test-theory correlation for the maximum transverse stress is shown in Figure 11. Again, as shown in figures 9 and 10, both two-dimensional models predict good results as compared with the test data. The two-dimensional model with interphase predicts better results than the model without interphase. Figure 11 shows the merit of the two-dimensional model. By using the same interphase modulus and thickness as used in the one-dimensional model, the two-dimensional model with interphase predicts the best results among all models.

COMPARISON OF ONE-DIMENSIONAL AND TWO-DIMENSIONAL RESULTS

Table I shows the stress concentration factor of the composite, with $E_f/E_m = 21.3$, $V_f = 0.65$, $t_i/R = .015$. The agreement between the one-dimensional model and two-dimensional model is excellent.

From the above comparisons, we conclude that the one-dimensional model is accurate enough to be used to predict the local stress of the composite under transverse loading. Also, the one-dimensional model can be used as a first approximation to estimate the interphase modulus and thickness. By using an iteration scheme and two-dimensional analysis, accurate interphase modulus and thickness estimates of the composite can be obtained.

APPLICATIONS

The one-dimensional model has been verified in the previous section as an adequate model to predict the local stress of the composite. Unless otherwise noted, the following analyses are based on one-dimensional theory.

Effect of Interphase Moduli and Thickness on Stress Concentration Factors of the Composite

Equation (16) was used to calculate the stress concentration factor at $Z = 0$, for various E_m/E_i and t_i/R. Figures 12 and 13 show the results.

Certain conclusions can be drawn: (1) with fixed t_i/R the softer the interphase, the lower the stress concentration factor, (2) with softer interphase and E_m/E_i fixed, the larger the interphase thickness the lower the stress concentration factor. For stiffer interphase (i.e., $E_m/E_i \leq 1.0$), the smaller the interphase thickness, the less the stress concentration factor.

Effect of E_f/E_m and E_m/E_i on the Maximum Stress Concentration Factor of the Composite

From Figure 14, one can see that the higher the E_f/E_m, the larger the stress concentration for all kinds of interphases. But, the degree of influence is decreased as the interphase becomes softer.

The Shift of the Critical Location and the Optimal Interphase (or Coating)

Equations (16), (18) and (19) were used to calculate the stress concentration factors at $Z = 0.0$ and $R + t_i \leq Z \leq L$, the results are shown in Figure 15. $\theta = 0^\circ$ represents the line $Z = 0$, while $\theta = 90^\circ$ represents line $X = 0.0$ and $R + t_i \leq Z \leq L$ As one can see from this figure, curve $(S.C.F.)\ \theta = 0^\circ$ and $(S.C.F.)\theta = 90^\circ$ intersect at $E_m/E_i = (E_m/E_i)_c$ and

$$(S.C.F.)_{\theta = 0^\circ} \geq (S.C.F.)_{\theta = 90^\circ}, \text{ for } E_m/E_i \leq (E_m/E_i)_c$$

$$(S.C.F.)_{\theta = 0^\circ} \leq (S.C.F.)_{\theta = 90^\circ}, \text{ for } E_m/E_i \geq (E_m/E_i)_c$$

Also, the two-dimensional analysis shows that for $E_m/E_i\ \sigma\ (E_m/E_i)_c$ $\sigma^{max}{}_L$ occurs at "b" as shown in Figure 16, where from Reference 8, $(E_m/E_i)_c$ can be expressed as follows

$$(E_m/E_i)_c = 1.0 + [1- (E_m/E_f)]/(t_i/R) \qquad (21)$$

Two-dimensional analysis had been performed in reference 8 and confirmed the transition of the critical location. From equation (21), one can conclude that (1) failure will occur at $\theta = 0^\circ$, if $E_m/E_i \leq E_{ic}$; (2) for $E_m/E_i \geq (E_m/E_i)_c$, failure will occur at $\theta = 90^\circ$ and $Z = R + t_i$; (3) for $E_m/E_i = (E_m/E_i)_c$, failure will occur at $\theta = 0^\circ$ and 90° simultaneously. Figure 16 shows the critical stress concentration factors at these three regions. It is evident that $E_m/E_i = (E_m/E_i)_c$ is the optimal interphase modulus to ensure the least stress concentration factor. Figure 17 shows curves of critical stress concentration factor for various interphase thicknesses. Curve a b c d constitutes the optimal interphase for the composite with fiber volume fraction equal to .65 and $E_f/E_m = 25.0$. For varying fiber volume fraction, the curves for optimal interphase are shown in Figure 18. These curves can be used to design the interphase or coating to obtain the maximum transverse tensile strength of the composite. Figure 19 shows how the interphase affects the transverse tensile strength as compared with the transverse tensile strength of the composite without interphase for various fiber volume fractions. It can be seen that the transverse tensile strength, for the composite with $E_f/E_m = 21.3$ and $t_i/R = .0286$, can be increased as much as 43%, 38% and 34% for $V_F = .65, .57$ and $.502$ respectively.

Effect of Interphase Properties on the Transverse Failure Modes of the Composite

One-dimensional models can not detect the location of failure because the local transverse tensile stress is assumed to be the same in the fiber, interphase and matrix at constant Z and any X as expressed in equation (5). For this purpose, the two-dimensional model must be used. Figure 20 shows the stress distribution in the interphase and matrix at Z = 0 as a function of interphase modulus. As shown in this figure, the assumption that the stress at the matrix and interphase is equal for one-dimensional theory is approximately correct. Also, for $1.0 \leq E_m/E_i \leq 5.0$, the local transverse tensile stress in the matrix is higher than in the interphase. Thus, in this range, matrix cracking is the failure mode. For $5.0 \leq E_m/E_i$, the stress in the interphase or interface is higher than the stress in the matrix. Thus, in this range, failure will occur either in the interphase or at the interface. Based on this idea, one can obtain the critical stress concentration factor and failure mode of the composite as a function of interphase modulus and thickness as shown in Figures 21 and 22.

Notice that the above results are based on the assumption that the strength in matrix, interphase and interface is the same. If the tensile strength of the matrix and interphase (or coating), and the disbonding strength of the interface are known, then similar design curves as shown in Figures 21 and 22 can also be produced to determine the transverse tensile strength and the failure mode as the function of interphase modulus and thickness.

CONCLUSIONS

Interphase thickness and modulus have significant influence on the transverse tensile strength of unidirectional fiber reinforced composites. A soft interphase is shown to reduce the stress concentration factor, hence increasing the transverse tensile strength of the composite. In order to increase transverse tensile strength of a composite, the interphase modulus should be decreased and/or the thickness of the interphase should be increased. The location for maximum stress concentration factor varies with interphase modulus and thickness, hence the mode and location of failures may be changed by changing these parameters.

RECOMMENDATIONS

It is recommended that micromechanical models, which include the interphase be further developed for composites with initial microcracks or fiber-matrix disbonds. These models can be used to determine: (1) the transverse tensile strength of the composite with existing microcracks or interface disbonds, (2) the effect of the interphase (or coating) on the fracture resistance of the composite, (3) the favorable failure mode (or failure location) by adjusting interphase properties so that the crack propagation can be arrested or slowed down. Also, we recommend that elastic-plastic matrix and interphase behavior should be included in the micromechanical models to determine the combined effects of interphase and material nonlinearity of the matrix on the transverse tensile strength of the composite.

REFERENCES

1. L. B. Greszczuk, "Theoretical and Experimental Studies on Properties and Behavior of Filamentary Composite," Society of Plastics Industry, 21st Annual Conference, Washington, D.C., 1966.

2. D. F. Adams and D. R. Doner, "Transverse Normal Loading of a Unidirectional Composite," Journal of Composite Materials, 1,152, 1967.

3. D. F. Adams, "A Micromechanics Analysis of the Influence of the Interface on the Performance of Polymer Matrix Composites," proceedings of the American Society for Composites, First Technical Conference, 1986.

4. L. T. Drzal, "Tough Composite Materials: Recent Developments," Noyes, Park Ridge, New Jersey, pp. 207-222, 1985.

5. M. R. Piggott, "Polymer Composite," B(5), pp. 291-296, 1987.

6. H. C. Tsai, A. M. Arocho, L. W. Gause, "Prediction of Fiber-Matrix Interphase Properties and Their Influence on Interface Stress, Displacement and Fracture Toughness of Composite Material," Journal of Materials Science and Engineering, A126, pp. 295-304, 1990.

7. D. F. Adams, and S. W. Tsai, "The Influence of Random Filament Packing on the Transverse Stiffness of Unidirectional Composite," Journal of Composite Materials, Volume 3, pp. 368-381, July 1969.

8. H. C. Tsai and A. M. Arocho, "Effect of the Fiber-Matrix Interphase on the Transverse Tensile Strength and Fracture Resistance of Composite Materials," NADC Technical Report, to be published.

9. A. M. Skuda, "Micromechanics of Failure of Reinforced Plastics," Failure Mechanics of Composites, p. 18, Volume 3, Handbook of Composites, North-Holland, 1986.

Table I - Comparison of 1-D and 2-D Analytical Results

($E_f/E_m = 21.3$, $V_F = 0.65$, $t_I/R = .015$)

S.C.F.

Em/EI \ METHOD	1.0	10.0	15.0	20.0	30.0	40.0	50.0
1-D	1.905	1.651	1.582	1.531	1.460	1.410	1.371
2-D	2.019	1.716	1.643	1.592	1.519	1.459	1.412
1-D/2-D	.944	.962	.963	.962	.961	.966	.971

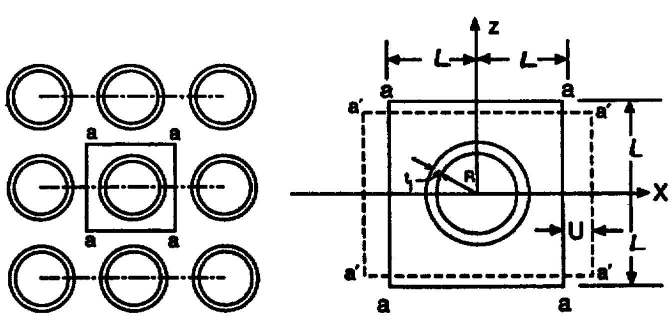

Figure 1 - Fiber Packing Geometries

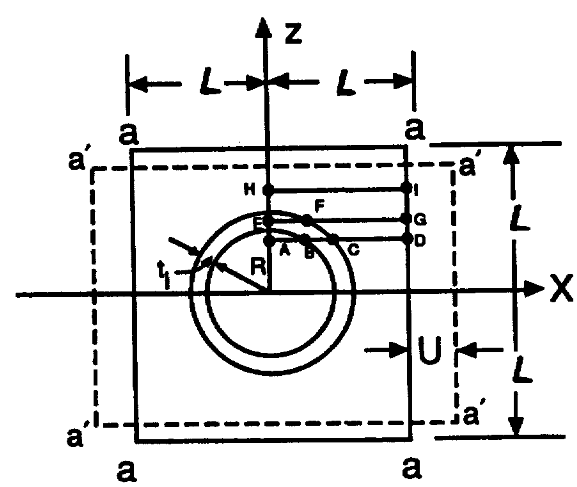

Figure 2 - One-Dimensional Model for the Composite Under Transverse Tensile Loading

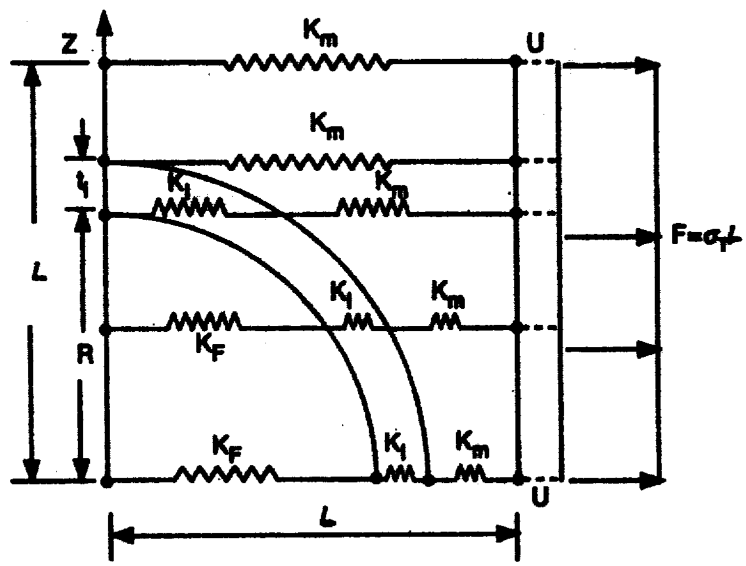

Figure 3 - Basic Concept for One-Dimensional Model

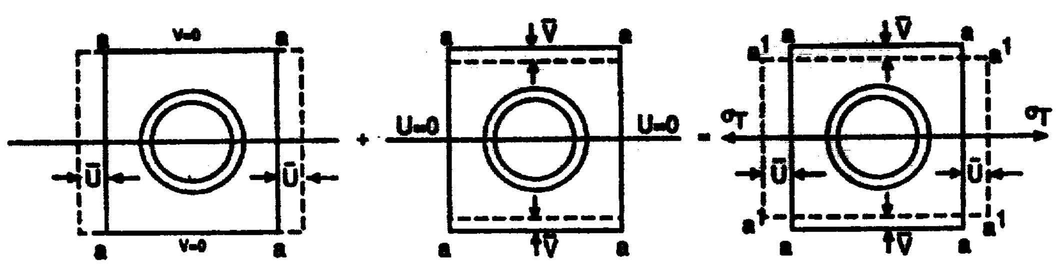

Figure 4 - Basic Concept for Two-Dimensional Model

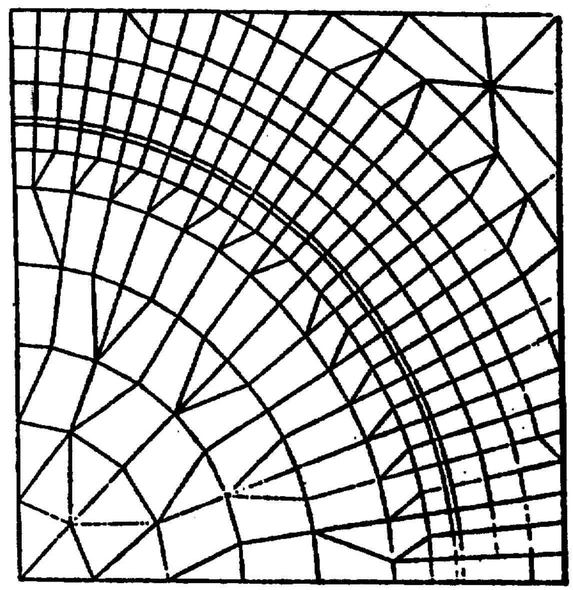

Figure 5 - Two-Dimensional Finite Element Model

A_L/E_p

$E_f = 10.0 \times 10^6$ psi , $\nu_f = 0.31$ $\quad V_f = 0.502$

$E_m = 0.47 \times 10^6$ psi , $\nu_m = 0.36$

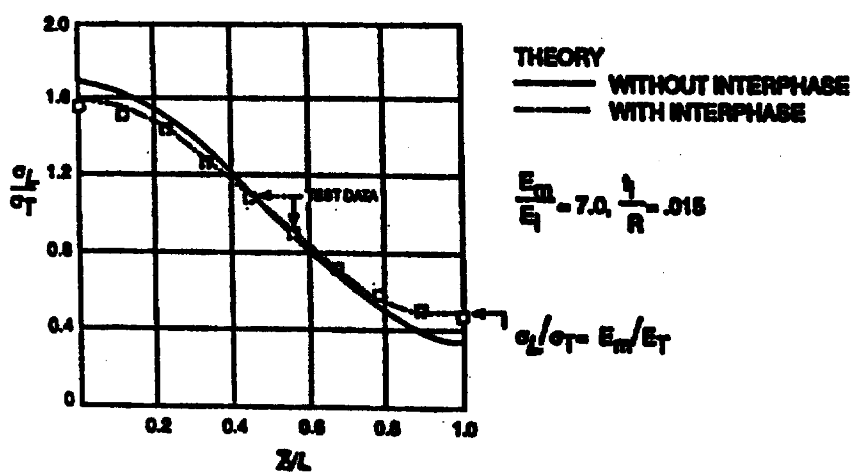

Figure 6 - Transverse Tensile Stress Distribution
Along X = L; Test vs. One-Dimensional Theory

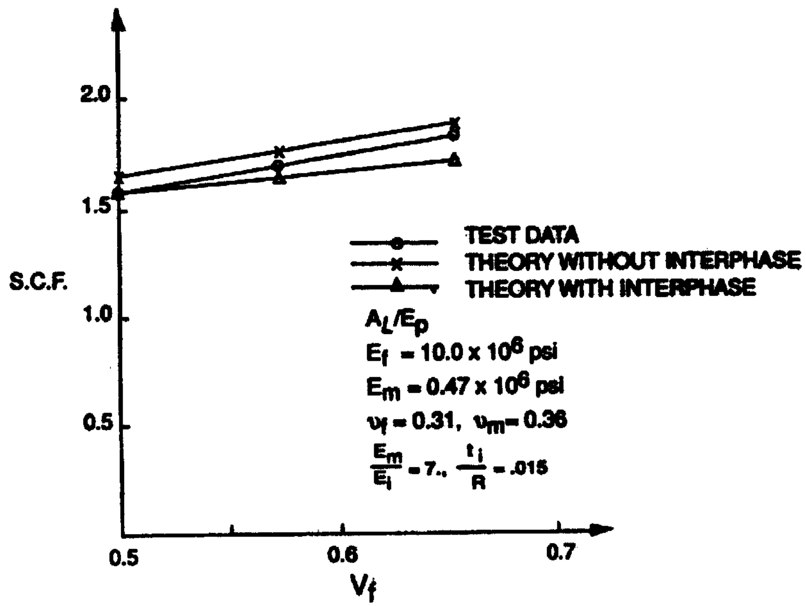

Figure 7 - Maximum Transverse Tensile Stress; Theory vs. Test

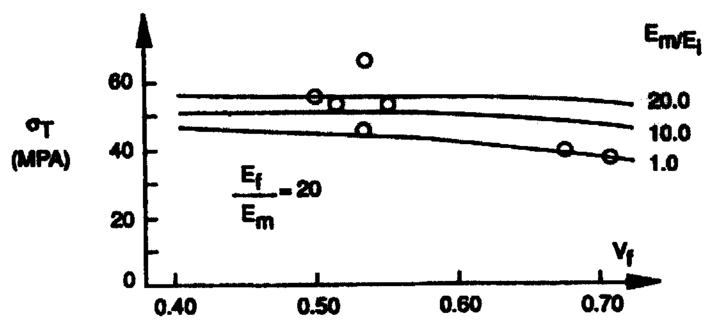

Figure 8 - Correlation of 1-D Theory With Test Data For Gl/Ep Composite

(σ_{mu} = 74.2 MPA, $t_i/_R$ = .0286)

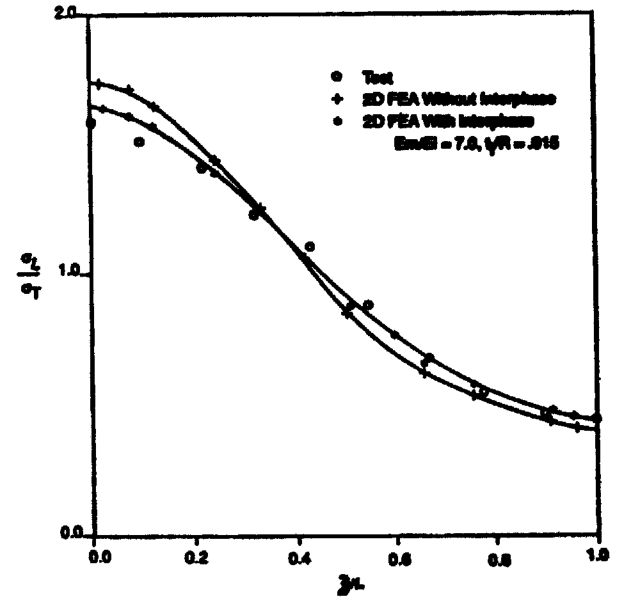

Figure 9 - Transverse Tensile Stress Distribution Theory vs. Test, V_f = .502

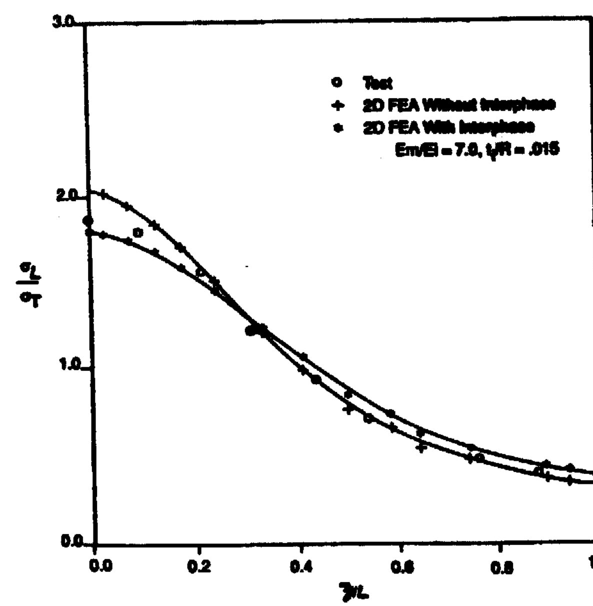

Figure 10 - Transverse Tensile Stress Distribution Theory vs. Test, V_f = .65

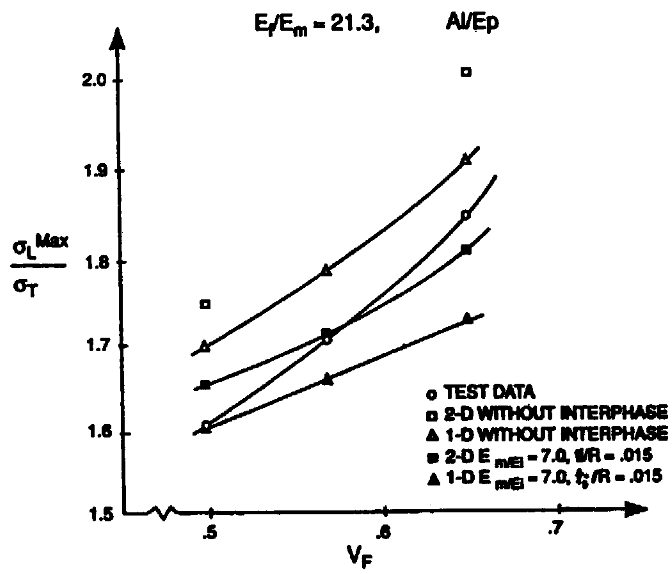

Figure 11 - Maximum Transverse Tensile Stress Theories vs. Test

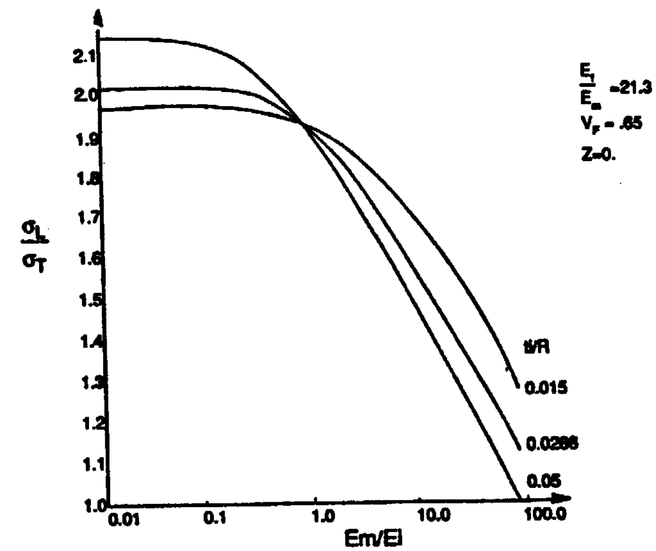

Figure 13 - $\frac{\sigma_L}{\sigma_T}$ vs. $\frac{E_m}{E_i}$ With Various Interphase Thickness

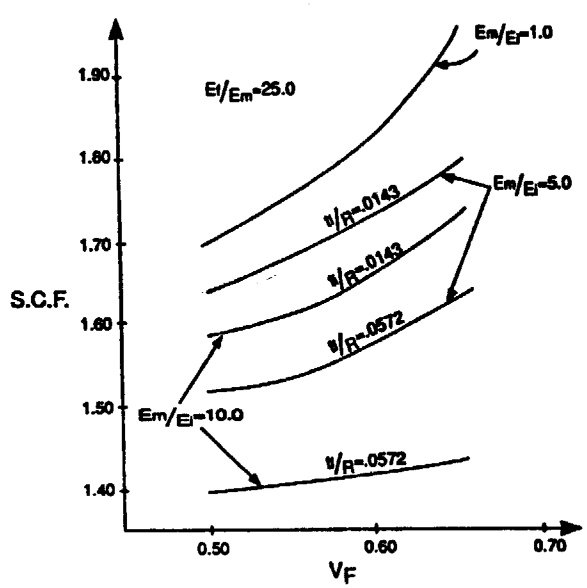

Figure 12 - Effect of Interphase Moduli and Thickness on S.C.F. of the Composite

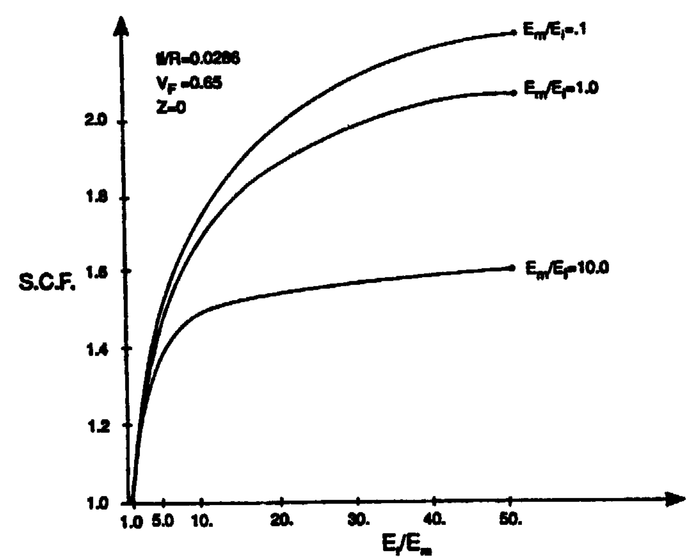

Figure 14 - Effect of E_f/E_m and E_m/E_i on the Maximum Stress Concentration Factor

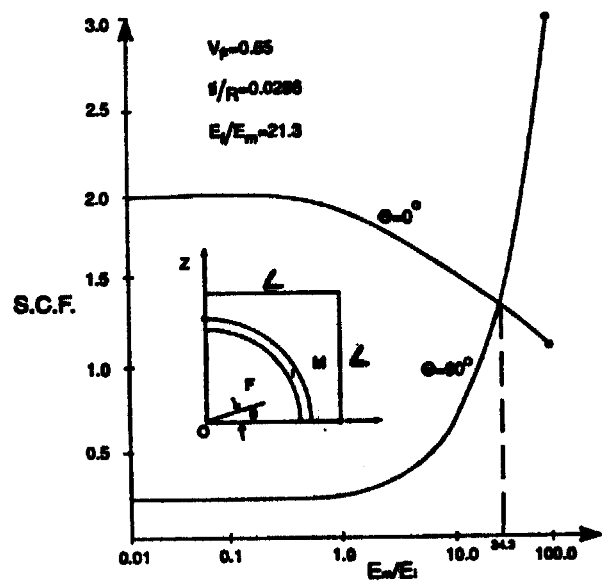

Figure 15 - The Shift of the Critical Location

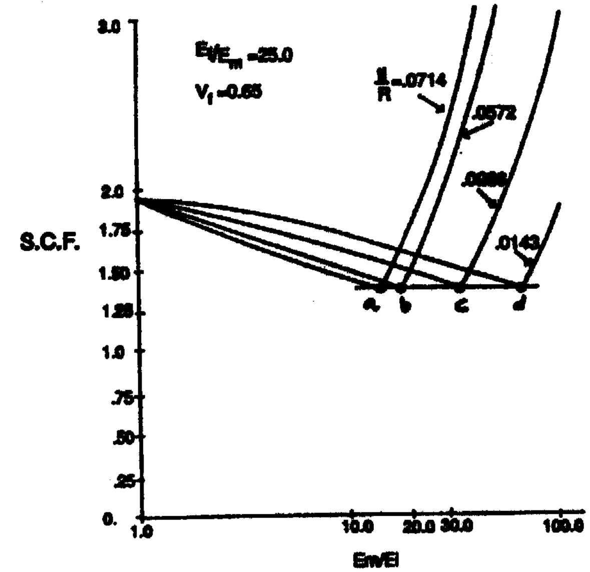

Figure 17 - Critical Stress Concentration Factors As Functions of Interphase Moduli and Thickness

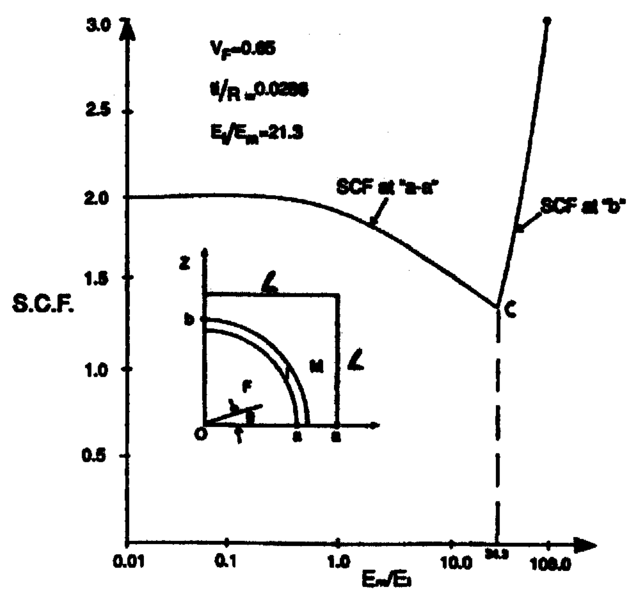

Figure 16 - Critical Stress Concentration Factors As Functions of Interphase Moduli

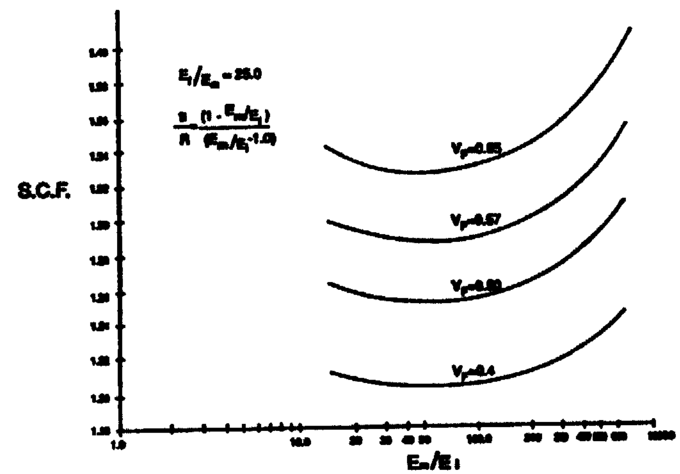

Figure 18 - Design Curves for Optimal Interphase Modulus and Thickness

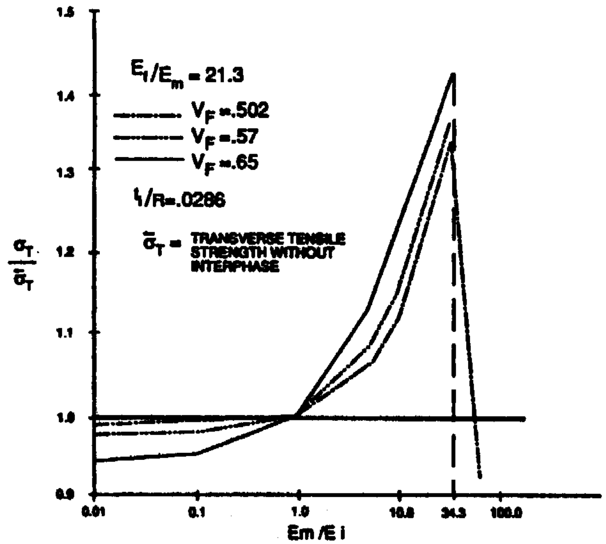

Figure 19 - Effect of Interphase Moduli on the Transverse Tensile Strength of Composite Material

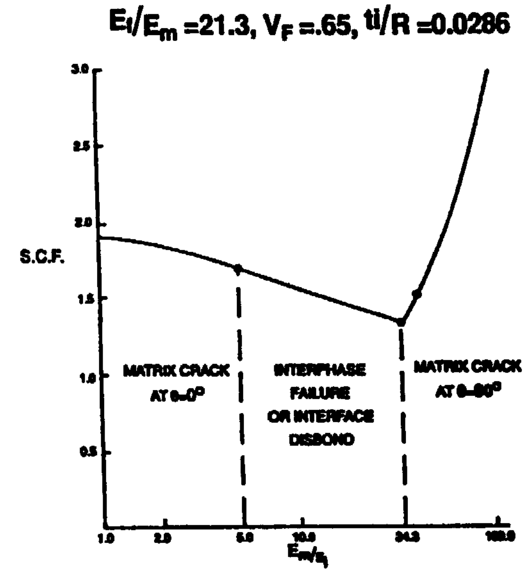

Figure 21 - Design Curve for Transverse Tensile Failure of the Composite Material

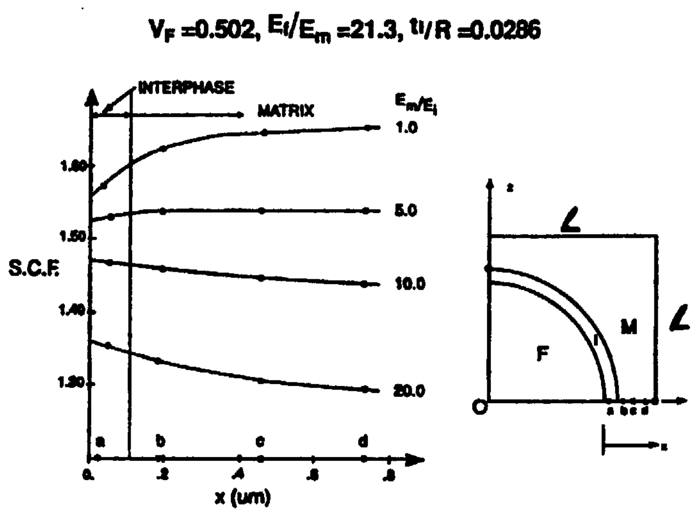

Figure 20 - Stress Distribution in the Interphase and Matrix at Z - 0.0, As Function of Interphase Moduli

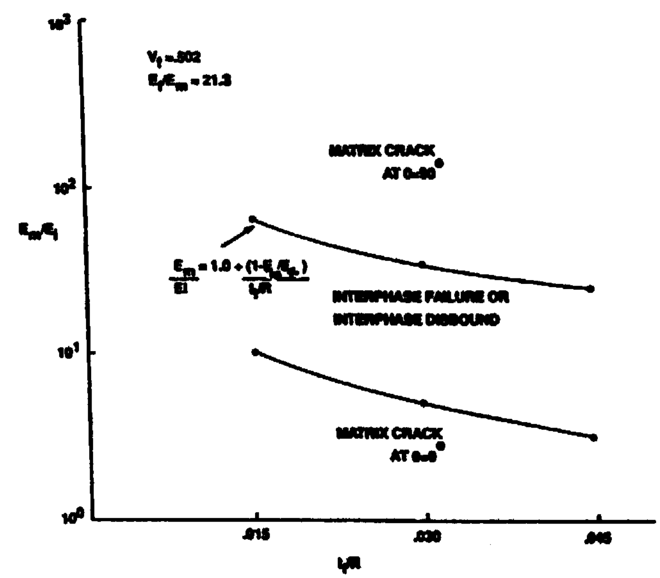

Figure 22 - Effect of Interphase Modulus and Thickness on the Transverse Failure Mode of the Composite

THIS PAGE INTENTIONALLY BLANK