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Testing the Performance of the Feedback Concatenated Decoder With a Nonideal Receiver

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One of the inherent problems in testing the feedback concatenated decoder (FCD) at our operating symbol signal-to-noise ratio (SSNR) is that the bit-error rate is so low that we cannot measure it directly through simulations in a reasonable time period. This article proposes a test procedure that will give a reasonable estimate of the expected losses even though the number of frames tested is much smaller than needed for a direct measurement. This test procedure provides an organized robust methodology for extrapolating small amounts of test data to give reasonable estimates of FCD loss increments at unmeasurable minuscule error rates.

Using this test procedure, we have run some preliminary tests on the FCD to quantify the losses due to the fact that the input signal contains multiplicative non-white non-Gaussian noises resulting from the buffered telemetry demodulator (BTD). Besides the losses in the BTD, we have observed additional loss increments of 0.3 to 0.4 dB at the output of the FCD for several test cases with loop signal-to-noise ratios (SNRs) lower than 20 dB. In contrast, these loss increments were less than 0.1 dB for a test case with the subcarrier loop SNR at about 28 dB. This test procedure can be applied to more extensive test data to determine thresholds on the loop SNRs above which the FCD will not suffer substantial loss increments.

I. Introduction

Thus far, the feedback concatenated decoder (FCD) has only been tested with signals corrupted by pure additive white Gaussian noise (AWGN). In reality, the FCD takes input from the output of a receiver, such as the buffered telemetry demodulator (BTD), which contains multiplicative non-Gaussian noise. The FCD is composed of a Viterbi decoder (VD) and a Reed–Solomon (RS) decoder, as shown in Fig. 1. The RS decoder decodes four different types of codewords with different error correction capabilities: $E = 47, 30, 15, 5$. In each eight-codeword frame, the single codeword with the highest correctability, $E = 47$, is decoded first. This decoded word is passed back to the Viterbi decoder, which redecodes its data utilizing the new information. Then the RS decoder is able to decode the single codeword with the next highest correctability, $E = 30$, and it feeds this word back to the Viterbi decoder for another redecoding. At the next stage, the two codewords with correctability $E = 15$ are decoded and finally, after one more Viterbi redecoding, the RS decoder decodes the final four codewords with correctability $E = 5$.

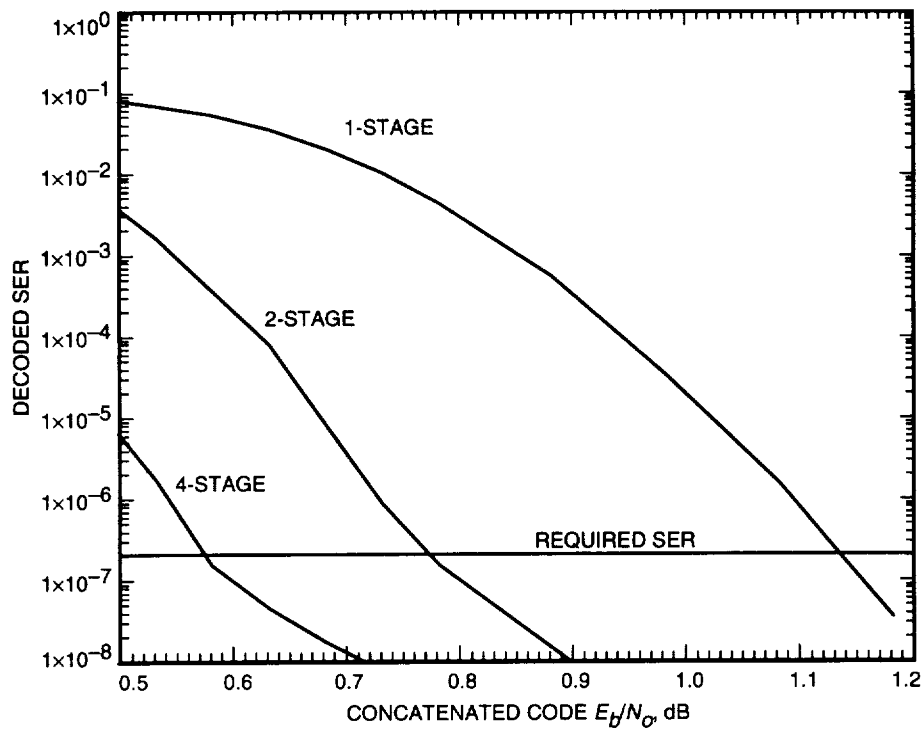

Fig. 8. Performance curves for one-, two-, and four-stage decoding with depth-8 interleaving and near-optimum redundancies.

beyond the requirement if further E_b/N_o is supplied. The same effect is evident but less noticeable for two-stage decoding. For one-stage decoding, the performance curve takes the traditional convex shape. The four-stage performance curve plunges most rapidly to the required SER level, reaching that point 0.56 dB more cheaply than one-stage decoding and 0.17 dB more cheaply than two-stage decoding. On this basis, the Galileo project selected four-stage decoding as the system for maximizing the possible data return.

References

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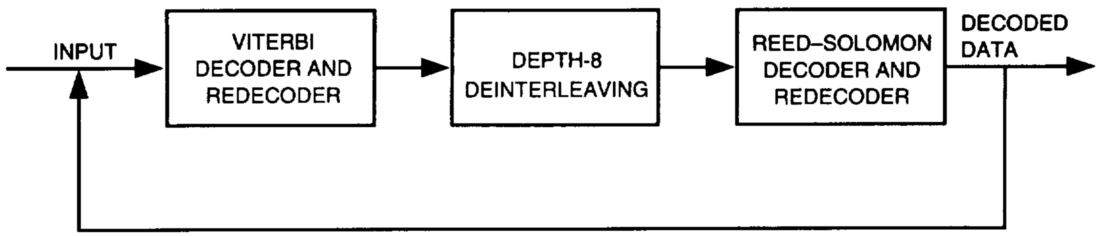

Fig. 1. The FCD structure.

Since the metrics in the Viterbi decoder are designed to be optimum for AWGN, they are not the optimum metrics for the actual BTD output; hence, there are additional losses in the Viterbi decoder. Similarly, the predicted performance of the four-stage RS decoder [1] is based on the error burst characteristics of a Viterbi decoder decoding symbols corrupted by pure AWGN and, hence, will be different for the actual BTD output. There are no analytical techniques for characterizing these losses; therefore, simulations are used to characterize the additional losses in the FCD due to the nonideal receiver (BTD).

The required error rate at the output of the FCD is extremely low. The required bit error rate (BER) is 10^{-7}, which corresponds to an 8-bit RS symbol error rate of 2×10^{-7} and an RS codeword error rate of 10^{-5} to 10^{-6}. To directly measure the FCD error rate, we would need to simulate several million codewords, which would take thousands of days using the current computing systems. In this article, we propose a test procedure that estimates FCD performance to the 10^{-7} level by applying sensitive extrapolation techniques to measurable hypothetical error rates for weaker RS codes (i.e., codes with lower correctability) within the same family of codes as the four actual RS codes used in the FCD.

II. Test Setup

We first generated an encoded data stream and modulated it with a suppressed carrier near baseband and four harmonics of a subcarrier (upper and lower sidebands) also at almost baseband. We then added white Gaussian noise to the modulated data and used the result as a test signal. Next we ran this test signal through the BTD, and at the BTD output, we measured the symbol error rate by comparing the hard symbols to the known test symbols. From this error rate, we computed the corresponding symbol signal-to-noise ratio (SSNR or E_s/N_o), assuming AWGN. We also made a second SSNR measurement from the split-symbol signal-to-noise ratio estimator built into the BTD. Finally, we fed the soft symbols obtained from the BTD to the FCD.

We decided to include the BTD in the test setup instead of modeling the BTD output with symbols containing multiplicative noises with a Tikhonov distribution. The reason is that the Tikhonov distribution is an appropriate assumption only for first-order loops, whereas the BTD actually uses second- or third-order tracking loops whose phase noise distribution is unknown.

We looked at the decoded output of the FCD and discarded any undecodable data before the receiver was in lock. From the in-lock decoded output, we counted how many 8-bit RS symbols the RS decoder corrected in each of its four stages of decoding. From the histogram of the numbers of corrected symbols, we estimated the performance of both the Viterbi decoder and the Reed–Solomon decoder in each decoding stage, and we used these measurements to estimate additional losses that show up at the output of the FCD but are not apparent at the output of the BTD. The analysis method for obtaining these estimates is described in the next section.

The test setup is shown in Fig. 2. This setup consists of a random information-bit generator, a carrier-subcarrier modulator, an AWGN generator, a receiver (BTD), and a decoder (FCD). The test signal does not have filtering effects on it; hence, it can be generated at a high speed. The speed is crucial in this case, since hundreds or thousands of frames need to be generated in a reasonable amount of time.

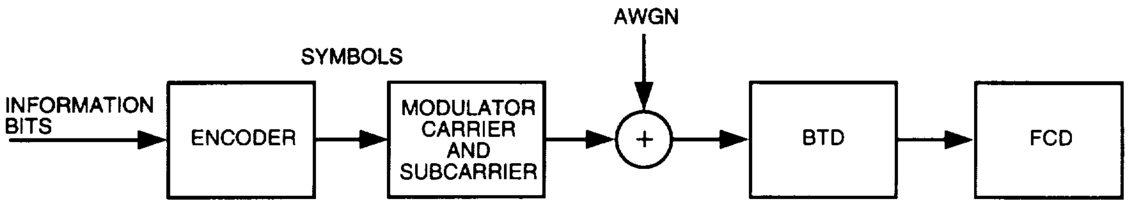

Fig. 2. Test setup.

The input to the receiver (BTD) is an encoded symbol stream on a suppressed carrier and the first four harmonics of a square-wave subcarrier at almost baseband. The size of the losses depends on the SSNR and the parameters of the carrier, subcarrier, and symbol synchronization loops, such as loop bandwidths and window sizes.

For our tests, we arbitrarily picked six sets of typical receiver parameters as examples, designated as cases A through F. In the first two cases, the SSNR is chosen to be −5 dB, which is a typical value in operation slightly above the design threshold where decoder errors are very rare. The loop SNRs are chosen to be about 20 and 18 dB for cases A and B, respectively. In cases C through F, the input SSNR is set at −5.5 dB. This is to push the effective bit SNR slightly below the design threshold, where the decoder may fail to decode. The loop SNRs are varied from about 20 to 16 dB, where below 16 dB the loops may have cycle slips.

Table 1 summarizes the receiver parameters associated with cases A through F, along with the corresponding estimates of losses at the output of the BTD before any decoding by the FCD. It is seen that BTD loss increments on the order of 0.3 dB are typical for all test cases except case D, which has a high subcarrier loop SNR of 28.5 dB and a resulting BTD loss increment under 0.1 dB.

Table 1. FCD test conditions.

Case	Loop BW, Hz		Window size	Loop SNR, dB	Carrier Doppler rate, mHz/s	E_s/N_o at BTD input, dB	E_s/N_o at BTD output, dB	BTD loss increment, dB
	Carrier	0.10		20.5				
A	Subcarrier	0.05	1.0	19.3	0.1	−5.22	−5.49	0.27
	Symbol	0.02	0.5	16.1				
	Carrier	0.17		18.2				
B	Subcarrier	0.06	1.0	18.6	0.1	−5.22	−5.54	0.32
	Symbol	0.01	0.5	18.0				
	Carrier	0.10		19.7				
C	Subcarrier	0.05	1.0	18.5	0.0	−5.72	−6.01	0.29
	Symbol	0.02	0.5	15.2				
	Carrier	0.08		20.7				
D	Subcarrier	0.04	0.25	27.9	0.0	−5.72	−5.79	0.07
	Symbol	0.01	0.25	21.6				
	Carrier	0.10		19.7				
E	Subcarrier	0.05	1.0	18.5	0.0	−5.72	−5.98	0.26
	Symbol	0.01	0.5	18.2				
	Carrier	0.23		16.1				
F	Subcarrier	0.05	1.0	18.5	0.0	−5.72	−6.06	0.34
	Symbol	0.02	0.5	15.2				

III. Classifying and Measuring the Losses

We classified the losses due to the nonideal receiver into several categories. The first category is the loss measured at the output of the receiver without any decoding; this is the BTD loss increment reported in Table 1. Any extra loss beyond the BTD loss increment that is measurable at the output of the full FCD is referred to as the FCD loss increment. The FCD loss increment is further subclassified into two types of stage-by-stage losses. The VD loss increment for a given decoding stage is the loss measured at the output of the Viterbi decoder assuming correct RS decoding in previous stages but without any Reed–Solomon decoding in succeeding stages; this loss is measured relative to the performance of a stand-alone Viterbi decoder operating with differing amounts of known information from stage to stage. The RS loss increment for a given stage is the loss measured at the output of the RS decoder assuming the observed average error rate from the Viterbi decoder for that stage; this loss is measured relative to the performance of a Reed–Solomon decoder operating with depth-8 interleaved symbols corrupted by pure AWGN.

The RS loss increment is referred to the FCD's performance with codewords interleaved to depth 8, not infinitely interleaved. As reported in [1], there is a 0.06- to 0.07-dB degradation due to finite depth-8 interleaving, but that loss is already accounted for in the FCD's performance baseline with an ideal BTD.

It should be emphasized that all the loss components evaluated in our tests arise from the nonideal noise originating in the receiver, and the various categories of loss increments estimate the successive degradations caused by the corrupted symbols as the processing moves further downstream from the receiver. Ideally, we would like to know the losses in each of the components, so that in the event of a fault, we can pinpoint where the fault may be. We also want to quantify the losses in smaller components so that we know where the losses are more significant and may need to be improved in the future.

A. BTD Loss Increment

The symbol SNR (SSNR or E_s/N_o) at the input to the BTD was -5.22 dB for cases A and B, and -5.72 dB for cases C, D, E, and F. These input SSNRs were achieved by keeping four harmonics from full-spectrum signals with SSNRs of -5.00 and -5.50 dB, respectively.

The SSNR at the output of the BTD was measured using the split symbol estimator. This estimate was also corroborated by measuring the hard-limited symbol error rate and looking up the corresponding SSNR on the standard performance curve for an uncoded AWGN channel. In all six test cases, the two SSNR estimation techniques gave almost identical estimates. The difference between the estimated output SSNR and the tested input SSNR is what we call the loss in the receiver or the BTD loss increment. Note that this definition of the BTD loss increment does not include the 0.22 dB lost before the BTD input due to using only four harmonics.

B. Stage-by-Stage VD Loss Increments

The effective bit SNR (BSNR or E_b/N_o) at the output of the Viterbi decoder for each decoding stage was estimated by counting the number of 8-bit Reed–Solomon code symbols corrected by the FCD in that stage. If it can be assumed that the FCD always decodes the truth data, then the observed symbol correction rate from the FCD equals the Viterbi decoder's output symbol error rate (SER) for 8-bit Reed–Solomon symbols. This is the output SER for a Viterbi decoder operating in a stand-alone mode but with different patterns of known symbols from previous RS decoding stages. The measured SER is mapped to a corresponding effective BSNR using the performance curve for a stand-alone Viterbi decoder for Galileo's (14,1/4) convolutional code, given a particular pattern of known 8-bit symbols from previous RS decoding stages (assumed successful); these Viterbi decoder reference curves were obtained in [1] and are reproduced here as Fig. 3. The VD loss increment for the given decoding stage is the difference between this measured effective BSNR and the BSNR computed from the estimated SSNR at

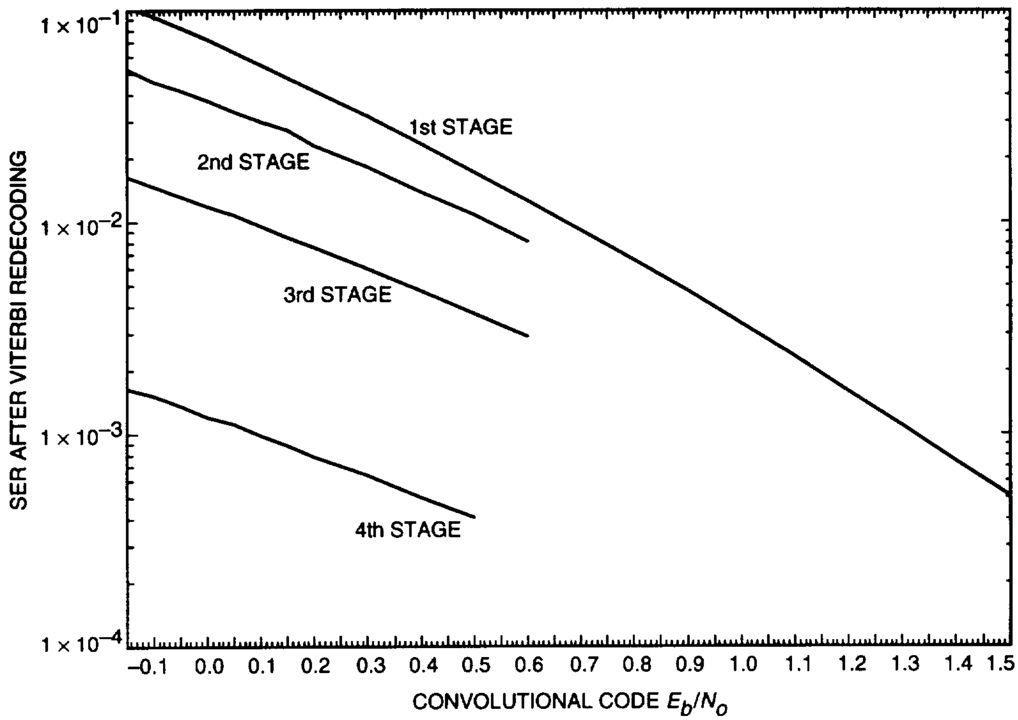

Fig. 3. Stage-by-stage Viterbi decoder performance for decoding Galileo's (14,1/4) convolutional code.

the output of the BTD. The values of BSNR used in these calculations are per bit at the output of the Viterbi decoder, and they do not include the 0.58-dB overhead to account for the average rate of the RS codewords.

C. Stage-by-Stage RS Loss Increments

The stage-by-stage RS loss increments cannot be measured directly using reasonable amounts of test data. They are estimated by a complicated method similar to that used in [1] for estimating the losses due to using depth-8 interleaving instead of infinite interleaving.

The four stages of the FCD are designed to be in "balance" with each other [1]. At the design operating point where the required error rate of 10^{-7} is just barely achieved, all four stages contribute comparable portions to the overall error rate. If the operating point is at a lower E_b/N_o than the design point, the performance of the RS code with the highest correctability, $E = 47$, deteriorates much more rapidly than the others, and so the error rate is dominated by errors from the first stage. If the operating point is at a higher E_b/N_o than the design point, the code with the lowest correctability, $E = 5$, improves very slowly relative to the others, and the error rate is dominated by errors from the fourth stage.

The effects on FCD performance of the non-AWGN introduced by the BTD must be evaluated stage by stage. If the design balance point is disturbed, the performance degradation will be dominated by that of the most affected stage.

1. Method for Estimating Losses Due to Depth-8 Interleaving. The analysis in [1] introduced a technique for estimating stage by stage the performance difference between a hypothetical FCD processing infinitely interleaved Reed–Solomon symbols and the actual FCD that must work with symbols interleaved only to depth 8. Depth-8 performance could be directly simulated only to an overall error rate of about 10^{-5} or 10^{-6}. Estimates of the design operating point required to produce a 10^{-7} error rate were obtained by extrapolation. The extrapolation method was to compare the simulated depth-8 data with an entire family of Reed–Solomon performance curves based on infinite interleaving, for all possible values of the error correction capability E of the code. The infinite-interleaving performance curves could be accurately calculated to error rates below 10^{-7}, and the depth-8 performance data were extrapolated to the 10^{-7} level by reference to the family of infinite-interleaving curves. This extrapolation was accomplished by first calculating "error magnification factors," relating (in the region where depth-8 data existed) the actual Reed–Solomon error correction capability to that of a code that would achieve the identical output error rate if its input symbols had the same input error rate but were infinitely interleaved. The error magnification factors were found to vary slowly and smoothly over the range of depth-8 data, and they could be extrapolated from the 10^{-5} level to the 10^{-7} level with a high degree of confidence.

2. Test Method for Estimating Losses Due to the BTD. In the present tests, we are trying to estimate 10^{-7} performance with much less data than was available in [1] for determining the effects of depth-8 interleaving. However, the basic extrapolation principle is the same. We first measure stage-by-stage FCD error rates, under the actual conditions introduced by the nonideal BTD, to the lowest error level that can be feasibly tested (in this case about 10^{-3} or 10^{-4}). Then all of the measured data are converted to equivalent error magnification factors by reference to the entire family of RS performance curves based on infinite interleaving; these reference curves are shown in Fig. 4. The magnification factors are extrapolated to the required 10^{-7} error level to give an estimate of the total degradation relative to infinite interleaving. Finally, the degradation due to depth-8 interleaving, already estimated in [1], is subtracted to give the net degradation due to the nonideal BTD.

The degradation measured in terms of error magnification factors is translated into an equivalent SNR loss by means of the calibration curves shown in Fig. 5. This figure plots the error magnification factor at an RS output SER of 10^{-7} versus the Viterbi decoder bit SNR that would achieve the same SER according to the stand-alone first-stage Viterbi decoder reference performance curve in Fig. 3, and assuming infinite interleaving. It is seen from Fig. 5 that the translation from magnification factors into SNR losses follows a nearly universal straight-line rule, regardless of the error correction capability E of the outer Reed–Solomon code. The calibration rule for all values of E greater than or equal to approximately 15 is that 8 dB of error magnification factor equals 1 dB of equivalent SNR loss. For E less than 15, this ratio drops drops off very gradually, staying above 6 to 1 for all values of E greater than or equal to 2.

A difference between these tests and the simulations in [1] is that for these tests the nonideal error rates were not directly measured, but instead were estimated without reference to known "truth" data. These estimates were obtained using histograms of Reed–Solomon symbol corrections reported by the FCD. A similar method[1] utilized only average symbol correction rates rather than entire histograms; this method allows accurate stage-by-stage measurement of the VD loss increment, but does not produce an estimate of the RS loss increment.

Suppose that a code with error correction capability E actually reports $e \leq E$ corrections for a given codeword. Then, assuming that this corrected codeword is not erroneous, any Reed–Solomon code with the same block length and correction capability $E' \geq e$ would have corrected a corresponding codeword with symbol errors in the same e places, whereas codes with correction capability $E' < e$ would have failed to decode (or possibly decoded incorrectly). By collecting a histogram of observed values of e for different decoded codewords, we can simultaneously estimate the RS decoded error rates for a whole family of codes with error correction capabilities $E' \leq E$. After noting RS output SER as a function of E', we look up the corresponding ideal error correction capabilities E^* that would achieve the same SER values under an infinite interleaving assumption. This yields the error magnification factors E'/E^*

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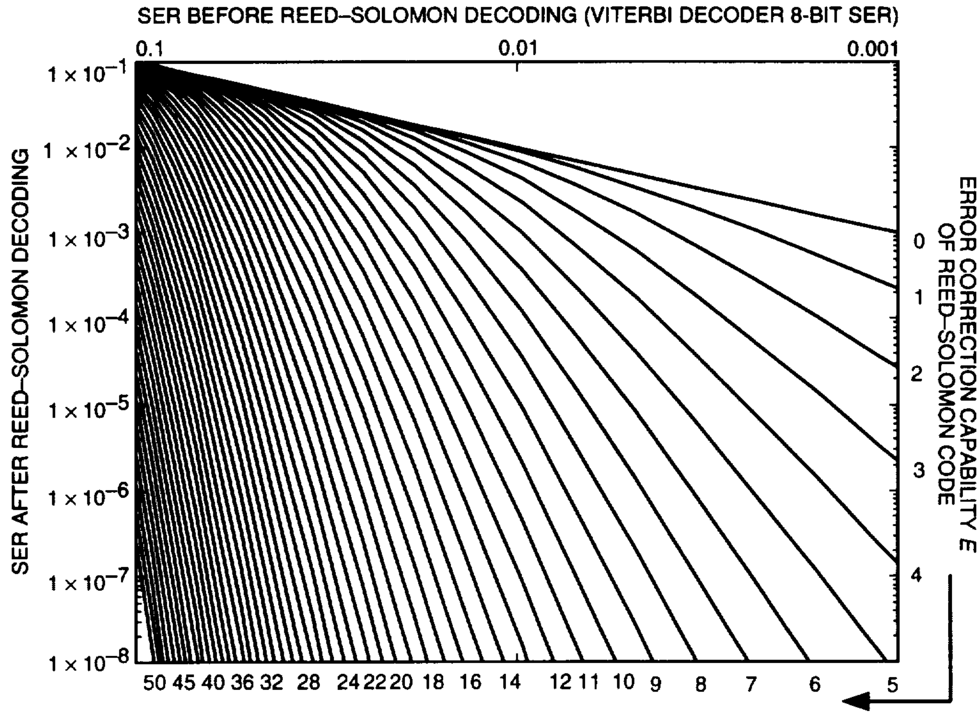

Fig. 4. Ideal Reed–Solomon performance curves for independent symbol errors (infinite interleaving).

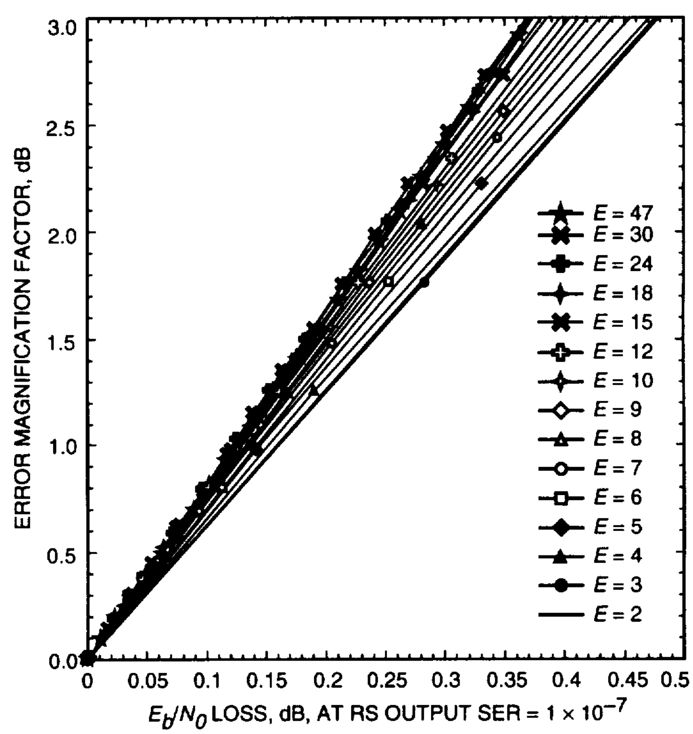

Fig. 5. Error magnification factor calibration curves for Galileo's (14,1/4) convolutional code concatenated with a blocklength-255 Reed–Solomon code.

as a function of SER. For the purposes of this computation, the equivalent ideal correctabilities E^* are determined as nonintegral values by interpolating between the discrete integer-valued curves in Fig. 4.

The process of measuring error magnification factors as a function of RS output SER must be repeated for each of the four stages separately. At each stage, error magnification factors are computed for hypothetical values of correctability E' less than or equal to the actual correctability of the RS code used in that stage. For this calculation, data are only collected from the specific codeword(s) designed to be corrected during that decoding stage. Very small values of E' are discarded if they are less than the code's block length (255) times the RS input average SER (i.e., the average output 8-bit SER from the Viterbi decoder), because they would not correspond to useful codes (even hypothetically) at the given input SER. Values of E' greater than or equal to the maximum number of corrected symbol errors e are also discarded because, for these values of correctability, there are insufficient data to detect an error rate greater than zero.

3. Test Results for Estimating the Nonideal BTD Effects. Figures 6 through 11 show, for cases A through F, the measured RS output SER for hypothetical correctabilities E' in each of the four decoding stages. The measured SERs for different values of E' are plotted as small circles at the same value of Viterbi decoder bit SNR. In the first stage, this is the effective VD BSNR after accounting for the BTD and VD loss increments. In stages 2 through 4, the horizontal coordinate plotted in Figs. 6 through 11 is an equivalent first-stage VD BSNR computed by looking up the output SER of the Viterbi *redecoder* on the first-stage VD performance curve in Fig. 3. Also shown in Figs. 6 through 11 is a family of reference performance curves assuming infinite interleaving and different values of correctability. The horizontal coordinate of the reference curves is similarly normalized to an equivalent first-stage BSNR. The figures show one small circle and one reference curve for each value of E' between the minimum and maximum values described above (and labeled explicitly in the figures).

Notice that the FCD test points represented by the small circles are generally displaced slightly to the right of the corresponding reference curves assuming infinite interleaving and AWGN. This same conclusion holds relative to the slightly degraded set of reference curves reported in [1] for depth-8 interleaving but still assuming ideal AWGN. The RS loss increment in the first stage is the horizontal displacement of the small circles from the depth-8 reference curves. For stages 2 through 4, this horizontal displacement represents the sum of the RS and VD loss increments for the given stage minus the VD loss increment for the first stage.

The RS loss increments that can be observed directly as horizontal displacements in Figs. 6 through 11 are for SERs several orders of magnitude higher than 10^{-7} and hypothetical values of correctability much lower than those of the actual four RS codes used in the FCD. The RS loss increment is extrapolated to the 10^{-7} level by first taking the SERs plotted as small circles and reinterpreting them as equivalent error magnification factors. The results are shown in Figs. 12 through 15. It is seen that the magnification factors for the first three stages approach or exceed 1 dB for output SERs in the 10^{-3} to 10^{-4} range. At the measured rate of increase of magnification factors between 10^{-2} and 10^{-4}, it is likely that the error magnification factors will be around 2 dB, and possibly as high as 3 dB, when the error rate is reduced to the order of 10^{-7}. In the fourth stage, the data are more difficult to extrapolate, but the magnification factors are somewhat lower than in the other three stages.

Since the error magnification factors are computed relative to an equivalent performance curve under an infinite interleaving assumption, the computation of the RS loss increment requires an adjustment to account for the portion of the error magnification that is due to depth-8 interleaving; this was already predicted and accounted for by the analysis in [1]. Figure 16 shows that the error magnification factors for depth-8 interleaving (assuming AWGN) are consistently below 0.5 dB and seem to approach 0.5 dB very reliably at 10^{-7} SER for all except the very lowest values of correctability. For small values of correctability, the extrapolated value of the magnification factor may be 0.1 to 0.2 dB smaller.

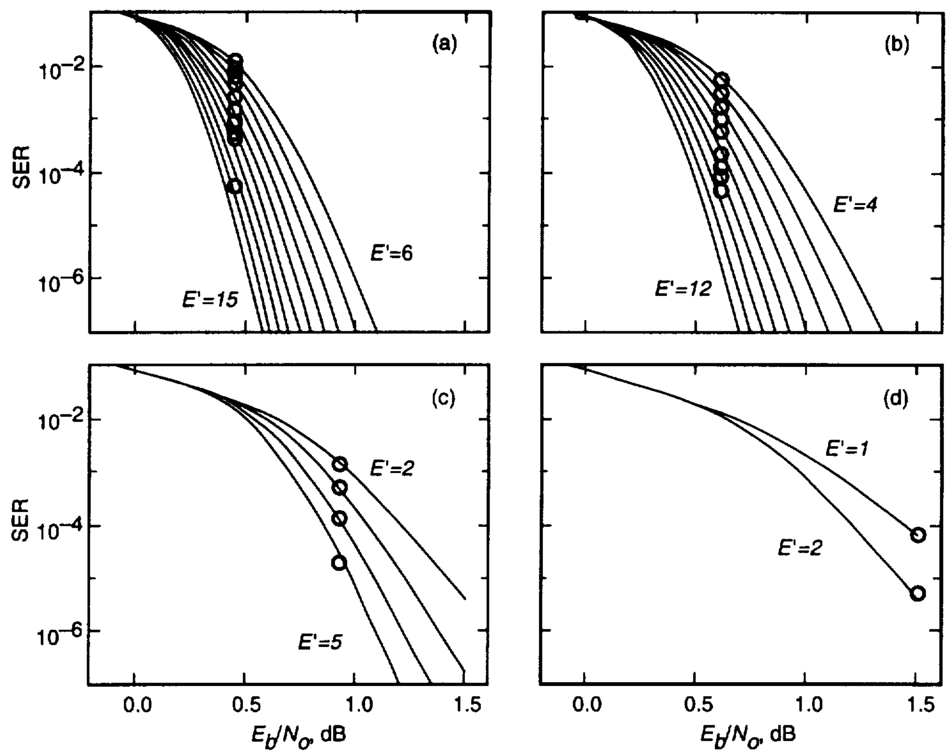

Fig. 6. Measured SER compared to ideal interleaving SER curves (case A): SER in (a) stage 1, (b) stage 2, (c) stage 3, and (d) stage 4.

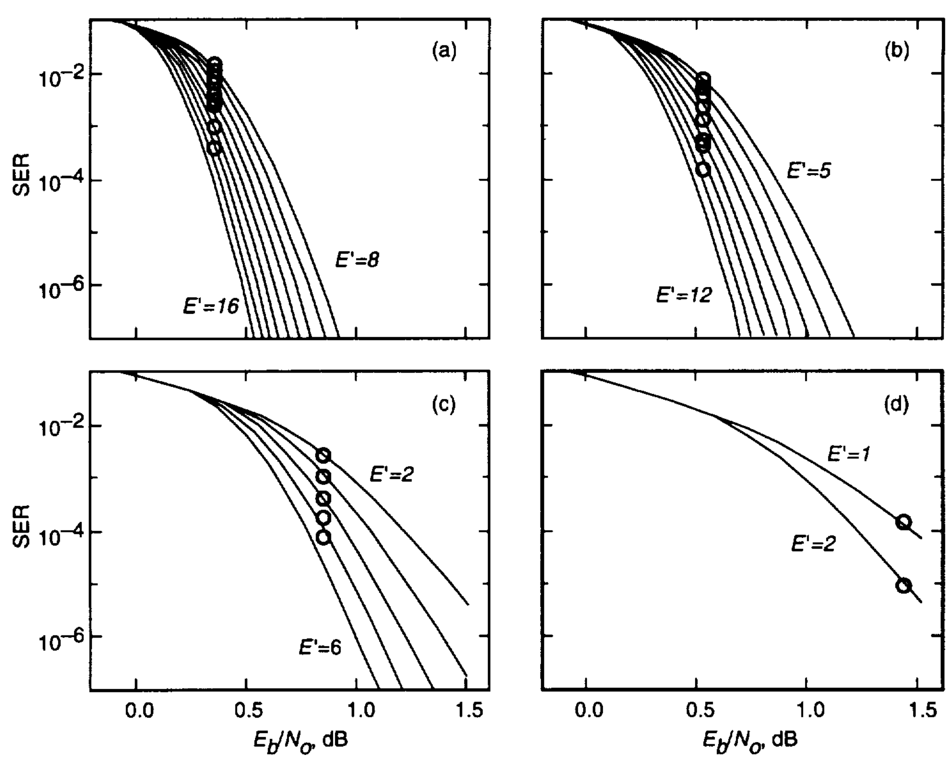

Fig. 7. Measured SER compared to ideal interleaving SER curves (case B): SER in (a) stage 1, (b) stage 2, (c) stage 3, and (d) stage 4.

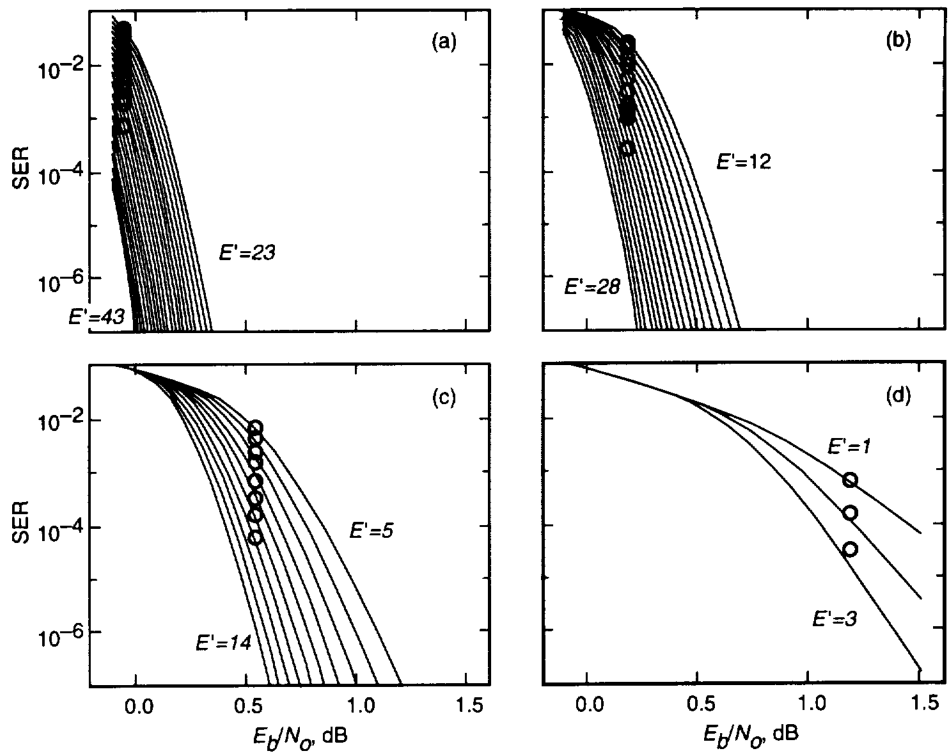

Fig. 8. Measured SER compared to ideal interleaving SER curves (case C): SER in (a) stage 1, (b) stage 2, (c) stage 3, and (d) stage 4.

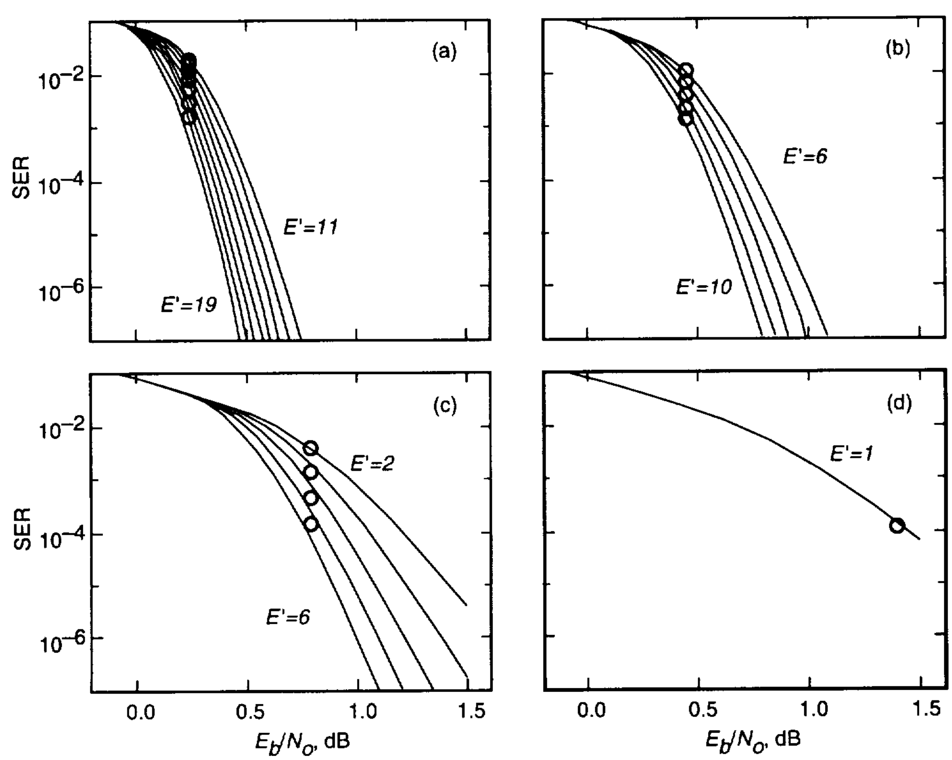

Fig. 9. Measured SER compared to ideal interleaving SER curves (case D): SER in (a) stage 1, (b) stage 2, (c) stage 3, and (d) stage 4.

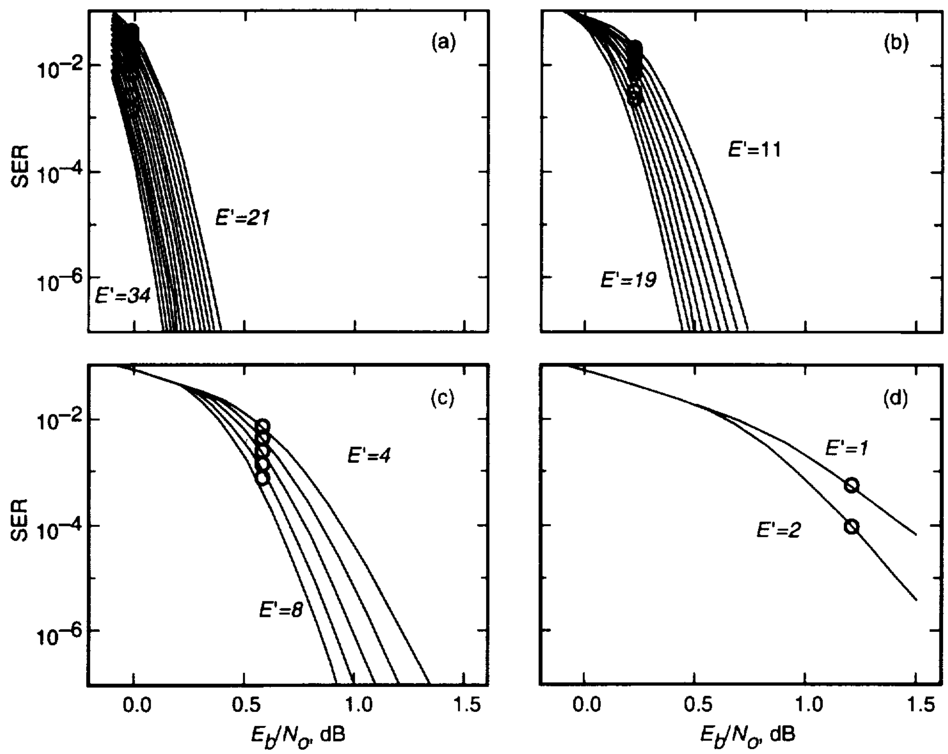

Fig. 10. Measured SER compared to ideal interleaving SER curves (case E): SER in (a) stage 1, (b) stage 2, (c) stage 3, and (d) stage 4.

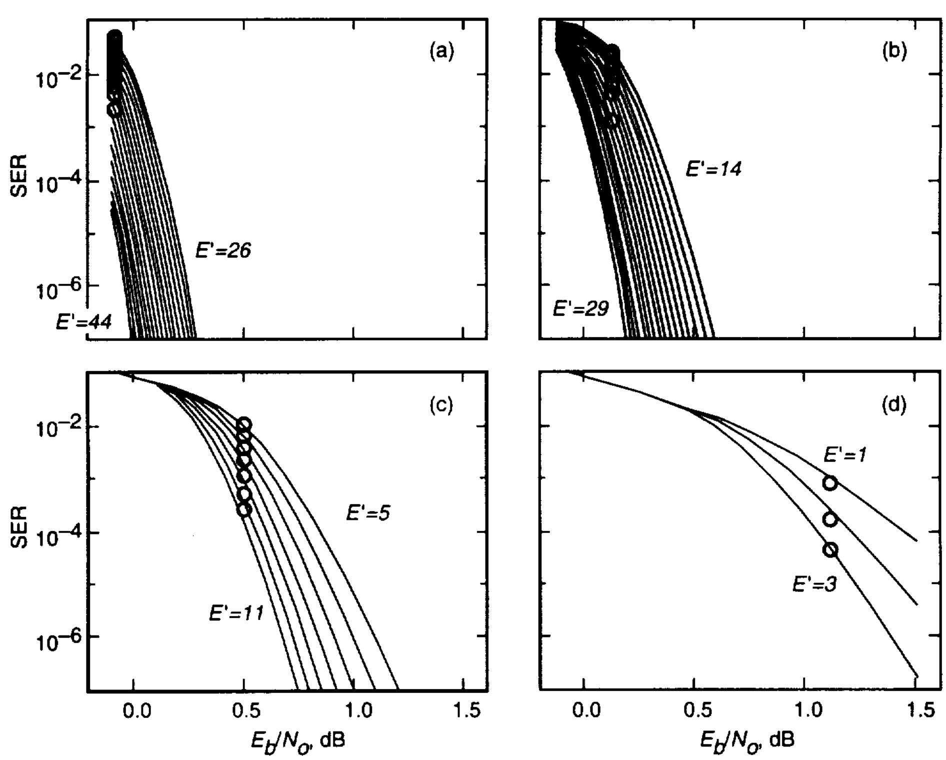

Fig. 11. Measured SER compared to ideal interleaving SER curves (case F): SER in (a) stage 1, (b) stage 2, (c) stage 3, and (d) stage 4.

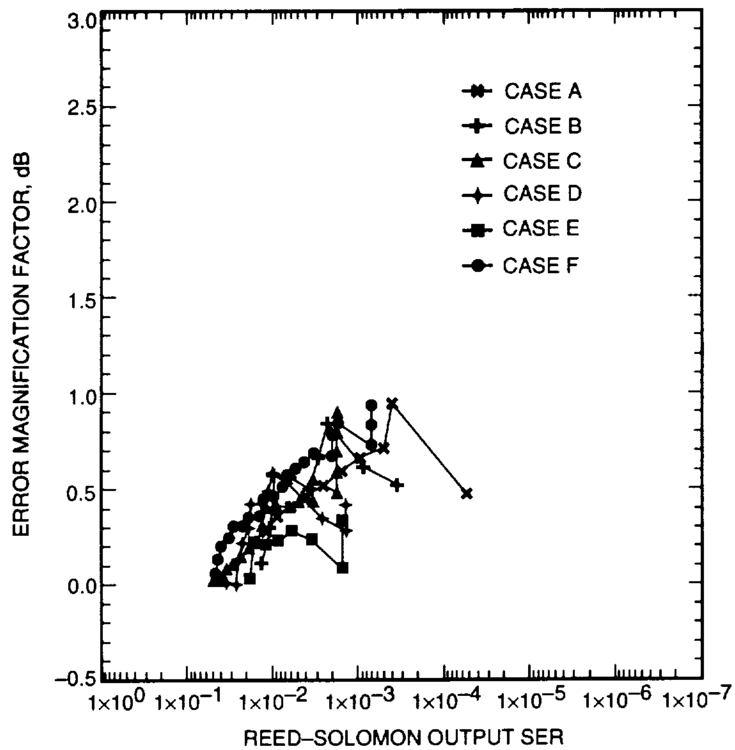

Fig. 12. Measured first-stage error magnification factors.

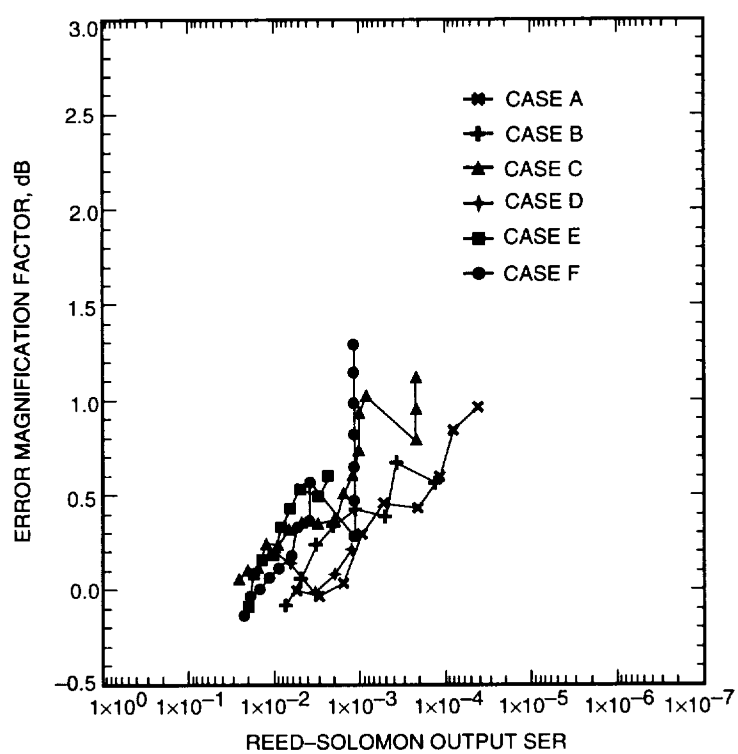

Fig. 13. Measured second-stage error magnification factors.

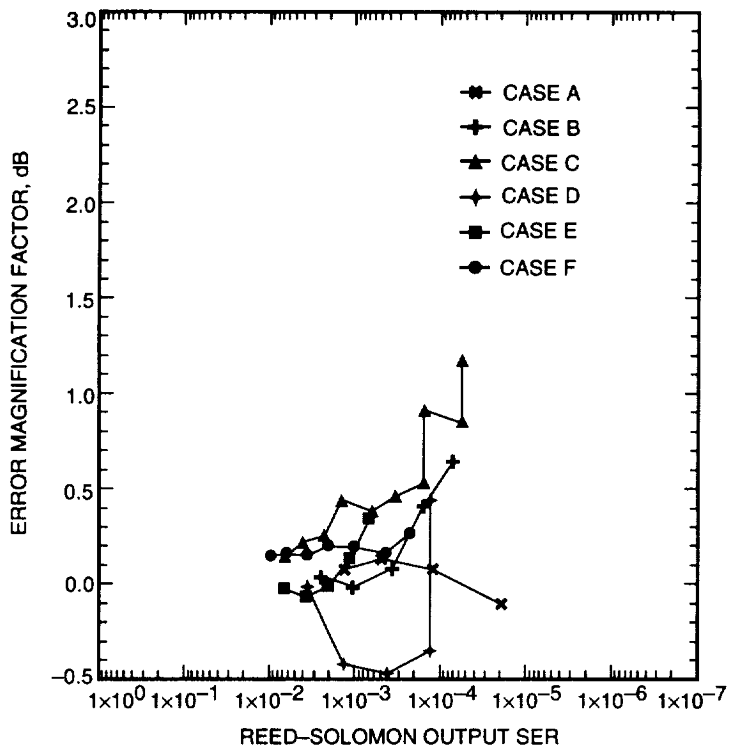

Fig. 14. Measured third-stage error magnification factors.

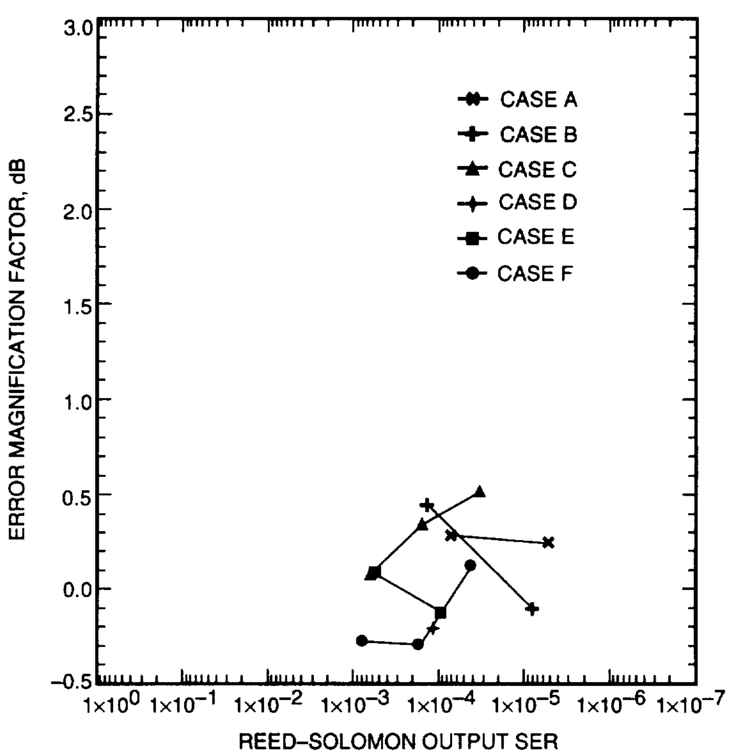

Fig. 15. Measured fourth-stage error magnification factors.

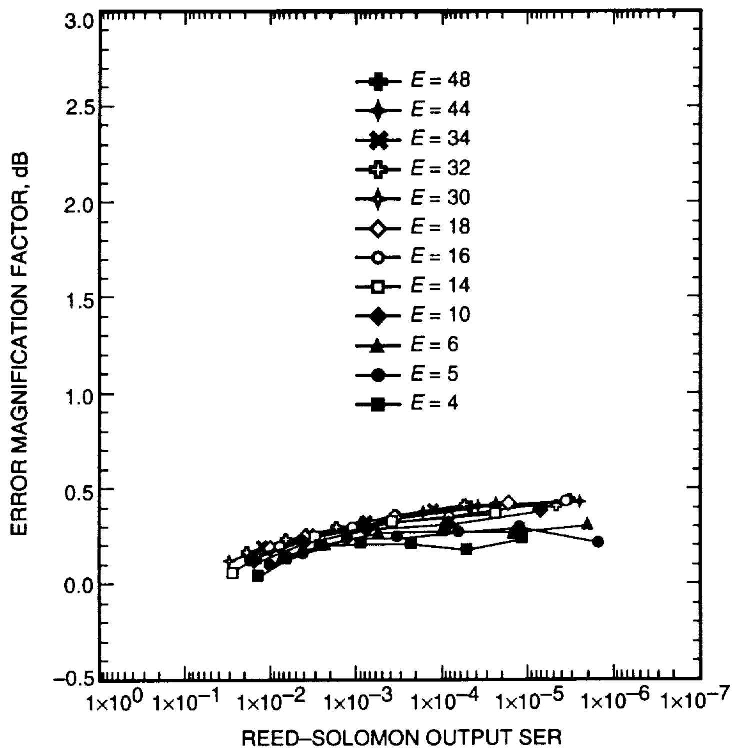

Fig. 16. Reference error magnification factors for depth-8 interleaving.

It should be noted that the reference curves show error magnification factors computed by varying the channel SNR but keeping the correctability fixed, while the test data curves show magnification factors computed by varying the hypothetical correctability at a fixed channel SNR for each of cases A through F. Thus, it is not legitimate to subtract the curves point by point. However, our extrapolation procedure still provides a good estimate, because the reference curves all cluster together and approach a very robust extrapolated value almost independent of correctability.

The final adjustment required to obtain the RS loss increment is to convert the error magnification factors into equivalent SNR losses according to the calibration curves in Fig. 5. This means dividing the net magnification factor (relative to the depth-8 AWGN reference) by 8 for stages with hypothetical correctabilities $E' \geq 15$, and by approximately 6 or 7 for stages with lower correctabilities. This results in estimated RS loss increments up to approximately 0.2 dB for the first three stages except for case D, and no more than approximately 0.1 dB for the fourth stage of all cases and for all stages of case D. However, it must be emphasized that these estimates are based on extrapolating some very ragged error magnification factor test data in Figs. 12 through 15 over three or four orders of magnitude in RS output SER, and the estimates might easily be off by 1 dB or so in magnification factor units, which is equivalent to a little more than 0.1 dB in SNR loss.

4. Discussion of the Extrapolation Method. If the error magnification factor extrapolations in Figs. 12 through 15 seem somewhat mysterious, here is a brief explanation in terms of the more understandable error rate measurements shown in Figs. 6 through 11. In the latter figures, the small circles represent hypothetical RS output SERs for codes with smaller correctabilities E' than the actual code's correctability. The desired but unmeasurable test datum is the small circle that would correspond to $E' = E_i$, where E_i is the actual codeword correctability in the ith decoding stage. We must try to estimate where this unmeasurable small circle might lie. The simplest method is to assume that it falls on the corresponding reference curve for the same value of correctability. The trail of small circles would be extended downward in a straight vertical line at the constant value of effective bit SNR shown in the figures, until the assumed reference curve is intersected. The reference curve may be the infinitely interleaved performance curve shown in the figures or the corresponding depth-8 performance curve (not shown). This method of extrapolating to an assumed reference curve precludes the detection of any deviation or loss relative to the reference.

We notice from Figs. 6 through 11 that the small circles begin to deviate more and more from their corresponding reference curves as the hypothetical correctability E' increases and the RS output SER gets smaller. The error magnification factors in Figs. 12 through 15 quantify this increasing deviation from the reference. By extrapolating along the trend of increasing magnification factors measurable for small values of E', we can obtain an estimate of how far above the reference curve the small circle would be either at the true value of correctability or at the value that yields an error rate around 10^{-7}. This produces an estimate of the loss relative to the assumed reference.

IV. Summary of Test Results

The measured VD and RS loss increments for cases A through F are reported in Table 2. The VD loss increments are mostly between 0.1 and 0.2 dB, except for case D, which has negligibly small VD loss increments. The RS loss increments estimated in the previous section range up to approximately 0.2 dB, not allowing for at least 0.1 dB possible error in extrapolating the data to the 10^{-7} SER level. The composite FCD loss increment, obtained roughly as the sum of the VD and RS loss increments for the most affected stage, is estimated to be approximately 0.3 to 0.4 dB for all cases except case D, for which the FCD loss increment is less than 0.1 dB. Again, the estimates of the composite FCD loss increment do not include the numerical uncertainties (positive or negative) in extrapolating the RS loss increments to the 10^{-7} SER level.

Cases C and F produced an effective operating point a few tenths of a dB lower than the design threshold required for a 10^{-7} error rate. As a result, the FCD failed to decode a few frames. In the previous section, we described our test procedure as if these undecodable frames never existed, and the numerical results in Table 2 are based on ignoring these frames. In the next section, adjustments are made to approximately account for the bias introduced by ignoring the undecodable frames. These adjustments add less than 0.1 dB to the composite FCD loss increment for cases C and F only.

V. Discussion of Test Results

A. Statistical Confidence in the Numerical Results

One of our concerns about the test results is that the measured error magnification factors for these tests jump around wildly, and, thus, it is much harder to confidently extrapolate the RS loss increments due to the nonideal BTD than the corresponding losses reported in [1] due to depth-8 interleaving. Some of this erratic behavior is purely statistical, as a result of the small number of frames tested. If error bars were shown in Figs. 12 through 15, they would lengthen dramatically proceeding from left to right as the SER decreases to the point of undetectability for the small number of frames tested.

Figure 17 illustrates how the statistical fluctuations can be smoothed out for the first stage by having eight times as much data. In Figs. 12 through 15, the only data used for the calculation of the magnification factors in a given stage came from the specific codeword(s) decoded during that stage. For example, the data for the first stage are from the observed RS symbol corrections in the single codeword with the highest correctability, $E = 47$. It would be equally valid to perform hypothetical first-stage decodings with correctabilities $E' \leq 47$ on all eight codewords, if it can be assumed that the correct symbols are eventually known in all eight codewords by the end of the fourth decoding stage. The data obtained from all eight codewords are plotted in Fig. 17. Notice the improved smoothness of the curves relative to those

Table 2. FCD test results.

Case	Number of decoded frames	BSNR at BTD output, dB	Stage	BSNR at VD output, dB	VD loss increment, dB	RS loss increment, dB	Composite FCD loss increment, dB
			1	0.43	0.10	0.10	
A	1184	0.53	2	0.42	0.11	0.15	0.3
			3	0.43	0.10	<0.1	
			4	0.40	0.13	<0.1	
			1	0.34	0.14	0.15	
B	372	0.48	2	0.32	0.16	0.15	0.3
			3	0.32	0.16	0.10	
			4	0.27	0.21	<0.1	
			1	−0.08	0.09	0.20	
C	491	0.01	2	−0.10	0.11	0.20	0.4
	(3 frames		3	−0.14	0.15	0.20	
	failed)		4	−0.17	0.18	0.10	
			1	0.21	0.02	<0.1	
D	100	0.23	2	0.24	−0.01	<0.1	<0.1
			3	0.24	−0.01	<0.1	
			4	0.22	0.01	<0.1	
			1	−0.05	0.09	0.10	
E	100	0.04	2	−0.06	0.10	0.20	0.3
			3	−0.08	0.12	0.20	
			4	−0.13	0.17	<0.1	
			1	−0.13	0.09	0.20	
F	99	−0.04	2	−0.16	0.12	0.25	0.4
	(1 frame		3	−0.20	0.16	0.10	
	failed)		4	−0.29	0.25	0.10	

in Fig. 12. This procedure cannot be repeated for stages two through four, because the output error characteristics from the Viterbi *redecoder* affect each codeword differently, depending on the placement of the codeword relative to codewords decoded in previous stages.

In Fig. 15, we observed a dearth of data for making extrapolations of fourth-stage error magnification factors. This is not a problem that can be cured by testing just a few more frames. When the decoder is operating at the design threshold and above, each fourth-stage codeword will report very few symbol corrections e. Values of e close to the code's correction capability, such as $e = 5$ or $e = 4$, will be highly unlikely. Thus, with reasonable amounts of test data, there may only be two or three distinct values of the hypothetical correctability E' for which any test results exist. It is difficult to justify extrapolations of the error magnification factor curves based on only two or three points. Fortunately, as pointed out earlier, the fourth-stage magnification factors seem to be somewhat more benign than those of the earlier stages, and an accurate extrapolation is not necessary if the overall FCD loss increment is dominated by the error magnifications in earlier stages.

Better estimates of the fourth-stage error magnification factor might be obtained by modifying the test procedure to more closely resemble the analysis in [1], fixing a particular value of correctability (e.g., $E' = 2$ or $E' = 3$) and running a series of tests with the same loop parameters but different SSNR values. In fact, because of the empirically observed near universality of the error magnification factor curves for similar values of E', testing different SSNR values is an appropriate way to merge more data into the magnification factor estimates for any stage.

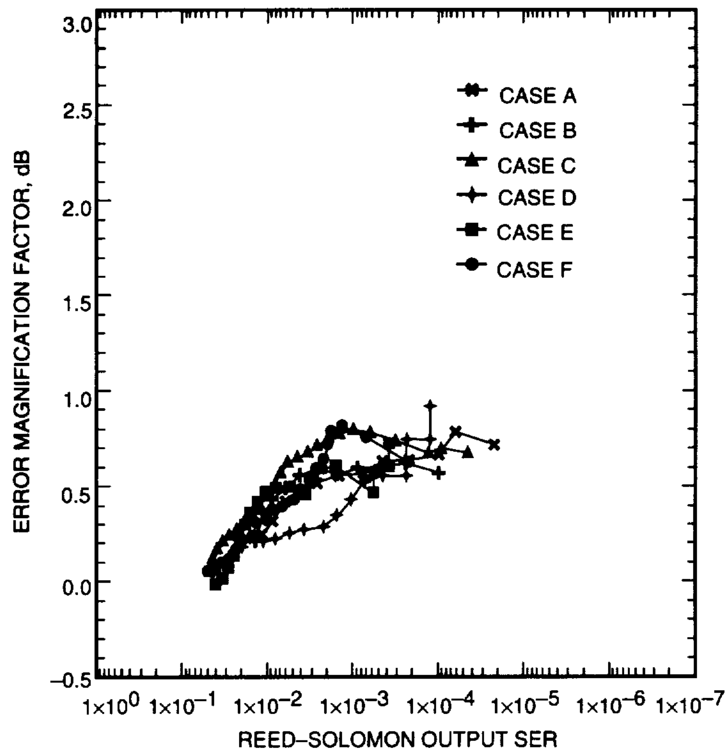

Fig. 17. First-stage error magnification factors using eight times as much data.

B. Tests Conducted Below Design Threshold

It was pointed out earlier that a few frames failed to decode for test cases C and F. As expected, these failures always happened on the first-stage codeword, because the effective operating point (accounting for all loss increments) was below the design threshold. The effective operating point for case E was also below threshold, but by luck no decoding failures occurred over the small sample size of 100 frames. In all of the analyses up to now, the data from undecodable frames have been completely ignored. There was no report from the FCD on what the correct symbols were, and exact SERs and error magnification factors cannot be computed. However, ignoring these frames biases the results optimistically. Figure 18 shows first-stage magnification factors computed for cases C and F by assuming that there were exactly 48 errors in the undecodable codewords. The magnification factors are increased relative to those reported in Fig. 12, reaching well above 1 dB at SERs near 10^{-3}. Extrapolated magnification factors of 3 dB or higher at a 10^{-7} SER are certainly imaginable based on these adjusted data.

We have consistently made the assumption that the codewords decoded by the FCD represent the truth data. This is a valid assumption as long as the test procedure is being applied at design threshold and above. Below the design threshold, there is the possibility of encountering undecodable frames, as in test cases C and F. One might also worry about incorrectly decoded codewords in the fourth stage, where the small correction capability, $E = 5$, implies that there is a non-negligible probability of making a decoding error. However, this should not happen unless the loss mechanism somehow concentrates its deleterious effects on the fourth decoding stage and, thus, dramatically disturbs the design balance point. In the usual circumstances, losses that drop the effective operating point below threshold will show up as detected decoding failures on the first stage, because first-stage decoding performance declines most sharply as the SNR drops below threshold.

It should be noted that the complicated test procedure described in this article is primarily intended for analyzing FCD performance when codeword errors are rare, i.e., at threshold or above. Below thresh-

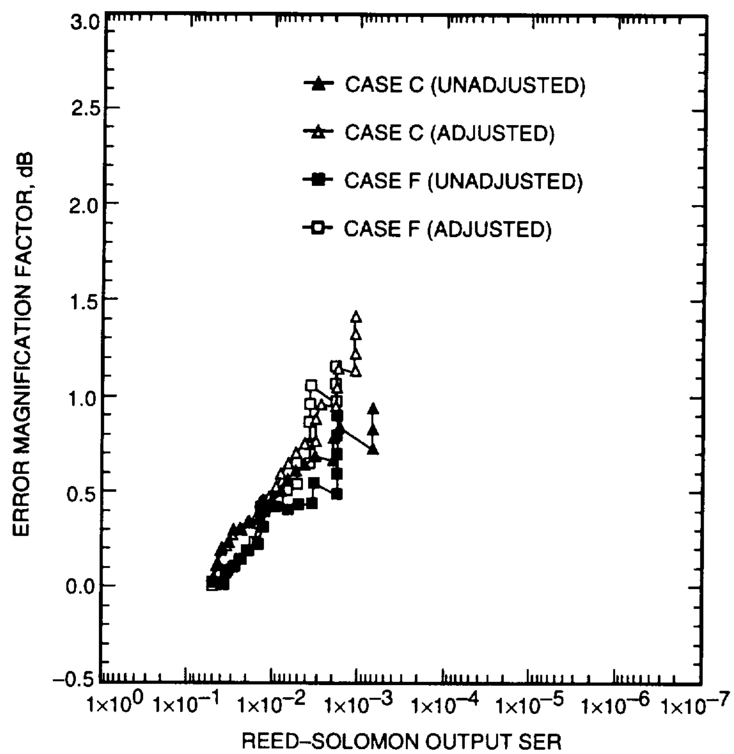

Fig. 18. First-stage error magnification factors adjusted for undecoded frames.

old, the FCD's performance deteriorates very rapidly, and there are sufficient codeword errors to make simple error counting tests reliable. Thus, the extra complications needed to account for undecodable or incorrectly decoded frames should not pose a problem in practice: at operating points where decoding failures are likely, a simpler test procedure should be substituted for the one described here.

Here is an illustration of how different the conclusions are for a test conducted below threshold. For case C, the observed first-stage codeword failure rate was 0.006, based on 3 failures out of 491 first-stage codewords. When a first-stage codeword fails, about 20 percent of the symbols are erroneous, so the RS output SER is around 10^{-3}. Due to the small number of observed undecodable words, this estimate is not highly accurate, but it still gives a ballpark number. From Table 2, the effective BSNR at the Viterbi decoder output is −0.08 dB, which is under the design threshold of 0.00 dB quoted in [1] for achieving a 10^{-7} SER. From Figs. 3 and 4, it is seen that four orders of magnitude in RS output SER are equivalent to about 0.17 dB of BSNR at the high slope of the first-stage code's performance curve. Therefore, the first-stage RS loss increment for case C is slightly less than 0.1 dB rather than the 0.2 dB quoted in Table 2. This apparent contradiction is resolved as follows. The calculations in this paragraph measure the RS loss increment at the actual test conditions for case C, i.e., at an operating point producing an SER around 10^{-3}. The calculations reported in Table 2 estimate how big the losses would be if the operating point had been adjusted to produce an SER around 10^{-7}. The calculations for 10^{-3} SER can be directly verified by reference to the error magnification curves in Figs. 12 and 16 without any need for extrapolation. The observed and reference error magnification factors at 10^{-3} SER are about 0.85 dB and 0.3 dB, respectively, translating into a net SNR loss of about 0.07 dB. This correlates well with the calculation based on just three codeword failures. The additional 0.1 dB of RS loss increment predicted in Table 2 for 10^{-7} SER results from extrapolating the magnification factors in Fig. 12, along their observed rate of increase, all the way to the 10^{-7} SER level. The increasing error magnification factors correspond to a slight flattening of the high-slope first-stage performance curve as compared to the reference ideal curves in Fig. 4. This flattening causes the RS loss increment to increase as the RS output SER is made smaller.

C. Combining the Results From All Four Stages

We have described a procedure for evaluating stage-by-stage FCD loss increments as the sum of stage-by-stage VD and RS loss increments, but we have not emphasized how to obtain the composite FCD loss increment taking into account all four stages. If the four stage-by-stage FCD loss increments are identical, then the composite loss increment is the same. The composite loss increment is no worse than the worst of the stage-by-stage loss increments, and it approaches this limit when one stage dominates the FCD's performance. Between these two extremes, it would be proper to calculate an average of the stage-by-stage loss increments by explicitly considering the effect of each stage on the overall SER or BER of the FCD. However, this complicated analysis would only improve the estimate over a narrow band of loss combinations, because the performance of the FCD passes very quickly into dominance by the performance of its weakest stage whenever its design balance point is disturbed.

We have also glossed over the precise error rate at which the FCD loss increment is evaluated. While Galileo has a very specific overall BER requirement of 1×10^{-7}, we have spoken more vaguely of reaching on each stage a target error rate on the order of 10^{-7}, and the error rates we have aimed at this target are 8-bit RS SERs rather than BERs. Given the several orders of magnitude range over which the error magnification factors must be extrapolated, there is no need to be more precise in specifying the exact target, since the overall BER is about half the overall 8-bit SER for a long-constraint convolutional code, and about one to two times the stage-by-stage 8-bit SERs.

Since our test procedure focuses on estimating individual stage-by-stage losses, it is also applicable to testing a simple one-stage concatenated decoder without feedback. The RS loss increments seen in our tests are qualitatively, if not quantitatively, similar to the RS loss increments that would be measured if the (14, 1/4) convolutional code were concatenated with the standard 16-error-correcting RS outer code and asked to perform at a 10^{-7} SER level with nonideal input from the BTD.

VI. Conclusion

This article presents a test procedure that tests the performance of the FCD when the resulting BER is very low (10^{-7}) and cannot be measured directly through simulations in a reasonable amount of time. Using this test procedure, we have tested the FCD taking the input from the BTD which contains multiplicative colored non-Gaussian noises. The preliminary test results show that there are about 0.3- to 0.4-dB loss increments in the FCD when the loop SNRs are lower than 20 dB as compared to analytical results assuming AWGN. In one test case, where we had the subcarrier-loop SNR around 28 dB, the loss increment in the FCD was less than 0.1 dB.

The numerical test results reported in this article are rough estimates due to the small amount of test data and test cases that were run. However, the test procedure described herein should be used as a template for conducting more extensive performance tests on the FCD in the future. This template provides an organized robust methodology for extrapolating small amounts of test data to give reasonable estimates of FCD loss increments at unmeasurable minuscule error rates.

Reference

[1] S. Dolinar and M. Belongie, "Enhanced Decoding for the Galileo Low-Gain Antenna Mission: Viterbi Redecoding With Four Decoding Stages," *The Telecommunications and Data Acquisition Progress Report 42-121, January–March 1995*, Jet Propulsion Laboratory, Pasadena, California, pp. 96–109, May 15, 1995.

Appendix

Step-by-Step Test Procedure

Follow this procedure to test the FCD at operating points that produce an output BER of around 10^{-7}

(1) Choose a set of loop parameters for testing. Based on the preliminary results in Tables 1 and 2 or on more extensive similar test results, guess a value of SSNR that will produce an output BER around 10^{-7}. Generate a number of frames of encoded data, modulate a carrier and a subcarrier with the data, add channel noise, and feed the resulting test signal through the BTD. Pass the output of the BTD through the FCD and note the results of the decoding.

(2) Estimate the SSNR at the output of the BTD using the split symbol estimator $\widehat{SSNR}$. Compute the BTD loss increment, in dB, as $\Delta L_{BTD} = 10\log_{10} SSNR/\widehat{SSNR}$.

(*) Repeat steps 3 through 10 for the output from each individual decoding stage, $i = 1, 2, 3, 4$. For these steps, an ith stage codeword is defined as a codeword with correctability E_i, where $E_1 = 47$, $E_2 = 30$, $E_3 = 15$, and $E_4 = 5$.

(3) Observe the number of corrected symbols e_i in each ith-stage codeword in each frame. If any ith-stage codeword is undecodable, record this event as $e_i = E_i + 1$, but be aware that, if this event occurs frequently, the test procedure is being used outside its intended range.

(4) Compute the VD output SER, $SER_{VD}(i)$, for ith-stage codewords as the sum of all the observed values of e_i divided by 255 times the total number of ith-stage codewords.

(5) Look up the measured value of $SER_{VD}(i)$ on the Viterbi decoder performance curve for the ith stage for Galileo's (14,1/4) code (Fig. 3) and interpolate to find the corresponding value of BSNR. Compute the VD loss increment, in dB, as $\Delta L_{VD}(i) = 10\log_{10}(4 \times \widehat{SSNR}/BSNR)$.

(6) Compute output SERs, $SER_{RS}(i, E')$, for RS codes with hypothetical correctabilities E' greater than $255 \times SER_{VD}(i)$ and strictly less than the maximum value of e_i observed in step 3: $SER_{RS}(i, E')$ is computed as the sum of the observed values of e_i for only those ith-stage codewords with $e_i > E'$, divided by 255 times the total number of ith-stage codewords.

(7) Compute a lookup table of ideal RS output SERs, $SER^*_{RS}(i, E)$, for RS codes with varying correctabilities E facing independent symbol errors occurring with rate $SER_{VD}(i)$. This table generates the ideal RS performance curves shown in Fig. 4. Be sure that the lookup table encompasses sufficient values of E for the interpolation in the next step.

(8) For each value of hypothetical correctability E' determined in step 6, interpolate using the lookup table in step 7 to find an equivalent ideal correctability E^* such that $SER^*_{RS}(i, E^*) = SER_{RS}(i, E')$. Be sure to perform this interpolation based on logarithms of error rates, e.g., for "linear" interpolation,

$$E^* = E^*_0 + \frac{\log[SER^*_{RS}(i, E^*_0)/SER_{RS}(i, E')]}{\log[SER^*_{RS}(i, E^*_0)/SER^*_{RS}(i, E^*_0 + 1)]}$$

where E_0^* is the largest value of E for which $SER_{RS}^*(i,E) \geq SER_{RS}(i,E')$. For each value of E', compute a corresponding ith-stage error magnification factor, measured in dB, as $MF(i,E') = 10\log_{10} E'/E^*$.

(9) Plot $MF(i,E')$ versus $SER_{RS}(i,E')$, varying the parameter E', to obtain a curve like those in Figs. 12 through 15. Use good engineering judgment to extrapolate these curves to the desired RS output SER level around 10^{-7}.

(10) Subtract 0.5 dB, or a little less, from the extrapolated error magnification factor obtained in step 9. Then divide by 8, or a little less, to get a corresponding SNR loss. The result is the RS loss increment, $\Delta L_{RS}(i)$, for stage i, compared to the reference performance derived in [1] for depth-8 interleaving and AWGN. The values to subtract or divide by depend on the values E' contributing to the error magnification factor curve: Subtract 0.5 dB and divide by 8 when E' is approximately 15 or greater, and reduce these calibration values slightly to 0.4 or 0.3 dB and 7 or 6 when E' is smaller. Since different values of E' contribute to the same error magnification factor curve, the exact calibration requires an exercise of good judgment.

(11) The FCD loss increment for the ith decoding stage is the sum of the ith-stage VD and RS loss increments (measured in dB). The composite FCD loss increment for all decoding stages is approximately the largest of the stage-by-stage FCD loss increments.