

BOSON LOCALIZATION AND UNIVERSALITY IN $\text{YBa}_2\text{Cu}_{3-x}\text{M}_x\text{O}_{7-\delta}$

A. Kallio, V. Apaja and S. Pöykkö

University of Oulu, Department of Theoretical Physics, Linnanmaa, SF-90570 Oulu, Finland
(June 22, 1994)

Abstract

We consider a two component mixture of charged fermions on neutralizing background with all sign combinations and arbitrarily small mass ratios. In the two impurity limit for the heavier component we show that the pair forms a bound state for all charge combinations. In the lowest order approximation we derive a closed form expression $V_{eff}(r)$ for the binding potential which has short-range repulsion followed by attraction. In the classical limit, when the mass of embedded particles is large $m_2 \gg m$, we can calculate from $V_{eff}(r)$ also the cohesive energy E and the bond length R of a metallic crystal such as lithium. The lowest order result is $R = 3.1 \text{ \AA}$, $E = -0.9 \text{ eV}$, not entirely different from the experimental result for lithium metal. The same interaction for two holes on a parabolic band with $m_2 > m$ gives the quantum mechanical bound state which one may interpret as a boson or local pair in the case of high- T_c and heavy fermion superconductors. We also show that for compounds of the type $\text{YBa}_2\text{Cu}_{3-x}\text{M}_x\text{O}_{7-\delta}$ one can understand most of the experimental results for the superconducting and normal states with a single temperature dependent boson breaking function $f(T)$ for each impurity content x governing the decay of bosons into pairing fermions. In the normal state $f(T)$ turns out to be a linear, universal function, independent of the impurity content x and the oxygen content δ . We predict with universality a depression in $T_c(x)$ with slight down bending in agreement with experiment. As a natural consequence of the model the bosons become localized slightly above T_c due to the Wigner crystallization, enhanced with lattice local field minima. The holes remain delocalized with a linearly increasing concentration in the normal state, thus explaining the rising Hall density. The boson localization temperature T_{BL} shows up as a minimum in the Hall density R_{ab}^{-1} . We also give explanation for very recently observed scaling of temperature dependent Hall effect in $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$.

I. INTRODUCTION

Despite great efforts, high- T_c superconductivity HTS is still lacking a convincing theoretical explanation that everybody can accept. There is ample experimental evidence for high- T_c , heavy fermions HF and Chevrel compounds that the properties of these systems cannot be understood with the most simple BCS model alone. More than anything the normal state properties in all these compounds deviate from BCS, which above the transition temperature T_c would predict a normal metal. The anomalies appear in the three main experiments: the NMR, the Raman and Hall experiments and to some extent in the thermal conductivity and surface resistivity.

The Hall-density ($n_H = 1/Re_c$) in the ab -plane is diminishing just above T_c and shows a minimum at $T_{BL} > T_c$ and beyond the minimum it rises linearly. In the case of 123 the Hall coefficients in different direction have opposite signs $R_{ab} > 0$, $R_c < 0$ [1]. This, more than anything, requires existence of charge carriers of more than one type in these compounds. Furthermore the NMR relaxation rate $W(T)$ below T_c does not show a shoulder predicted by

BCS. At very low temperatures the relaxation rate $W(T)$ exhibits power laws with very small exponent like 1.5 as compared with exponential BCS-behaviour or linear Korringa-behaviour [2]. The Korringa law is not obeyed in the normal states. The Raman scattering shows no coherence peak at $\omega = 2\Delta$ but continuum background [3], unexplainable within the BCS.

In the case of 123-compounds all the anomalies mentioned above can be understood in terms of doubly charged boson model where the bosons exist up to temperature $T_B \approx 300 - 500$ K, much higher than T_c . Below this temperature the bosons are in chemical equilibrium with respect to reaction $B^{++} \rightleftharpoons h^+ + h^+$, where the holes are supposed to be tied to a band with parabolic dispersion relation. The chemical equilibrium can quite generally be represented by a boson breaking function $f(T)$ such that the number density of bosons and fermions are

$$\begin{aligned} n_B(T) &= n_0 f(T) \\ n_h(T) &= 2n_0 [1 - f(T)], \end{aligned} \quad (1)$$

which simply contains the charge conservation. Just like dissociation in chemistry takes place in a solvent, here the equilibrium reaction takes place in the sea of electrons.

From the Hall-experiments we have deduced that $f(T)$ is nearly linear function above T_c [4]. If the bosons and holes are tied to the ab -plane they both contribute to Hall-density and $R_{ab} > 0$. The minimum in the Hall-density is explained by the Wigner crystallization (and hence by localization) of bosons above a temperature $T_{BL} > T_c$, since their density gets smaller than the critical density for Wigner crystallization. Similarly near $T = 0$ the density of the holes gets small since $1 - f(T) \rightarrow 0$. The model therefore predicts antiferromagnetic transitions in the superconducting states. Such transitions are also often observed. Although we cannot claim that they all are connected with the proposed Wigner crystallization of the pairing fermions, it is difficult to understand why they occur near $T = 0$. In c -direction the Hall density is dominated by the background electrons, called spectators.

The plan of the paper is as follows: First we show that the existence of bosons is feasible. In fact we will calculate the relative wave function of such boson and its binding energy in the case of two charged particles with heavy mass embedded in the electron gas with neutralizing background. We do this by applying the hypernetted chain (HNC) method of classical statistical mechanics which has previously been shown to be very accurate for electron gas. In chapter III we consider boson localization and in chapter IV two types of universal behaviour in high- T_c superconductors.

II. SPECTATOR FERMION BINDING OF BOSONS

To introduce the method we repeat the steps needed to calculate energy/particle of the electron gas. Here the relevant density parameter is the r_s -value defined by the volume taken by one electron $n^{-1} = \frac{4}{3}\pi(r_s a_0)^3$, where a_0 is Bohr radius $= \hbar^2/me^2$. Another relevant quantity is the radial distribution function $g(\mathbf{r}_{12})$ which gives the relative probability of finding an other electron at the position \mathbf{r}_2 , if there is one at the point \mathbf{r}_1 . For uniform electron liquid $g(\mathbf{r}_{12})$ is independent of the angles \mathbf{r}_{12} . The corresponding probability amplitude $\psi(r_{12})$ is $\sqrt{g(r_{12})}$. With Coulomb interaction $v_c(r)$ the potential energy can be calculated exactly form

$$\frac{V}{N} = \frac{1}{2}n \int d^3r [g(r) - 1]v_c(r), \quad (2)$$

provided that function $g(r)$ is calculated from the exact ground state wave function Ψ by

$$n^2 g(r_{12}) = N(N-1) \frac{\int |\Psi|^2 d\tau_2}{\int |\Psi|^2 d\tau}, \quad (3)$$

where N is number of particles and $d\tau = d^3r_1 d^3r_2 d\tau_2$. For interactions other than Coulomb force one has $g(r)V(r)$ inside of the integrand in Eq. (2). The reason for $(g-1)v_c$ is the screening. Since $g(r) \rightarrow 1$ at large distances, this also makes the integral in Eq. (2) to converge.

Another function which, in the case of quantum liquids, is intimately connected with neutron scattering experiments, is the liquid structure factor $S(k)$ which is obtained by Fourier transform of $g(r) - 1$

$$S(k) - 1 = n \int e^{i\mathbf{k}\cdot\mathbf{r}} [g(r) - 1] d^3r. \quad (4)$$

Using the HNC-method one can calculate also the kinetic energy as functional of $S(k)$ and $g(r)$. Without repeating here all the relevant steps needed we give the final result: One can write the Euler-Lagrange equation into a form of "Schrödinger equation" for the probability amplitude $\psi = \sqrt{g}$ [5]

$$-\frac{\hbar^2}{m} \nabla^2 \psi(r) + [v_c(r) + w(r) + w_e(r)] \psi(r) = 0, \quad (5)$$

where $v_c(r)$ is the Coulomb potential and the boson induced potential $w(r)$ is the Fourier transform of

$$w(k) = -\frac{\hbar^2 k^2}{4m} \left[\frac{S-1}{S} \right]^2 (2S+1). \quad (6)$$

The potential $w_e(r)$ contains higher order terms and corrections due to the Fermi statistics which are fairly small in the range $r_s > 3$. Eq. (5) is the zero energy limit of a Schrödinger equation with zero scattering length: $\psi(r) \rightarrow 1$ for large r . Since $w(k)$ is a functional of $S(k)$, the "Schrödinger equation" is highly non-linear. Simplest way to solve Eq. (5) formally is to define a correction function R by

$$\frac{\nabla^2 \psi}{\psi} = \frac{1}{2} \nabla^2 (g-1) + R(r), \quad (7)$$

which allows one to obtain the solution by Fourier transform of Eq. (5) (with $\gamma = 4\pi n e^2 m / \hbar^2$)

$$S(k) = \frac{k^2}{\sqrt{4\gamma + k^4 + 4k^2(m/\hbar^2 w_e(k) - R(k))}} \approx \frac{k^2}{\sqrt{4\gamma + k^4}} \quad (8)$$

In the range $r_s > 3$, which we are here interested in, both $R(k)$ and $w_e(k)$ are small, hence latter form, so-called bosonic uniform limit approximation, is quite accurate. In the uniform limit approximation by Eq. (8) two features are exact: It gives the correct k^2 -behaviour for small k and produces the correct plasma frequency. Also the energy, shown in Fig. 1 compares favorably with the Monte Carlo results of Ceperley [6], which in turn is very close to the fermi hypernetted chain (FHNC) [7] result of Zabolitzky [8], which would correspond to a proper treatment of w_e and $R(k) \neq 0$ in our case. The Fermi correlations were included as a potential w_e , for which Eq. (5) gives exactly the free Fermi gas radial distribution function in the $r_s \rightarrow 0$ -limit.

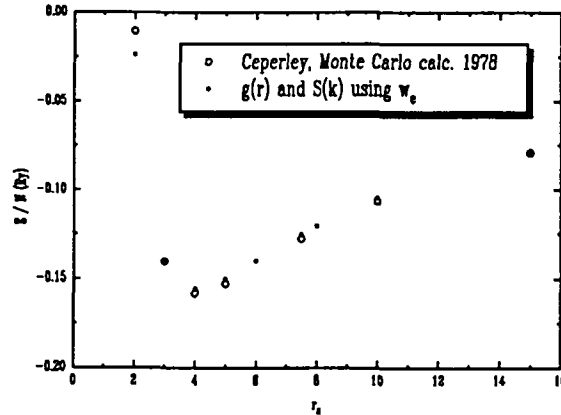


FIG. 1.

Total energy of electron gas calculated using the functions $g(r)$ and $S(k)$ solved from Eqs. (4) and (5), as compared with the Monte Carlo results of Ceperley [6]. Fermi correlation energy was added within the Lado-approximation.

The effect of screening comes out in a vivid way from Eq. (5): The free space Coulomb interaction is changed into an effective interaction $V_{eff} = v_c + w$ due to the other electrons around. In fact for large r , v_c is completely cancelled by $w(r)$ and hence $V_{eff}(r)$ has finite range.

With these preliminaries we are ready to examine what happens if we embed charged particles into the electron gas. The same treatment with radial distribution functions can be generalized for a mixture: One obtains a system of three coupled equations of the type Eq. (5). Calling the second component mass m_2 , density n_2 and charge $Q_2 = Ze$ one would now need three radial distribution functions $g_{11}(r) = g(r)$ (= the electron gas) $g_{12}(r)$ and $g_{22}(r)$ and the corresponding probability amplitudes $\psi_{ij} = \sqrt{g_{ij}}$, which satisfy the following three coupled Euler-Lagrange equations

$$-\frac{\hbar^2}{\mu_{ij}}\nabla^2\psi_{ij} + \left(\frac{Q_i Q_j}{r} + w_{ij}\right)\psi_{ij} = 0, \quad i, j = 1, 2 \quad (9)$$

with $\mu_1 = m$, $\mu_2 = m_2$ and $\mu_{12} = mm_2/(m + m_2) = m/(1 + M)$. The detailed expressions for w_{ij} are to be found in Ref. [9]. It turns out that the set (9) has again a solution if the mass ratio $M \approx 1$, but for $M < .1$ there seems to be no solution numerically [10]. For these reason it was thought that this approach cannot be applied to a problem like the metallic hydrogen. However, since the method works so well in the single component case, the reason for this inadequacy was not fully understood earlier. Our purpose here is to show that the reason for instability is appearance of bound state in channels ψ_{22} or ψ_{12} which one simply has to treat more properly. In what follows we will show that the proper treatment is to go first to one and two impurity limits $n_2/n \rightarrow 0$. In this limit one obtains a decoupling of the three equations: First equation reduces to the background electron gas described before. For the impurity, one obtains the liquid structure factor $S_{12}(k)$ from $S(k)$ of the electron gas by

$$S_{12} = \frac{4S^2(k)}{1 + MS(k)} \left[\frac{Z\gamma}{k^4} + \frac{1}{2}(1 + M)\frac{R_{12}(k)}{k^2} \right] \approx -\frac{4Z\gamma S^2(k)}{k^4(1 + MS(k))} \quad (10)$$

The small correction function $R_{12}(r)$ has analogous meaning as the one in Eq. (7) with the definition

$$\frac{\nabla^2\psi_{12}}{\psi_{12}} = \frac{1}{2}\nabla^2(g_{12} - 1) + R_{12}(r) \quad (11)$$

and in all Fourier transforms the background electron density n is used. For small mass ratio $M \approx 0$ we obtain in the uniform limit for both $R = R_{12} = 0$ very simple form

$$S_{12}(k) = -\frac{4Z\gamma}{4\gamma + k^4}, \quad (12)$$

showing that the sign of S_{12} is opposite to charge $Q_2 = Ze$. The third component in this limit decouples also and we obtain a Schrödinger equation for the pair

$$-\frac{\hbar^2}{m_2}\nabla^2\psi_{22} + \left(\frac{Z^2Q_2^2}{r} + w_{22}\right)\psi_{22} = E\psi_{22} \quad (13)$$

with induced potential $w_{22}(k)$ in k -space

$$w_{22}(k) = -\frac{\hbar^2 k^2}{4m} \left[\frac{S_{12}}{S} \right]^2 (1 + 2MS). \quad (14)$$

With Eq. (13) we can also treat the two impurity scattering with $E \geq 0$ and the bound states, if any, with $E < 0$. In the case of finite density n_2 one would have $E = 0$ and hence $g_{12}(r) \rightarrow 1$, for large r . Here, however, we have used the formalism simply to calculate the induced potential $w_{22}(r)$ since there is no reason why $w_{22}(r)$ would have different expression for $E \neq 0$ than for $E = 0$. Eq. (13) has earlier been used by Owen [11] to calculate the Landau parameters for $^4\text{He}+^3\text{He}$ mixture and the expression is the same with different notation. Hence eq. (13) is

a normal, linear Schrödinger equation in the medium with induced potential calculated from HNC. For $M \rightarrow 0$, $w_{22}(k)$ is always attractive in k -space, irrespective of the sign of S_{12} and hence of the impurity charge. If one goes to finite density n_2 and Eq. (13) has a bound state, one naturally has an instability in the two fermion mixture.

Writing $Q_2^2 = Z^2 e^2$ one can now calculate $w_{22}(r)$ in the uniform limit for both $S_{12}(k)$ and $S(k)$ and small M to be (with $b = \gamma^{1/4}$) inverse transform of $4\hbar^2 \gamma^2 / [mk^2(4\gamma + k^4)^{-1}]$ which is

$$w_{22}(r) = -\frac{Z^2 e^2}{r} (1 - e^{-br} \cos br) \quad (15)$$

and the effective potential becomes simply

$$V_{eff}(r) = \frac{Z^2 e^2}{r} e^{-br} \cos br, \quad (16)$$

showing the screening property explicitly. Notably this lowest order result is different from RKKY-type of interaction coming from Kohn-Luttinger instability [12].

This simple theory can now be applied to variety of instances. In the classical limit $M = 0$, the equilibrium distance R of the two impurities is determined by the minimum of the effective potential which is close to $bR = \pi$ and hence

$$R = 1.26 r_s^{3/4} \text{ \AA}. \quad (17)$$

The corresponding cohesive energy of the pair is

$$E = \frac{1}{2} V_{eff}(R) \approx -0.246 Z^2 r_s^{-3/4} \text{ eV}. \quad (18)$$

Simplest case to compare with experiment here is lithium with $Z = 3$ and $r_s = 3.25$. We obtain now $R = 3.1 \text{ \AA}$ and $E = -0.9 \text{ eV}$ as compared with the experimental values 3.4 \AA and -1.6 eV . Clearly we don't expect our result to agree with experiment in lowest order approximation but the magnitudes are correct. The calculation for lithium could in fact be performed in two ways: One could take the impurity to be He^+ -ion with modified Coulomb interaction $v_c(r)$ with $Z = 1$ or else take $Z = 3$ and improve on the $S_{12}(k)$, since $S(k)$ can be calculated accurately using FHNC. By taking $Z = 1$ one avoids the exciton singularity in the 12-channel and in principle such a calculation can be performed with desired accuracy, which however is not the point here. In the original set both bound state singularities show up and we can interpret this as a sign of good accuracy rather than a deficiency of the set (9). The remedy is to treat the two fermion bound states as the second component i.e. use boson fermion mixture instead but with modified Coulomb interaction. This is exactly what has been done so far within the spectator fermion model of the new superconductors [4]

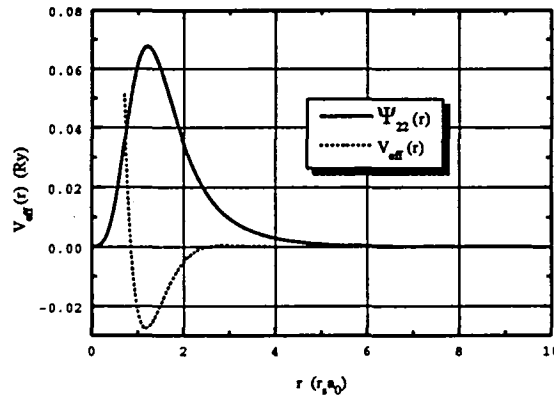


FIG. 2.

Effective potential (16) and wavefunction $\psi_{22}(r)$ for a pair of electrons or holes forming a bound boson, with $r_s = 4$ and $M = .1$.

Next we return to the high- T_c case by assuming that the impurity holes or electrons belong to a parabolic band with finite effective mass such that $M < 1$. The problem of embedding two heavy holes with effective mass m_2 is not different from the previous impurity problem and the same Schrödinger equation (13) has to be solved for the probability amplitude ψ_{22} . In Fig. 2 we show a representative case for a pair of holes or electrons forming a bound 1S_0 -boson, using the effective potential of Eq. (16). The size is about what is needed in ceramic superconductors: Few Ångströms and the binding $2\Delta \approx 500 - 1000$ K, depending upon r_s and M . The calculation shows that the existence of such a boson or a local pair is feasible. The bosons would survive up to temperatures $k_B T_B \sim \Delta$, where T_B is the boson formation temperature. Hence we predict $T_B \sim 250 - 500$ K, which is about what one needs in high- T_c [4] or heavy fermion [13] superconductors. Furthermore the theory predicts existence of all possible combinations $[B^{++}, e^-]$, $[B^{--}, e^-]$, $[B^{++}, h^+]$ and $[B^{--}, h^+]$, since the effective interaction is always attractive. These define the electron-hole liquid (EHL) and spectator fermion (SFS) superfluid models [4]. The superfluidity of the composite bosons can be shown at $T = 0$ by calculating the superfluid fraction n_c from the one particle density matrix. Such a calculation also uses formally the impurity liquid structure factor [14] where an impurity is this time embedded into the boson system.

III. BOSON LOCALIZATION

It is well known that charged particles (bosons or fermions) on a smooth, neutralizing background become localized into a Wigner crystal (WC) at densities lower than $r_s \approx 170$ [15,16]. The limiting density n_{WC} depends upon the mass and charge of the particles. In the case of ion background additional localization is caused by ion sites, lattice defects etc.. Hence WC-localization puts an ultimate lower limit for the boson density that can still lead to superfluidity. For bosons with $Z = 2$ the limiting density turns out to be

$$n_{WC} > 2.09 \left(\frac{m_B}{m_e} \right)^3 10^{19} \text{cm}^{-3}. \quad (19)$$

We use the boson mass $m_B = 2m_e$, hence the WC-localization limit is $n_B > 0.16 \cdot 10^{21} \text{cm}^{-3}$. This should be compared with the experimental boson density $n_0 \approx 8 \cdot 10^{21} \text{cm}^{-3}$. We anticipate that the lattice and the lattice defects localize additional portion of the bosons. The corresponding limit for the holes with $m_h = m_e$ is only $3 \cdot 10^{17} \text{cm}^{-3}$, and we predict that the holes localize at extremely low temperatures where $n_h(T) \approx 0$. This is important for understanding of the antiferromagnetism appearing in many high- T_c and HF-compounds [13]. In what follows the holes produced in the boson decay obeying the same rule dictated by $1 - f(T)$ regardless whether they are produced from localized or delocalized bosons. Since at T_B all bosons have decayed, somewhere in between T_c and T_B there must exist a temperature T_{BL} where they all become localized. In the present calculation we will treat the localization phenomenologically by writing the number density of delocalized bosons to be

$$n_{BDL}(T) = \begin{cases} \alpha n_0 f(T) \xi(T) \approx \alpha n_0 f(T) (1 - T/T_{BL}), & \text{if } T < T_{BL} \\ 0, & \text{if } T > T_{BL}. \end{cases} \quad (20)$$

Here the coefficient α determines what fraction of bosons are localized already at $T = 0$. Since the localized bosons hardly show up, we cannot determine α very accurately, and the exact behaviour of the localization factor is not known. We believe that in reality $\alpha < 1$, and suggest that the localization is the main source for sample dependence observed in high- T_c materials, even for single crystals.

In the normal state only the delocalized bosons give contribution to the specific heat, Hall coefficient, resistivity, etc. The only place where the localized bosons may show up are the magnetic experiments such as NMR, where they can give contribution to the orbital shift and thereby also to the relaxation rate $1/T_1$.

The issue of boson localization can come up only in models like EHL or SFS, where the bosons exist in the normal state and suffer boson breaking. We therefore predict that HTS and HF form a laboratory where the phenomenon of localization can be studied experimentally in

details. In our earlier calculations it was difficult to understand why in the Hall effect (above T_c) only the holes contribute to the Hall-density and not the bosons. Now the observed behaviour is fully understood, including the minimum which in our model should occur near the localization temperature T_{BL} . It would otherwise be very difficult to understand why the Hall density would diminish in the range $T_c < T \leq T_{BL}$, when the temperature is increased.

IV. UNIVERSALITY

Since the dispersion relation of the fermionic excitations is unknown and likewise the band structure details, we will use the simplest possible approach of taking into account only the concentration dependencies of the experimental quantities. The temperature dependencies of concentrations in turn are determined by the boson breaking function $f(T)$ which by our earlier discussions [17] is assumed to be linear above T_c . In the normal state we write

$$f(T) = f_c \frac{1 - T/T_B}{1 - T_c/T_B}, \quad T_c \leq T \leq T_B \quad (21)$$

The average number of holes and mobile bosons in the normal state are again given by Eqs. (1) and (20). In Eq. (21) the parameters f_c , T_c and T_B define a function which, if continued to $T = 0$, would predict that a fraction of bosons is localized already at zero temperature.

In the case of single crystal of 123 we have deduced from the experiments (Hall-effect) the values of the main parameters to be $f_c \approx 0.6$, and $T_B \equiv 280$ K. We assume that T_B is independent of impurities and likewise the linear function in Eq. (21) is assumed to be universal. These two parameters are sufficient to describe also the normal state of the impurity systems.

The idea behind universality is as follows: In the non-impurity case the boson density at $T = 0$ is n_0 . The presence of plane coppers is vital for the boson formation. In the impurity case $x > 0$ a fraction of plane coppers are replaced by impurities such as Zn. Hence near the impurity sites bosons may not be formed. Therefore the boson density at $T = 0$ is diminished by a fraction ν to be $n_B(0, x) = \nu(x)n_0$. Correspondingly the density of holes is increased to $n_h(0, x) = 2n_0[1 - \nu(x)]$. In the normal state this shift of chemical equilibrium is reflected as the decrease of temperatures $T_c(\nu)$ and $T_{BL}(\nu)$, while T_B should be characteristic to each compound. As a first approximation we use the same linear function for all x , which gives us the universality.

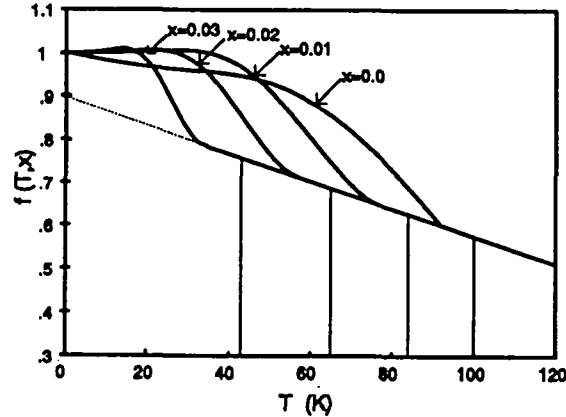


FIG. 3.

The boson breaking functions $f(T, x)$ for $\text{YBa}_2(\text{Cu}_{1-x}\text{Zn}_x)_3\text{O}_7$ with four values of x and $T_B = 280$ K. The vertical lines indicate the localization temperatures $T_{BL}(x)$. According to the universality idea the $f(T)$'s above T_c fall on the same line, which has been continued down to $T = 0$ (dotted line).

The idea of universal linear $f(T)$ above T_c is illustrated in Fig. 3. In fact we have gone one step further by assuming that the universality is true also for different oxygen contents δ , which controls the density n_0 in $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$. In Fig. 4 we compare the Hall density calculated using a universal $f(T)$ with the one reported by Jones *et al.* [18]. The result supports the idea of universal $f(T)$ above T_c : In the range $0 < \delta < 0.5$ the data is quite well reproduced with n_0 as the only free parameter.

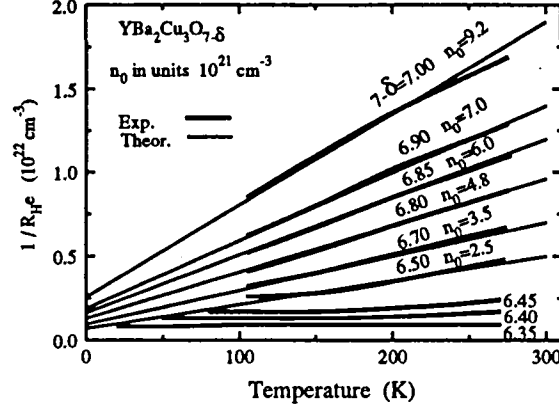


FIG. 4.

The Hall density with several oxygen contents in YBCO thin films. The short thick curves represent the experimental data of Ref. [18]. The theoretical lines were calculated using the relation $(eR_H)^{-1} = 2n_0[1 - f(T)]$ with the same linear $f(T)$ for all oxygen contents. The only changing parameter is n_0 . The $f(T)$ is given by Eq. (21) with the fixed parameters $T_c = 92$ K, $T_B = 300$ K, and $f_c = 0.6$. This result demonstrates that $f(T)$ above T_c is fairly independent of the oxygen content.

The universality allows us to calculate T_c , the localization temperature T_{BL} , and f_c as a function of impurity content x from Eqs.

$$\begin{aligned} \nu f(T_{BL}(\nu)) &= f(T_{BL}(1)) \\ \nu f(T_c(\nu)) &= f(T_c(1)) \end{aligned} \quad (22)$$

where ν is related to the impurity concentration x by $1 - \nu \approx 7x$ (determined from the NMR experiment of Ref. [2]). Eqs. (22) are based on the fact that localization or the superfluid transitions take place at a fixed boson density for the same crystalline background. Using Eqs. (21) and (22) one obtains the following formula for the T_c depression:

$$T_c(\nu) = \nu^{-1}[T_c(1) - (1 - \nu)T_B], \quad (23)$$

or

$$T_c(x) = \frac{T_c(0) - 5x T_B}{1 - 5x}, \quad (24)$$

where $T_B \approx 300$ K and factor 5 comes from the valence counting for 123 discussed by Harashina *et al.* [20]: The Zn impurity atoms replace only coppers in the ab plane, and $\frac{2}{3}7 \approx 5$. This gives quantitative agreement with experiment [20]. The pole appearing in Eq. (24) for unphysical negative values of $T_c(x)$ causes down bending of the curve which is observable in the case of $\text{La}_{2-y}\text{Sr}_y\text{Cu}_{1-x}\text{Zn}_x\text{O}_4$ in the data of the same group. Also the T_c depression of Pr and Ce substituted Bi2122 seems to bend down in a similar fashion [21]. Harashina *et al.* give an extensive discussion of the "spin gap", which they conclude to be connected with localization of holes. Here we associate the minimum in the Hall coefficient R_{ab} with boson localization, which explains in a simple way the existence of the minimum in R_{ab} near $T_{BL} \approx 100$ K in 123 [20]. From the universality idea we obtain for the localization temperature $T_{BL}(\nu)$ the relation

$$T_{BL}(\nu) - T_c(\nu) = \nu^{-1}[T_{BL}(1) - T_c(1)]. \quad (25)$$

With increasing impurity content x (decreasing ν) the separation between the localization temperature T_{BL} and T_c increases. The universality is illustrated in Fig. 3 for 123 impurity systems.

Another kind of universality in HTS has been considered by Schneider and Keller [23], who assumed that T_c has a parabolic maximum at the optimum condensate density. They found out that the rescaled T_c plotted against the muon-spin-relaxation rate of many HTS fall on a single curve. Zhang and Sato [24] have proposed that when the rescaled T_c is plotted against the hole concentration in the CuO_2 -planes a universal doping curve emerges for many cuprate superconductors. Instead of the usual parabolic form, they obtain a plateau around maximum T_c . As pointed out by Awana and Narlikar [25], the weakness of such plots are the ambiguities in estimating the hole concentration. Schneider and Keller further calculate the pressure derivative $d \ln T_c / dP$ and the isotope effect coefficient $-m d \ln T_c / dm$ vs. T_c . The present model obeys these universalities: the doping behaviour of T_c has been derived in Ref. [26], and the pressure derivatives were plotted in Fig. 4 of Ref. [13]. The pressure derivatives of both hole and electron doped HTS come out correctly from SFS.

In Ref. [26] the derivation of doping curves for $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$ was based on the formula $n_0 + n_e = \bar{n} = \text{constant}$ while n_0 and n_e are changed (notice the different notation). At the time of Ref. [26] the Landau damping of the sound mode was thought to be the reason why $T_c(x_B)$ is zero outside the boson concentration region $x_1 < x_B < x_2$. Now we suggest that the critical boson concentrations x_1 and x_2 correspond to the localization limits: the underdoping limit x_1 and the overdoping limit x_2 correspond to boson localization, the latter one because too few spectators remain. The localization gives the same sound velocity exponent $1/2$ at both critical points [4], and we obtain a T_c formula slightly simpler than the one in Ref. [26]

$$T_c(x_B) \approx A [(x_B - x_1)(x_2 - x_B)]^{5/8}, \quad (26)$$

where A is approximately independent of concentration x_B .

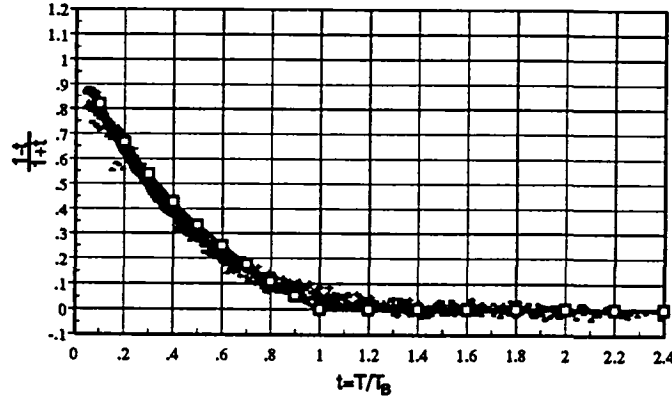


FIG. 5.

Scaling of T-dependent Hall coefficient R_H white squares respect the result from (28).

Since both n_0 and n_e change in doping we expect that also T_B , T_c and f_c in Eq. (21) will change with x . However if one uses $t = T/T_B$ with $f(t)$ universal, we may write the Hall-coefficient for $T > T_{BL}$ $R_H(t) = R_H^\infty / (1 - f(t))$, therefore

$$\frac{R_H(t) - R_H^\infty}{R_H^\infty} = \frac{f(t)}{1 - f(t)} \approx \frac{\hat{f}_c(1 - t)}{1 - \hat{f}_c(1 - t)} \approx \frac{\hat{f}_c(1 - t/\mu)^\mu}{1 - \hat{f}_c(1 - t/\mu)^\mu}, \quad (27)$$

which is also universal, if in Eq. (21) the quantity $\hat{f}_c = f(T_c(x)) / (1 - T_c(x)/T_B(x))$ is independent of x .

Recently Hwang *et al.* [27] have reported a scaling of the in-plane temperature dependent Hall coefficient $R_H(T)$. They rescaled $R_H(T)$ in a form $[R_H(T/T^*) - R_H^\infty] / R_H^*$, giving the same T/T^* dependence for all x . R_H^* and T^* are x dependent parameters. If we identify T_B with $1.8T^*$, we obtain using $\hat{f}_c = .5$ the curve

$$[R_H(t) - R_H^\infty]/R_H^\infty = \frac{1-t}{1+t}, \quad (28)$$

which is shown in Fig. 5 to be in rather good agreement with experiment. The last, non-linear form in Eq (27) with critical exponent $\mu > 1$ would give good agreement without rescaling of t .

V. CONCLUSIONS

We conclude that simple microscopic theory predicts the possibility of boson formation in the situation where the background electron or hole liquid (the spectators) have smaller effective mass than the pairing fermions which below temperature T_B combine pairwise into bosons. Since the approximate close form effective potential $V_{eff}(r)$ by Eq. (16) gives a reasonable value for the unit cell of Li-metal and also the cohesive energy we believe that it is fairly accurate also in the quantum mechanical boson problem for one to add the effects of unisotropies present in ceramic superconductors. The knowledge of boson wave function enables one to calculate the density of states and hence $f(T)$ from the boson decay at finite temperature by simply calculating the scattering states from Eq. (13) with $E > 0$.

The present theory of boson binding is different from BCS phonon coupling and also from the bipolaron model.

- [1] N. P. Ong, in: *Physical Properties of High-Temperature Superconductors II*, ed. D. M. Ginsberg (World Scientific, Singapore, 1990) p. 459.
- [2] K. Ishida, Y. Kitaoka, T. Yoshitomi, N. Ogata, T. Kamino and K. Asayama, *Physica C* **179** (1991) 29.
- [3] C. Thomsen and M. Cardona, in: *Physical Properties of High-Temperature Superconductors I*, ed. D. M. Ginsberg (World Scientific, Singapore, 1989) p. 409.
- [4] A. Kallio, V. Apaja, X. Xiong and S. Pöykkö, *Physica C* **219** (1994) 340-362.
- [5] A. Kallio and R. A. Smith, *Phys. Lett.* **68B** (1977) 315; R. Abe, *Progr. Theor. Phys.* **19** (1958) 407.
- [6] D. Ceperley, *Phys. Rev. B* **18** (1978) 3126.
- [7] S. Fantoni and S. Rosati, *Nuovo Cim.* **25** (1975) 593; *Nuovo Cim. Lett.* **16** (1976) 531.
- [8] J. G. Zabolitzky, *Phys. Rev. B* **22** (1980) 2353.
- [9] P. Pietiläinen and A. Kallio, *Phys. Rev. B* **27** (1983) 224.
- [10] T. Chakraborty, A. Kallio, L.J. Lantto and P. Pietiläinen, *Phys. Rev. B* **27** (1983) 3061.
- [11] J. C. Owen, *Phys. Rev. Lett.* **47** (1981) 311.
- [12] W. Kohn and J.H. Luttinger, *Phys. Rev. Lett.* **15** (1965) 524.
- [13] A. Kallio and X. Xiong, *Physica C* **199** (1992) 340; A. Kallio, S. Pöykkö and V. Apaja, submitted to *Physica C*.
- [14] M. Puoskari and A. Kallio, *Phys. Rev. B* **30** (1984) 152.
- [15] E. P. Wigner, *Phys. Rev.* **46** (1934) 1002.
- [16] K. Mouloupoulos, and N. W. Ashcroft, *Phys. Rev. Lett.* **69** (1992) 2555.
- [17] A. Kallio, V. Apaja, and X. Xiong, *Physica C* **191** (1992) 23.
- [18] E. C. Jones, D. K. Christen, J. R. Thompson, R. Feenstra, S. Zhu, D. H. Lowndes, J. M. Phillips, M. P. Siegal and J. D. Budai, *Phys. Rev. B* **47** (1993) 8986.
- [19] T. R. Chien, D. A. Brawner, Z. Z. Wang, and N. P. Ong, *Phys. Rev. B* **43** (1991) 6242; N. P. Ong, in *Physical Properties of High Temperature Superconductors II*, Ed. D. M. Ginsberg (World Scientific, Singapore, 1990) p. 459.
- [20] H. Harashina, T. Nishikawa, T. Kiyokura, S. Shamoto, M. Sato, and K. Kakurai, *Physica C* **212** (1993) 142.
- [21] V. P. S. Awana, S. K. Agarwal, A. V. Narlikar, and M. P. Das, *Phys. Rev. B* **48** (1993) 1211.
- [22] V. Apaja, and A. Kallio, *Physica C* **209** (1993) 519.
- [23] T. Schneider, and H. Keller, *Phys. Rev. Lett.* **69** (1992) 3374.
- [24] H. Zhang, and H. Sato, *Phys. Rev. Lett.* **70** (1993) 1697.
- [25] V. P. S. Awana, and A. V. Narlikar, *Phys. Rev. Lett.* **71** (1993) 303.
- [26] A. Kallio, X. Xiong, *Phys. Rev. B* **43** (1991) 5564.
- [27] H.Y. Hwang, B. Batlogg, H. Takagi, H.L. Kao, J. Kwo, R.J. Cava, J.J. Krajewski and W.F. Peck Jr., *Phys. Rev. Lett.* **72** (1994) 2636.