RELIABILITY ANALYSIS OF COMPOSITE STRUCTURES

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ABSTRACT

A probabilistic static stress analysis methodology has been developed to estimate the reliability of a composite structure. Closed form stress analysis methods are the primary analytical tools used in this methodology. These structural mechanics methods are used to identify independent variables whose variations significantly affect the performance of the structure. Once these variables are identified, scatter in their values is evaluated and statistically characterized. The scatter in applied loads and the structural parameters are then fitted to appropriate probabilistic distribution functions. Numerical integration techniques are applied to compute the structural reliability. The predicted reliability accounts for scatter due to variability in material strength, applied load, fabrication and assembly processes. The influence of structural geometry and mode of failure are also considerations in the evaluation. Example problems are given to illustrate various levels of analytical complexity.

INTRODUCTION

Application of composite materials to primary aircraft structures requires proven certification procedures to demonstrate their reliability. The development of certification procedures for primary composite structures must recognize the inherent characteristics of composites. One of these characteristics is the scatter in static strength and fatigue life data. Because of the higher static strength and fatigue life data scatter in composites (as compared to metals), the structural reliability provided by the conventional deterministic certification approach would be different for composite and metallic structures. The variation of static strength reliability with the material data scatter can be seen in Figure 1. In this figure, the material strength distribution is characterized by a two-parameter Weibull distribution. The reliability at design limit load is computed based on a static design with a factor of safety of 1.5 and a margin of safety of 0.0. Reliabilities based on both A-basis and B-basis designs are shown in the figure. As shown in the figure, for a Weibull shape parameter α of 20 (typical for composites) the B-basis reliability is 0.99996832, and at $\alpha = 25$ (typical for aluminum alloys) it is 0.99999583. Although these reliabilities appear to be equally high, significant difference exists in the risk of the

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design or the probability of failure. The risk for composite structures is 3.2×10^{-5} as compared to 4.2×10^{-6} for metals. This example indicates that in order for composite structure to achieve the same level of reliability as metallic structures a probabilistic based reliability analysis methodology is needed. The importance of the probabilistic approach is more obvious when fatigue life of composites is considered, because of the even higher data scatter observed in composite fatigue tests.

In addition to the scatter in strength and life data, another factor that affects the reliability of a structure is the uncertainty in the applied load. These uncertainties are conventionally covered by a factor of safety. A 1.5 factor of safety, traditionally used in aircraft structural design, generally provides a very high level of reliability. However, this deterministic approach cannot be used to assess the risk involved in a structural design. Thus, trade studies cannot be conducted to optimize the risk of failure. A more desirable approach is the probabilistic approach that utilizes the statistical distribution of the applied loads and the material strength. The probability of failure can be computed by integrating the overlapping region of these two distributions. Using this technique, the risk of failure of a structure can be estimated and the risk can be minimized within the limits of cost and performance needs. The conventional factor of safety approach and the probabilistic approach are illustrated in Figure 2.

There have been several efforts to develop reliability analysis methods for composite structures (References 1 through 7). The work in Reference 1 concentrated on the evaluation of certification approaches relating to structural reliability. An integrated impact damage tolerance reliability analysis method was proposed in Reference 2. Attempts were made in Reference 3, 4 and 5 to statistically characterize the applied load distribution. References 6 and 7 proposed a micromechanics approach to characterize the scatter in the mechanical properties.

In this paper, a comprehensive probabilistic static stress analysis methodology is proposed. The methodology integrates structural mechanics methods, scatter in material properties, uncertainties in applied loads and the variability in fabrication and assembly processes into a single analysis package to estimate the reliability of a composite structure. The various levels of analytical complexity are illustrated by example problems.

ANALYSIS APPROACH

Structural reliability analysis methods depend on the structural configuration and the anticipated mode of failure and, therefore, are problem specific. In this section, a general approach is first outlined. Example problems are then used to illustrate various levels of analytical complexities.

The general approach for reliability analysis of composite structures can be summarized below.

1. Select static analysis method

2. Identify scatter parameters

3. Characterize applied load distribution statistics

4. Compute structural reliability

Closed form stress analysis methods are the primary analytical tools used in the present methodology. Static stress analysis provides results that describe the general response of a structure in response to applied loads. In addition, for reliability analysis, static analysis methods can also be used to identify independent variables and failure modes. Only those variables whose scatter significantly affects the performance of the structure are selected for reliability analysis. These variables are then statistically characterized based on experimental data. The probabilistic distributions of the structural parameters are finally incorporated into the static stress analysis method for reliability computations.

The macromechanics approach recommended in Reference 1 is used here for scatter parameter characterization. This approach is selected over the micromechanics approach proposed in References 6 and 7 because it significantly reduces the number of primitive variables. Furthermore, extensive database at the macromechanics level exists in the literature to verify the analysis.

A second data item that requires statistical characterization is the applied loads. In the case of existing aircraft types, the basic source of loads data is the flight loads recorder data generated from the aircraft type. The loads data are generally available in form of exceedance function. The exceedance functions must be transformed into a probabilistic function for reliability analysis. Attempts were made in References 3, 4 and 5 to fit the exceedance function into a probabilistic function.

The exceedance data are fitted into a two-parameter Weibull distribution in References 3 and 5. The Weibull shape parameter for air combat maneuvers missions for the F-16 wing structures is found to be between 8 and 10 in Reference 5. In Reference 3, the cumulative probability of exceeding a given load in one lifetime was defined by a Weibull function with a shape parameter of 6 and a 0.001 probability of exceeding design ultimate load (DUL). This distribution of applied load is shown in Figure 3. Both the cumulative probability of exceedance and the probability density are shown in the figure. As shown in the figure, the probability that an applied load exceeds the design limit load (DLL) in the lifetime of an aircraft is approximately 0.55. The figure also indicates that during the lifetime of the aircraft, the most frequently occurred (modal) load is approximately 1.05DLL. Such a load distribution may be reasonable for a military aircraft but rather severe for a civil airplane. A less severe distribution has a Weibull shape

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parameter of two and retains the assumption that there is a 0.001 chance of an applied load exceeding DUL. This distribution gives approximately 0.05 probability that the flight load exceeds DLL and the modal load is approximately 0.4DLL. This approach is used here for applied load distribution characterization.

Once the key structural variables and the applied load are statistically characterized and the probabilistic distributions are incorporated into the selected stress analysis method, the structural reliability can be evaluated numerically. The principle of reliability analysis was shown in Figure 2. The risk or probability of failure is assessed by integrating the area of the shaded region of the joint probability distribution shown in the figure. However, in reality, more than two random variables are encountered in the analysis, and these variables may or may not be totally independent. Special numerical techniques are required for these analyses. Several reliability algorithms are available (References 1, 3-5 and 8-11). In References 3-5 the reliability or the probability of failure is obtained by direct integration of the joint probability function as shown in Figure 2. This approach is used in the present paper.

Structures With Single Variable and Single Failure Mode

As a first example, consider a simple tensile composite element with cross-sectional area A and axial Young's modulus E subjected to axial load P. The element is designed based on B-basis allowable strain (ϵ_{ALL}) with zero margin of safety at design ultimate load (DUL) and the factor of safety is 1.5. That is $\epsilon_{DUL} = \epsilon_{ALL}$ and $\epsilon_{DLL} = \epsilon_{ALL}/1.5$. Assuming that the design allowable is derived from a two-parameter Weibull distribution of strength data, from the definition of the B-basis allowables the Weibull scale parameter (with a 95% confidence) is given by

$$\beta = \frac{1.5}{[-\ln(0.9)]^{1/\alpha}} \tag{1}$$

Notice that the strains are normalized with respect to the design limit load (DLL). The value of α is determined from statistical analysis of the material strength data. For the commonly used graphite/epoxy composite $\alpha = 20$ is a reasonable estimate (Reference 1). The reliability of the structure subjected to a discrete applied load P is then given by

$$R = Exp\left[-\left(\frac{x}{\beta}\right)^{\alpha}\right] \tag{2}$$

where $x = (P/AE)/\epsilon_{DLL}$ is the normalized strain. The reliability of the structure subjected to a distributed load over the lifetime of the structure is

$$R = 1 - \int_0^\infty f(\epsilon) \, F(P) \, dP \tag{3}$$

where

F(P) is the probability of occurrence of the applied load at level P.

It should be noted that the integral on the right-hand side of equation (3) is the risk or the probability of failure. The integral is evaluated by numerical integration. The reliability of the structure thus evaluated is shown in Figure 4. The load distribution used in this evaluation is based on the assumptions used in Reference 3, i.e. the load distribution is described by a two-parameter Weibull distribution and the probability for applied load exceeding DUL is 0.001. The figure shows the reliability of the tensile element with a 95% confidence. As shown in the figure the reliability is generally higher than 0.99. The figure also indicates that the reliability increases with material strength shape parameter (α_s). However, based on the load distributions assumed, the reliability decreases as the load shape parameter (α_L) increases.

The influence of the factor of safety (FS) on the reliability of the structure was examined for this simple structure. The results are shown in Figure 5. These results were obtained for α_s - 20 and α_L - 6. The figure shows that the reliability increases rapidly with FS for FS less than 1.4. The risk or the probability of failure (P_f) is plotted in terms of FS in the insert of Figure 5. The figure shows that the FS increases linearly with the negative order of magnitude of the risk. This trend indicates that the weight increases at an exponential rate with reducing risk. Also shown in Figure 5 is the reliability of the structure operated under the discrete applied DLL. It can be seen that this reliability is significantly higher than that predicted by distributed load.

Structures With Multiple Variables and Single Failure Mode

The reliability evaluation of an open hole element is a second example for illustration purposes. The reliability computation in this example is more complicated because of the increased complexity of the stress analysis procedure and, therefore, more scatter variables involved. The strength prediction of an open hole under uniaxial tension loading has been investigated by many authors. For illustrative purposes, the average stress criteria suggested in Reference 12 is selected here.

In Reference 12 the static strength of an orthotropic plate with a circular hole under uniaxial tensile loading can be approximated by

$$\epsilon_f - \epsilon_o \cdot 2(1 - \xi)/[2 - \xi^2 - \xi^4 + (K - 3)(\xi^6 - \xi^8)] \qquad (4)$$

where

ϵ_f is the static failure strain of the plate with a hole

ϵ_o is the unnotched failure strain of the laminate

K   is the stress concentration factor

$\xi = R/(R+a_0)$ with R is the radius of the hole and a_{op} is a characteristic length

Equation (4) states that the plate failure occurs when average strain (stress) within the region characterized by a_0 reaches the unnotched strength of the laminate. Equation (4) suggests that the reliability of the hole strength can be evaluated in two separate parts: the unnotched strength and the hole quality. The unnotched strength (ϵ_0), as can be seen from equation (4), is independent of the hole geometry and quality. The reliability contributed by this part can be treated in the same manner as in the previous example. The second part or the hole quality is more complex because of the number of variables involved. These variables are: hole size (R), elastic properties (K is a function of the elastic moduli in an orthotropic plate) and the characteristic length (a_0). For the purpose of this illustration, the hole size and the elastic properties are assumed to be within the design tolerance limits. Thus, the influence of these variables on the reliability is negligible. This is a reasonable assumption because small variations in hole size and moduli have a negligible effect on the strength scatter. The major contributing parameter for the strength scatter is the hole quality characterized by a_0.

By considering only the scatter of the unnotched strength and the characteristic length in the reliability evaluation, the probability that a hole strength exceeds a certain strain ϵ_f is then given by

$$p(\epsilon_f) = \text{Exp}\left[-\left(\frac{\epsilon_0}{\beta_s}\right)^{\alpha_s}\right] \cdot \text{Exp}\left[-\left(\frac{a_0}{\beta_a}\right)^{\alpha_a}\right] \tag{5}$$

In equation (5), both the unnotched strength and the characteristic length distributions are assumed to be Weibull. Equation (5) was used, together with the loads distribution and numerical integration technique described previously, to evaluate the reliability of a structure with an open hole. The results are shown in Figure 6. The figure shows the results for a plate with a 1/4 inch diameter hole with a stress concentration factor of 4.75. Three values of average characteristic length (a_0) are shown; they are 0.10, 0.12 and 0.15 inch. The value of a_0 used in the design analysis was 0.10 inch, which gave a design notch strength of $0.545\epsilon_{ALL}$ at DUL. The lifetime reliability of an unnotched structure based on the same load distribution is 0.99848. Figure 6 shows that the hole quality scatter may significantly affect the reliability of the structure when the scatter in hole quality is large (small α_a).

Structures With Competing Failure Modes

A third example here illustrates the effects of competing failure modes on the reliability of a structure. In general, the probability of failure for a structure with N failure modes is given by

$$P_f = P_{f1} + P_{f2} + \ldots\ldots + P_{fN} = \sum_1^N P_{fi} \leq 1.0 \tag{6}$$

where p_{fi} is the probability of failure due to ith failure mode.

The reliability is then

$$R = 1 - p_f \geq 0.0 \tag{7}$$

Consider a beam-column with compression and buckling as two competing failure modes. The beam-column is designed to buckle at DUL and the prebuckling strain at DUL equals the compression design allowable ϵ_{ALL}, which is derived from a Weibull distribution. Then the probability of failure at applied strain ϵ due to compression is

$$p_{fc} = 1 - Exp\left[-\left(\frac{\epsilon}{\beta_s}\right)^{\alpha_s}\right] \tag{8}$$

The buckling strain is

$$\epsilon_{cr} = k \frac{(2\Pi)2}{L} \frac{t^2}{12(1-\upsilon_{xy}\upsilon_{yx})} \tag{9}$$

where

L   is the length of the beam

t   is the thickness of the beam

υ_{xy} and υ_{yx} are the Poisson's ratios

k   is a coefficient depending upon the boundary condition

The scatter variables in equation (9) are k, L, t, υ_{xy} and υ_{yx}. In the present example, only the thickness variation is considered. Then the probability of failure, at applied strain ϵ, due to buckling is given by

$$p_{fb} = 1 - Exp\left\{ - \left[\frac{1}{\beta_t} \frac{\sqrt{12\ (1-\upsilon_{xy}\upsilon_{yx})}}{k}\epsilon\ \left(\frac{L}{2\Pi}\right) \right]^{\alpha_t} \right\} \tag{10}$$

where α_t and β_t are the Weibull shape and scale parameter for the thickness distribution.

The results of a clamped beam-column are shown in Figure 7. The beam-column is designed for $\epsilon_{DUL} = 0.009$ ($= \epsilon_{ALL}$) with a factor of safety of 1.5. The coefficient k = 1.030629 for clamped ends. Figure 7 shows the results for a fixed strength scatter ($\alpha_s = 20$) and variable load and thickness scatters. The nominal thickness of the beam is 0.132 inch. The reliability of the beam without taking buckling into consideration is also shown in the figure. The

results indicate that the reliability may be significantly influenced by the buckling failure at higher α_L and lower α_t. In the practical range (α_L between 2 to 6 and α_t approximately 40) the reliability for compression failure only ranges from 0.99848 to 0.99951. These values reduced to 0.98942 to 0.99812 when buckling is considered as a competing failure mode.

SUMMARY

A probabilistic static stress analysis and structural reliability prediction approach have been outlined. This approach uses conventional structural mechanics methods and associates probabilistic distributions with the most significant variables. In addition, the uncertainties in the applied loads is also considered in the structural reliability evaluation. The level of complexity in the reliability analysis depends on the complexity of the structure and the number of competing failure modes. Therefore, the actual analysis method is structure specific. However, the overall approach outlined in this paper is generic and can be applied to composite structures.

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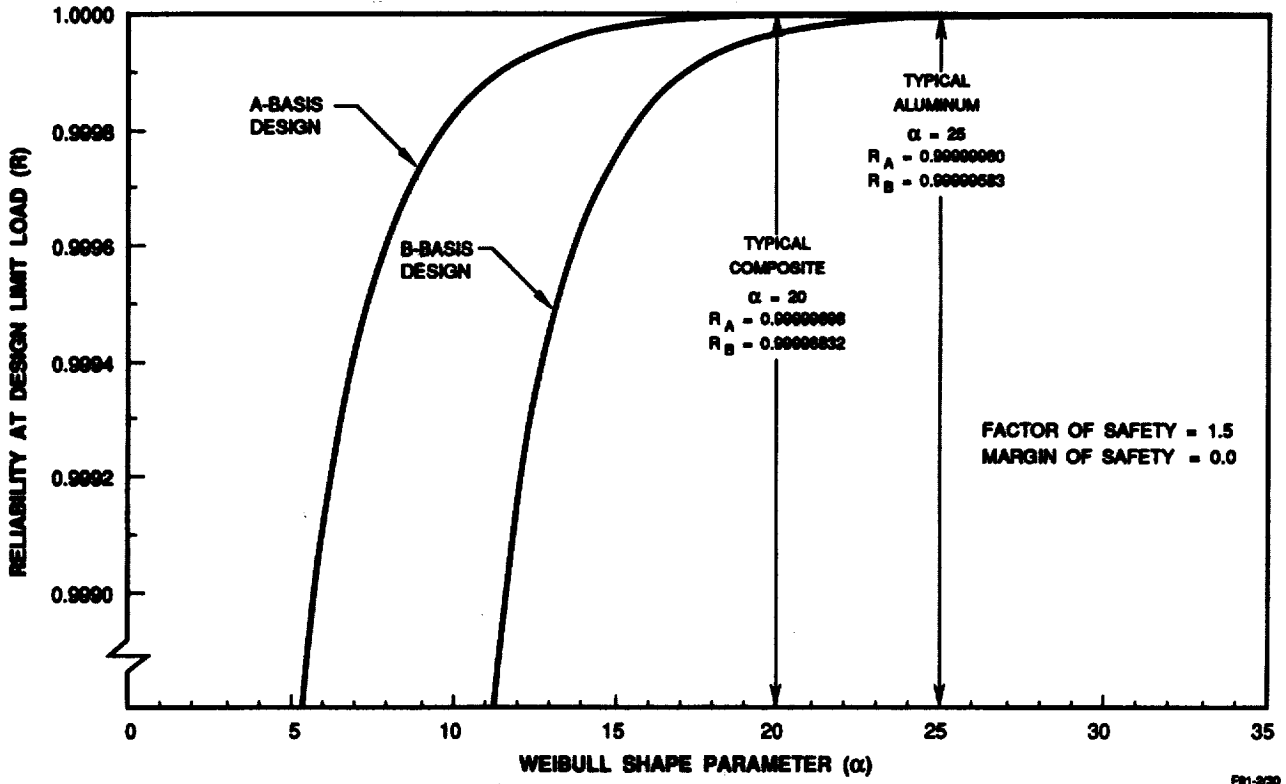
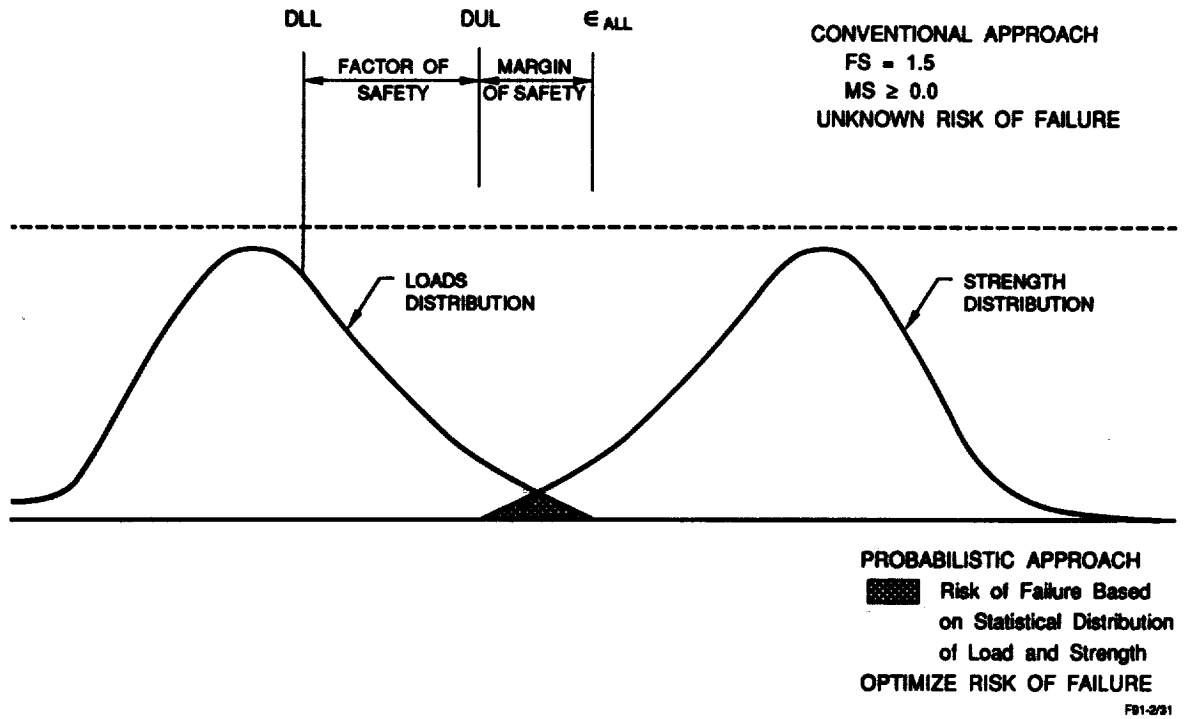Figure 1. Variation of Structural Reliability With Strength Scatter.

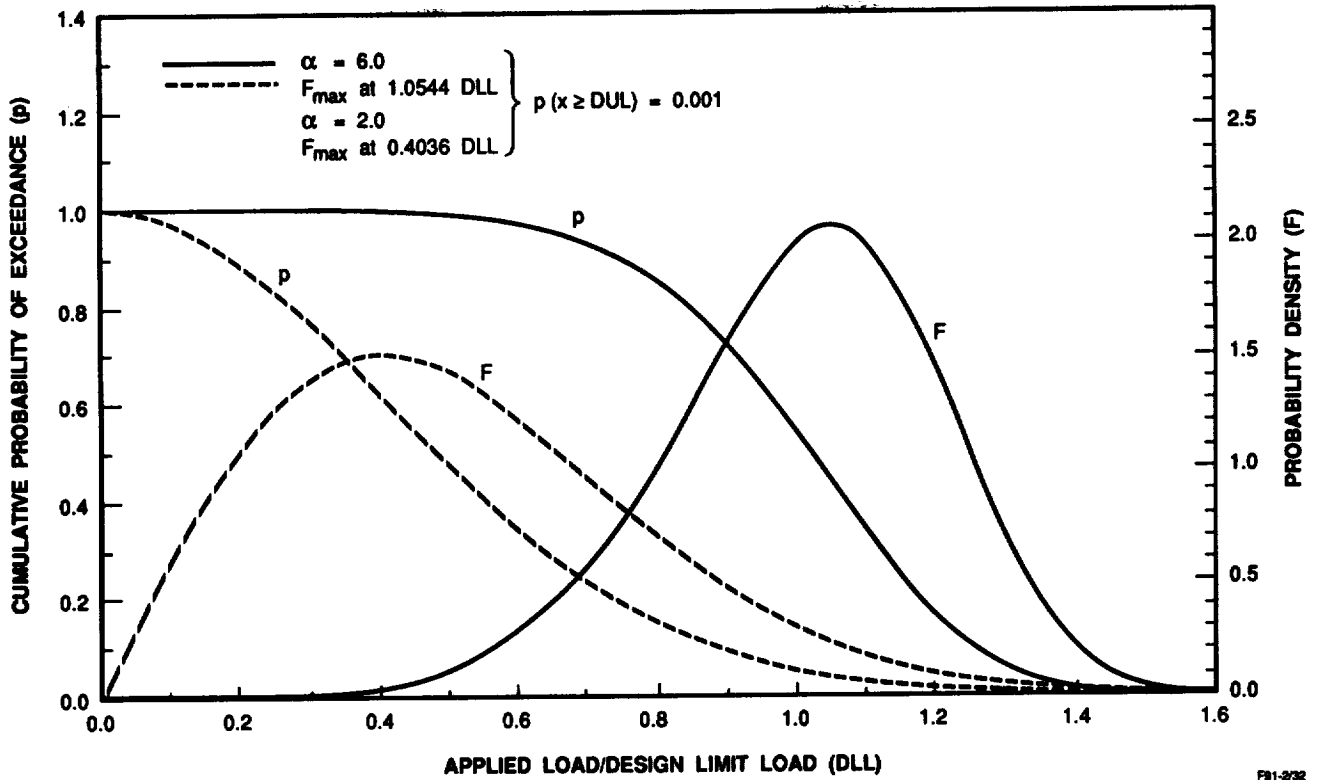Figure 2. Static Strength Certification Approaches.

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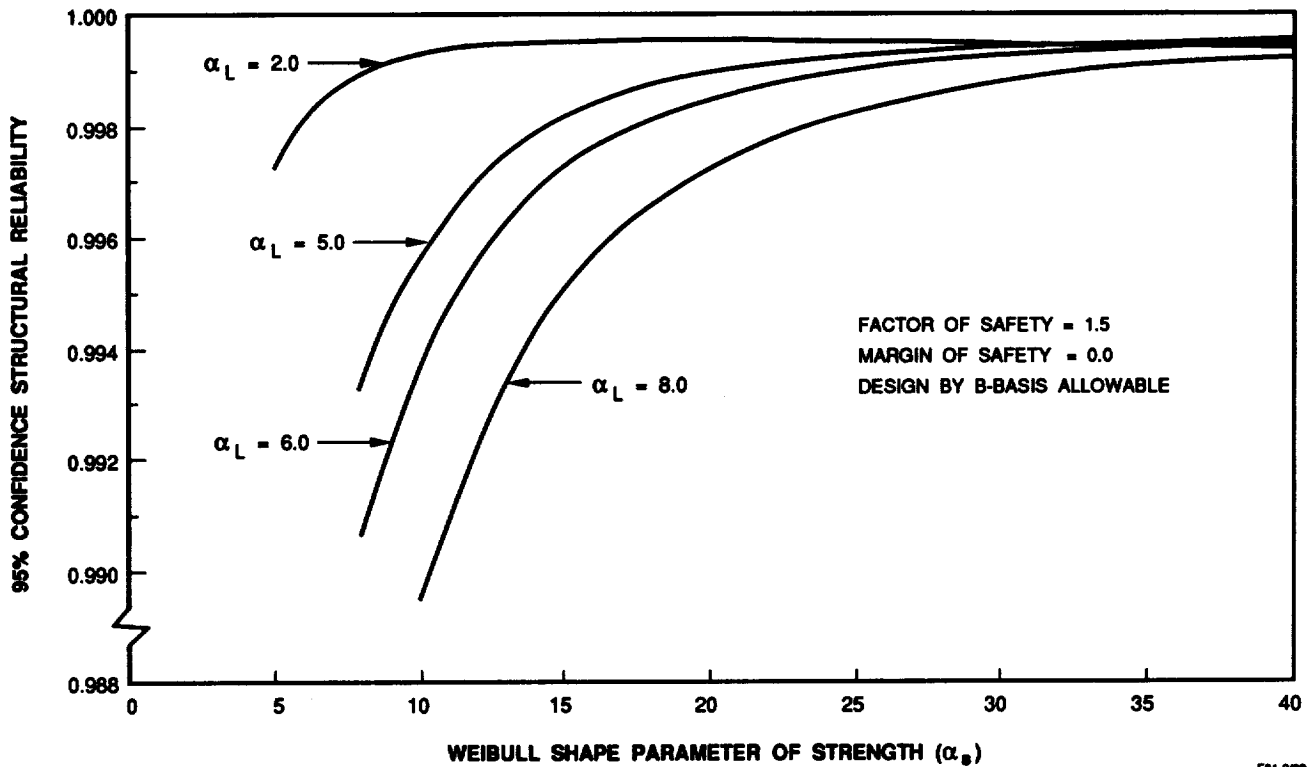Figure 3. Probabilistic Distribution of Applied Loads.

Figure 4. Influence of Weibull Shape Parameter on the Reliability of a Tensile Element.
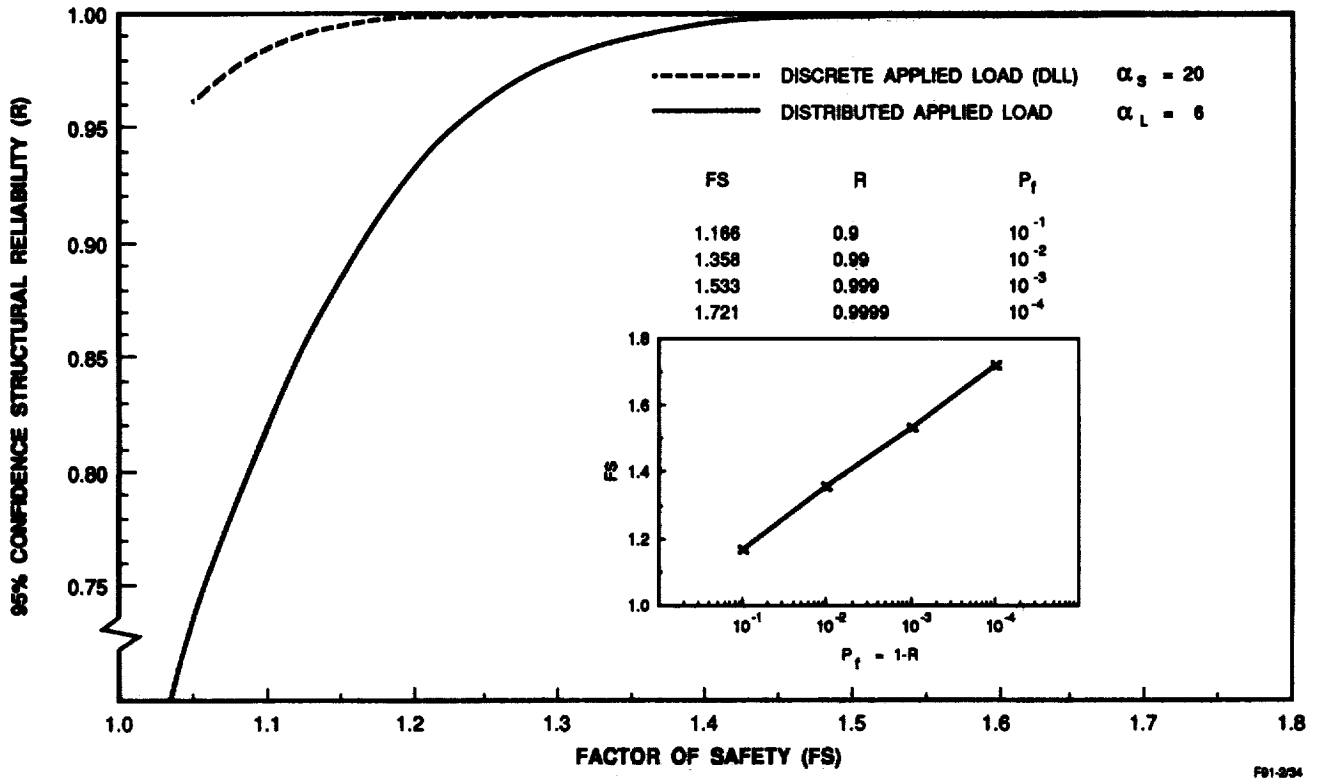
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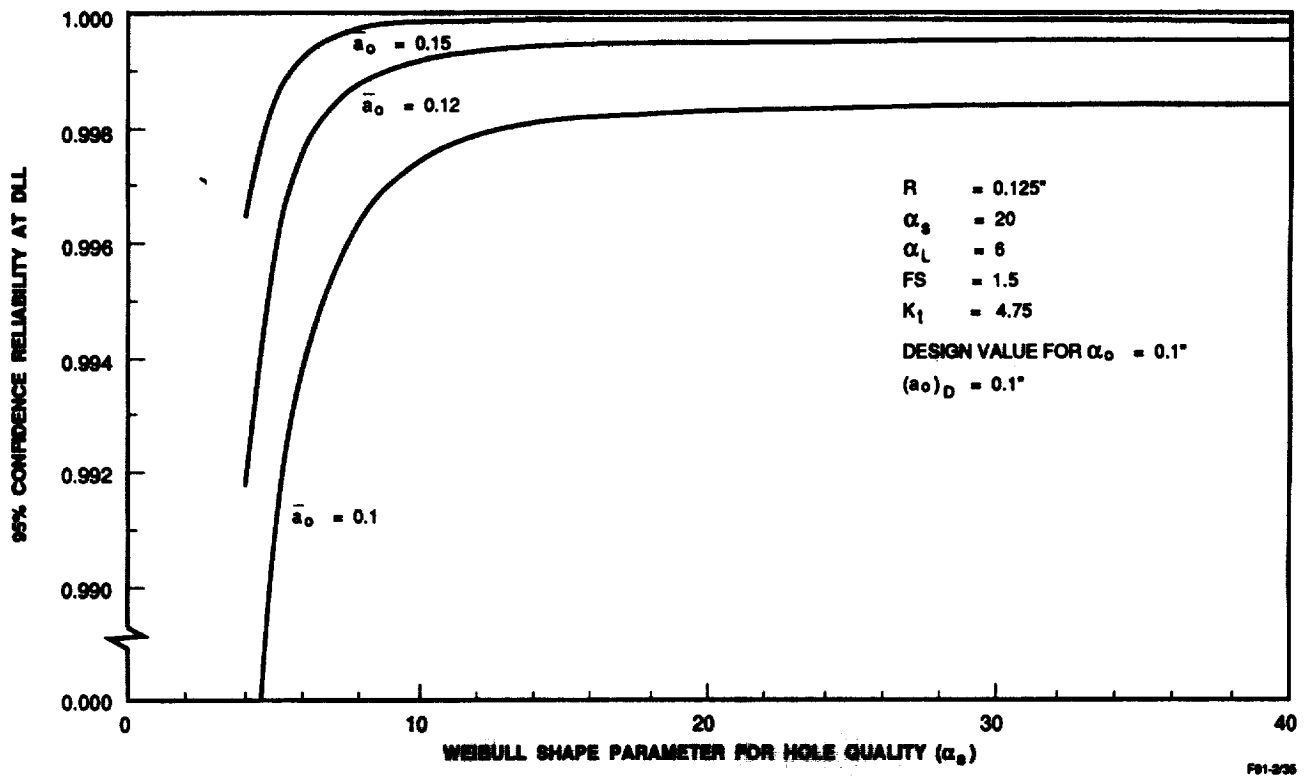Figure 5. Influence of Factor of Safety on the Reliability of a Tensile Element.

Figure 6. Influence of Hole Quality Scatter on the Reliability of a Structure With an Open Hole.
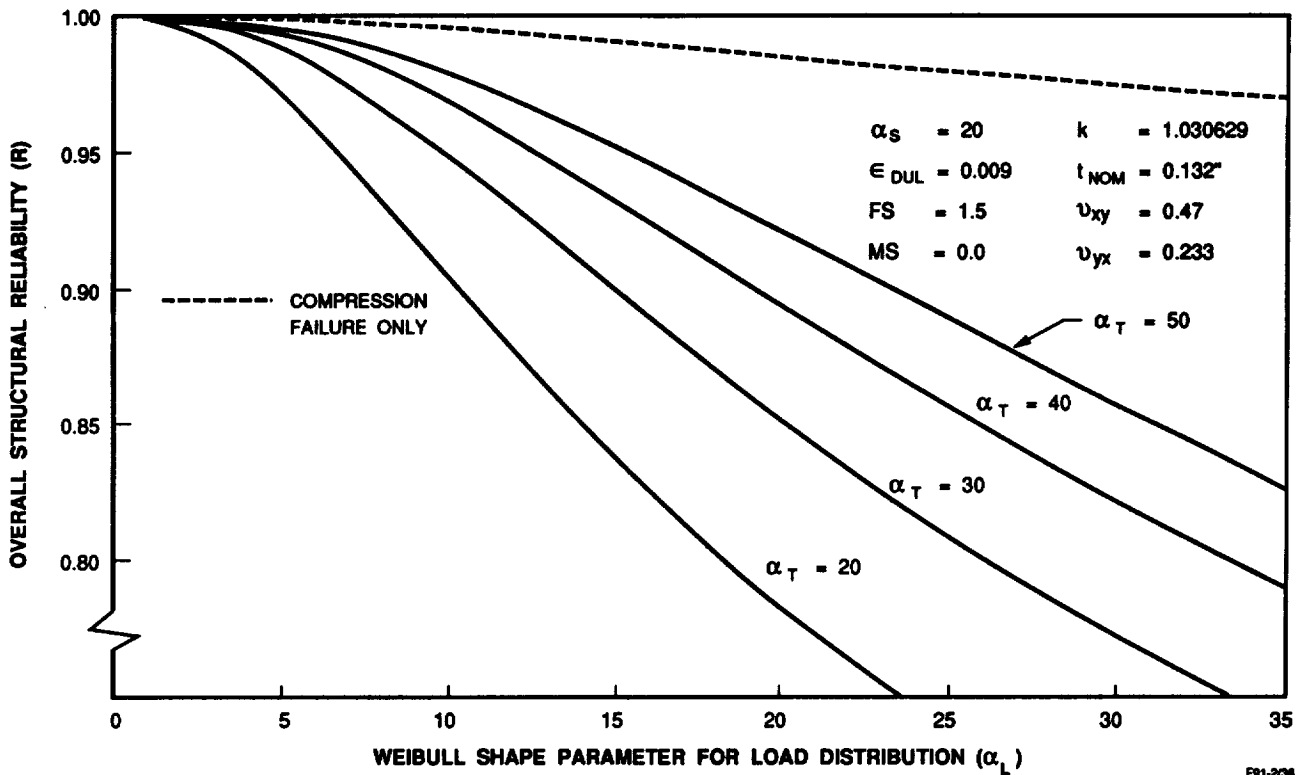
Figure 7. Structural Reliability of a Beam-Column With Compression and Buckling as Competing Failure Modes.

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