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**MULTIDISCIPLINARY DESIGN OPTIMIZATION USING**  
**MULTIOBJECTIVE FORMULATION TECHNIQUES**

**BY**

**ADITI CHATTOPADHYAY, ASSOCIATE PROFESSOR**

**AND**

**NARAYANAN S. PAGALDIPTI, GRADUATE RESEARCH ASSOCIATE**

**DEPARTMENT OF MECHANICAL AND AEROSPACE ENGINEERING**  
**ARIZONA STATE UNIVERSITY**  
**TEMPE, ARIZONA 85287-6106**

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## TABLE OF CONTENTS

<b>1.</b>	<b>Abstract.....</b>	<b>1</b>
<b>2.</b>	<b>Introduction.....</b>	<b>1</b>
<b>3.</b>	<b>Objectives .....</b>	<b>5</b>
<b>4.</b>	<b>Discrete Semi-Analytical Aerodynamic Sensitivity Analysis.....</b>	<b>6</b>
<b>5.</b>	<b>Grid Sensitivity Technique .....</b>	<b>7</b>
<b>6.</b>	<b>Problem Formulation Using Multilevel Decomposition.....</b>	<b>8</b>
<b>7.</b>	<b>Analysis.....</b>	<b>10</b>
<b>8.</b>	<b>Optimization.....</b>	<b>11</b>
<b>9.</b>	<b>Application.....</b>	<b>12</b>
	<b>9.1 Aircraft Configuration .....</b>	<b>12</b>
	<b>9.2 Two Level Optimization Problem .....</b>	<b>13</b>
	<b>9.3 Results and Discussion .....</b>	<b>14</b>
<b>10.</b>	<b>Significance of the Research.....</b>	<b>17</b>
<b>11.</b>	<b>References.....</b>	<b>18</b>

## LIST OF TABLES

<b>Table 1. Grid Sensitivity of the Three-Dimensional Hyperbolic Grid.....</b>	<b>22</b>
<b>Table 2. Sensitivity of the Drag Coefficient.....</b>	<b>22</b>
<b>Table 3. Sensitivity of the Lift Coefficient.....</b>	<b>22</b>
<b>Table 4. Aerodynamic Design Variables.....</b>	<b>23</b>
<b>Table 5. Structural Design Variables.....</b>	<b>23</b>

## LIST OF FIGURES

<b>Figure 1. Coordinate Systems .....</b>	<b>24</b>
<b>Figure 2. Delta Wing-Body Configuration.....</b>	<b>24</b>
<b>Figure 3. Wing Cross-Section and Wing Spar (Box Beam).....</b>	<b>25</b>
<b>Figure 4. Comparison of CPU Time for the Sensitivity of Hyperbolic Grid....</b>	<b>25</b>
<b>Figure 5. Comparison of CPU Time for Aerodynamic Sensitivity Analysis ....</b>	<b>26</b>
<b>Figure 6. Comparison of CPU Time With Design Variables .....</b>	<b>26</b>
<b>Figure 7. Iteration History of Drag Coefficient .....</b>	<b>27</b>
<b>Figure 8. Iteration History of Lift Coefficient .....</b>	<b>27</b>
<b>Figure 9. Iteration History of Aircraft Weight .....</b>	<b>28</b>

## 1. ABSTRACT

This report addresses the development of a multidisciplinary optimization procedure using an efficient semi-analytical sensitivity analysis technique and multilevel decomposition for the design of aerospace vehicles. A semi-analytical sensitivity analysis procedure is developed for calculating computational grid sensitivities and aerodynamic design sensitivities. Accuracy and efficiency of the sensitivity analysis procedure is established through comparison of the results with those obtained using a finite difference technique. The developed sensitivity analysis techniques are then used within a multidisciplinary optimization procedure for designing aerospace vehicles. The optimization problem, with the integration of aerodynamics and structures, is decomposed into two levels. Optimization is performed for improved aerodynamic performance at the first level and improved structural performance at the second level. Aerodynamic analysis is performed by solving the three-dimensional parabolized Navier Stokes equations. A nonlinear programming technique and an approximate analysis procedure are used for optimization. The procedure developed is applied to design the wing of a high speed aircraft. Results obtained show significant improvements in the aircraft aerodynamic and structural performance when compared to a reference or baseline configuration. The use of the semi-analytical sensitivity technique provides significant computational savings.

## 2. INTRODUCTION

Analysis and design of aerospace vehicles are associated with complex multidisciplinary couplings. The development of an efficient optimization procedure for the design of aircraft must incorporate the interactions between disciplines such as aerodynamics, dynamics, aeroelastic stability, structures, controls and acoustics. However, the validity of the designs obtained using optimization techniques depends strongly upon the accuracy of the analysis procedures used and it is essential to integrate sufficiently comprehensive analysis procedures within the closed-loop optimization. Such procedures are computationally intensive, and therefore, can be prohibitive within an optimization environment. For example, it is essential to use a comprehensive

aerodynamic analysis procedure to solve the complex flow field associated with high speed aircraft. Over the past few years, Computational Fluid Dynamics (CFD) has evolved rapidly with the development of numerous numerical algorithms. Although accurate detailed analyses of many complex flow fields are now possible using supercomputers, viscous-compressible flow simulations of wing-body configurations can require several CPU hours per steady-state solution. Therefore, the use of such comprehensive analytical procedures for design optimization can be prohibitively expensive if a gradient-based technique is used.

Sensitivity analysis, in which the derivative of a system performance function (e.g., the lift or drag of an aircraft) with respect to a design variable (e.g., a parameter controlling the wing planform) is calculated, is an essential ingredient in design optimization. A widely used technique for performing aerodynamic sensitivity analysis is the method of finite differences. The use of this method is associated with several calls to the flow analysis routine. Although this technique is conceptually simple, the associated computational cost is prohibitive when used in an optimization problem involving a large number of design variables. Therefore, it is necessary to develop efficient techniques to calculate aerodynamic sensitivities, so that advanced CFD codes may be more useful as practical design tools in multidisciplinary optimization environments.

Two popular alternatives to the finite difference technique are the direct differentiation approach and the adjoint variable approach. These techniques are widely used in structural sensitivity calculations [1, 2]. In both techniques, the actual governing equations are differentiated with respect to the design variables using chain rule. The direct differentiation approach yields a large system of equations involving the desired sensitivities that can be solved directly. In the adjoint variable approach, adjoint variables are obtained as the solution to an adjoint problem. The adjoint variables are then used to calculate the sensitivities. These two techniques are equivalent and yield identical results for the sensitivities. More recently, there has been widespread interest in using these techniques for calculating aerodynamic sensitivities. Carlson and Elbanna [3] have used the direct differentiation technique to differentiate the discretized transonic small perturbation equations and obtain aerodynamic sensitivities. Baysal et al. [4, 5] have performed discrete

sensitivity analysis using the Euler equations. Taylor et al. [6] and Newman et al. [7] have developed a semi-analytical sensitivity analysis procedure for the thin-layer Navier-Stokes equations using an incremental strategy. Jameson et al. [8] have proposed a continuous sensitivity approach using the adjoint variable method to calculate aerodynamic sensitivities. In the continuous sensitivity approach, the governing equations are differentiated prior to their discretization. The sensitivities are calculated using a numerical algorithm similar to the one used for obtaining the flow solution. Therefore, the continuous sensitivity approach needs to be modified, depending upon the governing equations that are differentiated. In the discrete sensitivity approach, the discretized governing algebraic equations are differentiated. Although there is a need for solving a large system of equations, this procedure can easily be adapted to different analysis procedures. In the present research, the aerodynamic sensitivities have been calculated by directly differentiating the discretized governing parabolized Navier Stokes (PNS) equations [9]. The discrete direct differentiation approach has been adopted over the continuous sensitivity approach because the CFD procedure used in this research to solve the PNS equations is based on a finite volume approach. This finite volume algorithm is more readily amenable to the direct differentiation approach than the continuous sensitivity approach.

Two main ingredients in an aerodynamic sensitivity analysis procedure are: (1) the calculation of the sensitivities of the discretized flow variables and (2) the calculation of the sensitivities of the computational grid with respect to the aerodynamic design variables. It has been well recognized that the sensitivities of the flow variables are dependent upon the sensitivities of the computational grid [4-9]. However, in most of the aforementioned work, brute force finite difference techniques were used to calculate the grid sensitivities. Very few formal investigations have been reported on the development of analytical or semi-analytical techniques for computing grid sensitivities. High quality elliptic and hyperbolic grid generation codes are often used for generating meshes for aircraft configurations [10]. The use of the finite difference method for calculating grid sensitivities can be computationally prohibitive in such situations. Korivi et al. [11] developed a grid sensitivity analysis wherein the Jacobian matrix of the entire grid with respect to the grid

points on the boundary of the domain is calculated. The sensitivities of the surface grid points are calculated using an elastic membrane analogy to represent the computational domain, and the surface grid sensitivities are calculated from a structural analysis code using the finite elements method. Extension of this technique to complex three dimensional flow fields can be extremely complicated and time consuming. Further, the use of an additional structural analysis code increases computing time. Sadrehaghighi et al. [12] proposed an analytical approach for calculating grid sensitivities in which algebraic grid generation is performed using transfinite interpolation and surface parameterization in terms of design variables. The transfinite interpolation equations are analytically differentiated to obtain the grid sensitivities. The most general parameterization of the boundaries would require the specification of every grid point on the boundary. This, however, is impractical from a computational point of view. A quasi-analytical parameterization is used in Ref. 12 which allows the aircraft component to be specified by a relatively smaller number of parameters. However, the technique does not offer a great amount of generality because most CFD codes use complex grids which are generated using methods based on partial differential equations.

In the present research, the grid sensitivity parameters are efficiently calculated, without any loss of generality and complexity, by directly differentiating the elliptic and hyperbolic grid generation equations [13]. This results in a large system of equations which can be solved readily to yield the grid sensitivities. The technique developed is not restricted to two dimensional problems and can be applied to three dimensional problems without any additional effort. The developed grid sensitivity technique is then used in conjunction with the semi-analytical aerodynamic sensitivity analysis procedure developed by Chattopadhyay and Pagaldipti [9], for calculating aerodynamic design sensitivities within a multidisciplinary optimization procedure.

The development of the computationally efficient semi-analytical sensitivity analysis techniques allows the integration of comprehensive CFD-based codes within realistic multidisciplinary optimization studies. The necessity of multidisciplinary coupling in successful design optimization has been recognized. Recently, attempts have been made in coupling two or



more of these disciplines [14-18] in which optimization was performed by addressing all the design criteria in a single level. This “all-at-once” optimization procedure, in which all the disciplines are coupled inside a single loop and optimization is performed based on criteria involving every discipline, can be inefficient and time consuming. Decomposition techniques are often used to simplify such complex optimization problems into a number of sub-problems. Multilevel decomposition techniques have been applied to problems based on a single discipline [19-24] in structural applications. Recently, attempts have been made to use these techniques for multidisciplinary optimization of rotary wing aircraft. Adelman et al. [25] developed a two-level procedure for performing integrated aerodynamic, dynamic and structural optimization of rotor blades, based on the multilevel optimization strategy described in Ref. 22. Chattopadhyay et al. [26] developed a three-level procedure for optimization of helicopter rotor blades with the integration of aerodynamics, dynamics, aeroelastic stability, and structures. In a multidisciplinary design problem, the number of levels in a multilevel decomposition procedure typically depends upon the number of disciplines involved. Individual optimization is performed at each level using analysis procedures pertaining to that level. Optimal sensitivity parameters are exchanged between the levels to provide the necessary coupling. An optimal design is obtained when each individual level is converged and overall convergence is achieved. Therefore, the speed of obtaining a fully converged result depends upon the strength of coupling between the various levels. In the present research, the developed semi-analytical sensitivity techniques are used within a multilevel optimization procedure for designing aerospace vehicles with the coupling of aerodynamics and structures.

### **3. OBJECTIVES**

The objectives of the current research project are as follows.

- (a) Development of a semi-analytical approach for calculating aerodynamic design sensitivities.
- (b) Development of a semi-analytical approach for calculating grid sensitivities to be used within the aerodynamic sensitivity analysis.

- (c) Development of a multiobjective, multilevel optimization procedure for aerospace vehicles with the integration of the necessary disciplines such as aerodynamics and structures.

These objectives have all been achieved successfully, details of which are furnished in the following sections.

#### 4. DISCRETE SEMI-ANALYTICAL AERODYNAMIC SENSITIVITY ANALYSIS

The sensitivities of the aerodynamic coefficients of the aircraft with respect to its relevant geometric parameters are calculated in the present research using a discrete, direct differentiation approach. This approach is described in detail here. In general, an aerodynamic performance coefficient,  $C_j$ , depends on the steady-state flow variables,  $Q^*$ , the vector of computational grid coordinates,  $X$ , and, sometimes, explicitly on the vector of independent design variables,  $\Phi$ . Mathematically,

$$C_j = C_j(Q^*(\Phi), X(\Phi), \Phi) \quad (1)$$

The derivative of  $C_j$  with respect to the  $i^{\text{th}}$  design variable,  $\phi_i$ , is expressed as follows.

$$\frac{dC_j}{d\phi_i} = \left\{ \frac{\partial C_j}{\partial Q^*} \right\}^T \left\{ \frac{\partial Q^*}{\partial \phi_i} \right\} + \left\{ \frac{\partial C_j}{\partial X} \right\}^T \left\{ \frac{\partial X}{\partial \phi_i} \right\} + \frac{\partial C_j}{\partial \phi_i} \quad (2)$$

In Eq. 2, the terms  $\left\{ \frac{\partial C_j}{\partial Q^*} \right\}$ ,  $\left\{ \frac{\partial C_j}{\partial X} \right\}$  and  $\frac{\partial C_j}{\partial \phi_i}$  are easily calculated knowing the explicit dependence of  $C_j$  on  $Q^*$ ,  $X$  and  $\phi_i$ . The term  $\left\{ \frac{\partial Q^*}{\partial \phi_i} \right\}$ , which represents the sensitivity of the steady state flow variables with respect to the  $i^{\text{th}}$  design variable, is calculated using the direct differentiation technique. In the discrete sensitivity approach, the discretized flow equations are directly differentiated, as described next. The discretized flow equations which model the flow can be written as follows.

$$\{R(Q^*(\Phi), X(\Phi), \Phi)\} = \{0\} \quad (3)$$

Equation 3, differentiated with respect to  $\phi_i$ , yields

$$\left\{ \frac{dR}{d\phi_i} \right\} = \left\{ \frac{\partial R}{\partial Q^*} \right\}^T \left\{ \frac{\partial Q^*}{\partial \phi_i} \right\} + \left\{ \frac{\partial R}{\partial X} \right\}^T \left\{ \frac{\partial X}{\partial \phi_i} \right\} + \frac{\partial R}{\partial \phi_i} = \{0\} \quad (4)$$

Equation 4 represents a set of linear algebraic equations in  $\frac{\partial Q^*}{\partial \phi_i}$  which can be solved easily. It is to be noted that the terms  $\left\{\frac{\partial R}{\partial Q^*}\right\}$ ,  $\left\{\frac{\partial R}{\partial X}\right\}$  and  $\frac{\partial R}{\partial \phi_i}$  in Eq. 4, can be calculated easily, knowing the explicit dependence of  $\{R\}$  on  $Q^*$ ,  $X$  and  $\phi_i$ .

## 5. GRID SENSITIVITY TECHNIQUE

The term  $\left\{\frac{\partial X}{\partial \phi_i}\right\}$  appearing in Eqs. 2 and 4 represents the grid sensitivity vector which must be computed semi-analytically. The semi-analytical grid sensitivity approach is illustrated on a hyperbolic grid generator. In general, a three dimensional hyperbolic grid generation code generates a two-dimensional grid at various stations along the longitudinal direction by solving the following equations [10].

$$\frac{\partial y \partial y}{\partial \eta \partial \zeta} + \frac{\partial z \partial z}{\partial \eta \partial \zeta} = 0 \quad (5)$$

$$\frac{\partial y \partial z}{\partial \eta \partial \zeta} - \frac{\partial z \partial y}{\partial \eta \partial \zeta} = F(\eta, \zeta) \quad (6)$$

where the xyz coordinate system is a Cartesian coordinate system fitted to the body and  $\xi\eta\zeta$  coordinate system is the computational domain used by the CFD procedure for aerodynamic analysis [Fig. 1]. In Eq. 6,  $F(\eta, \zeta)$  is a known function approximating the Jacobian of transformation between the xyz and the  $\xi\eta\zeta$  coordinate systems. Equations 5 and 6 are discretized and solved numerically to obtain the grid vector  $X$ . The grid sensitivity vector,  $\left\{\frac{\partial X}{\partial \phi_i}\right\}$ , can be obtained by directly differentiating Eqs. 5 and 6 with respect to  $\phi_i$  after their discretization, as follows.

$$\frac{\partial y}{\partial \eta} \left( \frac{dy_{j+1,k}}{d\phi_i} - \frac{dy_{j,k}}{d\phi_i} \right) \frac{1}{(\zeta_{j+1} - \zeta_j)} + \frac{\partial y}{\partial \zeta} \left( \frac{dy_{j,k+1}}{d\phi_i} - \frac{dy_{j,k}}{d\phi_i} \right) \frac{1}{(\eta_{k+1} - \eta_k)} + \frac{\partial z}{\partial \eta} \left( \frac{dz_{j+1,k}}{d\phi_i} - \frac{dz_{j,k}}{d\phi_i} \right) \frac{1}{(\zeta_{j+1} - \zeta_j)} + \frac{\partial z}{\partial \zeta} \left( \frac{dz_{j,k+1}}{d\phi_i} - \frac{dz_{j,k}}{d\phi_i} \right) \frac{1}{(\eta_{k+1} - \eta_k)} = 0 \quad (7)$$

$$\frac{\partial y}{\partial \eta} \left( \frac{dz_{j+1,k}}{d\phi_i} - \frac{dz_{j,k}}{d\phi_i} \right) \frac{1}{(\zeta_{j+1} - \zeta_j)} + \frac{\partial z}{\partial \zeta} \left( \frac{dy_{j,k+1}}{d\phi_i} - \frac{dy_{j,k}}{d\phi_i} \right) \frac{1}{(\eta_{k+1} - \eta_k)} - \frac{\partial z}{\partial \eta} \left( \frac{dy_{j+1,k}}{d\phi_i} - \frac{dy_{j,k}}{d\phi_i} \right) \frac{1}{(\zeta_{j+1} - \zeta_j)} - \frac{\partial y}{\partial \zeta} \left( \frac{dz_{j,k+1}}{d\phi_i} - \frac{dz_{j,k}}{d\phi_i} \right) \frac{1}{(\eta_{k+1} - \eta_k)} = \frac{dF}{d\phi_i} \quad (8)$$

Equations 7 and 8 represent a system of equations which can be solved readily to yield the grid sensitivity vector,  $\left\{ \frac{\partial X}{\partial \phi_i} \right\}$ .

The grid sensitivity and the aerodynamic sensitivity techniques yield large systems of algebraic equations characterized by sparse coefficient matrices. Direct methods for solving these systems of equations are inefficient and iterative techniques have been used in this work. The systems of equations are solved using the successive over relaxation (SOR) scheme. A detailed description of this iterative technique can be found in Ref. 10 and is not repeated here.

## 6. PROBLEM FORMULATION USING MULTILEVEL DECOMPOSITION

This section describes the multilevel decomposition technique useful in breaking up coupled and complex optimization problems into simpler optimization problems. The multilevel decomposition procedure is illustrated through a two-level formulation. Each level is a multiobjective optimization problem characterized by a vector of objective functions, constraints and design variables. During optimization at a particular level, it is essential to maintain the objective functions and design variables of lower levels close to their optimum values. Therefore, constraints are imposed on the perturbations to the lower level objective functions and design variables to prevent significant changes. These parameters are called optimal sensitivity derivatives, and they establish the necessary link between the various levels of optimization. The multilevel decomposition procedure is outlined below.

Level 1:

Minimize

$$F_i^1(\phi^1) \quad i = 1, \dots, \text{NOBJ}^1$$

subject to

$$g_k^1(\phi^1) \leq 0 \quad k = 1, \dots, \text{NC}^1$$

$$\sum_{i=1}^{NDV^1} \frac{\partial F_j^{2*}}{\partial \phi_i^1} \Delta \phi_i^1 \leq \epsilon_{2j} \quad j = 1, \dots, NOBJ^2$$

$$\phi_i^{1L} \leq \phi_i^1 \leq \phi_i^{1U} \quad i = 1, \dots, NDV^1$$

$$\phi_j^{2L} \leq \phi_j^{2*} + \sum_{i=1}^{NDV^1} \frac{\partial \phi_j^{2*}}{\partial \phi_i^1} \Delta \phi_i^1 \leq \phi_j^{2U} \quad j = 1, \dots, NDV^2$$

where  $F^1$  and  $F^2$  are the objective function vectors at levels 1 and 2 respectively,  $g^1$  and  $g^2$  are the corresponding constraint vectors and  $\phi^1$  and  $\phi^2$  are the corresponding design variable vectors. The quantity  $\epsilon_{2j}$  is a tolerance on the change in the  $j^{\text{th}}$  objective of level 2 during optimization at level 1. Superscripts L and U represent lower and upper bounds respectively, and superscript \* represents optimum values obtained at level 2. Finally, the quantities,  $\frac{\partial F_j^{2*}}{\partial \phi_i^1}$  and  $\frac{\partial \phi_j^{2*}}{\partial \phi_i^1}$  are the optimal sensitivity parameters of level 2 objective function and design variable vectors, respectively, with respect to the level 1 design variables.

### Level 2:

Minimize

$$F_i^2(\phi^{1*}, \phi^2) \quad i = 1..NOBJ^2$$

subject to

$$g_k^2(\phi^{1*}, \phi^2) \leq 0 \quad k = 1..NC^2$$

$$\phi_i^{2L} \leq \phi_i^2 \leq \phi_i^{2U} \quad i = 1..NDV^2$$

where  $\phi^{1*}$  is the optimum design variable vector from level 1. This vector is kept fixed during optimization at level 2. The optimization procedure cycles through the two levels before global convergence is achieved. A "cycle" is defined as one complete sweep through the two levels of

optimization. Optimization at an individual level also requires several "iterations" before local convergence is achieved. Cycling between the two levels is necessary to account for the coupling between the objective functions, constraints, and design variables pertaining to the levels.

## 7. ANALYSIS

The parabolized Navier-Stokes equations (PNS equations) have been used for the evaluation of three-dimensional, supersonic, viscous flow fields. The PNS equations are obtained from the full Navier-Stokes equations based on the following assumptions: (a) steady state, (b) the streamwise viscous gradients are neglected, and (c) the streamwise pressure gradient in the subsonic portion of the viscous flow near the body surface are approximated. The inviscid region of the flow field must be supersonic and the streamwise velocity component must be positive everywhere. Thus streamwise flow separation is not allowed but crossflow separation is allowed. Efficiency in computational time and memory requirements are achieved because the equations can be solved using a space-marching technique. The computational procedure used in this study, as implemented in the code, UPS3D [27] integrates the PNS equations using an implicit, approximately factored, finite-volume algorithm where the crossflow inviscid fluxes are evaluated by Roe's flux-difference splitting scheme [28]. The UPS3D code also has the capability to calculate the inviscid flow field by solving the PNS equations without the viscous terms. The upwind algorithm is used to improve the resolution of the shock waves over that obtained with the conventional central differencing schemes. A hyperbolic computational grid is used with 75 grid points along the circumferential ( $\eta$ ) direction, 80 grid points along the normal ( $\zeta$ ) direction, and 31 grid points along the longitudinal ( $\xi$ ) direction. Further refinement of the grid does not change the flow solution significantly.

The aircraft wing structural analysis is performed using an inhouse code. The code is capable of analyzing multicelled box beams of arbitrary cross-section and tapered planform. The wing section is represented by a diamond airfoil (Fig. 2). The principal load carrying member in the wing is modeled by an isotropic box beam with a rectangular cross-section and unequal wall

thicknesses. The wall thicknesses are represented as fractions of local chord ( $c$ ) as shown in Figure 2. The aircraft weight ( $W$ ) is calculated as the sum of the weight of the box beam ( $W_{\text{box}}$ ) and the weight of the skin ( $W_{\text{skin}}$ ). The stresses ( $\sigma$ ) are calculated using thin wall theory.

## 8. OPTIMIZATION

A gradient based optimization technique based on the method of feasible directions [29] has been used to solve the optimization problems at levels 1 and 2. Since the optimization process requires several evaluations of the objective function and the constraints before an optimum design is obtained, the process can be very expensive if actual analyses are performed for each function evaluation. The objective function and constraints at levels 1 and 2 are, therefore, approximated using a two-point exponential approximation [30] based on the design variable values from the optimizer and the sensitivities of these functions for the current and the previous design points. This approximation is given by the following expression.

$$\hat{F}(\Phi) = F(\Phi_1) + \sum_{n=1}^{\text{NDV}} \left[ \left( \frac{\phi_n}{\phi_{1n}} \right)^{p_n} - 1.0 \right] \frac{\phi_{1n}}{p_n} \frac{\partial F}{\partial \phi_n}(\Phi_1) \quad (9)$$

where  $\hat{F}(\Phi)$  is the approximation of the function  $F(\Phi)$  in the neighborhood of the current design variable vector,  $\Phi_1$ . The quantity  $\phi_n$  is the  $n^{\text{th}}$  design variable from the design variable vector  $\Phi$ . NDV is the total number of design variables. The approximate values for the constraints,  $\hat{g}_j(\Phi)$ , are similarly calculated. The exponent  $p_n$  is defined as

$$p_n = \frac{\log_e \left\{ \frac{\frac{\partial F}{\partial \phi_n}(\Phi_0)}{\frac{\partial F}{\partial \phi_n}(\Phi_1)} \right\}}{\log_e \left\{ \frac{\Phi_{0n}}{\Phi_{1n}} \right\}} + 1.0 \quad (10)$$

where  $\Phi_1$  refers to the design variable vector from the current cycle and  $\Phi_0$  denotes the design variable vector from the previous cycle. Equation 9 indicates that in the limiting case of  $p_n = 1$ , the

expansion is identical to the traditional first order Taylor series and when  $p_n = -1$ , the two-point exponential approximation reduces to the reciprocal expansion form. Therefore, the exponent ( $p_n$ ) can be interpreted as a “goodness of fit” parameter which explicitly determines the trade-offs between traditional and reciprocal Taylor series based expansions.  $p_n$  is chosen to be within the interval,  $-1 \leq p_n \leq 1$  thus resulting in a hybrid approximation technique.

## 9. APPLICATION

A two level optimization procedure has been developed for simultaneous improvement of aerodynamic and structural characteristics of high speed aircraft. From aerodynamics point of view, the aircraft drag coefficient ( $C_D$ ) must be minimized while maintaining the lift coefficient ( $C_L$ ) above a desired level. From a structural perspective, it is desirable to minimize the aircraft weight ( $W$ ) while maintaining the stresses in the load carrying members within material limits. A semi-analytical technique is developed and used within the optimization procedure for calculating sensitivities of aerodynamic coefficients and the computational grid used in the aerodynamic analysis. The developed two level optimization procedure and sensitivity analysis technique are applied to a delta wing-body configuration related to high speed aircraft.

### 9.1 Aircraft Configuration

The two level optimization procedure and the semi-analytical sensitivity analysis techniques are applied to the design optimization of a delta wing-body configuration illustrated in Figure 2. This center body is characterized by a nose region where the radius increases parabolically from zero to its maximum value ( $r_m$ ). Beyond the point where maximum radius is reached, the body remains a cylinder through the length of the aircraft. The delta wing is characterized by a leading edge sweep ( $\lambda$ ), root chord ( $c_o$ ) and a wing span ( $w_s$ ). The wing cross-section is a diamond airfoil [Fig. 1] with a thickness-to-chord ratio denoted  $t_c$ . The wing leading edge sweep, root chord, wing span and thickness-to-chord ratio are used as design variables within the aerodynamic optimization at level 1. The load carrying structural member of the wing is modeled as a single



celled, isotropic box beam [Fig. 3]. The beam width-to-chord ratio ( $w_c$ ), the horizontal wall thickness-to-chord ratio ( $t_1$ ) and the vertical wall thickness-to-chord ratio ( $t_2$ ) are used as design variables during the structural optimization at level 2.

## 9.2 Two Level Optimization Problem

In a typical aircraft design, the aerodynamic performance criteria dictate the planform shape of the aircraft wing. After designing the planform, the wing load carrying member is designed to carry the loads arising from aerodynamics, inertia and gravity. Keeping this in mind, the two level optimization procedure improves the aircraft aerodynamic performance at level 1 and the structural performance at level 2. These optimization problems are stated as follows.

### Level 1 (Aerodynamics):

Minimize

Drag coefficient ( $C_D$ )

subject to the constraints

$$C_L \geq C_{L_{\min}} \quad (\text{Lift coefficient constraint})$$

$$W \leq W^* \quad (\text{Optimal weight constraint})$$

$$\phi_{j2}^L \leq \phi_{j2}^* + \sum_{i=1}^{NDV_1} \frac{\partial \phi_{j2}^*}{\partial \phi_{i1}} \Delta \phi_{i1} \leq \phi_{j2}^U \quad j = 1..NDV_2 \quad (\text{Structural design variables constraint})$$

$$\phi_{i1}^L \leq \phi_{i1} \leq \phi_{i1}^U \quad i = 1..NDV_1 \quad (\text{Side constraints on aerodynamic design variables})$$

where  $\phi_1$  denotes the vector of aerodynamic design variables,  $\phi_2$  denotes the structural design variable vector,  $NDV_1$  and  $NDV_2$  denote the number of aerodynamic and structural design variables respectively and the superscripts L and U denote lower and upper bounds on the design variables respectively.

## Level 2 (Structures):

Minimize

Weight (W)

subject to the constraints

$$\sigma \leq \sigma_{\text{all}} \quad (\text{Stress constraints on wing load carrying member})$$

$$\phi_{i2}^L \leq \phi_{i2} \leq \phi_{i2}^U \quad i = 1..NDV_2 \quad (\text{Side constraints on structural design variables})$$

where  $\sigma$  is the vector of stresses at specific locations on the aircraft wing and  $\sigma_{\text{all}}$  is the corresponding vector of material limits. During the structural optimization at the second level, the aerodynamic design variables of level 1 ( $\phi_1$ ) are held fixed at their optimum values. The quantities,  $\frac{\partial W^*}{\partial \phi_{i1}}$  and  $\frac{\partial \phi_{j2}^*}{\partial \phi_{i1}}$  are the sensitivities of the optimum values of the second level (structural) objective function and the design variables with respect to first level (aerodynamic) design variables. These optimal sensitivity derivatives provide the necessary coupling between the two disciplines.

### 9.3 Results and Discussion

The two level optimization procedure and the semi-analytical sensitivity analysis techniques are applied to the design optimization of the delta wing-body configuration [Fig. 2]. Results from the grid sensitivity technique and the aerodynamic sensitivity analysis are presented first followed by results obtained from the two level optimization. The grid sensitivity technique is applied to the three-dimensional hyperbolic grid around the delta wing-body configuration. The values of the design variables used are: wing root chord ( $c_o$ ) = 7.08 m, leading edge sweep ( $\lambda$ ) = 66.0 degrees, wing span ( $w_s$ ) = 2.96 m and the wing thickness-to-chord ratio ( $t_c$ ) = 0.052. The computational grid includes 75 grid points in the circumferential ( $\eta$ ) direction, 31 grid points in the longitudinal ( $\xi$ ) direction and 80 grid points in the normal ( $\zeta$ ) direction. The grid sensitivities obtained from the direct differentiation technique and finite difference technique are presented in Table 1. As shown, there is excellent agreement between the two techniques. Further, a comparison of the CPU time

shows a 42 percent reduction achieved for one complete grid sensitivity analysis using the developed procedure [Fig. 4]. This clearly demonstrates the significant computational savings achievable by using this approach, especially in a formal design optimization procedure where several such design sensitivity analysis are necessary.

Results obtained from the aerodynamic sensitivity analysis procedure are presented next. The analysis is performed for an angle of attack of 5 degrees and a flight Mach number of 2.5. The sensitivities of the drag coefficient ( $C_D$ ) and the lift coefficient ( $C_L$ ), calculated using the direct differentiation technique as well as the finite difference technique, are presented in Tables 2 and 3 respectively. It must be noted that column 3 in Tables 2 and 3 presents the results of the semi-analytical aerodynamic sensitivity approach with finite difference grid sensitivity while column 4 presents the results of the semi-analytical aerodynamic sensitivity approach with semi-analytical grid sensitivity. As shown, the results from both techniques are in excellent agreement. For one complete sensitivity analysis, the direct differentiation technique with finite difference grid sensitivity calculations results in a 30 percent reduction in computing time from the finite difference technique [Fig. 5]. The semi-analytical sensitivity analysis technique with semi-analytical grid sensitivity calculations yields a 45 percent reduction in computing time from the finite difference approach [Fig. 5]. This further illustrates the efficiency of the discrete semi-analytical technique for grid sensitivity calculations. In order to investigate the trend in CPU savings, the results are plotted for varying number of design variables. Figure 6 compares the computing time required, for one complete sensitivity analysis, using the finite difference technique, the semi-analytical aerodynamic sensitivity analysis using finite difference grid sensitivity, and the semi-analytical aerodynamic sensitivity analysis using semi-analytical grid sensitivity, as a function of the number of design variables. It is observed that, although all three curves follow a near linear relationship with the number of design variables, their slopes are quite different. The design sensitivity analysis with semi-analytical grid sensitivity calculations results in the minimum slope. This indicates that, although, as expected, the computation time increases almost linearly with the increase in the number of design variables, the increase is less significant in the semi-analytical

approaches and least in the case where analytical grid sensitivities are used. This further illustrates the efficiency of the grid sensitivity analysis technique.

The results obtained from the two level optimization procedure are presented next. The reference values for the aerodynamic design variables are the same as those used to demonstrate the grid and aerodynamic sensitivity analysis. The load carrying structural member is modeled as an isotropic box beam made of 2014-T6 Aluminum alloy. The reference values of structural design variables are: spar width-to-chord ratio ( $w_c$ ) = 0.5, spar horizontal wall thickness-to-chord ratio ( $t_1$ ) = 0.0015 and vertical wall thickness-to-chord ratio ( $t_2$ ) = 0.0075. The iteration histories of the drag coefficient and the lift coefficient of the aircraft, during the level 1 aerodynamic optimization, are presented in Figs. 7 and 8 respectively. Significant improvements are observed in both the quantities. The drag coefficient decreases by 5.5 percent and the lift coefficient increases by 5.43 percent. The lift coefficient is maintained at its improved value throughout the optimization procedure because, the lift coefficient constraint, imposed during the aerodynamic optimization, remains an active constraint. The optimizer, based on the method of feasible directions, remains on this constraint boundary, as it improves the objective function value. The constraint imposed on the aircraft weight, at level 1, is well satisfied during the level 1 optimization. Table 4 compares the reference and the optimum values of the aerodynamic design variables used in the aerodynamic optimization. The root chord is increased significantly from its reference value (8.76 percent), whereas the wing thickness-to-chord ratio is decreased significantly (40.29 percent). The wing span and the leading edge sweep are maintained close to their reference values (2.94 percent and 1.01 percent respectively). The reduction in the wing thickness-to-chord ratio decreases the form drag of the diamond airfoil section. The optimization is driven by this reduction in drag. The increase in the wing planform area, caused by the increase in the root chord and the wing span, helps improve the lift of the aircraft in spite of the decrease in the wing thickness-to-chord ratio.

The iteration history of the aircraft weight is presented in Fig. 9. Considerable reduction (18.13 percent) in the aircraft weight is observed from the reference to the optimum. The shear and the normal stresses at the blade root remain well within the allowable limits of the chosen

Aluminum alloy. Table 5 compares the reference and optimum values of the structural design variables used in the structural optimization. The optimum spar width to chord ratio, horizontal and vertical wall thickness to chord ratios decrease significantly from their reference values (60 percent, 60 percent and 65.2 percent respectively). As mentioned above, the optimum configuration has an increased planform area compared to the reference configuration. It must be noted that the optimum weight is still lower than the reference due to the significant reduction in the wall thicknesses.

## **10. SIGNIFICANCE OF THE RESEARCH**

An efficient, semi-analytical sensitivity analysis technique, based on the direct differentiation approach, has been developed for calculating aerodynamic and grid sensitivities. A multidisciplinary optimization procedure for designing wing-body configurations has been developed based on a two level decomposition. The aircraft aerodynamic and structural design requirements have been coupled within this two level optimization procedure. The following observations are made.

1. The results from the semi-analytical grid sensitivity and aerodynamic sensitivity analysis techniques compare very well with those obtained using a finite difference approach.
2. The developed sensitivity analysis techniques yield significant savings in computational time and allow the use of comprehensive CFD within gradient based optimization procedures.
3. The semi-analytical aerodynamic sensitivity technique in conjunction with the developed grid sensitivity procedure shows the minimum increase in CPU time with increase in total number of design variables.
4. The two level decomposition optimization procedure yields significant reductions in the aircraft drag and weight while improving the lift.
5. The reduction in the aircraft drag is predominantly due to the significant reduction in the wing thickness-to-chord ratio. The improvement in the lift is due to the increased planform area caused by the increases in the wing root chord and wing span.

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Table 1. Grid sensitivity of the three-dimensional hyperbolic grid

Grid point (x, y, z)	Design variable	Finite difference grid sensitivity method	Direct differentiation grid sensitivity method
(0.300, 0.044, 0.008)	Sweep ( $\lambda$ )	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
	Root chord ( $c_o$ )	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
	Wing span ( $w_s$ )	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
	Thickness/chord ( $t_c$ )	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)
(16.406, 13.297, 2.753)	Sweep ( $\lambda$ )	(0.0, 0.0338, 0.3930)	(0.0, 0.0338, 0.3930)
	Root chord ( $c_o$ )	(0.0, 0.2886, 0.3356)	(0.0, 0.2884, 0.3355)
	Wing span ( $w_s$ )	(0.0, 0.8014, 0.9320)	(0.0, 0.8013, 0.9319)
	Thickness/chord ( $t_c$ )	(0.0, 46.640, 54.240)	(0.0, 46.550, 54.132)

Table 2. Sensitivity of the drag coefficient, ( $dC_D/d\phi_i$ )

Design variable	Finite Difference	Semi-analytical method (with finite difference grid sensitivity)	Semi-analytical method (with semi-analytical grid sensitivity)
Sweep ( $\lambda$ )	-0.0097712	-0.0106882	-0.0106453
Root chord ( $c_o$ )	0.0356022	0.0353581	0.0353786
Wing span ( $w_s$ )	0.0239097	0.0210748	0.0229120
Thickness/chord ( $t_c$ )	1.4250140	1.4219202	1.4820189

Table 3. Sensitivity of the lift coefficient, ( $dC_L/d\phi_i$ )

Design variable	Finite difference	Semi-analytical method (with finite difference grid sensitivity)	Semi-analytical method (with semi-analytical grid sensitivity)
Sweep ( $\lambda$ )	-0.0943048	-0.0920100	-0.0880802
Root chord ( $c_o$ )	0.0775282	0.0727289	0.0755285
Wing span ( $w_s$ )	0.0695290	0.0628465	0.0647937
Thickness/chord ( $t_c$ )	-1.8981815	-1.7628147	-1.8495492

Table 4. Aerodynamic design variables

Design variable	Reference	Optimum
Sweep ( $\lambda$ )	66.00 deg.	65.33 deg.
Root Chord ( $c_0$ )	7.08 m	7.70 m
Wing Span ( $w_s$ )	2.96 m	3.047 m
Thickness-to-chord ratio ( $t_c$ )	0.05200	0.03105

Table 5. Structural design variables

Design variable	Reference	Optimum
Spar width-to-chord ratio ( $w_c$ )	0.5	0.2
Horizontal wall thickness-to-chord ratio ( $t_1$ )	0.0015	0.0006
Vertical wall thickness-to-chord ratio ( $t_2$ )	0.0075	0.00261

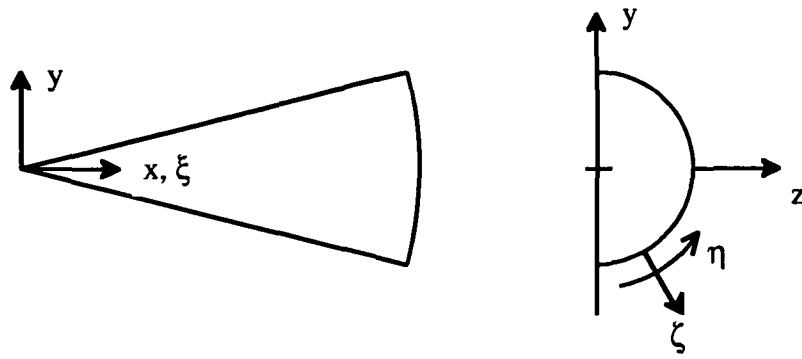
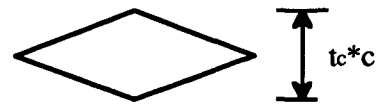


Figure 1. Coordinate systems



Diamond airfoil (Section AA)

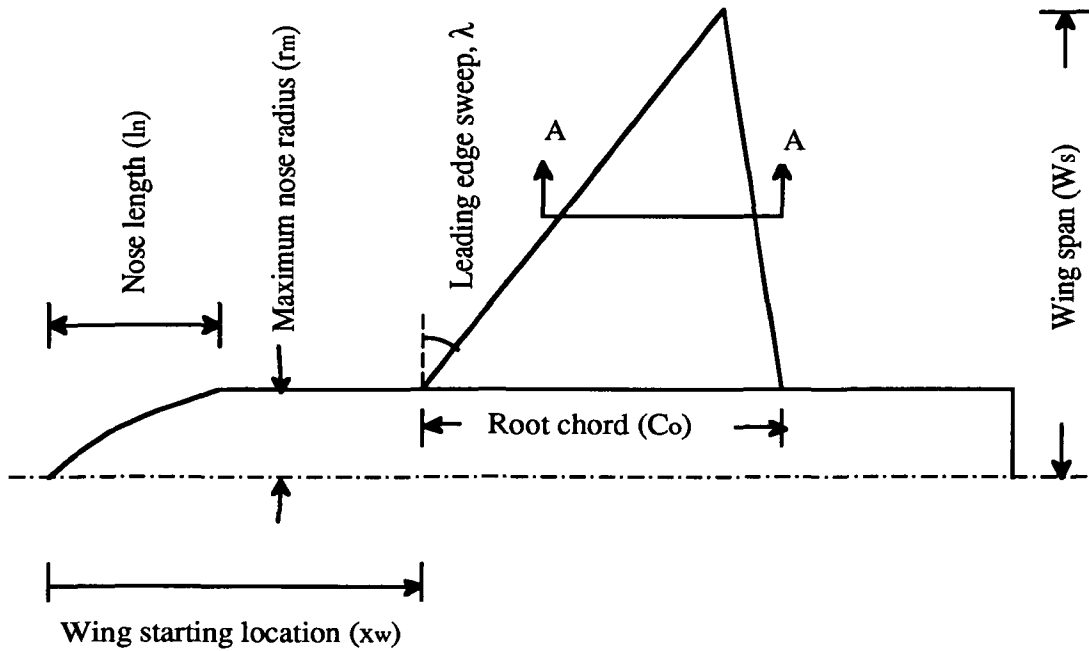


Figure 2. Delta wing-body configuration

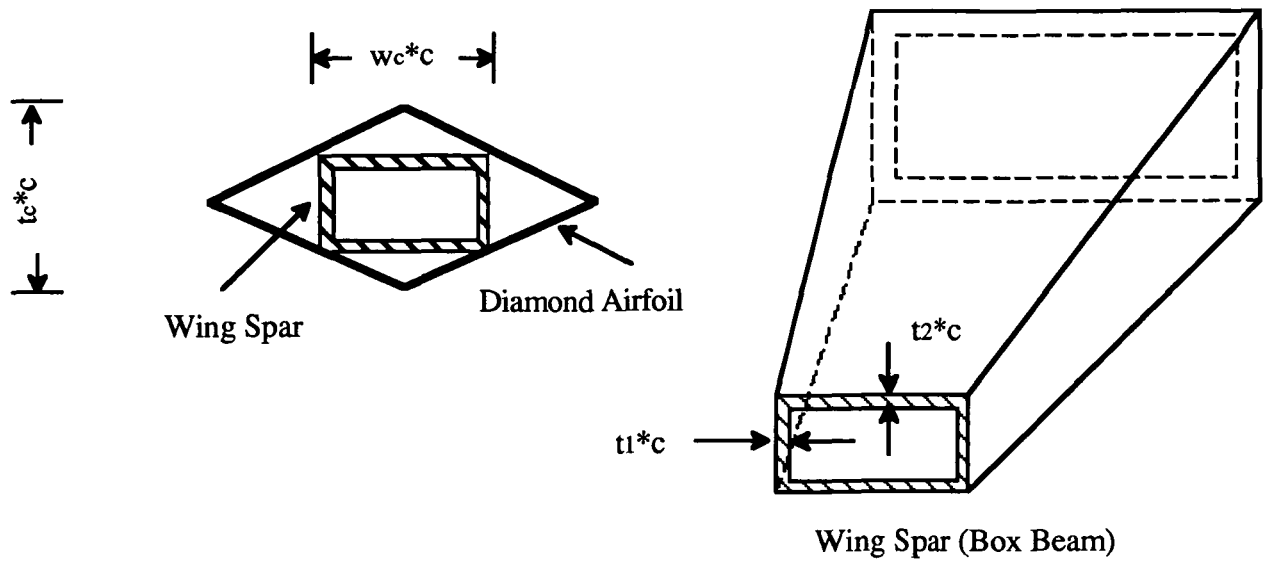


Figure 3. Wing cross-section and wing spar (box beam)

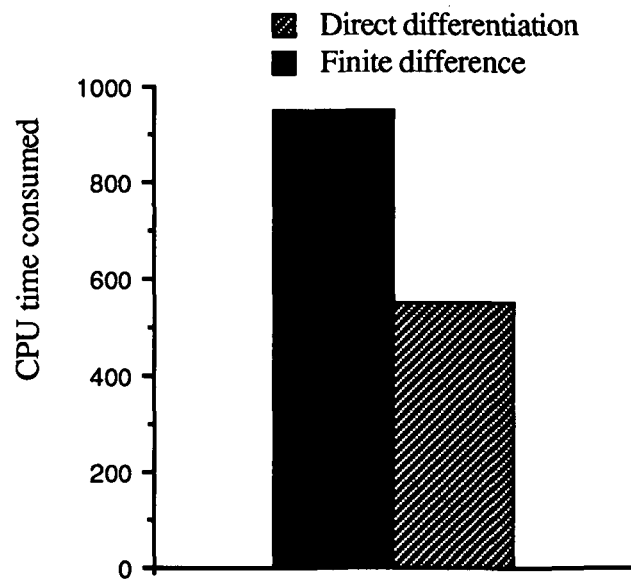


Figure 4. Comparison of CPU time for the sensitivity of hyperbolic grid

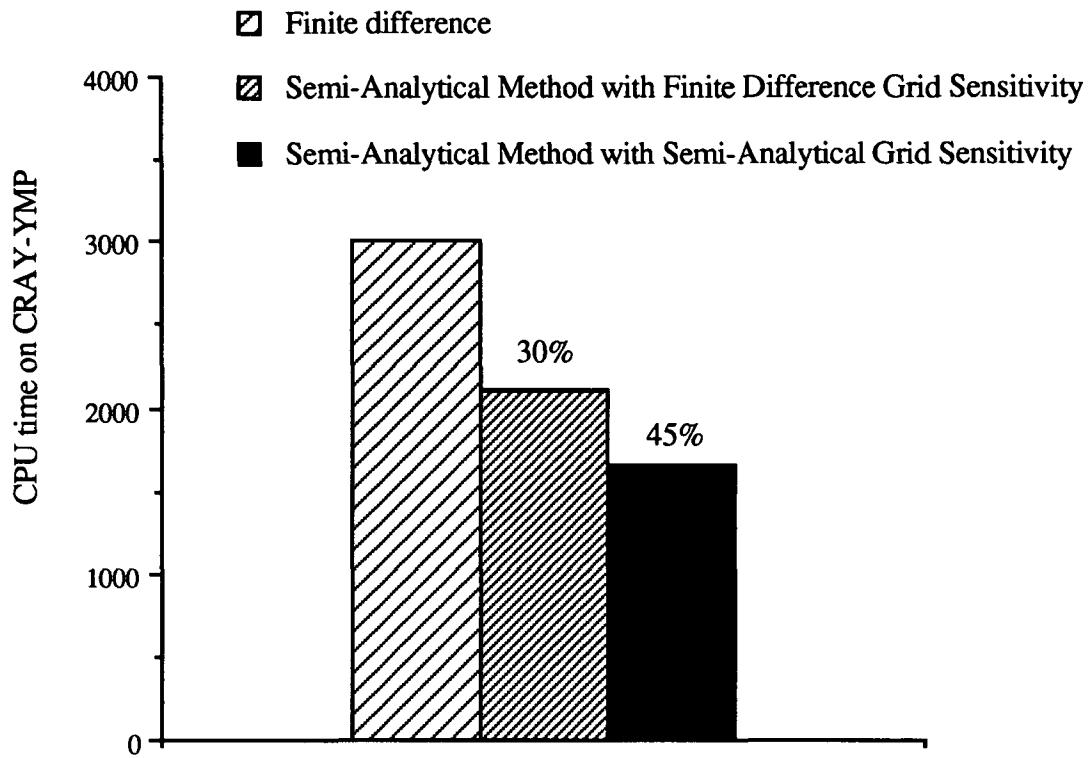


Figure 5. Comparison of CPU time for aerodynamic sensitivity analysis

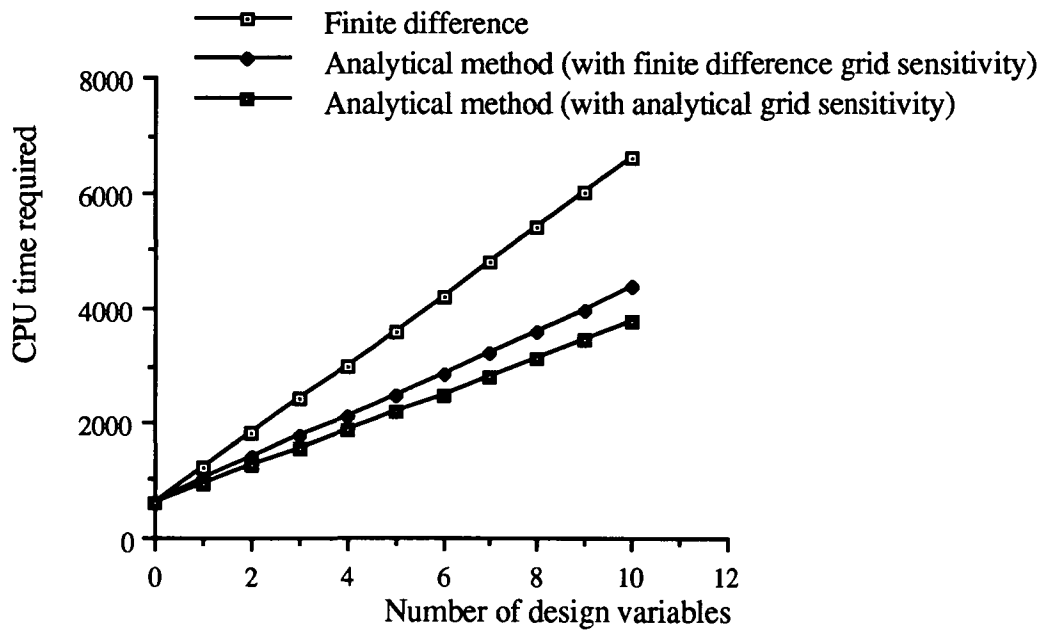


Figure 6. Comparison of CPU time with design variables

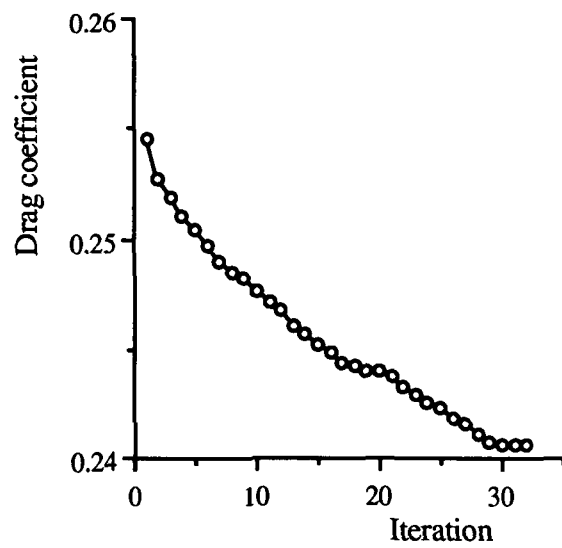


Figure 7. Iteration history of drag coefficient

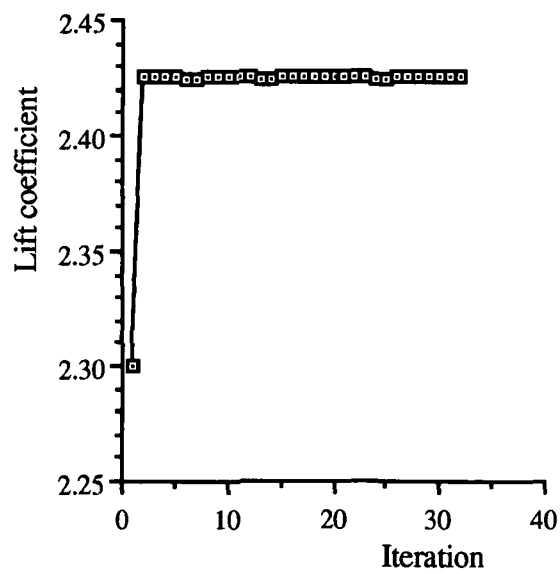


Figure 8. Iteration history of lift coefficient

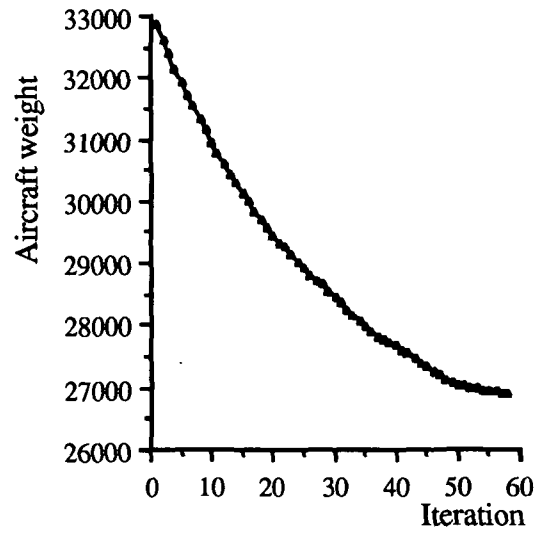


Figure 9. Iteration history of aircraft weight