3 ESTIMATION OF SPATIALLY RESTRICTED LET USING TRACK STRUCTURE MODELS

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The spatial distribution of energy deposition is an important determinant in the formation of biologically significant lesions. It has been widely realized that Linear Energy Transfer (LET) being an average quantity is not sufficient to describe the situation at a submicroscopic scale. To remedy this to some extent "energy-cut-off" values are sometimes used but since they are related to secondary electron energy and only indirectly to their range they are also not adequate although they may be easily calculated (ICRU 1970). "Range-restricted LET" appears to be better but its determination is usually quite involved. Xapsos (1992) suggested a semi-empirical approximation based on a modified Bethe-formula which contains a number of assumption which are difficult to verify. A simpler and easier way is to use existing beam-models which describe energy deposition around an ions path (see e. g. Kiefer and Kost 1988 and references therein). They all agree that the energy density (i. e. energy deposited per unit mass) decreases with the inverse square of the distance from the track centre. This simple dependence can be used to determine the fraction of total LET which is deposited in a cylinder of a given radius. As an example our own beam model (Kiefer and Straaten 1986) is used. Energy density depends on distance x (measured in m) from the track centre according to the formula

$$\rho = C\frac{Z^{*2}}{\beta^2}\frac{1}{x^2} \tag{1}$$

where Z^* is the effective ion energy, β its velocity relative to that of light in vacuo and x the distance from the track centre. The coefficient $C = 0.78 eV/m$ for water, the energy density is then given in eV/m^3. Total LET (LET_∞) is obtained by integration over all concentric shells from a lower limit x_0 to the penumbra radius x_p

$$LET_\infty = 2\pi C\frac{Z^{*2}}{\beta^2}\int\frac{1}{x}dx = 2\pi C\frac{Z^{*2}}{\beta^2}ln\frac{x_p}{x0} \tag{2}$$

The lower limit x_0 is not defined and is chosen so that the correct LET-value is obtained:

$$x_0 = x_p exp(-\frac{LET_\infty}{2\pi}C\frac{Z^{*2}}{\beta^2}) \tag{3}$$

The range-restricted $LET_\Delta(r)$ within a radius r can be calculated in an analogous way

$$LET_\Delta = 2\pi C\frac{Z^{*2}}{\beta^2}ln\frac{r}{x_0} \tag{4}$$

The fraction f_r of total energy deposition within the cylinder is then

$$fr = \frac{LET_\Delta}{LET_\infty} = \frac{ln\frac{r}{x_0}}{ln\frac{x_p}{x_0}} \tag{5}$$

This can be rewritten using equ. (3) as

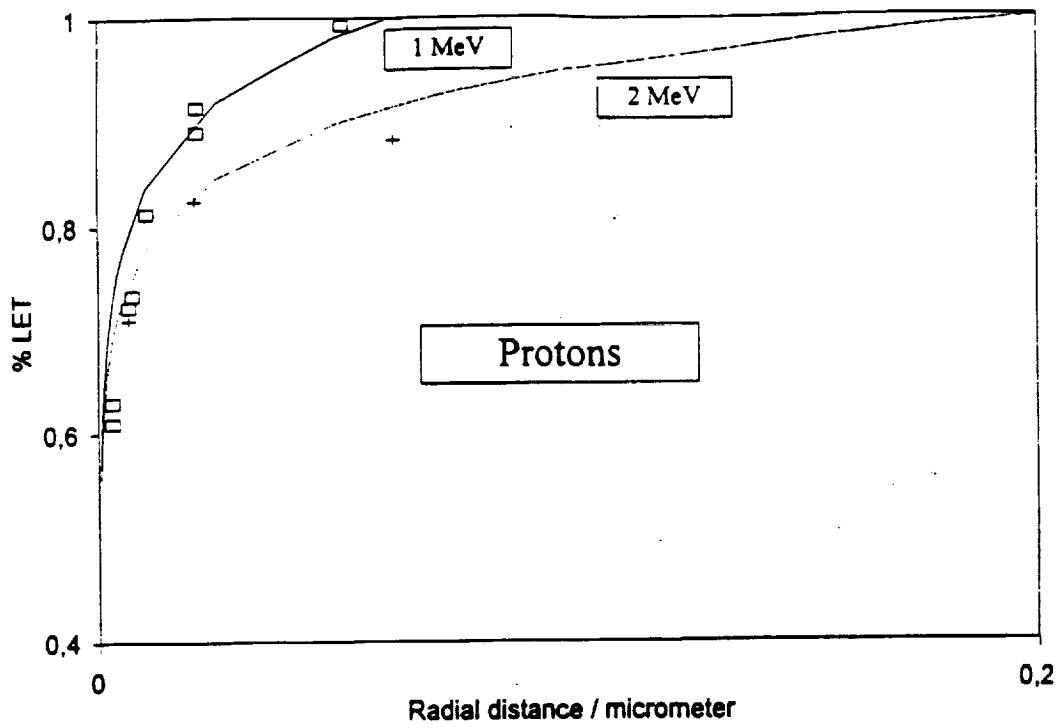$$LET_\Delta = 1 - 2\pi C\frac{\frac{Z^{*2}}{\beta^2}}{LET}ln\frac{x_p}{r} \tag{6}$$

Figure 4:

Since LET scales with $\frac{Z^{*2}}{\beta^2}$ the term before the logarithm is independent of ion charge and changes only with its specific energy. As an example figure 1 displays a comparison between the present theoretical approach and measurements of Wingate and Baum (1976). It is seen that the differences are quite small and give credence to the calculations.

Within the framework of our beam model equ. (6) can also be written in another form. Since the penumbra radius x_p depends only on the ion specific energy E

$$x_p = 0.0616E^{1.7} \tag{7}$$

it takes the form

$$f_r = 1 - 2\pi C \frac{Z^{*2}}{\beta^2 LET_\infty}(1.7lnE - lnr - 4.135) \tag{8}$$

which may be easier for some calculations.

The advantage of the here suggested way to determine range restricted LET is not only the simplicity of calculation but rather more that it starts with a beam model which is compatible with experimental data. No further assumptions are necessary than the $\frac{1}{r^2}$-dependence of the energy density -which is well supported by measurements- and the penumbra extension. The latter, however, is not very critical since it is contained only in a logarithmic term.

References

[1] ICRU 16, Int. Commission on Radiation Units and Measurements (1970): Linear energy transfer. ICRU: Washington.

[2] Kiefer, J.; Kost M. (1988): Fine structure of energy deposition - introductory remarks, in: Quantitative mathematical models in radiation biology (J. Kiefer, Ed.), SPRINGER: Berlin - Heidelberg, pp.29-40

[3] Kiefer, J.; Straaten, H. (1986): A model of ion track structure based on classical collision dynamics, Phys. Med. Biol. 31, 1201-1209

[4] Xapsos, M. A. (1992): A spatially restricted linear energy transfer, Rad. Res. 132,282

[5] Wingate, C. L.; Baum, J. W. (1976): Measured radial distribution of dose and LET for alpha and proton beams in hydrogen and tissue-equivalent gas. Rad. Res. 65,1-19