HEAVY FERMION BEHAVIOUR EXPLAINED BY BOSONS

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Abstract

Conventional heavy fermion theories require existence of massive fermions. We show that heavy fermion phenomena can also be simply explained by existence of bosons with moderate mass but temperature dependent concentration below the formation temperature T_B, which in turn is close to room temperature. The bosons B^{++} are proposed to be in chemical equilibrium with a system of holes h^+: $B^{++} \rightleftharpoons h^+ + h^+$. This equilibrium is governed by a boson breaking function $f(T)$, which determines the decreasing boson density and the increasing fermion density with increasing temperature. Since HF-compounds are hybridized from minimum two elements, we assume in addition existence of another fermion component h_s^+ with temperature independent density. This spectator component is thought to be the main agent in binding the bosons in analogy with electronic or muonic molecules. Using a linear boson breaking function we can explain temperature dependence of the giant linear specific heat coefficient $\gamma(T)$ coming essentially from bosons. The maxima in resistivity, Hall coefficient and susceptibility are explained by boson localization effects due to the Wigner crystallization. The antiferromagnetic transitions in turn are explained by similar localization of the pairing fermion system when their density $n_h(T_{FL})$ becomes lower than n_{WC}, the critical density of Wigner crystallization. The model applies irrespective whether a compound is superconducting or not. The same model explains the occurrence of low temperature antiferromagnetism also in high-T_c superconductors. The double transition in UPt$_3$ is proposed to be due to the transition of the pairing fermion liquid from spin polarized to unpolarized state.

I. INTRODUCTION

Despite of the great efforts to understand heavy fermion (HF) properties [1–3] in terms of conventional Fermi liquid theories [4–6], the Kondo model [7] or unconventional order parameter models [8,9], the theories have not provided a clear overall picture of the most dramatic properties: the large value and the temperature dependence of specific heat coefficient $\gamma(T)$, the resistance maximum at low temperatures, the Hall coefficient and the susceptibility maxima near the same temperature. Likewise the superconducting states exhibit power law behaviors of several quantities such as the specific heat near $T = 0$ and an additional linear term in the specific heat. From the above reviews the following picture emerges: In the HF-compounds the f-electrons show localized behaviour at high temperatures and become delocalized at low temperatures. The point of division for the two behaviours is the Kondo temperature T_K.

The purpose here is to show that the spectator fermion superfluid model (SFS), originally proposed for high-T_c compounds [10,11] and the superfluid states of heavy fermions [12,13], also provides a global picture which connects the superfluid state with the normal state and, at the same time, explains the properties of non-superconducting HF-compounds within the same framework with bosons decaying into pairing fermions, the holes. Since the boson density for $T \gg T_c$ and the pairing fermion density for $T \ll T_c$ both become small, they get localized due to Wigner crystallization [11,14]. In what follows we intend to show that the boson localization temperature $T_{BL} < T_B$ is in fact the coherence temperature corresponding to maxima of Hall coefficient and susceptibility. Similarly the pairing fermion Wigner crystallization

temperature is $T_{FL} = T_N$, the Néel temperature, since the fermion spins are antiferromagnetically ordered for $T < T_{FL}$. Both temperatures T_{BL} and T_{FL} are logical consequences of the SFS-model, not an extra input. In the case of high-T_c compounds of the type $RBa_2Cu_3O_{7-x}$, 3D-antiferromagnetism is known to exist together with superconductivity [15,16], with the Néel temperatures in the range 0.5 K< T_N <2.2 K. For HF-compound UPt$_3$ the low temperature peak at $T_{FL} \approx 20$ mK observed by Schuberth et al. [17] is here proposed to be also an example of Wigner crystallization of the pairing holes. Immediately above T_{FL} the holes are proposed to be polarized up to the lower temperature T_c^- of the double peak of UPt$_3$ as a consequence of the ordering of the pairing hole liquid, rather than having unconventional order parameter for the superfluid. The sequence of temperatures $T_{FL} = T_N$, T_c^- and T_c^+ is in agreement with MC-calculations by Ceperley et al. [18,19].

The plan of the paper is as follows. In chapter II we propose that the giant linear specific heat coefficient $\gamma(T)$ is a consequence of the bosons as charge carriers in these compounds. In chapter III we show how the maxima in Hall coefficient and susceptibility follow from the boson localization. Knowing the boson localization temperature $T_{BL} > T_c$, one can estimate the pairing fermion localization temperature $T_{FL} = T_N$, which is discussed in chapter IV. The Wigner crystal has two transverse electronic sound modes, and in the fermion localization case also spin waves with a gap. The longitudinal plasmon of fermion Wigner crystal gives a spectrum with temperature dependent gap below T_N, similar to the spin wave spectrum.

II. THE NORMAL STATE SPECIFIC HEAT

We follow closely the discussion of SFS carried recently [14] through in details for the high-T_c compound $YBa_2Cu_{3-x}M_xO_{7-\delta}$ and proceed to show that the same model can explain also many of the heavy fermion features mentioned above. We adopt here the same simple principle proved successful in the case of 123-compounds: The temperature dependence of an experimental quantity is simply determined by the concentration dependence of that same quantity, since carrier densities are temperature dependent according to the charge conservation in the chemical equilibrium

$$n_B(T) = n_0 f(T)$$
$$n_h(T) = 2n_0[1 - f(T)]$$
$$n_s(T) = \text{constant},$$

$$(1)$$

with $f(0) = 1$ and $f(T_B) = 0$. These equations are quite general and form a cornerstone for SFS. In particular we don't need to assume statistical mechanics of non-interacting particles to their validity. The holes with temperature dependent density $n_h(T)$ are treated in the superfluid and normal states like conventional charge carriers. Their density of states can in principle be calculated from the two-fluid formalism [14]. The fact that their density of states is temperature dependent can be looked upon as coming from the existence of bosons. With the concentration rule we may proceed to connect various experiments without knowing the exact band structure and the binding mechanism of the bosons [11,14]. In the normal states we use a simple linear function $f(T) = 1 - T/T_B$. Since T_c's are zero or very small in comparison with the boson formation temperature T_B we can take $f(T_c) \approx f(0) = 1$, as a first approximation. These features come out of an explicit fit of the model parameters to experiments in the case of UPt$_3$ and URu$_2$Si$_2$, to be discussed later in more detail. We assume the existence of bosons also in the non-superconducting cases. The picture proposed here is very close to ionization of atoms in gas. There the fraction of non-ionized atoms is given [20] by $f(T) = 1 - [1 + (P/P_0)(T_0/T)^{5/2}e^{2\Delta/k_BT}]^{-1/2}$, which is linear in a range of temperatures.

Following the concentration rule, the internal energy is taken to be a sum of a boson contribution and the pairing and the spectator fermion contributions

$$u(T) = \frac{3}{2}k_B n_0 T f(T) + \frac{1}{2}\gamma_h T^2[1 - f(T)] + \frac{1}{2}\gamma_s T^2,$$

$$(2)$$

where γ_h and γ_s are linear specific heat coefficients at $T = 0$ for the two fermion components. We will use the expression for noninteracting bosons, and therefore Eq. (2) represents the simplest

approximation one can think of. For $f = 0$ this reduces to the conventional two fermion result and for $f = 1$ we obtain the free boson and the spectator contributions. In the normal state this expression is supposed to be valid only down to finite temperatures near T_c where the superfluid fluctuations become important. In the superconducting state one may use the expression (2), except that the boson part is replaced by the acoustic plasmon contribution to be discussed later. In the BCS-like situation ($T_B = T_c$, no plasmon sound) we have $f(T) = n_s(T)$, where the superfluid fraction behaves exponentially, $n_s \approx 1 - e^{\kappa(1-1/t)}$, with $t = T/T_c$, Eq.(2) reproduces the typical BCS result with a step in the specific heat at T_c. The expression (2) is therefore reasonable in the three special cases $f = 1$, $f = 0$ and $f = n_s$. We anticipate that the calculation of boson breaking function $f(T)$ is in principle possible once we know the details of the boson binding, the band structure and the chemical lattice. So far we are content to assume that the spectators play an essential role in the binding [14]. In view of the fact that the compounds we are working with are rather complicated, we therefore should use a formalism, simple enough, to enable one to relate various experiments with a reasonable accuracy as was done in Ref. [14] for the 123-case. In what follows we intend to show that also the heavy fermions can be treated in this spirit and the results turn out to be equally good.

Eq.(2) gives for the normal state the specific heat coefficient

$$\gamma(T) = \frac{c(T)}{T} = \frac{3}{2}\frac{k_B n_0}{T}(1 - 2\frac{T}{T_B}) + \gamma_s + \frac{3}{2}\gamma_h\frac{T}{T_B}, \tag{3}$$

using the simple linear function for $f(T)$. At low temperatures the boson contribution dominates, since $T_B \approx 300 - 500$ K, and at low temperatures ($T < 20$ K) we obtain

$$\gamma(T) = \gamma_s + \frac{A}{T}. \tag{4}$$

This also shows, how the spectators give the linear term in the specific heat in the superconducting state at $T = 0$. At the temperature T_B the linear term reads

$$\gamma(T) = \gamma_s + \frac{3}{2}\gamma_h. \tag{5}$$

Above T_B we obtain the normal metallic behaviour $\gamma(T) = \gamma_s + \gamma_h$. Immediately below T_B we obtain linear behaviour and a step at T_B. We anticipate that $m_h^* >> m_s^*$, therefore the contribution from the pairing fermions γ_h should dominate at room temperature. On the other hand, already at much lower temperature the boson contribution gets small as compared with fermion parts, therefore one may be able to separate the contributions from γ_h and γ_s, since the latter one can be obtained at least in the superconducting situation from the measurements near $T = 0$. Near the localization temperatures T_{BL} and T_{FL} these expressions have to be modified in a way to be discussed later. In particular the boson term cannot continue down to zero temperature but is intruded by the superconducting or the antiferromagnetic transition. Since these are ordered states, the boson contribution is replaced below the ordering temperature by the plasmon sound or spin wave contributions, which will give finite entropy.

We have compared the electronic specific heat coefficient $\gamma(T)$ from Eq. (4) with experiments in Fig. 1 a) for UBe_{13} and $CeCu_2Si_2$ and in Fig. 1 b) for $CeAl_3$ and $CeCu_6$. These figures are based on figures (6) and (16) of Stewart [2]. It is seen that this simple theory gives a satisfactory explanation for the observed behaviour of $\gamma(T)$ in these four cases. The coefficients A come out to be nearly the same $A \approx 1400$ mJ/Kmol corresponding to boson densities of the order 10^{21}cm^{-3}. One obtains this order of magnitude also from the the analysis of the Hall-data, as we shall see. The model therefore proposes that the large size of $\gamma(T)$ is due to the bosons rather than fermions. This means also that the fermions need not be excessively heavy, which is desirable, since band structure calculations do not produce excessively narrow bands required by the conventional heavy fermion theories (see Ref. [4], p. 121). In fact the bandwidths obtained from band calculation are two orders of magnitude larger.

The compounds in Fig. 1 a) become superconducting at very low temperatures, but those in Fig. 1 b) do not. This simple theory should apply to both types. As we shall see the present model predicts a glassy antiferromagnetic transition at very low temperatures so that $\gamma(T)$ is cut off. For $CeAl_3$ this is clearly visible (see Ref. [2], Fig. 15) but for $CeCu_6$ less so. At

present we are not able to determine the fermion effective masses very accurately. There is some uncertainty associated with the possible non-linearities in $f(T)$ which no doubt can be estimated easily, but goes beyond the scope of this paper. Clearly Eqs. (2)-(5) cannot be valid down to zero temperature, as mentioned above. For superconducting compounds $\gamma(T)$ near T_c is modified by superconducting fluctuations and below T_c the plasmonic contribution replaces the free boson part. For non-superconducting compounds the superconducting fluctuations may exist near $T = 0$. The ordered transitions at T_c or $T_{FL} = T_N$ seem to offer a natural cut off for most HF-compounds.
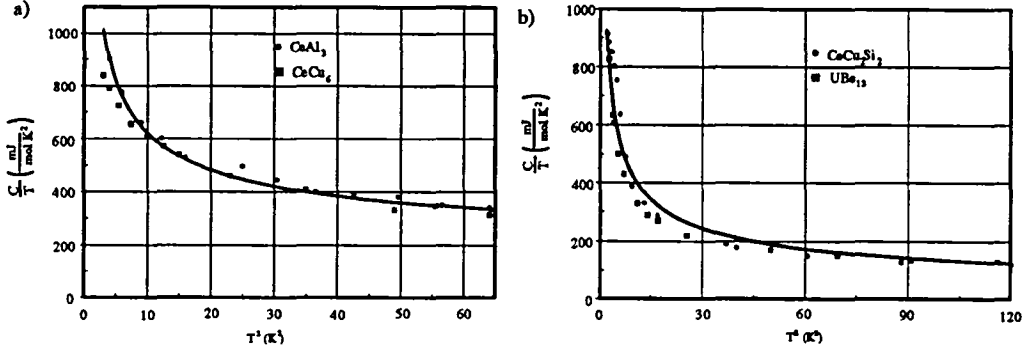

FIG. 1. The electronic specific heat coefficient $\gamma(T)$ for a) UBe$_{13}$ and CeCu$_2$Si$_2$ and b) CeAl$_3$ and CeCu$_6$. Solid lines presents our theoretical result from Eq. (4) with parameters: a) $A = 1300$mJ/molK and $\gamma_s \cong 5$mJ/molK^2 and b) $A = 1500$mJ/molK and $\gamma_s \cong 140$mJ/molK^2. Experimental points are from Figs. 6 and 16 in Ref. [2].

III. BOSON LOCALIZATION

A logical consequence of the SFS model is the Wigner crystallization (WC) of bosons, because at a certain temperature T_{BL}, which lies in between T_c and T_B, their density diminishes below the critical value corresponding to $r_s = 170$ [19]. The localization temperature T_{BL} can be estimated from the condition for the critical density,

$$n_{WC} = n_0 f(T_{BL}) \approx n_0(1 - T_{BL}/T_B) \approx 0.16 \cdot 10^{21} \left(\frac{m_h}{m}\right)^3 \text{cm}^{-3}, \tag{6}$$

where m is the electron mass and the boson mass $m_B = 2m_h$. This order of magnitude estimate corresponds to a jellium background and may be inaccurate for backgrounds prevalent in the heavy fermions with spectators and the chemical lattice. Most likely, however, such a transition does take place, and the main consequence is that the bosons become gradually delocalized below T_{BL}. The measured carrier densities in HF-compounds are of the order of magnitude 10^{21}cm^{-3} [6], close to those required by Eq. (6). Above T_{BL} the bosons cease to be active charge carriers.

A fully consistent thermodynamical description of the localization is not attempted here for the following reasons: Above T_{BL} the bosons continue to decay into holes with approximately linear temperature dependence, as deduced from Hall-measurements. Because of this decay, the Wigner crystal is never complete. Since the boson localization phenomena occur at moderately low temperatures, the Wigner crystal is never in thermal equilibrium, and one expects a glassy behaviour. The situation is further complicated by the lattice which can cause partial localization of the bosons even below T_{BL}. This is presently the main uncertainty which clearly affects our calculation of the coefficient A in Eq. (4) with density n_0 deduced from the Hall-measurements.

The Monte-Carlo-calculations by Ceperley et al. [18,19] indicate, that the energy densities of Coulomb liquid and crystal states are not very different, therefore the associated specific heat jump may be small at elevated temperatures. For these reasons the liquid-crystal transition is very rounded and shows up most clearly in the Hall density which well above T_{BL} is determined solely by the holes. This explains the near linearity of $n_H(T)$ observed in virtually all compounds

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above the coherence temperature, which we associate here with T_{BL}. We actually do not exactly know what $f(T)$ to use between T_c and T_{BL}, therefore we have used the following purely phenomenological description: Divide the density of bosons below T_{BL} into localized and delocalized portions with a phenomenological function $\xi(T)$ such that the density of mobile bosons is

$$n_{BDL}(T) = n_0 f(T)\xi(T) \approx n_0(1 - T/T_B)\,\xi(T), \tag{7}$$

where $\xi = \alpha \leq 1$ for $T \ll T_{BL}$ and $\xi = 0$ for $T \gg T_{BL}$. Here $\alpha < 1$ means that a fraction $(1 - \alpha)$ of the bosons remain localized for $T \ll T_{BL}$. Localized bosons continue to produce holes with linear rate $f(T)$. This was previously [14] used successfully to explain the Hall effect in the high-T_c case. The existence of bosons in the normal state was also used to explain the "reservoir" effect for 123-compounds.

The function $\xi(T)$ can be obtained from the experimental Hall coefficient $R_{ab} \propto n_h(T)^{-1}$. The effective Hall density (in the ab-plane) reads

$$n_H = \begin{cases} 2n_0(1 - f(T)) \cong 2n_0\frac{T}{T_B}, & \text{for } T \gg T_{BL} \\ 2n_0(1 - f(T)) + 2n_0 f(T)\xi(T) \cong 2n_0\frac{T}{T_B} + 2n_0(1 - T/T_B)\xi(T), & \text{for } T \leq T_{BL}. \end{cases} \tag{8}$$

These equations together with the Hall-measurements can be used to fix T_B as well. The localization explains the minimum in n_H, a feature which seems to be as common for heavy fermions as it is for high-T_c superconductors [14]. We have shown for 123 compounds [14] that $f(T)$ is a universal function.

The Hall density in the normal state together with the experimental data of Lapierre *et al.* [21] and Schoenes *et al.* [22] is shown in Figs. 2 a) and 2 b). The theoretical curve in Fig. 2 a) for UPt$_3$ was calculated using Eqs. (8) with the phenomenological boson localization factor of the type

$$\xi(T) = \exp\left[3.5\left(\frac{T - 6\text{ K}}{T - 50\text{ K}}\right)\right] \quad, 6\text{ K} < T < 50\text{ K}. \tag{9}$$

Similar function was used also for URu$_2$Si$_2$ in Fig. 2 b).
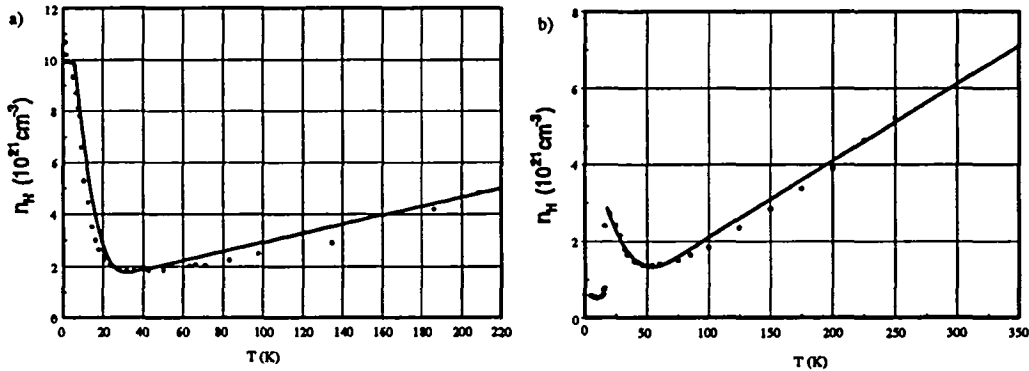

FIG. 2. a) The Hall density in the normal state for UPt$_3$ from Eq. (8) together with the experimental data of Lapierre *et al.* [21]. Theoretical result is obtained by using parameters: $T_B = 500$ K, $f_c = .88$, $n_0 = 4.95 \cdot 10^{21}\text{cm}^{-3}$ and $\alpha = 1$. b) The Hall density in the normal state for URu$_2$Si$_2$ from Eq. (8) together with the experimental data of Schoenes *et al.* [22]. Theoretical result is obtained by using parameters: $T_B = 350$ K, $f_c = .98$ and $n_0 = 5.7 \cdot 10^{21}\text{cm}^{-3}$ and boson localization factor $\xi(T) = .36 \cdot \exp\left[4\left(\frac{T-18\text{ K}}{T-110\text{ K}}\right)\right]$ for 18 K $< T <$ 110 K.

We overlook the further complication, that the spectators could bring in a more complicated two-band Hall behaviour. The fact that, Hall coefficient with $\mathbf{H} \perp \hat{c}$ is nearly constant whereas the one with $\mathbf{H} \parallel \hat{c}$ changes with temperature above T_{BL} [23], gives evidence that band mixing is small, therefore this point is not fully understood at present. The outcome of Eq. (8) for

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two compounds, UPt$_3$ in Fig. 2 a) and URu$_2$Si$_2$ in Fig. 2 b) gives us their T_B-values and the localization temperatures. The reason for selecting these two HF-compounds is that they represent the two main cases with respect to fermion localization: In UPt$_3$ the antiferromagnetic transition turns out to occur in the superconducting state and for URu$_2$Si$_2$ in the normal state. In both cases n_H-curves show slight upwards bending for $T > T_{BL}$ and the T_B-values are of the order 300-500 K. The localization temperatures are $T_{BL} \approx 30$ K for UPt$_3$ and approximately 50 K for URu$_2$Si$_2$.

Also the experimental specific heats show structure associated with T_{BL} and T_B. The approximate separation of the electronic part ΔC by Renker et $al.$ [24] shows a broad maximum for UPt$_3$ near 30-40 K and a peak near 300-500 K. We expect that the pairing fermion band and the spectator band rearrange themselves near T_B, which could also show up in the electronic specific heat. The broad maximum at 30-40 K can be interpreted in terms of Bose glass transition associated with boson localization. A qualitatively correct behaviour is obtained by taking a linear superposition of the Bose liquid and the crystal internal energies $\frac{3}{2}n_B(T)k_BT$ and $3n_B(T)k_BT$ for the boson liquid and solid, respectively, hence

$$\Delta C = \begin{cases} \frac{3}{2}n_0 k_B \frac{d}{dT}\left[Tf(T)(2 - \xi(T))\right] + \gamma_s T + \frac{3}{2}\gamma_h\left[1 - f(T)\right]T & \text{, for } T \leq T_B \\ (\gamma_s + \gamma_h)T & \text{, for } T > T_B \end{cases} \quad (10)$$

The total electronic specific heat from Eqs. (10) is compared with the data of Renker et $al.$ [24] in Fig. 3 for UPt$_3$. The agreement is rather striking. In particular the boson formation temperature T_B shows up clearly in both compounds as a peak with quadratic temperature dependence near T_B. Just above T_B we predict a jump by amount $\frac{1}{2}\gamma_h T_B$ in the specific heat. Unfortunately the data do not go high enough in temperature to show the jump. In the case of URu$_2$Si$_2$ one obtains from Renker et $al.$ [24] similar picture except for the peak near the antiferromagnetic transition at $T_N = 18$ K, which will be discussed later. In the latter case the jump at T_B shows up.
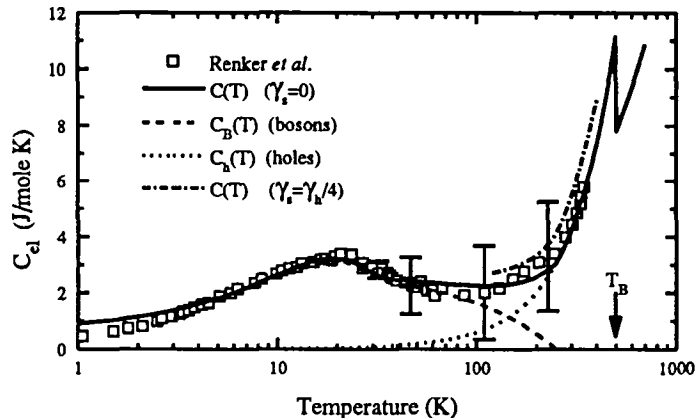

FIG. 3. The electronic specific heat for UPt$_3$ after Renker et $al.$ [24]. Solid line is our result from Eq. (10 with parameters: $\gamma_s = 0$ and $\gamma_h = 16$mJ/molK^2. Dash-dotted line is the result with $\gamma_s = 4$mJ/molK^2 and $\gamma_h = 16$mJ/molK^2.

The boson localization temperature T_{BL} limits the attainable transition temperature T_c, since $T_{BL} > T_c$. By Eq. (6) one should make the effective mass m_h small to increase T_{BL} and thereby the T_c. At the same time the background can also have a decisive effect. In Ref. [14] we found effective mass $m_h \sim m_e$ and the estimate for UPt$_3$ is 5-20 electron masses.

IV. FERMION LOCALIZATION AND THE ANTIFERROMAGNETISM

Since the Wigner crystallization takes place independent of the statistics, also the pairing holes must become localized at some temperature $T_{FL} < T_{BL}$, since their density vanishes at $T = 0$. Assuming that T_{BL} is known, we can actually calculate T_{FL} from Eqs. (1) since the

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bosons have double charge and mass. The crystallization densities of holes and bosons are therefore related by

$$2n_0[1 - f(T_{FL})] = \frac{n_0 f(T_{BL})}{64}Q.$$ (11)

The factor Q takes into account the fact that the background here is not jellium, which would correspond to $Q = 1$ and $r_s = 170$. For the superconducting compounds we have to make distinction between the two cases $T_{FL} < T_c$ and $T_{FL} > T_c$. It is well known that for fermions the spins are in antiferromagnetic arrangement in the Wigner crystal phase for jellium background. In this connection one should remember that for somewhat higher densities, hence here above T_{FL}, the holes may form a spin polarized liquid. The direction of spins may be strongly influenced by non-jellium background effects, but WC-localization offers a natural explanation for occurrence of antiferromagnetism in the HF-compounds. If the antiferromagnetic transition takes pla e above T_c we obtain from Eq. (11), by using the linear approximation for $f(T)$,

$$T_{FL} = \frac{T_B}{128}(1 - \frac{T_{BL}}{T_B})Q \approx \frac{T_B}{128}Q.$$ (12)

Depending upon the values of T_B and Q this gives $T_{FL} > 1$ K. The main source of inaccuracy in Eq. (11) is the factor Q and the sensitivity of the left hand side to possible non-linearities in $f(T)$, which show up in the experiments for the Hall densities illustrated in Figs. 2 a) and 2 b).

The fermion localization at $T_N = 18$ K explains also the sudden drop in n_H for URu_2Si_2 apparent in Fig. 2 b). The amount of the drop is larger than we would expect. This we believe is coming from spectators and for a quantitative calculation of the drop one should use a two-band formula. The main point we want to make is that the strange behaviour of URu_2Si_2 Hall coefficient is understood with the boson and fermion localizations. This includes also the different behaviour for UPt_3. Conversely the Hall density n_H is like a map where the localization effects can be read out.

To carry out a calculation of T_{FL} one must first determine $f(T)$ from experimental data on Hall effect, specific heat etc.. As an example we show how to calculate T_{FL} for UPt_3, where the antiferromagnetic transition supposedly occurs in the superfluid state. Another example we have been able to carry through with reasonable accuracy is the superconductor URu_2Si_2, where $T_N = T_{FL} \approx 18$ K in the normal state. We start the discussion with UPt_3. Using the Hall data, the specific heat data and the exponent relations derived by Kallio et al. [12] we have determined the boson breaking function $f(T)$ and the superfluid fraction $n_s(T)$ using the functional form $(t = T/T_c)$

$$f(t) = n_s(t) + f(T_c)t^{4-5\gamma/3}$$
$$n_s(t) = 1 - at^\gamma - (1 - a)e^{\kappa(1-1/t)}.$$ (13)

The philosophy behind these terms is of course that the functions $f(t)$ and $n_S(t)$ have direct connection with experiments and the exponent γ is limited by the two-fluid model to the range $0 \le \gamma \le 1.5$. For UPt_3 we use the specific heat data of Midgley et al. [25] and Hall density data of Lapierre et al. [21] to obtain for the parameters the values $\gamma = 1.42$, $f(T) = .88$, $a = .91$, and $\kappa = 3.5$. In the normal state we therefore obtain

$$f(T) = .88(1 - T/T_B) \qquad , T_B \cong 500 \text{ K.}$$ (14)

In calculating the specific heat we used Eq. (2) with the boson contribution replaced by the plasmon term [14]

$$U_{pl} = \frac{\pi^2}{30} \frac{(k_B T)^4}{[\hbar u_1(T)]^3},$$ (15)

where $u_1(T)$ is the plasmon sound velocity (see Eqs. (16) and (17)). The data for UPt_3 are extremely well fitted with these functions. As an example we show the specific heat in the superfluid state in Fig. 4 a). Because of the low value of T_c, $f(T_c)$ is nearly unity justifying our approximation $f(T) = 1 - T/T_B$ in Eq. (3). The value of exponent $\gamma = 1.42$ in Eq. (13) is close

to the one obtained for 123 in Ref. [14] ($\gamma = 1.3$) as well as the gap parameter $\kappa = 3.5$ instead of 3. The small coefficient .09 in front of the gap term makes $n_s(t)$ to have nearly power-law behaviour. The most dramatic difference between the HF- and high-T_c compounds is the pair breaking at T_c which gives $f_c \approx 1$ for former and $f_c \approx .6$ for the latter. For URu_2Si_2 it comes out close $f_c \cong .98$. The physical explanation for this is the low value of T_c for HF-compounds. The heavy fermions are therefore further away from the BCS than the high-T_c superconductors, in view of the fact that within the present framework $f_c = 0$ for BCS. Since the boson densities are not very different for the high-T_c and HF-compounds, simple Bose-Einstein model for heavy fermions would give, with moderate boson mass $m_B < 10m_e$, right order of magnitude for T_c [14,26]. Without boson breaking this model would, however, have difficulties in explaining the localization phenomena discussed here.
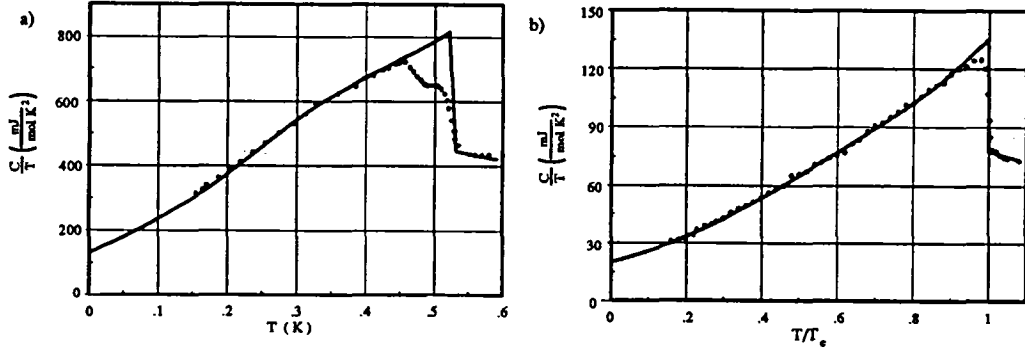

FIG. 4. a) The experimental specific heat for UPt_3 from Ref. [25] together with our theoretical result. For $f(t)$ we have used parameters: $f_c = .88$, $\gamma = 1.42$, $a = .91$ and $\kappa = 3.5$. The used spectator contribution $\gamma_s = 130 mJ/molK^2$ is much larger than the one obtained from Fig. 3. b) The experimental specific heat for URu_2Si_2 from Ref. [30] together with our theoretical result. For $f(t)$ we have used parameters: $f_c = .98$, $\gamma = 2.16$, $a = .987$ and $\kappa = 3.6$. The spectator contribution comes out to be $\gamma_s = 23 mJ/molK^2$.

Knowing the boson breaking function $f(T)$ for UPt_3 we can now calculate the Wigner crystallization temperature T_{FL} for the holes to be about 20 mK, close to the experimental value 18 mK found for the extra peak by Schuberth et $al.$ [17]. Our result is obtained from Eq. (11) with $Q = 1$. Since in the Wigner crystal for the holes the spins are in the antiferromagnetic arrangement, we propose that $T_{FL} = T_N$, the Néel temperature.

Although the properties of the antiferromagnetic transition may be affected by the chemical lattice, we may explain some properties, like the specific heat peak, by the Wigner crystal model below T_N. Since the transition takes place at low temperatures, we may use the sound mode approximation for the WC specific heat. The excitation modes of WC have been studied theoretically by Carr [27]. According to his calculations the Wigner crystal has two transversal sound modes, with the density dependent sound velocities $u_\perp \propto r_s^{-1/2}$. In the present case this translates into the temperature dependence [14]

$$u_\perp(T) = u_\perp(T_{FL}) \left[\frac{1 - f(T)}{1 - f(T_{FL})} \right]^{1/6} . \qquad (16)$$

In the case of pairing fermions forming a liquid, the longitudinal sound velocity $u_1(T)$ was determined by the relation

$$u_1(T) = u_1(T_c) \left[\frac{1 - f(T)}{1 - f(T_c)} \right]^{1/3} . \qquad (17)$$

Despite of the pairing fermion localization the two-fluid philosophy remains the same: The normal liquid density in the superfluid system consisting of bosons and holes, is obtained from the bosonic (sound modes, spin waves) and fermionic excitations (boson breaking). The latter ones are determined by $[1 - f(T)]$ which should be determined by the chemical equilibrium, but

we choose to follow the philosophy of Ref. [14] and try to determine $f(T)$ from experiments, if possible. One uncertainty remains: Are the sound mode or the spin waves the dominating feature in the bosonic excitations, which determine the specific heat in the superfluid state? The data on UPt$_3$ is insufficient to give an answer, because T_{FL} is so low. If the transition takes place in the normal state, we believe that immediately below T_{FL} the boson part in the specific heat will be dominated by the spin waves as compared with plasmonic contributions. This will be apparent in URu$_2$Si$_2$, to be discussed later. Much below T_{FL} the spin wave dominated specific heat would go to zero exponentially when $T \to 0$ since there is the gap, which experimentally turns out to be rather large. We expect this to be true also for non-superconducting compounds: The giant bosonic specific heat coefficient is cut off at $T_{FL} = T_N$, because in the antiferromagnetic state the bosonic coefficient has to be replaced by the plasmonic contribution. We of course do realize, that some of the HF-compounds, so far, show no order whatsoever, below the lowest temperatures measured. One such example is CeAl$_3$, with $\gamma(0) \approx$ 18.5 mJK^{-2}cm^{-3}. However, below 0.32 K for this compound the specific heat coefficient $\gamma(T)$ starts reducing, which can be interpreted as coming from a glassy antiferromagnetic transition (See Ref. [2], Fig. 15). One should also beware of that some of the antiferromagnetic transitions can have a different origin.

The antiferromagnetic Wigner crystal made of the holes is also never in equilibrium because their number density is changing with temperature and one has vacancies and/or the full WC-order can never be extended in practice. For this reason one should expect also here spin glass behaviour. In the high-T_c case the situation should be similar: The antiferromagnetic transition of holes should take place at about 1-2 K, if one uses Eq. (12) and the $f(T)$ determined in Ref. [14]. This is born out in the experiments on Bi$_2$Sr$_2$CaCu$_2$O$_8$ by Caspary et al. [28], who separated out the electronic specific heat in this temperature range and found broad maxima of $\gamma(T)$ at different magnetic fields. These maxima were interpreted as spin glass behaviour by Caspary et al., for different reasons though. Several other high-T_c compounds show antiferromagnetic transitions at low temperatures below T_c.

A further feature easily understood within the present model is that the antiferromagnetic specific heat peak should exist even in magnetic fields higher than H_{c2}, because the bosons still exist, and hence their decay into holes occurs independent of superconductivity. This was indeed observed in UPt$_3$ by Schuberth et al. [29].

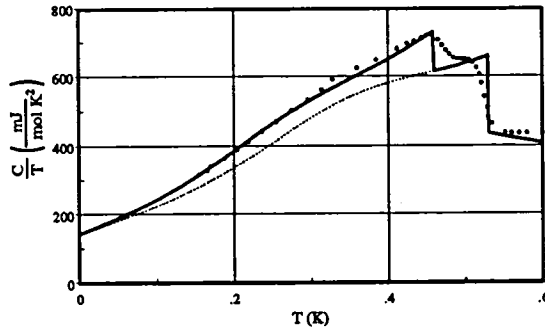FIG. 5. The experimental specific heat for UPt$_3$ from Ref. [25] together with our theoretical result, with double peak at T_c. For spin polarized liquid we have used parameters: $T_c^- = .47$ K, which corresponds $T_c^- = .904 \cdot T_c^+$ and $\kappa^- = 3.5$. For unpolarized liquid: $T_c^+ = .52$ K, $\kappa^+ = 4.3$. Exponent $\gamma = 1.41$ and coefficient $a = .91$ are the same for both liquids. The gray line shows our result for unpolarized liquid below T_c^-.

In the Monte Carlo calculations of Ceperley et al. [18,19] it was found that for $r_s < 100 - 170$ (hence $T > T_{FL}$ in our case) one obtains a spin polarized liquid and below $r_s = 60 - 75$ unpolarized liquid. Since the energy difference between the two liquids is very small and in UPt$_3$ we have chemical lattice different from jellium, the possibility exists that the double peak near $T_c \approx .5$ K is explained by the phase transition between the two types of pairing fermion liquids. In fact, if one makes use of $f(T)$ deduced from experiments and assigns for $T_{FL} = .02$ K the value $r_s = 170$, we obtain at T_c the value $r_s = 60$, which is close to the value 75 obtained by Ceperley et al. for the crossing of the polarized and unpolarized liquid

energies. This interpretation of the double peak would be in agreement with the domain structure found by Midgley et al. [25], since the energies for the two structures are close. The associated domain structure could also be a reason for the metamagnetism observed in several compounds. Since the T_c's are low it would also occur in the normal states. We stress that the present interpretation preserves $^1S_0$-order parameter for the superfluid. Actually the r_s-limits for Wigner crystal of fermions are numerically uncertain even for jellium background due to the fact that the GMC-method is strictly speaking "exact" only for Bose systems, as discussed by Ceperley et al. Since we have a different background, the right hand sides of Eqs. (11) and (12) can have sizable Q-factors, which can only be determined experimentally, owing to the complicated background and the glassy nature of the Wigner crystal transitions. We believe, however, that the order of the two localization temperatures remains always the same, i.e., $T_{BL} > T_N$.

V. CONCLUSIONS

If we identify the pairing holes belonging to f-band in the SFS case the following picture emerges: The f-electrons combine into bosons at high temperatures $T < T_B$ which remain localized down to temperature T_{BL}. In the temperature range $T_{BL} < T < T_B$ the chemical equilibrium requires existence of mobile f-holes with density $n_h \sim 1 - f(T) \approx T/T_B$, where $f(T)$ is a universal function. At very low temperatures the pairing fermions suffer a glassy antiferromagnetic transition at T_N. This explains the existence of superheavy quasiparticles in dHvA-experiments, which then, near T_N, measure the distribution of masses near a localization transition.

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