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APPLICATIONS OF SYMBOLIC COMPUTATION IN FRACTURE MECHANICS

A FINAL REPORT

by

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CALCULATE THE STRESSES OF ONE CRACK EMBEDDED IN ISOTROPIC PLATE

This program coded in Macsyma calculates the stresses of a crack embedded in isotropic plate by solving the governing equation with certain boundary conditions.

1. Define the governing equation

$$\frac{\partial^4}{\partial^4 x}F(x,y)+\frac{2\partial^4}{\partial^2 x\partial^2 y}F(x,y)+\frac{\partial^4}{\partial^4 y}F(x,y)=0, \tag{1}$$

where $F(x,y)$ is the stress function.

By Fourier transform the governing equation can be changed to an ODE with constant coefficients respect to y.

If $F(x,y)$ is evaluated by $\frac{1}{2\pi}\int_{-\infty}^{\infty}\phi(s,y)\exp^{-isx}\,ds$ the integration will be performed by Macsyma. Since only the integrand is involved during the calculation, we can omit the integral sign. The integral sign will be added later on.

2. Taking the Fourier transform yields the ODE

$$s^4\phi - s^2\phi'' + \phi'''' = 0. \tag{2}$$

3. Evaluating ϕ by $C\exp^{rys}$ in equation (2) the characteristic equation is obtained

$$r^4 - 2r^2 + 1 = 0. \tag{3}$$

4. Solve the characteristic equation.

5. If the roots of the characteristic equation are two identical real roots, the general solution of equation (2) is

$$(C_1' + C_2'y)\exp^{ys} + (C_3' + C_4'y)\exp^{-ys}. \tag{4}$$

6. Since the stress function $F(x,y)$ is bounded at infinity, the solution of the governing equation (1) is

$$F(x,y^+) = \frac{1}{2\pi}[(C_1 + C_2 y)\exp^{-|s|y}\exp^{-isx}], \qquad for \;\; y > 0, \tag{5}$$
$$F(x,y^-) = \frac{1}{2\pi}[(C_3 + C_4 y)\exp^{|s|y}\exp^{-isx}], \qquad for \;\; y < 0, \tag{6}$$

where $C_i (i = 1, ..., 4)$ are arbitrary functions of s.

Applying the continuity conditions C_3 and C_4 can be expressed in terms of C_1 and C_2.

7. Define equation

$$F(x,0^+) - F(x,0^-) = 0 \tag{7}$$

and by evaluating $F(x,y^+)$ and $F(x,y^-)$ at 0,we obtain

$$C_3 = C_1. \tag{8}$$

8. Define equation

$$\frac{\partial F(x,0^+)}{\partial y} - \frac{\partial F(x,0^-)}{\partial y} = 0 \tag{9}$$

by differentiating equuation (9) w.r.t. y and evaluating y at 0. Solving C_4 and get

$$C_4 = -2|s|C_1 + C_2. \tag{10}$$

Displacements u and v are obtained from Hooke's law

$$v = a_{12}F_x + a_{11}\int F_{xx}dy, \tag{11}$$

$$u = a_{11}\int F_{yy}dx + a_{12}F_x, \tag{12}$$

where a_{11} and a_{12} are coefficients of plane stress and plane strain.

9. Computing u^+ and u^- by Macsyma.

10. Computing v^+ and v^- by Macsyma.

If we introduce two auxiliary functions $f_1(x)$ and $f_2(x)$ properly, C_1 and C_2 can be written in terms of $f_1(x)$ and $f_2(x)$.

11. Define

$$f_1(x) = \frac{\partial}{\partial x}[u^+(x,0) - u^-(x,0)]. \tag{13}$$

After differentiating u^+ and u^- w.r.t. x and evaluating y at 0, $f_1(x)$ is simplified by Macsyma simplication functions. So

$$f_1(x) = -\frac{(2a_{11}C_2|s| - aa_{11}C_1s^2)\exp^{-isx}}{\pi}. \tag{14}$$

12. Define

$$f_2(x) = \frac{\partial}{\partial x}[v^+(x,0) - v^-(x,0)]. \tag{15}$$

After differentiating v^+ and v^- w.r.t. x and evaluating y at 0, $f_2(x)$ can not be simplified only by Macsyma simplification functions. There are terms such as

$$\frac{g(x,s)s^2 + h(x,s)s^3 + w(x,s)s|s|}{|s|}, \tag{16}$$

and these terms can be simplified by substitutions. For example, s^3 is substituted by $k|s|$, then k is substituted by $s|s|$. Although it is the same as $s^3 = s|s||s|$, the only difference is that terms like $\frac{s^3}{|s|}$ are simplified to $s|s|$. Finally we obtain

$$f_2(x) = -\frac{2ia_{11}C_1 s|s| \exp^{-isx}}{\pi}. \tag{17}$$

13. Taking the inverse Fourier transform on both sides of equation (15) (Notice that we omitted the integral sign during the calculation). C_1 is expressed in terms of $f_2(x)$,

$$C_1 = \frac{i}{4a_{11}|s|s} f_2(t) \exp^{ist}. \tag{18}$$

14. Taking the inverse Fourier transform on both sides of equation (14) (Notice that we omitted the integral sign during the calculation). With equation (18) C_2 is expressed in terms of $f_1(x)$ and $f_2(x)$,

$$C_2 = \frac{1}{4a_{11}}[\frac{-1}{|s|} f_1(t) \exp^{ist} + \frac{i}{s} f_2(t) \exp^{ist}]. \tag{19}$$

15. Calculating C_3 from equation (8), we have

$$C_3 = \frac{i}{4a_{11}|s|s} f_2(t) \exp^{ist}. \tag{20}$$

16. Calculating C_4 from equation (10), we have

$$C_4 = \frac{-1}{4a_{11}} [\frac{1}{|s|} f_1(t) \exp^{ist} + \frac{i}{s} f_2(t) \exp^{ist}]. \tag{21}$$

17. Stresses σ_{xx} σ_{yy} and τ_{xy} are defined as follows

$$\sigma_{xx} = \frac{\partial^2 F(x,y)}{\partial^2 y}; \tag{22}$$

$$\sigma_{yy} = \frac{\partial^2 F(x,y)}{\partial^2 x}; \tag{23}$$

$$\tau_{xy} = \frac{\partial}{\partial x}\frac{\partial}{\partial y} F(x,y). \tag{24}$$

18. Simplification of σ_{xx}.

A second derivative of $F(x,y)$ w.r.t. y and substitution of $C_i (i = 1, ..., 4)$ yields

$$-\frac{isf_2(t)y \exp^{-sy+isx-ist}}{4a_{11}} - \frac{sf_1(t)y \exp^{-sy+isx-ist}}{4a_{11}}$$

$$+\frac{if_2(t) \exp^{-sy+isx-ist}}{4a_{11}} + \frac{f_1(t) \exp^{-sy+isx-ist}}{2a_{11}}$$

$$+\frac{isf_2(t)y \exp^{-sy-isx+ist}}{4a_{11}} - \frac{sf_1(t)y \exp^{-sy-isx+ist}}{4a_{11}}$$

$$-\frac{if_2(t)\exp^{-sy-isx+ist}}{4a_{11}}+\frac{f_1(t)\exp^{-sy-isx+ist}}{2a_{11}}. \tag{25}$$

With integral signs, we know the stress σ_{xx} is

$$\int_{-a}^{a}\int_{0}^{\infty}\{-\frac{isf_2(t)y\exp^{-sy+isx-ist}}{4a_{11}}-\frac{sf_1(t)y\exp^{-sy+isx-ist}}{4a_{11}}$$

$$+\frac{if_2(t)\exp^{-sy+isx-ist}}{4a_{11}}+\frac{f_1(t)\exp^{-sy+isx-ist}}{2a_{11}}$$

$$+\frac{isf_2(t)y\exp^{-sy-isx+ist}}{4a_{11}}-\frac{sf_1(t)y\exp^{-sy-isx+ist}}{4a_{11}}$$

$$-\frac{if_2(t)\exp^{-sy-isx+ist}}{4a_{11}}+\frac{f_1(t)\exp^{-sy-isx+ist}}{2a_{11}}\}dsdt. \tag{26}$$

The function integrate in Macsyma can not be used directly to to perform the integration of equation (25) w.r.t. s, because it keeps calculating and never comes to an end. Also we know the fact

$$\int_0^\infty \exp^{(a\pm bi)s}\,ds=-\frac{1}{a\pm bi}; \tag{27}$$

$$\int_0^\infty s\exp^{(a\pm bi)s}\,ds=\frac{1}{(a\pm bi)^2}. \tag{28}$$

Substitution of $-\frac{1}{-y\pm(x-t)i}$ for all $\exp^{[-y\pm i(x-t)]s}$ terms and $\frac{1}{[-y\pm(x-t)i]^2}$ for all $y\exp^{[-y\pm i(x-t)]s}$ terms has the same effect as integration. So the following expression is obtained

$$\frac{1}{8\pi a_{11}}\{\frac{[if_2(t)-f_1(t)]y}{[i(t-x)-y]^2}+[-\frac{if_2(t)}{[-y-i(t-x)]^2}-\frac{f_1(t)}{[-y-i(t-x)]^2}]y$$

$$-\frac{2f_1(t)-if_2(t)}{i(t-x)-y}-\frac{if_2(t)+2f_1(t)}{-y-i(t-x)}\}. \tag{29}$$

Taking the common denominator with the same order of power, we ultimately obtain

$$\begin{aligned}\sigma_{xx}=\frac{1}{4\pi a_{11}}\{&f_2(t)(t-x)[\frac{-2y^2}{[y^2+(t-x)^2]^2}+\frac{1}{y^2+(t-x)^2}]\\&-yf_1(t)[\frac{y^2-(t-x)^2}{[y^2+(t-x)^2]^2}-\frac{2}{y^2+(t-x)^2}]\}.\end{aligned} \tag{30}$$

Finally by adding the integral sign, we have the exact answer

$$\begin{aligned}\sigma_{xx}=\frac{1}{4\pi a_{11}}\int_{-a}^{a}\{&f_2(t)(t-x)[\frac{-2y^2}{[y^2+(t-x)^2]^2}+\frac{1}{y^2+(t-x)^2}]\\&-yf_1(t)[\frac{y^2-(t-x)^2}{[y^2+(t-x)^2]^2}-\frac{2}{y^2+(t-x)^2}]\}dt.\end{aligned} \tag{31}$$

19. Following the same procedures to simplify σ_{yy} with slight change, we obtain

$$\begin{aligned}\sigma_{yy}=\frac{1}{4\pi a_{11}}\int_{-a}^{a}\{&f_2(t)(t-x)[\frac{2y^2}{[y^2+(t-x)^2]^2}+\frac{1}{y^2+(t-x)^2}]\\&+yf_1(t)\frac{y^2-(t-x)^2}{[y^2+(t-x)^2]^2}\}dt.\end{aligned} \tag{32}$$

20. Following the same procedures to simplify τ_{xy} with slight change, we obtain

$$\begin{aligned}\tau_{xy}=\frac{-1}{4\pi a_{11}}\int_{-a}^{a}\{&f_1(t)(t-x)[\frac{2y^2}{[y^2+(t-x)^2]^2}-\frac{1}{y^2+(t-x)^2}]\\&-yf_2(t)\frac{y^2-(t-x)^2}{[y^2+(t-x)^2]^2}\}dt.\end{aligned} \tag{33}$$

CALCULATE THE STRESSES OF TWO CRACKS EMBEDDED IN ISOTROPIC PLATE

According to local coordinate system the stresses for one crack is already given. For two cracks, the relation between the local coordinate systems x_1, y_1 and x_2, y_2 are

$$r_{1x} + x_1 \cos\varphi_1 - y_1 \sin\varphi_1 = r_{2x} + x_2 \cos\varphi_2 - y_2 \sin\varphi_2 \tag{1}$$

$$r_{1y} + x_1 \sin\varphi_1 + y_1 \cos\varphi_1 = r_{2y} + x_2 \sin\varphi_2 + y_2 \cos\varphi_2 \tag{2}$$

the transformation equations for plane stress are

$$\sigma_{x_2x_2} = \sigma_{x_1x_1} \cos^2\theta + \sigma_{y_1y_1} \sin^2\theta + \tau_{x_1y_1} \sin 2\theta \tag{3}$$

$$\sigma_{y_2y_2} = \sigma_{x_1x_1} \sin^2\theta + \sigma_{y_1y_1} \cos^2\theta - \tau_{x_1y_1} \sin 2\theta \tag{4}$$

$$\tau_{x_2y_2} = -\frac{1}{2}[\sigma_{x_1x_1} - \sigma_{y_1y_1}] \sin 2\theta + \tau_{x_1y_1} \cos 2\theta \tag{5}$$

where $\theta = \varphi_2 - \varphi_1$.

Thus, the total stress for the first crack is the summation of the stress of the first crack and the transformed stress from the second crack. The total stress of the second crack is vice versa. Let $x_i = a_i\xi$ and $t_i = a_i\tau$, where $i = (1,2)$, by applying the boundary conditions, the total stress for crack 1 can be expressed

$$\sigma 1^T_{xy} = \frac{1}{4a_{11}}\{\frac{1}{\pi}\int_{-1}^{1} \frac{f_{11}}{\tau - \xi} d\tau + \int_{-1}^{1} k_{21}^{11} f_{21} d\tau + \int_{-1}^{1} k_{22}^{11} f_{22} d\tau\} \tag{6}$$

$$\sigma 1^T_{yy} = \frac{1}{4a_{11}}\{\frac{1}{\pi}\int_{-1}^{1} \frac{f_{12}}{\tau - \xi} d\tau + \int_{-1}^{1} k_{21}^{12} f_{21} d\tau + \int_{-1}^{1} k_{22}^{12} f_{22} d\tau\} \tag{7}$$

where $k_{\alpha\beta}^{nm}$ are Fredholm kernels $(\alpha, \beta, n, m = 1, 2)$

$$k_{21}^{11} = \frac{a_2}{\pi}\{\frac{\sin 2\theta(p_4 - a_1\xi\sin\theta)[(a_2\tau - p_3 - a_1\xi\cos\theta)^2 - (p_4 - a_1\xi\sin\theta)^2]}{[(p_4 - a_1\xi\sin\theta)^2 + (a_2\tau - p_3 - a_1\xi\cos\theta)^2]^2}$$

$$+\frac{2\cos 2\theta(a_2\tau - p_3 - a_1\xi\cos\theta)(p_4 - a_1\xi\sin\theta)^2}{[(p_4 - a_1\xi\sin\theta)^2 + (a_2\tau - p_3 - a_1\xi\cos\theta)^2]^2}$$

$$+\frac{\sin 2\theta(p_4 - a_1\xi\cos\theta) - \cos 2\theta(a_2\tau - p_3 - a_1\xi\cos\theta)}{(p_4 - a_1\xi\sin\theta)^2 + (a_2\tau - p_3 - a_1\xi cos\theta)^2}\} \tag{8}$$

$$k_{22}^{11} = \frac{a_2}{\pi}\{\frac{\cos 2\theta(p_4 - a_1\xi\sin\theta)[(a_2\tau - p_3 - a_1\xi\cos\theta)^2 - (p_4 - a_1\xi\sin\theta)^2]}{[(p_4 - a_1\xi\sin\theta)^2 + (a_2\tau - p_3 - a_1\xi\cos\theta)^2]^2}$$

$$-\frac{2\sin 2\theta(a_2\tau - p_3 - a_1\xi\cos\theta)(p_4 - a_1\xi\sin\theta)^2}{[(p_4 - a_1\xi\sin\theta)^2 + (a_2\tau - p_3 - a_1\xi\cos\theta)^2]^2}\} \tag{9}$$

$$k_{21}^{12} = \frac{a_2}{\pi}\{\frac{2\sin 2\theta(a_2\tau - p_3 - a_1\xi\cos\theta)(p_4 - a_1\xi\sin\theta)^2}{[(p_4 - a_1\xi\sin\theta)^2 + (a_2\tau - p_3 - a_1\xi\cos\theta)^2]^2}$$

$$-\frac{\cos 2\theta(p_4 - a_1\xi\sin\theta)[(a_2\tau - p_3 - a_1\xi\cos\theta)^2 - (p_4 - a_1\xi\sin\theta)^2]}{[(p_4 - a_1\xi\sin\theta)^2 + (a_2\tau - p_3 - a_1\xi\cos\theta)^2]^2}$$

$$+\frac{2\sin^2\theta(p_4 - a_1\xi\cos\theta) - \sin 2\theta(a_2\tau - p_3 - a_1\xi\cos\theta)}{(p_4 - a_1\xi\sin\theta)^2 + (a_2\tau - p_3 - a_1\xi cos\theta)^2}\} \tag{10}$$

$$k_{22}^{12} = \frac{a_2}{\pi}\{\frac{\sin 2\theta(p_4 - a_1\xi\sin\theta)[(a_2\tau - p_3 - a_1\xi\cos\theta)^2 - (p_4 - a_1\xi\sin\theta)^2]}{[(p_4 - a_1\xi\sin\theta)^2 + (a_2\tau - p_3 - a_1\xi\cos\theta)^2]^2}$$

$$+\frac{2\cos 2\theta(a_2\tau - p_3 - a_1\xi\cos\theta)(p_4 - a_1\xi\sin\theta)^2}{[(p_4 - a_1\xi\sin\theta)^2 + (a_2\tau - p_3 - a_1\xi\cos\theta)^2]^2}$$

$$+\frac{(a_2\tau - p_3 - a_1\xi\cos\theta)}{(p_4 - a_1\xi\sin\theta)^2 + (a_2\tau - p_3 - a_1\xi cos\theta)^2}\} \tag{11}$$

where

$$p_3 = (r_{1y} - r_{2y})\sin\varphi_2 + (r_{1x} - r_{2x})\cos\varphi_2$$

$$p_4 = (r_{1y} - r_{2y})\cos\varphi_2 - (r_{1x} - r_{2x})\sin\varphi_2$$

The total stress for crack 2 can be expressed

$$\sigma 2_{xy}^T = \frac{1}{4a_{11}}\{\frac{1}{\pi}\int_{-1}^{1}\frac{f_{21}}{\tau - \xi}d\tau + \int_{-1}^{1} k_{11}^{21} f_{11} d\tau + \int_{-1}^{1} k_{12}^{21} f_{12} d\tau\} \tag{12}$$

$$\sigma 2_{yy}^T = \frac{1}{4a_{11}}\{\frac{1}{\pi}\int_{-1}^{1}\frac{f_{22}}{\tau - \xi}d\tau + \int_{-1}^{1} k_{11}^{22} f_{11} d\tau + \int_{-1}^{1} k_{12}^{22} f_{12} d\tau\} \tag{13}$$

where $k_{\alpha\beta}^{nm}$ are Fredholm kernels $(\alpha, \beta, n, m = 1, 2)$

$$k_{11}^{21} = \frac{a_1}{\pi}\{\frac{-\sin 2\theta(p_2 + a_2\xi\sin\theta)[(a_1\tau - p_1 - a_2\xi\cos\theta)^2 - (p_2 + a_2\xi\sin\theta)^2]}{[(p_2 + a_2\xi\sin\theta)^2 + (a_1\tau - p_1 - a_2\xi\cos\theta)^2]^2}$$

$$\frac{2\cos 2\theta(a_1\tau - p_1 - a_2\xi\cos\theta)(p_2 + a_2\xi\sin\theta)^2}{[(p_2 + a_2\xi\sin\theta)^2 + (a_1\tau - p_1 - a_2\xi\cos\theta)^2]^2}$$

$$+\frac{-\sin 2\theta(p_2 + a_2\xi\sin\theta) - \cos 2\theta(a_1\tau - p_1 - a_2\xi\cos\theta)}{(p_2 + a_2\xi\sin\theta)^2 + (a_1\tau - p_1 - a_2\xi cos\theta)^2}\} \tag{14}$$

$$k_{12}^{21} = \frac{a_1}{\pi}\{\frac{\cos 2\theta(p_2 + a_2\xi\sin\theta)[(a_1\tau - p_1 - a_2\xi\cos\theta)^2 - (p_2 + a_2\xi\sin\theta)^2]}{[(p_2 + a_2\xi\sin\theta)^2 + (a_1\tau - p_1 - a_2\xi\cos\theta)^2]^2}$$

$$\frac{2\sin 2\theta(a_1\tau - p_1 - a_2\xi\cos\theta)(p_2 + a_2\xi\sin\theta)^2}{[(p_2 + a_2\xi\sin\theta)^2 + (a_1\tau - p_1 - a_2\xi\cos\theta)^2]^2}\} \tag{15}$$

$$k_{11}^{22} = \frac{a_1}{\pi}\{\frac{-2\sin 2\theta(a_1\tau - p_1 - a_2\xi\cos\theta)(p_2 + a_2\xi\sin\theta)^2}{[(p_2 + a_2\xi\sin\theta)^2 + (a_1\tau - p_1 - a_2\xi\cos\theta)^2]^2}$$

$$-\frac{\cos 2\theta(p_2 + a_2\xi\sin\theta)[(a_1\tau - p_1 - a_2\xi\cos\theta)^2 - (p_2 + a_2\xi\sin\theta)^2]}{[(p_2 + a_2\xi\sin\theta)^2 + (a_1\tau - p_1 - a_2\xi\cos\theta)^2]^2}$$

$$+\frac{2\sin^2\theta(p_2 + a_2\xi\sin\theta) + \sin 2\theta(a_1\tau - p_1 - a_2\xi\cos\theta)}{(p_2 + a_2\xi\sin\theta)^2 + (a_1\tau - p_1 - a_2\xi cos\theta)^2}\} \tag{16}$$

$$k_{12}^{22} = \frac{a_1}{\pi}\{\frac{-\sin 2\theta(p_2 + a_2\xi\sin\theta)[(a_1\tau - p_1 - a_2\xi\cos\theta)^2 - (p_2 + a_2\xi\sin\theta)^2]}{[(p_2 + a_2\xi\sin\theta)^2 + (a_1\tau - p_1 - a_2\xi\cos\theta)^2]^2}$$

$$+\frac{2\cos 2\theta(a_1\tau - p_1 - a_2\xi\cos\theta)(p_2 + a_2\xi\sin\theta)^2}{[(p_2 + a_2\xi\sin\theta)^2 + (a_1\tau - p_1 - a_2\xi\cos\theta)^2]^2}$$

$$+\frac{(a_1\tau - p_1 - a_2\xi\cos\theta)}{(p_2 + a_2\xi\sin\theta)^2 + (a_1\tau - p_1 - a_2\xi cos\theta)^2}\} \tag{17}$$

where

$$p_1 = (r_{2y} - r_{1y}) \sin \varphi_1 + (r_{2x} - r_{1x}) \cos \varphi_1$$

$$p_2 = (r_{2y} - r_{1y}) \cos \varphi_1 - (r_{2x} - r_{1x}) \sin \varphi_1$$

CALCULATION OF STRESS INTENSITY FACTORS OF n CRACKS EMBEDDED IN ISOTROPIC PLATE

A formula to calculate the stresses for each crack according to its local coordinate system were developed, i.e.:

$$\sigma_{xx} = \frac{1}{4\pi a_{11}} \int_{-a}^{a} \{f_2(t)(t-x)[\frac{-2y^2}{[y^2+(t-x)^2]^2} + \frac{1}{y^2+(t-x)^2}]$$
$$- y f_1(t)[\frac{y^2-(t-x)^2}{[y^2+(t-x)^2]^2} - \frac{2}{y^2+(t-x)^2}]\}dt. \tag{1}$$

$$\sigma_{yy} = \frac{1}{4\pi a_{11}} \int_{-a}^{a} \{f_2(t)(t-x)[\frac{2y^2}{[y^2+(t-x)^2]^2} + \frac{1}{y^2+(t-x)^2}]$$
$$+ y f_1(t)\frac{y^2-(t-x)^2}{[y^2+(t-x)^2]^2}\}dt. \tag{2}$$

$$\tau_{xy} = \frac{-1}{4\pi a_{11}} \int_{-a}^{a} \{f_1(t)(t-x)[\frac{2y^2}{[y^2+(t-x)^2]^2} - \frac{1}{y^2+(t-x)^2}]$$
$$- y f_2(t)\frac{y^2-(t-x)^2}{[y^2+(t-x)^2]^2}\}dt. \tag{3}$$

The relations between two coordinate systems x', y' and x, y are given as

$$x' = (r_y - r_{y'})\sin\varphi' + (r_x - r_{x'})\cos\varphi' + x\cos(\varphi - \varphi') - y\sin(\varphi - \varphi') \tag{4}$$

$$y' = (r_y - r_{y'})\cos\varphi' - (r_x - r_{x'})\sin\varphi' + x\sin(\varphi - \varphi') + y\cos(\varphi - \varphi') \tag{5}$$

and the transformation equations for plane stresses are

$$\sigma_{x'x'} = \sigma_{xx}\cos^2(\varphi' - \varphi) + \sigma_{yy}\sin^2(\varphi' - \varphi) + \tau_{xy}\sin 2(\varphi' - \varphi) \tag{6}$$

$$\sigma_{y'y'} = \sigma_{xx}\sin^2(\varphi' - \varphi) + \sigma_{yy}\cos^2(\varphi' - \varphi) - \tau_{xy}\sin 2(\varphi' - \varphi) \tag{7}$$

$$\tau_{x'y'} = -\frac{1}{2}[\sigma_{xx} - \sigma_{yy}]\sin 2(\varphi' - \varphi) + \tau_{xy}\cos 2(\varphi' - \varphi) \tag{8}$$

Label τ'_{xy} and σ'_{yy} are the stresses transformed by $\tau_{x'y'}$ and $\sigma_{y'y'}$ from x', y' coordinate system to x, y coordinate system. Let $x = a\xi$, $t = a\tau$, and $y = 0$, where $-1 \leq \xi\tau \leq 1$. After simplification we get

$$\tau'_{xy} = \frac{1}{4a_{11}}\{\int_{-1}^{1} k_1(\tau, \xi, r_x, r_{x'}, r_y, r_{y'}, \varphi, \varphi', a, a')f'_1 d\tau$$
$$+ \int_{-1}^{1} k_2(\tau, \xi, r_x, r_{x'}, r_y, r_{y'}, \varphi, \varphi', a, a')f'_2 d\tau\} \tag{9}$$

$$\sigma'_{yy} = \frac{1}{4a_{11}}\{\int_{-1}^{1} k_3(\tau, \xi, r_x, r_{x'}, r_y, r_{y'}, \varphi, \varphi', a, a')f'_1 d\tau$$
$$+ \int_{-1}^{1} k_4(\tau, \xi, r_x, r_{x'}, r_y, r_{y'}, \varphi, \varphi', a, a')f'_2 d\tau\} \tag{10}$$

Where

$$p_1 = (r_y - r_{y'})\sin\varphi' + (r_x - r_{x'})\cos\varphi' \tag{11}$$

$$p_2 = (r_y - r_{y'})\cos\varphi' - (r_x - r_{x'})\sin\varphi' \tag{12}$$

and k_i are Fredholm kernels ($i = 1, ..., 4$) and $\theta = \varphi' - \varphi$.

$k_1(\tau, \xi, r_x, r_{x'}, r_y, r_{y'}, \varphi, \varphi', a, a')$

$$= \frac{a'}{\pi}\{\frac{-\sin 2\theta(p_2 + a\xi\sin\theta)[(a'\tau - p_1 - a\xi\cos\theta)^2 - (p_2 + a\xi\sin\theta)^2]}{[(p_2 + a\xi\sin\theta)^2 + (a'\tau - p_1 - a\xi\cos\theta)^2]^2}$$

$$\frac{2\cos 2\theta(a'\tau - p_1 - a\xi\cos\theta)(p_2 + a\xi\sin\theta)^2}{[(p_2 + a\xi\sin\theta)^2 + (a'\tau - p_1 - a\xi\cos\theta)^2]^2}$$

$$+ \frac{-\sin 2\theta(p_2 + a\xi\sin\theta) - \cos 2\theta(a'\tau - p_1 - a\xi\cos\theta)}{(p_2 + a\xi\sin\theta)^2 + (a'\tau - p_1 - a\xi cos\theta)^2}\} \tag{13}$$

$$k_2(\tau,\xi,r_x,r_{x'},r_y,r_{y'},\varphi,\varphi',a,a')$$

$$= \frac{a'}{\pi}\{\frac{\cos 2\theta(p_2+a\xi\sin\theta)[(a'\tau-p_1-a\xi\cos\theta)^2-(p_2+a\xi\sin\theta)^2]}{[(p_2+a\xi\sin\theta)^2+(a'\tau-p_1-a\xi\cos\theta)^2]^2}$$

$$\frac{2\sin 2\theta(a'\tau-p_1-a\xi\cos\theta)(p_2+a\xi\sin\theta)^2}{[(p_2+a\xi\sin\theta)^2+(a'\tau-p_1-a\xi\cos\theta)^2]^2}\} \tag{14}$$

$$k3(\tau,\xi,r_x,r_{x'},r_y,r_{y'},\varphi,\varphi',a,a')$$

$$= \frac{a'}{\pi}\{\frac{-2\sin 2\theta(a'\tau-p_1-a\xi\cos\theta)(p_2+a\xi\sin\theta)^2}{[(p_2+a\xi\sin\theta)^2+(a'\tau-p_1-a\xi\cos\theta)^2]^2}$$

$$-\frac{\cos 2\theta(p_2+a\xi\sin\theta)[(a'\tau-p_1-a\xi\cos\theta)^2-(p_2+a\xi\sin\theta)^2]}{[(p_2+a\xi\sin\theta)^2+(a'\tau-p_1-a\xi\cos\theta)^2]^2}$$

$$+\frac{2\sin^2\theta(p_2+a\xi\sin\theta)+\sin 2\theta(a'\tau-p_1-a\xi\cos\theta)}{(p_2+a\xi\sin\theta)^2+(a'\tau-p_1-a\xi cos\theta)^2}\} \tag{15}$$

$$k_4(\tau,\xi,r_x,r_{x'},r_y,r_{y'},\varphi,\varphi',a,a')$$

$$= \frac{a'}{\pi}\{\frac{-\sin 2\theta(p_2+a\xi\sin\theta)[(a'\tau-p_1-a\xi\cos\theta)^2-(p_2+a\xi\sin\theta)^2]}{[(p_2+a\xi\sin\theta)^2+(a'\tau-p_1-a\xi\cos\theta)^2]^2}$$

$$+\frac{2\cos 2\theta(a'\tau-p_1-a\xi\cos\theta)(p_2+a\xi\sin\theta)^2}{[(p_2+a\xi\sin\theta)^2+(a'\tau-p_1-a\xi\cos\theta)^2]^2}$$

$$+\frac{(a'\tau-p_1-a\xi\cos\theta)}{(p_2+a\xi\sin\theta)^2+(a'\tau-p_1-a\xi cos\theta)^2}\} \tag{16}$$

For two cracks, the total stresses for crack one are

$$\tau_{xy}^{1T} = \frac{1}{4a_{11}}\{\frac{1}{\pi}\int_{-1}^{1}\frac{f_{11}}{\tau-\xi}d\tau + \int_{-1}^{1}k_1(\tau,\xi,r_{x_1},r_{x_2},r_{y_1},r_{y_2},\varphi_1,\varphi_2,a_1,a_2)f_{21}d\tau$$
$$+\int_{-1}^{1}k_2(\tau,\xi,r_{x_1},r_{x_2},r_{y_1},r_{y_2},\varphi_1,\varphi_2,a_1,a_2)f_{22}d\tau\} \qquad (17)$$

$$\sigma_{yy}^{1T} = \frac{1}{4a_{11}}\{\frac{1}{\pi}\int_{-1}^{1}\frac{f_{12}}{\tau-\xi}d\tau + \int_{-1}^{1}k_3(\tau,\xi,r_{x_1},r_{x_2},r_{y_1},r_{y_2},\varphi_1,\varphi_2,a_1,a_2)f_{21}d\tau$$
$$+\int_{-1}^{1}k_4(\tau,\xi,r_{x_1},r_{x_2},r_{y_1},r_{y_2},\varphi_1,\varphi_2,a_1,a_2)f_{22}d\tau\} \qquad (18)$$

The total stresses for crack two are

$$\tau_{xy}^{2T} = \frac{1}{4a_{11}}\{\frac{1}{\pi}\int_{-1}^{1}\frac{f_{21}}{\tau-\xi}d\tau + \int_{-1}^{1}k_1(\tau,\xi,r_{x_2},r_{x_1},r_{y_2},r_{y_1},\varphi_2,\varphi_1,a_2,a_1)f_{11}d\tau$$
$$+\int_{-1}^{1}k_2(\tau,\xi,r_{x_2},r_{x_1},r_{y_2},r_{y_1},\varphi_2,\varphi_1,a_2,a_1)f_{12}d\tau\} \qquad (19)$$

$$\sigma_{yy}^{2T} = \frac{1}{4a_{11}}\{\frac{1}{\pi}\int_{-1}^{1}\frac{f_{22}}{\tau-\xi}d\tau + \int_{-1}^{1}k_3(\tau,\xi,r_{x_2},r_{x_1},r_{y_2},r_{y_1},\varphi_2,\varphi_1,a_2,a_1)f_{11}d\tau$$
$$+\int_{-1}^{1}k_4(\tau,\xi,r_{x_2},r_{x_1},r_{y_2},r_{y_1},\varphi_2,\varphi_1,a_2,a_1)f_{12}d\tau\} \qquad (20)$$

Where σ_{yy}^{iT} and τ_{xy}^{iT} $(i=1,2)$ should be equal to the far field stresses transformed to each local coordinate system in opposite direction,

$$\sigma_{yy}^{iT} = -[\sigma_{xx}^{0}\sin^2\varphi_i + \sigma_{yy}^{0}\cos^2\varphi_i - \tau_{xy}^{0}\sin 2\varphi_i] \qquad (21)$$

$$\tau_{xy}^{iT} = -[-\frac{1}{2}[\sigma_{xx}^{0} - \sigma_{yy}^{0}]\sin 2\varphi_i + \tau_{xy}^{0}\cos 2\varphi_i] \qquad (22)$$

So we get four integral equations which can be solved numerically.

$$\frac{1}{\pi}\int_{-1}^{1}\frac{f_{11}}{\tau-\xi}d\tau+\int_{-1}^{1}k_1(\tau,\xi,r_{x_1},r_{x_2},r_{y_1},r_{y_2},\varphi_1,\varphi_2,a_1,a_2)f_{21}d\tau$$
$$+\int_{-1}^{1}k_2(\tau,\xi,r_{x_1},r_{x_2},r_{y_1},r_{y_2},\varphi_1,\varphi_2,a_1,a_2)f_{22}d\tau=4a_{11}\tau_{xy}^{1T} \tag{23}$$

$$\frac{1}{\pi}\int_{-1}^{1}\frac{f_{12}}{\tau-\xi}d\tau+\int_{-1}^{1}k_3(\tau,\xi,r_{x_1},r_{x_2},r_{y_1},r_{y_2},\varphi_1,\varphi_2,a_1,a_2)f_{21}d\tau$$
$$+\int_{-1}^{1}k_4(\tau,\xi,r_{x_1},r_{x_2},r_{y_1},r_{y_2},\varphi_1,\varphi_2,a_1,a_2)f_{22}d\tau=4a_{11}\sigma_{yy}^{1T} \tag{24}$$

$$\frac{1}{\pi}\int_{-1}^{1}\frac{f_{21}}{\tau-\xi}d\tau+\int_{-1}^{1}k_1(\tau,\xi,r_{x_2},r_{x_1},r_{y_2},r_{y_1},\varphi_2,\varphi_1,a_2,a_1)f_{11}d\tau$$
$$+\int_{-1}^{1}k_2(\tau,\xi,r_{x_2},r_{x_1},r_{y_2},r_{y_1},\varphi_2,\varphi_1,a_2,a_1)f_{12}d\tau=4a_{11}\tau_{xy}^{2T} \tag{25}$$

$$\frac{1}{\pi}\int_{-1}^{1}\frac{f_{22}}{\tau-\xi}d\tau+\int_{-1}^{1}k_3(\tau,\xi,r_{x_2},r_{x_1},r_{y_2},r_{y_1},\varphi_2,\varphi_1,a_2,a_1)f_{11}d\tau$$
$$+\int_{-1}^{1}k_4(\tau,\xi,r_{x_2},r_{x_1},r_{y_2},r_{y_1},\varphi_2,\varphi_1,a_2,a_1)f_{12}d\tau=4a_{11}\sigma_{yy}^{2T} \tag{26}$$

For n cracks, we get 2n integral equations

$$\frac{1}{\pi}\int_{-1}^{1}\frac{f_{11}}{\tau-\xi}d\tau+\int_{-1}^{1}k_1(\tau,\xi,r_{x_1},r_{x_2},\cdots)f_{21}d\tau+\int_{-1}^{1}k_2(\tau,\xi,r_{x_1},r_{x_2},\cdots)f_{22}d\tau$$
$$\cdots+\int_{-1}^{1}k_1(\tau,\xi,r_{x_1},r_{x_n},\cdots)f_{n1}d\tau+\int_{-1}^{1}k_2(\tau,\xi,r_{x_1},r_{x_n},\cdots)f_{n2}d\tau=4a_{11}\tau_{xy}^{1T} \tag{27}$$

$$\frac{1}{\pi}\int_{-1}^{1}\frac{f_{12}}{\tau-\xi}d\tau+\int_{-1}^{1}k_3(\tau,\xi,r_{x_1},r_{x_2},\cdots)f_{21}d\tau+\int_{-1}^{1}k_4(\tau,\xi,r_{x_1},r_{x_2},\cdots)f_{22}d\tau$$
$$\cdots+\int_{-1}^{1}k_3(\tau,\xi,r_{x_1},r_{x_n},\cdots)f_{n1}d\tau+\int_{-1}^{1}k_4(\tau,\xi,r_{x_1},r_{x_n},\cdots)f_{n2}d\tau=4a_{11}\sigma_{yy}^{1T} \tag{28}$$

$$\int_{-1}^{1} k_1(\tau,\xi,r_{x_2},r_{x_1},\cdots)f_{11}d\tau + \int_{-1}^{1} k_2(\tau,\xi,r_{x_2},r_{x_1},\cdots)f_{12}d\tau + \frac{1}{\pi}\int_{-1}^{1}\frac{f_{21}}{\tau-\xi}d\tau$$
$$\cdots + \int_{-1}^{1} k_1(\tau,\xi,r_{x_2},r_{x_n},\cdots)f_{n1}d\tau + \int_{-1}^{1} k_2(\tau,\xi,r_{x_2},r_{x_n},\cdots)f_{n2}d\tau = 4a_{11}\tau_{xy}^{2T} \quad (29)$$

$$\int_{-1}^{1} k_3(\tau,\xi,r_{x_2},r_{x_1},\cdots)f_{11}d\tau + \int_{-1}^{1} k_4(\tau,\xi,r_{x_2},r_{x_1},\cdots)f_{12}d\tau + \frac{1}{\pi}\int_{-1}^{1}\frac{f_{22}}{\tau-\xi}d\tau$$
$$\cdots + \int_{-1}^{1} k_3(\tau,\xi,r_{x_2},r_{x_n},\cdots)f_{n1}d\tau + \int_{-1}^{1} k_4(\tau,\xi,r_{x_2},r_{x_n},\cdots)f_{n2}d\tau = 4a_{11}\sigma_{yy}^{2T} \quad (30)$$

$$\cdots$$

$$\int_{-1}^{1} k_1(\tau,\xi,r_{x_n},r_{x_1},\cdots)f_{11}d\tau + \int_{-1}^{1} k_2(\tau,\xi,r_{x_n},r_{x_1},\cdots)f_{12}d\tau$$
$$+ \int_{-1}^{1} k_1(\tau,\xi,r_{x_n},r_{x_2},\cdots)f_{21}d\tau + \int_{-1}^{1} k_2(\tau,\xi,r_{x_n},r_{x_2},\cdots)f_{22}d\tau$$
$$\cdots \frac{1}{\pi}\int_{-1}^{1}\frac{f_{n1}}{\tau-\xi}d\tau = 4a_{11}\tau_{xy}^{nT} \quad (31)$$

$$\int_{-1}^{1} k_3(\tau,\xi,r_{x_n},r_{x_1},\cdots)f_{11}d\tau + \int_{-1}^{1} k_4(\tau,\xi,r_{x_n},r_{x_1},\cdots)f_{12}d\tau$$
$$+ \int_{-1}^{1} k_3(\tau,\xi,r_{x_n},r_{x_2},\cdots)f_{21}d\tau + \int_{-1}^{1} k_4(\tau,\xi,r_{x_n},r_{x_2},\cdots)f_{22}d\tau$$
$$\cdots \frac{1}{\pi}\int_{-1}^{1}\frac{f_{n2}}{\tau-\xi}d\tau = 4a_{11}\sigma_{yy}^{nT} \quad (32)$$

For the case of straight cracks, we use the Lobatto-Chebyshev method to solve these integral equations. Let

$$\tau_p = \cos\frac{(p-1)\pi}{m-1} \qquad for \quad p = 1,...,m \quad (33)$$

The corresponding weights are

$$w_1 = w_m = \frac{\pi}{2(n-1)} \qquad w_r = \frac{\pi}{2(m-1)} \qquad for \quad r = 1, ..., m-1 \tag{34}$$

The collocation points are

$$\xi_j = \cos\frac{(2j-1)\pi}{2m-2} \qquad for \quad j = 1, ..., m-1 \tag{35}$$

For n cracks we get 2nm system of equations

$$\sum_{p=1}^{m}[\frac{w_p f_{11}(\tau_p)}{\pi(\tau_p - \xi_j)} + k_1(\tau_p, \xi_j, r_{x_1}, r_{x_2}, \cdots) f_{21}(\tau_p) w_p + k_2(\tau_p, \xi_j, r_{x_1}, r_{x_2}, \cdots) f_{22}(\tau_p) w_p$$

$$\cdots + k_1(\tau, \xi, r_{x_1}, r_{x_n}, \cdots) f_{n1}(\tau_p) w_p + k_2(\tau, \xi, r_{x_1}, r_{x_n}, \cdots) f_{n2}(\tau_p) w_p] = 4a_{11}\tau_{xy}^{1T}$$

$$for \quad j = 1, ..., m-1 \tag{36}$$

$$\sum_{p=1}^{m}[\frac{w_p f_{12}(\tau_p)}{\pi(\tau_p - \xi_j)} + k_3(\tau_p, \xi_j, r_{x_1}, r_{x_2}, \cdots) f_{21}(\tau_p) w_p + k_4(\tau_p, \xi_j, r_{x_1}, r_{x_2}, \cdots) f_{22}(\tau_p) w_p$$

$$\cdots + k_3(\tau_p, \xi_j, r_{x_1}, r_{x_n}, \cdots) f_{n1}(\tau_p) w_p + k_4(\tau_p, \xi_j, r_{x_1}, r_{x_n}, \cdots) f_{n2}(\tau_p) w_p] = 4a_{11}\sigma_{yy}^{1T}$$

$$for \quad j = 1, ..., m-1 \tag{37}$$

$$\sum_{p=1}^{m}[k_1(\tau_p, \xi_j, r_{x_1}, r_{x_2}, \cdots) f_{21}(\tau_p) w_p + k_2(\tau_p, \xi_j, r_{x_1}, r_{x_2}, \cdots) f_{22}(\tau_p) w_p + \frac{w_p f_{21}(\tau_p)}{\pi(\tau_p - \xi_j)}$$

$$\cdots + k_1(\tau, \xi, r_{x_1}, r_{x_n}, \cdots) f_{n1}(\tau_p) w_p + k_2(\tau, \xi, r_{x_1}, r_{x_n}, \cdots) f_{n2}(\tau_p) w_p] = 4a_{11}\tau_{xy}^{2T}$$

$$for \quad j = 1, ..., m-1 \tag{38}$$

$$\sum_{p=1}^{m}[k_3(\tau_p, \xi_j, r_{x_1}, r_{x_2}, \cdots) f_{21}(\tau_p) w_p + k_4(\tau_p, \xi_j, r_{x_1}, r_{x_2}, \cdots) f_{22}(\tau_p) w_p + \frac{w_p f_{22}(\tau_p)}{\pi(\tau_p - \xi_j)}$$

$$\cdots + k_3(\tau_p, \xi_j, r_{x_1}, r_{x_n}, \cdots) f_{n1}(\tau_p) w_p + k_4(\tau_p, \xi_j, r_{x_1}, r_{x_n}, \cdots) f_{n2}(\tau_p) w_p] = 4a_{11}\sigma_{yy}^{2T}$$

$$for \quad j = 1, ..., m-1 \tag{39}$$

$$\cdots$$

$$\sum_{p=1}^{m}[k_1(\tau_p,\xi_j,r_{x_n},r_{x_1},\cdots)f_{11}(\tau_p)w_p+k_2(\tau_p,\xi_j,r_{x_1},r_{x_2},\cdots)f_{12}(\tau_p)w_p$$
$$+k_1(\tau,\xi,r_{x_1},r_{x_n},\cdots)f_{21}(\tau_p)w_p+k_2(\tau_p,\xi_j,r_{x_1},r_{x_n},\cdots)f_{22}(\tau_p)w_p$$
$$\cdots+\frac{w_p f_{n1}(\tau_p)}{\pi(\tau_p-\xi_j)}]=4a_{11}\tau_{xy}^{nT}$$
$$for \quad j=1,...,m-1 \tag{40}$$

$$\sum_{p=1}^{m}[k_3(\tau_p,\xi_j,r_{x_n},r_{x_1},\cdots)f_{11}(\tau_p)w_p+k_4(\tau_p,\xi_j,r_{x_n},r_{x_1},\cdots)f_{22}(\tau_p)w_p$$
$$+k_3(\tau_p,\xi_j,r_{x_n},r_{x_2},\cdots)f_{21}(\tau_p)w_p+k_4(\tau_p,\xi_j,r_{x_n},r_{x_2},\cdots)f_{22}(\tau_p)w_p$$
$$\cdots+\frac{w_p f_{n2}(\tau_p)}{\pi(\tau_p-\xi_j)}]=4a_{11}\sigma_{yy}^{nT}$$
$$for \quad j=1,...,m-1 \tag{41}$$

$$\sum_{p=1}^{m} f_{11}(\tau_p)w_p=0 \tag{42}$$

$$\sum_{p=1}^{m} f_{12}(\tau_p)w_p=0 \tag{43}$$

$$\sum_{p=1}^{m} f_{21}(\tau_p)w_p=0 \tag{44}$$

$$\sum_{p=1}^{m} f_{22}(\tau_p)w_p=0 \tag{45}$$

$$\cdots$$

$$\sum_{p=1}^{m} f_{n1}(\tau_p)w_p=0 \tag{46}$$

$$\sum_{p=1}^{m} f_{n2}(\tau_p) w_p = 0 \tag{47}$$

We write the system of equations in matrix form

$$[A](f) = (p) \tag{48}$$

Let

$$\begin{pmatrix} \frac{w_p}{\pi(\tau_p - \xi_j)} \\ p = 1, ..., m \\ j = 1, ..., m-1 \\ w_p \\ p = 1, ..., m \end{pmatrix}_{m*m} = \begin{pmatrix} \frac{w_1}{\pi(\tau_1 - \xi_1)} & \cdots & \frac{w_m}{\pi(\tau_m - \xi_1)} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \frac{w_1}{\pi(\tau_1 - \xi_{m-1})} & \cdots & \frac{w_m}{\pi(\tau_m - \xi_{m-1})} \\ w_1 & \cdots & w_m \end{pmatrix}_{m*m}$$

and

$$\begin{pmatrix} w_p k_i(\tau_p, \xi_j, \\ r_{x_a}, r_{x_b}, ...) \\ p = 1, ..., m \\ j = 1, ..., m-1 \end{pmatrix}_{(m-1)*m}$$

$$= \begin{pmatrix} w_1 k_i(\tau_1, \xi_1, r_{x_a}, r_{x_b}, ...) & \cdots & w_m k_i(\tau_m, \xi_1, r_{x_a}, r_{x_b}, ...) \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ w_1 k_i(\tau_1, \xi_{m-1}, r_{x_a}, r_{x_b}, ...) & \cdots & w_m k_i(\tau_m, \xi_{m-1}, r_{x_a}, r_{x_b}, ...) \end{pmatrix}_{(m-1)*m}$$

where $i = 1, ...4$; $a, b \in 1, ...n$; and $a \neq b$.

The matrix [A] is

2nm *2nm	1 … m	m+1 … 2m	2m+1 … 3m	3m+1 … 4m	…	2(n-1)m+1 … 2(n-1)m+m	2(n-1)m+m+1 … 2nm
1 m-1	$\frac{w_p}{\pi(\tau_p-\zeta_j)}$ $p = 1, \ldots, m$ $j = 1, \ldots, m-1$	0	$w_p k_1(\tau_p, \zeta_j,$ $\tau_{z_1}, \tau_{z_2}, \ldots)$ $p = 1, \ldots, m$ $j = 1, \ldots, m-1$	$w_p k_2(\tau_p, \zeta_j,$ $\tau_{z_1}, \tau_{z_2}, \ldots)$ $p = 1, \ldots, m$ $j = 1, \ldots, m-1$	…	$w_p k_1(\tau_p, \zeta_j,$ $\tau_{z_1}, \tau_{z_n}, \ldots)$ $p = 1, \ldots, m$ $j = 1, \ldots, m-1$	$w_p k_2(\tau_p, \zeta_j,$ $\tau_{z_1}, \tau_{z_n}, \ldots)$ $p = 1, \ldots, m$ $j = 1, \ldots, m-1$
m	w_p $p = 1, \ldots, m$	0	0	0	…	0	0
m+1	0	$\frac{w_p}{\pi(\tau_p-\zeta_j)}$ $p = 1, \ldots, m$ $j = 1, \ldots, m-1$	$w_p k_3(\tau_p, \zeta_j,$ $\tau_{z_1}, \tau_{z_2}, \ldots)$ $p = 1, \ldots, m$ $j = 1, \ldots, m-1$	$w_p k_4(\tau_p, \zeta_j,$ $\tau_{z_1}, \tau_{z_2}, \ldots)$ $p = 1, \ldots, m$ $j = 1, \ldots, m-1$	…	$w_p k_3(\tau_p, \zeta_j,$ $\tau_{z_1}, \tau_{z_n}, \ldots)$ $p = 1, \ldots, m$ $j = 1, \ldots, m-1$	$w_p k_4(\tau_p, \zeta_j,$ $\tau_{z_1}, \tau_{z_n}, \ldots)$ $p = 1, \ldots, m$ $j = 1, \ldots, m-1$
2m	0 $p = 1, \ldots, m$	w_p	0	0	…	0	0
2m+1 3m-1	$w_p k_1(\tau_p, \zeta_j,$ $\tau_{z_2}, \tau_{z_1}, \ldots)$ $p = 1, \ldots, m$ $j = 1, \ldots, m-1$	$w_p k_2(\tau_p, \zeta_j,$ $\tau_{z_2}, \tau_{z_1}, \ldots)$ $p = 1, \ldots, m$ $j = 1, \ldots, m-1$	$\frac{w_p}{\pi(\tau_p-\zeta_j)}$ $p = 1, \ldots, m$ $j = 1, \ldots, m-1$	0	…	$w_p k_1(\tau_p, \zeta_j,$ $\tau_{z_2}, \tau_{z_n}, \ldots)$ $p = 1, \ldots, m$ $j = 1, \ldots, m-1$	$w_p k_2(\tau_p, \zeta_j,$ $\tau_{z_2}, \tau_{z_n}, \ldots)$ $p = 1, \ldots, m$ $j = 1, \ldots, m-1$
3m	0	0	w_p $p = 1, \ldots, m$	0	…	0	0
3m+1 4m-1	$w_p k_3(\tau_p, \zeta_j,$ $\tau_{z_2}, \tau_{z_1}, \ldots)$ $p = 1, \ldots, m$ $j = 1, \ldots, m-1$	$w_p k_4(\tau_p, \zeta_j,$ $\tau_{z_2}, \tau_{z_1}, \ldots)$ $p = 1, \ldots, m$ $j = 1, \ldots, m-1$	0	$\frac{w_p}{\pi(\tau_p-\zeta_j)}$ $p = 1, \ldots, m$ $j = 1, \ldots, m-1$	…	$w_p k_3(\tau_p, \zeta_j,$ $\tau_{z_2}, \tau_{z_n}, \ldots)$ $p = 1, \ldots, m$ $j = 1, \ldots, m-1$	$w_p k_4(\tau_p, \zeta_j,$ $\tau_{z_2}, \tau_{z_n}, \ldots)$ $p = 1, \ldots, m$ $j = 1, \ldots, m-1$
4m	0	0	0	w_p $p = 1, \ldots, m$	…	0	0
	. . .	. . .	. . .	. . .	. . .	. . .	. . .
2(n-1)m 2(n-1)m +m-1	$w_p k_1(\tau_p, \zeta_j,$ $\tau_{z_n}, \tau_{z_1}, \ldots)$ $p = 1, \ldots, m$ $j = 1, \ldots, m-1$	$w_p k_2(\tau_p, \zeta_j,$ $\tau_{z_n}, \tau_{z_1}, \ldots)$ $p = 1, \ldots, m$ $j = 1, \ldots, m-1$	$w_p k_1(\tau_p, \zeta_j,$ $\tau_{z_n}, \tau_{z_2}, \ldots)$ $p = 1, \ldots, m$ $j = 1, \ldots, m-1$	$w_p k_2(\tau_p, \zeta_j,$ $\tau_{z_n}, \tau_{z_2}, \ldots)$ $p = 1, \ldots, m$ $j = 1, \ldots, m-1$	…	$\frac{w_p}{\pi(\tau_p-\zeta_j)}$ $p = 1, \ldots, m$ $j = 1, \ldots, m-1$	0
2(n-1)m +m	0	0	0	0	…	w_p $p = 1, \ldots, m$	0
2(n-1)m +m+1 2nm-1	$w_p k_3(\tau_p, \zeta_j,$ $\tau_{z_n}, \tau_{z_1}, \ldots)$ $p = 1, \ldots, m$ $j = 1, \ldots, m-1$	$w_p k_4(\tau_p, \zeta_j,$ $\tau_{z_n}, \tau_{z_1}, \ldots)$ $p = 1, \ldots, m$ $j = 1, \ldots, m-1$	$w_p k_3(\tau_p, \zeta_j,$ $\tau_{z_n}, \tau_{z_2}, \ldots)$ $p = 1, \ldots, m$ $j = 1, \ldots, m-1$	$w_p k_4(\tau_p, \zeta_j,$ $\tau_{z_n}, \tau_{z_2}, \ldots)$ $p = 1, \ldots, m$ $j = 1, \ldots, m-1$	…	0	$\frac{w_p}{\pi(\tau_p-\zeta_j)}$ $p = 1, \ldots, m$ $j = 1, \ldots, m-1$
2nm	0	0	0	0	…	0	w_p $p = 1, \ldots, m$

$$
(f) = \begin{pmatrix}
\begin{pmatrix} f_{11}(\tau_1) \\ \cdot \\ \cdot \\ \cdot \\ f_{11}(\tau_{m-1}) \\ f_{11}(\tau_m) \end{pmatrix}_{m*1} \\
\begin{pmatrix} f_{12}(\tau_1) \\ \cdot \\ \cdot \\ \cdot \\ f_{12}(\tau_{m-1}) \\ f_{12}(\tau_m) \end{pmatrix}_{m*1} \\
\cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\
\begin{pmatrix} f_{n1}(\tau_1) \\ \cdot \\ \cdot \\ \cdot \\ f_{n1}(\tau_{m-1}) \\ f_{n1}(\tau_m) \end{pmatrix}_{m*1} \\
\begin{pmatrix} f_{n2}(\tau_1) \\ \cdot \\ \cdot \\ \cdot \\ f_{n2}(\tau_{m-1}) \\ f_{n2}(\tau_m) \end{pmatrix}_{m*1}
\end{pmatrix}_{2nm*1}
$$

$$(p) = 4a_{11} \begin{pmatrix} \begin{pmatrix} \tau_{xy}^{1T} \\ \cdot \\ \cdot \\ \cdot \\ \tau_{xy}^{1T} \\ 0 \end{pmatrix}_{m*1} \\ \begin{pmatrix} \sigma_{yy}^{1T} \\ \cdot \\ \cdot \\ \cdot \\ \sigma_{yy}^{1T} \\ 0 \end{pmatrix}_{m*1} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \begin{pmatrix} \tau_{xy}^{nT} \\ \cdot \\ \cdot \\ \cdot \\ \tau_{xy}^{nT} \\ 0 \end{pmatrix}_{m*1} \\ \begin{pmatrix} \sigma_{yy}^{nT} \\ \cdot \\ \cdot \\ \cdot \\ \sigma_{yy}^{nT} \\ 0 \end{pmatrix}_{m*1} \end{pmatrix}_{2nm*1}$$

In order to calculate matrix [A], we calculate diagonal block matrix

$$\begin{pmatrix} \frac{w_p}{\pi(\tau_p - \xi_j)} \\ p = 1, ..., m \\ j = 1, ..., m-1 \\ w_p \\ p = 1, ..., m \end{pmatrix}_{m*m}$$

and fill it into diagonal blocks in matrix [A].

Calculate the k_i block matrix and fill them into proper places in [A] for $i = 1, ..., 4$.

$k_i(block) =$

$$\begin{pmatrix} 0 & \begin{matrix} w_p k_i(\tau_p, \xi_j, \\ r_{x_1}, r_{x_2}, ...) \\ p = 1, ..., m \\ j = 1, ..., m-1 \end{matrix} & \cdots & \begin{matrix} w_p k_i(\tau_p, \xi_j, \\ r_{x_1}, r_{x_{n-1}}, ...) \\ p = 1, ..., m \\ j = 1, ..., m-1 \end{matrix} & \begin{matrix} w_p k_i(\tau_p, \xi_j, \\ r_{x_1}, r_{x_n}, ...) \\ p = 1, ..., m \\ j = 1, ..., m-1 \end{matrix} \\ \vdots & \vdots & & \vdots & \vdots \\ \begin{matrix} w_p k_i(\tau_p, \xi_j, \\ r_{x_n}, r_{x_1}, ...) \\ p = 1, ..., m \\ j = 1, ..., m-1 \end{matrix} & \begin{matrix} w_p k_i(\tau_p, \xi_j, \\ r_{x_n}, r_{x_1}, ...) \\ p = 1, ..., m \\ j = 1, ..., m-1 \end{matrix} & \cdots & \begin{matrix} w_p k_i(\tau_p, \xi_j, \\ r_{x_n}, r_{x_{n-1}}, ...) \\ p = 1, ..., m \\ j = 1, ..., m-1 \end{matrix} & 0 \end{pmatrix}_{(nm-2n)*nm}$$

After solving the system of equations, we get the stress intensity factors

$$k_1^{(j)}(1) = \frac{1}{4a_{11}} f_{j2}(1) \tag{49}$$

$$k_2^{(j)}(1) = \frac{1}{4a_{11}} f_{j1}(1) \tag{50}$$

$$k_1^{(j)}(-1) = -\frac{1}{4a_{11}} f_{j2}(-1) \tag{51}$$

$$k_2^{(j)}(-1) = -\frac{1}{4a_{11}} f_{j1}(-1) \tag{52}$$

where $j = 1, ..., n$.

FORTRAN PROGRAM FOR CALCULATING THE STRESSES OF n CRACKS EMBEDDED IN ISOTROPIC PLATE

Main Program

1) Call the subroutine **indata** to input all the informations of each crack and the far field stress.

2) Select the collocation points.

3) Call the subroutine **lobatto** to calculates the abscissae weights and collocation points by using Lobatto-Chebyshev integral technique.

4) Call subroutine **solsy** to construct matrix A and right hand side of the system equations

5) Call system subroution **dlslrg** to solve the large system of equations.

6) Calculate the stress intensity factors.

Subroutine indata

An interface with the user.

1) Tell user to input the elastic modulus E.

2) Tell user to input the position ratio ν.

3) Let the user to choose the problem of plane stress or plane strain.

4) Input the far field stress information.

5) Do loop from 1 to n to input the position, angle and length of each crack.

6) Display all the parameters in the tables for user to make changes according to the index number.

Plot all cracks proportionally on the screen.

1) Initialize the crack representation. The ith crack is a line segment composed of number i.
2) Calculate the maximum absolute value of the x−axis and y−axis involved in the real problem to decide the proportion of the graph which will be displayed according to the real problem.
3) Do loop from 1 to n plot the center and tips of the ith crack and fill the middle with the number i for ith crack.

Subroutine lobatto

1) Calculation of the abscissae τ.
2) Calculation of the weights w.
3) Calculation of the collocation points.

Subroutine solsy

1) Calculate diagonal blocks of block matrix A.
2) Using the formulas of the four kernels derived from macsyma to calculate the (mn-2n)*nm block matrices, where n is the number of cracks, m is the collocation points.
3) Fill block matrix A with the four block matrices in proper positions.

4) Calculate the right hand side of the system of equations by using the boundary conditions.

5) Call the system subroutine **dlslag** to solve the system of equations.

6) Calculate the stress intendity factors.

```
*********10********20********30********40********50********60********70*
************************************************************************
**                                ABT                                 **
************************************************************************
************************************************************************
*                                                                      c
*        Ten or less Cracks in the isotropic infinite plate            c
*                                                                      c
************************************************************************
************************************************************************
*
       double precision gt(50),gc(50),w(50)
       double precision fun(5000),a(5000,5000),p(5000)
       double precision rx(10),ry(10),gf(10),al(10)
       integer ncr
       double precision E,gn
       double precision gs0xx,gs0yy,gt0xy
       double precision a11
       integer n
*
*
       common/property/ a11
       common /numcrack/ ncr
       common /matprop/ E, gn
       common /load/ gs0xx, gs0yy, gt0xy
*
       external lslrg
       common /worksp/ rwksp
       real rwksp(9000000)
       call iwkin(9000000)
*
        print *,'##################################################'
        print *,'######    The Problem of Less Than 11 Cracks   ######'
        print *,'######       in  Infinite Isotropic Plate      ######'
        print *,'##################################################'
       write(*,104)
************************************************************
*       to change max number of cracks more than 10
*       the dimension of rx,ry,gf,al,... must be increased
************************************************************
*
*---------------------------------------------------------------------
*       reading the problem data
*---------------------------------------------------------------------
*       parameter (ncr=2)
       print *,'Input the number of cracks less than 11'
       read(*,*)ncr
        print *,'what your input is  ',ncr
       write(*,104)
       call indata(rx,ry,gf,al)
*
*VVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVV
*                 here we put an expert logic for the
*                  optimum collocation points number
*^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^
*
       write(*,*)'input number of collocation points n (less then 50)'
       read(*,*)n
       print *,'what your input is  ',n
*
************************************************************************
*      Here we put the choice of collocation technique (later)
************************************************************************
*
************************************************************************
*      calculating the collocation points, abbscissae and weigths
```

```
*        using Lobatto-Tchebyshev integral technique
***********************************************************************
         call lobatto(n,gt,gc,w)
*
        write(*,104)
 104    format(3x,/)
**********************************************************************
*        solve the system of equations and
*        calculate the stress intensity factor
***********************************************************************
        call solsy(n,rx,ry,gf,al,gt,gc,w)
        stop
        end
*
************************************************************************
************************************************************************
**********************************************************************
*
*       Subroutine for the problem setup
*       - type of the problem (plain stress or plain strain)
*       - number of cracks     (ncrack=2)  <this should be send from symb>
*       - material properties (E, Nu=gn)
*       - geometry of the crack locations (rx,ry,fi=gf)
*       - crack length         (half of a crack=al)
*       - far field loading condition in global coordinates
*                0                 0             0
*        (  Sigma   = gs0xx    Sigma   = gs0yy  Tau    = gt0xy  )
*               xx                yy             xy
*
*********************************************************************
************************************************************************
************************************************************************
         subroutine indata(rx,ry,gf,al)
*********************************************************************
        character*1 line(1:70,1:30)
        character*1 c(10),ans
        integer x,ncr,i,j,k,l,k1,l1,l11,kkk,num
        double precision rx(ncr),ry(ncr),gf(ncr),al(ncr),E,gn
        double precision gs0xx,gs0yy,gt0xy
        double precision all,xx,g(ncr)
*
        double precision max,maxx,maxy,mxx,mxy
*
        common/property/ all
        common/numcrack/ncr
        common/matprop/E,gn
        common/load/gs0xx,gs0yy,gt0xy
************************************************************************
*
*        Input data
*
************************************************************************
*
        pi=dacos(-1.0d0)
*
*
*
         print *,'******************************************************'
        write(*,103)
         print *,'Input the elastic modulus E '
         read(*,*)E
         print *,'what your input is   ',E
        write(*,103)
         print *,'******************************************************'
        write(*,103)
*
```

```
       print *,'Input Possion ratio v '
       read(*,*)gn
       print *,'what your input is  ',gn
      write(*,103)
       print *,'******************************************************'
      write(*,103)
*
       print *,'Choose the problem of plane stress or plane strain'
       print *,'For plane stress input 1'
       print *,'For plane strain input an integer other than 1'
       read(*,*)x
       IF (x .EQ. 1)THEN
          a11=E
          print *,'what you select is plane stress problem'
       ELSE
          a11=E/(1-gn**2)
          print *,'what you select is plane strain problem'
       END IF
      write(*,103)
       print *,'******************************************************'
      write(*,103)
*
       print *,'####################################################'
       print *,'#####  Input the far field stress information  #####'
       print *,'####################################################'
      write(*,103)
*
       print *,'******************************************************'
      write(*,103)
       print *,'Input the far field stress field gs0xx'
       read(*,*)gs0xx
       print *,'what your input is  ',gs0xx
      write(*,103)
       print *,'******************************************************'
      write(*,103)
*
       print *,'Input the far field stress field gs0yy'
       read(*,*)gs0yy
       print *,'what your input is  ',gs0yy
      write(*,103)
       print *,'******************************************************'
      write(*,103)
*
       print *,'Input the far field stress field gt0xy'
       read(*,*)gt0xy
       print *,'what your input is  ',gt0xy
      write(*,103)
       print *,'******************************************************'
      write(*,103)
*
*
*
      do 200 i=1,ncr
       print *,'####################################################'
       print *,'#####  Input the information of the crack #',i,'    ##'
       print *,'####################################################'
      write(*,103)
       print *,'******************************************************'
      write(*,103)
       print *,'Input the crack position vector rx of crack #',i
       read(*,*)rx(i)
       print *,'what your input is  ',rx(i)
      write(*,103)
       print *,'******************************************************'
      write(*,103)
*
```

```
         print *,'Input the crack position vector ry of crack #',i
         read(*,*)ry(i)
         print *,'what your input is  ',ry(i)
        write(*,103)
         print *,'****************************************************'
        write(*,103)
*
         print *,'Input the crack rotation gf of crack #',i
         read(*,*)gf(i)
         print *,'what your input is  ',gf(i)
         g(i)=gf(i)
        gf(i)=(2*gf(i)*pi)/360
*        gf(1)=dacos(dcos(gf(1)))
        print *,'In arc gf(i)=',gf(i)
        write(*,103)
         print *,'****************************************************'
        write(*,103)
*
         print *,'Input half of the length of the crack #',i
         read(*,*)al(i)
         print *,'what your input is  ',al(i)
        write(*,103)
         print *,'****************************************************'
        write(*,103)
 200    continue
*
 103     format(3x,/)
*
*********10********20********30********40********50********60********70*
*
*         Print out a table of input data
*
*********10********20********30********40********50********60********70*
*
 400     print *,'****************************************************'
         print *,'*      *             *'
         print *,'*  1   *      E      * ', E
         print *,'*      *             *'
         print *,'****************************************************'
         print *,'*      *             *'
         print *,'*  2   *      v      * ', gn
         print *,'*      *             *'
         print *,'****************************************************'
         print *,'*      *             '
         IF (x .EQ. 1)THEN
         print *,'*  3   *  Problem of plane stress'
         ELSE
         print *,'*  3   *  Problem of plane strain'
         END IF
         print *,'*      *             '
         print *,'****************************************************'
        write(*,103)
         print *,'Do you want to change any of the value above? '
         print *,'Answer y for yes, n for no. '
         read(*,*)ans
            IF (ans .EQ. 'y')THEN
            print *,'Input the number to represent the element'
            print *,'you want to change. '
            read(*,*)num
            IF (num .EQ. 1)THEN
            print *,'Input the elastic modulus E '
            read(*,*)E
            print *,'what your input is  ',E
            write(*,103)
            END IF
            IF (num .EQ. 2)THEN
```

```
        print *,'Input Possion ratio v '
        read(*,*)gn
        print *,'what your input is  ',gn
        write(*,103)
        END IF
        IF (num .EQ. 3)THEN
        print *,'Choose the problem of plane stress or plane strain'
        print *,'For plane stress input 1'
        print *,'For plane strain input an integer other than 1'
        read(*,*)x
        IF (x .EQ. 1)THEN
          a11=E
          print *,'what you select is plane stress problem'
         ELSE
          a11=E/(1-gn**2)
           print *,'what you select is plane strain problem'
         END IF
         write(*,103)
         END IF
         goto 400
      END IF
    write(*,103)
401  print *,'**************************************************'
     print *,'*      *        0    *'
     print *,'*  4   *   Sigma     * ', gs0xx
     print *,'*      *        xx   *'
     print *,'**************************************************'
     print *,'*      *        0    *'
     print *,'*  5   *   Sigma     * ', gs0yy
     print *,'*      *       yy    *'
     print *,'**************************************************'
     print *,'*      *       0     *'
     print *,'*  6   *    Tau      * ', gt0xy
     print *,'*      *       yy    *'
     print *,'**************************************************'
    write(*,103)
     print *,'Do you want to change any of the value above? '
     print *,'Answer y for yes, n for no. '
     read(*,*)ans
        IF (ans .EQ. 'y')THEN
        print *,'Input the number to represent the element'
        print *,'you want to change. '
        read(*,*)num
        IF (num .EQ. 4)THEN
           print *,'Input the far field stress field gs0xx'
           read(*,*)gs0xx
           print *,'what your input is  ',gs0xx
           write(*,103)
        END IF
        IF (num .EQ. 5)THEN
           print *,'Input the far field stress field gs0yy'
           read(*,*)gs0yy
           print *,'what your input is  ',gs0yy
           write(*,103)
        END IF
        IF (num .EQ. 6)THEN
           print *,'Input the far field stress field gt0xy'
           read(*,*)gt0xy
           print *,'what your input is  ',gt0xy
         write(*,103)
         END IF
      goto 401
      END IF
*     pause '******* Hit the key <return> to continue *******'
    write(*,103)
    do 201 i=1,ncr
```

```
        j=11+4*(i-1)
 402    print *,'*****************************************************'
        print *,'*      *              *'
        print *,'* ',j,'  *   rx(',i,' )   * ', rx(i)
        print *,'*      *              *'
        print *,'*****************************************************'
        print *,'*      *              *'
        print *,'* ',j+1,'  *   ry(',i,' )   * ', ry(i)
        print *,'*      *              *'
        print *,'*****************************************************'
        print *,'*      *              *'
        print *,'* ',j+2,'  *   gf(',i,' )   * ', g(i)
        print *,'*      *              *'
        print *,'*****************************************************'
        print *,'*      *              *'
        print *,'* ',j+3,'  *   al(',i,' )   * ', al(i)
        print *,'*      *              *'
        print *,'*****************************************************'
       write(*,103)
        print *,'Do you want to change any of the value above? '
        print *,'Answer y for yes, n for no. '
        read(*,*)ans
           IF (ans .EQ. 'y')THEN
           print *,'Input the number to represent the element'
           print *,'you want to change. '
           read(*,*)num
           IF (num .EQ. j)THEN
            print *,'Input the #',i,' crack position vector rx(',i,' )'
               read(*,*)rx(i)
               print *,'what your input is  ',rx(i)
               write(*,103)
           END IF
           IF (num .EQ. j+1)THEN
            print *,'Input the #',i,' crack position vector ry(',i,' )'
              read(*,*)ry(i)
              print *,'what your input is  ',ry(i)
              write(*,103)
           END IF
           IF (num .EQ. j+2)THEN
             print *,'Input the #',i,' crack rotation gf(',i,' )'
             read(*,*)gf(i)
             print *,'what your input is  ',gf(i)
             g(i)=gf(i)
             gf(i)=(2*gf(i)*pi)/360
            print *,'In arc gf(i)=',gf(i)
            write(*,103)
            END IF
           IF (num .EQ. j+3)THEN
               print *,'Input half of the length of the #',i,' crack'
               read(*,*)al(i)
               print *,'what your input is  ',al(i)
               write(*,103)
            END IF
         goto 402
         END IF
 201     continue
       write(*,103)
*
       write(*,103)
       write(*,103)
*
*
*
********10*******20*******30*******40*******50*******60*******70*
*
*         Initialize the screen with all blank
```

```
*
********10*******20*******30*******40*******50*******60*******70*
        do 3 i = 1,70
           do 3 j=1,30
           line(i,j) = ' '
 3         continue
********10*******20*******30*******40*******50*******60*******70*
*
*        Create a coordinate on the screen
*
********10*******20*******30*******40*******50*******60*******70*
        do 5 i=1,70
           line(i,15)='-'
 5         continue
           line(5,15)='|'
           line(15,15)='|'
           line(25,15)='|'
           line(35,15)='|'
           line(45,15)='|'
           line(55,15)='|'
           line(65,15)='|'
         do 6 i=1,30
           line(35,i)='|'
 6      continue
        line(35,5)='-'
        line(35,10)='-'
        line(35,15)='|'
        line(35,20)='-'
        line(35,25)='-'
*
********10*******20*******30*******40*******50*******60*******70*
*
*        Initialize the crack representation
*
********10*******20*******30*******40*******50*******60*******70*
        c(1)='1'
        c(2)='2'
        c(3)='3'
        c(4)='4'
        c(5)='5'
        c(6)='6'
        c(7)='7'
        c(8)='8'
        c(9)='9'
        c(10)='$'
*
********10*******20*******30*******40*******50*******60*******70*
*
*        Calculate the maximum value of x-axis
*        and y-axis involved in the real problem
*
********10*******20*******30*******40*******50*******60*******70*
*
        max=0
        do 8 i=1,ncr
           if ( abs(rx(i))+al(i) .GT. max ) then
              max=abs(rx(i))+al(i)
              end if
           if ( abs(ry(i))+al(i) .GT. max ) then
              max=abs(ry(i))+al(i)
              end if
 8      continue
*
********10*******20*******30*******40*******50*******60*******70*
*
*        Calculate the crack center on the screen
```

```
*
*********10********20********30********40********50********60********70*
        do 10 i=1,ncr
        k=INT(rx(i)*(30/max))
*        print *,'k=',k
        l=INT(30*ry(i)/max)/2
        line(35+k,15-l)=c(i)
*********10********20********30********40********50********60********70*
*
*        Calculate right end of the crack on the screen
*
*********10********20********30********40********50********60********70*
        k=INT((rx(i)+dcos(gf(i))*al(i))*30/max)
        l=INT(30*(ry(i)+dsin(gf(i))*al(i))/max/2)
        line(35+k,15-l)=c(i)
*
*********10********20********30********40********50********60********70*
*
*        Plot the line between crack center and the right end (x-axis)
*
*********10********20********30********40********50********60********70*
*
        if (rx(i)*30/max .LT. k ) then
           kkk=1
        k1=rx(i)*(30/max)+1
        do while (k1 .LT. k)
           l1=(30*ry(i)/max+kkk*dtan(gf(i)))/2
           line(35+k1,15-l1)=c(i)
           k1=k1+1
           kkk=kkk+1
          end do
          else
             kkk=1
        k1=rx(i)*(30/max)-1
        do while (k1 .GT. k)
*           l1=-14*ry(i)/maxy-(k1-rx(i)*(34/maxy))*14/34*dtan(gf(i))
*           l1=14*(ry(i)-kkk*maxx/34*dtan(gf(i)))/2/maxy
           l1=(30*ry(i)/max-kkk*dtan(gf(i)))/2
           line(35+k1,15-l1)=c(i)
           k1=k1-1
           kkk=kkk+1
          end do
          end if
*
*********10********20********30********40********50********60********70*
*
*        Plot the line between crack center and the right end (y-axis)
*
*********10********20********30********40********50********60********70*
*
        if (ry(i)*30/max/2 .LT. l ) then
           kkk=2
        l1=ry(i)*30/max/2+1
        do while (l1 .LT. l)
           k1=30*rx(i)/max+kkk/dtan(gf(i))
           line(35+k1,15-l1)=c(i)
           l1=l1+1
           kkk=kkk+2
          end do
          else
          kkk=2
        l1=ry(i)*30/max/2-1
        do while (l1 .GT. l)
           k1=30*rx(i)/max-kkk/dtan(gf(i))
           line(35+k1,15-l1)=c(i)
           l1=l1-1
```

```
          kkk=kkk+2
         end do
         end if
*
*********10********20********30********40********50********60********70*
*
*        Calculate the left end of the crack on the screen
*
*********10********20********30********40********50********60********70*
*
        k=INT((rx(i)-dcos(gf(i))*al(i))*30/max)
        l=INT(30*(ry(i)-dsin(gf(i))*al(i))/max/2)
        line(35+k,15-l)=c(i)
*
*********10********20********30********40********50********60********70*
*
*        Plot the line between crack center and the left end (y-axis)
*
*********10********20********30********40********50********60********70*
*
        if ( l .LT. ry(i)*30/max/2 ) then
          kkk=2
        l1=ry(i)*30/max/2-1
        do while (l .LT. l1)
           k1=30*rx(i)/max-kkk/tan(gf(i))
           line(35+k1,15-l1)=c(i)
           kkk=kkk+2
           l1=l1-1
           end do
           else
          kkk=2
        l1=ry(i)*30/max/2+1
        do while (l .GT. l1)
           k1=30*rx(i)/max+kkk/tan(gf(i))
           line(35+k1,15-l1)=c(i)
           kkk=kkk+2
           l1=l1+1
           end do
              end if
*
*********10********20********30********40********50********60********70*
*
*        Plot the line between crack center and the left end (x-axis)
*
*********10********20********30********40********50********60********70*
*
           if ( rx(i)*30/max .LT. k ) then
           kkk=1
        k1=rx(i)*(30/max)+1
        do while (k1 .LT. k)
           l1=(ry(i)*30/max+kkk*dtan(gf(i)))/2
           line(35+k1,15-l1)=c(i)
           k1=k1+1
           kkk=kkk+1
         end do
         else
           kkk=1
        k1=rx(i)*(30/max)-1
        do while (k1 .GT. k)
           l1=(ry(i)*30/max-kkk*dtan(gf(i)))/2
           line(35+k1,15-l1)=c(i)
           k1=k1-1
           kkk=kkk+1
         end do
         end if
*
```

```
*
 10     continue
*
*
*********10********20********30********40********50********60********70*
*
*        Show the graph on the screen
*
*********10********20********30********40********50********60********70*
*
*
        do 4 j=1,30
        print *,(line(i,j),i = 1,70)
 4      continue
*
*********10********20********30********40********50********60********70*
*        print *,'rx(1)=',rx(1),'ry(1)=',ry(1)
*        print *,'rx(2)=',rx(2),'ry(2)=',ry(2)
*        print *,'gf(1)=',tan(gf(1)),'gf(2)=',tan(gf(2))
*        print *,'al(1)=',al(1),'al(2)=',al(2)
*********10********20********30********40********50********60********70*
*
*      the end of the subroutine of input data
*
*********10********20********30********40********50********60********70*
* 11     format(1x,61a1,')')
       write(*,103)
        pause '******* Hit the key <return> to continue *******'
       write(*,103)
        return
        end
*
************************************************************************
************************************************************************
************************************************************************
*
*     Subroutine for collocation points, abscissaes, and weights
*     using Lobatto-Tchebyshev integral formula
*
************************************************************************
************************************************************************
************************************************************************
      subroutine lobatto(n,gt,gc,w)
      integer n
*
      double precision gt(n),gc(n-1),w(n),pi
*
************************************************************************
*
*       Calculation of the tau - abscissae
*
************************************************************************
       pi=dacos(-1.0d0)
*       print *,'pi',pi
*       write(*,105)
        do 10 j=1,n
          gt(j)=dcos(((j-1)*pi)/(n-1))
 10       continue
************************************************************************
*
*       Calculation of the weights
*
************************************************************************
       w(1)=pi/(2*(n-1))
       w(n)=w(1)
       do 20 i=2,n-1
```

```
      w(i)=pi/(n-1)
 20     continue
**********************************************************************
*
*       Calculation point  x
*
**********************************************************************
        do 30 j=1,n-1
           gc(j)=dcos(((2*j-1)*pi)/(2*n-2))
*           print *,'gc',gc(j)
 30        continue
*
 105     format(3x,/)
*
         return
         end
**********************************************************************
**********************************************************************
**********************************************************************
*
*     Subroutine for solving the system of integral equations
*
**********************************************************************
**********************************************************************
**********************************************************************
      subroutine solsy(n,rx,ry,gf,al,gt,gc,w)
**********************************************************************
**********************************************************************
*
        integer i,j,k,l,n,ncr
        double precision kr1(ncr*n,ncr*n),kr2(ncr*n,ncr*n)
        double precision kr3(ncr*n,ncr*n),kr4(ncr*n,ncr*n)
        double precision rx(ncr),ry(ncr),gf(ncr),al(ncr),gt(n),gc(n-1)
        double precision w(n),E,gn,gs0xx,gs0yy,gt0xy
        double precision fun(2*ncr*n),a(2*ncr*n,2*ncr*n),p(2*ncr*n)
        double precision ki1p1(ncr),ki1m1(ncr),ki2p1(ncr),ki2m1(ncr)
        double precision pi,ww,wwm,part1,part2
        double precision gh,p1a1a2,p2a2
        double precision p1,p2
        double precision gsyiyi(ncr),gtxiyi(ncr)
        double precision pixi(ncr),qixi(ncr)
        double precision a11
*
        double precision r1x,r1y,r2x,r2y,gf1,gf2,a1,a2
*
*
       common/property/ a11
       common/numcrack/ncr
       common/matprop/E,gn
       common/load/gs0xx,gs0yy,gt0xy
*
*
       pi=dacos(-1.0d0)
*
*
*
*       print *,'gs0xx',gs0xx
*       print *,'gs0yy',gs0yy
*       print *,'gt0xy',gt0xy
*       print *,'E',E
*       print *,'v',gn
*       print *,'ncr',ncr
*       print *,'rx(1)',rx(1)
*       print *,'rx(2)',rx(2)
*       print *,'n',n
*       print *,'gt',gt(1)
```

```
*       do 66 j=1,n-1
*       print *,'gc',gc(j)
*       print *,'all',all
* 66    continue
*       print *,'w',w(1)
*
**********************************************************************
*
*       Calculation of the matrix [A]
*
**********************************************************************
**********************************************************************
*
*       Initialize the matrix [kr1]
*
**********************************************************************
*
      do 292 i=1,ncr*n
         do 293 j=1,ncr*n
            kr1(i,j)=0
 293     continue
 292  continue
*
**********************************************************************
*
*       Initialize the matrix [kr2]
*
**********************************************************************
*
      do 294 i=1,ncr*n
         do 295 j=1,ncr*n
            kr2(i,j)=0
 295     continue
 294  continue
*
**********************************************************************
*
*       Initialize the matrix [kr3]
*
**********************************************************************
*
      do 296 i=1,ncr*n
         do 297 j=1,ncr*n
            kr3(i,j)=0
 297     continue
 296  continue
*
**********************************************************************
*
*       Initialize the matrix [kr4]
*
**********************************************************************
*
      do 298 i=1,ncr*n
         do 299 j=1,ncr*n
            kr4(i,j)=0
 299     continue
 298  continue
*
**********************************************************************
*
*       Initialize the matrix [A] and [P]
*
**********************************************************************
*
      do 300 i=1,2*ncr*n
```

```
          do 301 j=1,2*ncr*n
             a(i,j)=0
 301      continue
          p(i)=0
 300    continue
*
************************************************************************
*
*       Calculate the diagonal elements  of matrix [A]
*
************************************************************************
*
        do 305 k=0,(2*ncr-1)
         do 40 i=1,n-1
            do 50 j=1,n
               a(k*n+i,k*n+j)=w(j)/(pi*(gt(j)-gc(i)))
*
*               print *,'adiagonal=',a(k*n+i,k*n+j)
*
 50            continue
 40            continue
 305    continue
*
*
        do 306 k=0,(2*ncr-1)
         do 60 i=1,n
            a((k+1)*n,k*n+i)=w(i)
*            print *,'adiagdown=',a((k+1)*n,k*n+i)
 60         continue
 306   continue
*
*
*
************************************************************************
*
*********       kr1        *****     (3,1)     *****    k2111     *********
*
************************************************************************
*
         do 310 k=1,ncr
            do 311 l=1,ncr
*
               if (k .NE. l) then
*
              r2x=rx(k)
              r2y=ry(k)
              gf2=gf(k)
              a2=al(k)
**
              r1x=rx(l)
              r1y=ry(l)
              gf1=gf(l)
              a1=al(l)
*
              gh=gf2-gf1
*
         p1=(r2y-r1y)*dsin(gf1)+(r2x-r1x)*dcos(gf1)
*
         p2=(r2y-r1y)*dcos(gf1)-(r2x-r1x)*dsin(gf1)
*
         do 78 i=1,n-1
            do 79 j=1,n
*
               p1a1a2 = -p1+a1*gt(j)-a2*dcos(gh)*gc(i)
               p2a2 = p2+a2*dsin(gh)*gc(i)
               ww = p2a2**2+p1a1a2**2
```

```
             wwm = p1a1a2**2-p2a2**2
             part1 = 2*dcos(2*gh)*p1a1a2*p2a2**2-dsin(2*gh)*p2a2*wwm
             part1 = part1/ww**2
             part2 = (-dsin(2*gh)*p2a2-dcos(2*gh)*p1a1a2)/ww
*
             kr1((k-1)*n+i,(l-1)*n+j)=(a1/pi)*(part1+part2)*w(j)
*             print *,'kr1=',kr1((k-1)*n+i,(l-1)*n+j)
*
 79         continue
 78      continue
*
          end if
*
 311     continue
 310    continue
*
*
**********************************************************************
*
********        kr2        ******    (3,2)    *****    k2112      **********
*
**********************************************************************
*
        do 312 k=1,ncr
           do 313 l=1,ncr
*
              if (k .NE. l) then
*
             r2x=rx(k)
             r2y=ry(k)
             gf2=gf(k)
             a2=al(k)
**
             r1x=rx(l)
             r1y=ry(l)
             gf1=gf(l)
             a1=al(l)
*
             gh=gf2-gf1
*
        p1=(r2y-r1y)*dsin(gf1)+(r2x-r1x)*dcos(gf1)
*
        p2=(r2y-r1y)*dcos(gf1)-(r2x-r1x)*dsin(gf1)
*
        do 80 i=1,n-1
           do 81 j=1,n
*
              p1a1a2 = -p1+a1*gt(j)-a2*dcos(gh)*gc(i)
              p2a2 = p2+a2*dsin(gh)*gc(i)
              ww = p2a2**2+p1a1a2**2
              wwm = p1a1a2**2-p2a2**2
              part1 = dcos(2*gh)*p2a2*wwm+2*dsin(2*gh)*p1a1a2*p2a2**2
              part1 = part1/ww**2
*
              kr2((k-1)*n+i,(l-1)*n+j)=(a1/pi)*part1*w(j)
*              print *,'kr2=',kr2((k-1)*n+i,(l-1)*n+j)
*
 81         continue
 80      continue
*
          end if
*
 313   continue
 312   continue
*
*
```

```
************************************************************************
*
********       kr3      *****   (4,1)    *****      k2211      **********
*
************************************************************************
*
        do 314 k=1,ncr
           do 315 l=1,ncr
*
              if (k .NE. l) then
*
             r2x=rx(k)
             r2y=ry(k)
             gf2=gf(k)
             a2=al(k)
**
             r1x=rx(l)
             r1y=ry(l)
             gf1=gf(l)
             a1=al(l)
*
             gh=gf2-gf1
*
        p1=(r2y-r1y)*dsin(gf1)+(r2x-r1x)*dcos(gf1)
*
        p2=(r2y-r1y)*dcos(gf1)-(r2x-r1x)*dsin(gf1)
*
        do 82 i=1,n-1
           do 83 j=1,n
*
              p1a1a2 = -p1+a1*gt(j)-a2*dcos(gh)*gc(i)
              p2a2 = p2+a2*dsin(gh)*gc(i)
              ww = p2a2**2+p1a1a2**2
              wwm = p1a1a2**2-p2a2**2
              part1 = -dcos(2*gh)*p2a2*wwm-2*dsin(2*gh)*p1a1a2*p2a2**2
              part1 = part1/ww**2
              part2 =  (2*dsin(gh)**2*p2a2+dsin(2*gh)*p1a1a2)/ww
*
              kr3((k-1)*n+i,(l-1)*n+j)=(a1/pi)*(part1+part2)*w(j)
*              print *,'kr3=',kr3((k-1)*n+i,(l-1)*n+j)
*
 83        continue
 82     continue
*
          end if
*
 315  continue
 314  continue
*
*
*
************************************************************************
*
********       kr4      ******   (4,2)    *****      k2212      *********
*
************************************************************************
*
        do 316 k=1,ncr
           do 317 l=1,ncr
*
              if (k .NE. l) then
*
             r2x=rx(k)
             r2y=ry(k)
             gf2=gf(k)
             a2=al(k)
```

```
**
              r1x=rx(l)
              r1y=ry(l)
              gf1=gf(l)
              a1=al(l)
*
              gh=gf2-gf1
*
         p1=(r2y-r1y)*dsin(gf1)+(r2x-r1x)*dcos(gf1)
*
         p2=(r2y-r1y)*dcos(gf1)-(r2x-r1x)*dsin(gf1)
*
         do 84 i=1,n-1
            do 85 j=1,n
*
               p1a1a2 = -p1+a1*gt(j)-a2*dcos(gh)*gc(i)
               p2a2 = p2+a2*dsin(gh)*gc(i)
               ww = p2a2**2+p1a1a2**2
               wwm = p1a1a2**2-p2a2**2
               part1 = 2*dcos(2*gh)*p1a1a2*p2a2**2-dsin(2*gh)*p2a2*wwm
               part1 = part1/ww**2
               part2 = p1a1a2/ww
*
               kr4((k-1)*n+i,(l-1)*n+j)=(a1/pi)*(part1+part2)*w(j)
*               print *,'kr4=',kr4((k-1)*n+i,(l-1)*n+j)
*
 85         continue
 84      continue
*
           end if
*
 317  continue
 316  continue
*
*
***********************************************************************
*
*        Combine kr1,kr2,kr3 and kr4 to calculate matrix [A]
*
***********************************************************************
********      Input  kr1 to [A]     ************************************
*
      do 350 k=0,ncr-1
         do 351 l=0,ncr-1
*
             if (k .NE. l) then
*
        do 352 i=1,n-1
           do 353 j=1,n
*
             a(2*k*n+i,2*l*n+j)=kr1(k*n+i,l*n+j)
*
 353   continue
 352  continue
*
           end if
*
 351  continue
 350  continue
*
*
********      Input  kr2 to [A]     ************************************
*
      do 354 k=0,ncr-1
         do 355 l=0,ncr-1
*
```

```fortran
               if (k .NE. l) then
*
         do 356 i=1,n-1
            do 357 j=1,n
*
               a(2*k*n+i,2*l*n+n+j)=kr2(k*n+i,l*n+j)
*
 357   continue
 356  continue
*
          end if
*
 355  continue
 354  continue
*
*
********      Input  kr3 to [A]     ************************************
*
      do 358 k=0,ncr-1
         do 359 l=0,ncr-1
*
               if (k .NE. l) then
*
         do 360 i=1,n-1
            do 361 j=1,n
*
               a(2*k*n+n+i,2*l*n+j)=kr3(k*n+i,l*n+j)
*
 361   continue
 360  continue
*
          end if
*
 359  continue
 358  continue
*
*
********      Input  kr4 to [A]     ************************************
*
      do 362 k=0,ncr-1
         do 363 l=0,ncr-1
*
               if (k .NE. l) then
*
         do 364 i=1,n-1
            do 365 j=1,n
*
               a(2*k*n+n+i,2*l*n+n+j)=kr4(k*n+i,l*n+j)
*
 365   continue
 364  continue
*
          end if
*
 363  continue
 362  continue
*
*
*************************************************************************
*************************************************************************
*************************************************************************
*************************************************************************
*************************************************************************
*
*        print *,'matrix a='
*        do 200 j=1,4*n,6
```

```fortran
*           do 201 i=1,4*n
*        print 400,a(i,j),a(i,j+1),a(i,j+2),a(i,j+3),a(i,j+4),a(i,j+5)
* 400    format (1x,f10.6,3x,f10.6,3x,f10.6,3x,f10.6,3x,f10.6,3x,f10.6)
* 201            continue
*                print *,'new column'
* 200            continue
*
**********************************************************************
*
*       Calculation of the right hand side of the systems equations {P}
*
**********************************************************************
*
      do 370 i=1,ncr
*
        gsyiyi(i)=gs0xx*dsin(gf(i))**2+gs0yy*dcos(gf(i))**2
        gsyiyi(i)=gsyiyi(i)-gt0xy*dsin(2*gf(i))
        gtxiyi(i)=-(gs0xx-gs0yy)/2*dsin(2*gf(i))+gt0xy*dcos(2*gf(i))
*
        pixi(i)=-gsyiyi(i)
        qixi(i)=-gtxiyi(i)
*
 370  continue
*
*
*
      do 380 k=0,ncr-1
         do 381 i=1,n-1
            p(2*k*n+i)=-4*a11*qixi(k+1)
 381     continue
 380  continue
*
      do 382 k=0,ncr-1
         do 383 i=1,n-1
            p(2*k*n+n+i)=-4*a11*pixi(k+1)
 383     continue
 382  continue
*
*
*
**********************************************************************
*
*       Solve the system of equations [A]{F}={P}
*
**********************************************************************
       call dlslrg(2*ncr*n,a,2*ncr*n,p,1,fun)
*       call wrrrn('fun',1,4*n,fun,1,0)
*
**********************************************************************
*
*       Calculate the stress intensity factor
*
**********************************************************************
*
       do 390 k=0,ncr-1
          ki1p1(k+1)=fun(2*k*n+n+1)/4/a11
          ki1m1(k+1)=-fun(2*(k+1)*n)/4/a11
          ki2p1(k+1)=fun(2*k*n+1)/4/a11
          ki2m1(k+1)=-fun(2*k*n+n)/4/a11
 390   continue
*
       do 391 k=0,ncr-1
          print *,'For crack #',k+1
*
       print 501,ki1m1(k+1),ki1p1(k+1)
 501   format (1x,'k1(-1)=',e12.6,3x,'k1(1)=',e12.6)
```

```
      print 502,ki2m1(k+1),ki2p1(k+1)
 502  format (1x,'k2(-1)=',e12.6,3x,'k2(1)=',e12.6)
*
 391  continue
*
*
      return
      end
```

```
/* solve the ODE get r */
ff:cc*%e^(r*y*s)$
fq:factor(diff(ff*%e^(-%i*s*x),x,4)+2*diff(ff*%e^(-%i*s*x),x,2,y,2)+diff(ff*%e^(-%i

/* solve the characteristic equation */
q1:solve(fq*%e^(-r*y*s+%i*s*x),'r)$
/* it should be two identical real roots */
if length(q1)#2 then 'error1$
if rhs(q1[1])#-rhs(q1[2]) then 'error2$
r:abs(rhs(q1[1]))$

/* the solution of the ODE at y>0 */
fxyp:(cc1+cc2*y)*%e^(-ev(r)*abs(s)*y-%i*s*x)$
/* the solution of the ODE at y<0 */
fxym:(cc3+cc4*y)*%e^(ev(r)*abs(s)*y-%i*s*x)$

/* calculate stresses gs[i] */
block(gs[1]:diff(fxyp,y,2),gs[2]:diff(fxyp,x,2),gs[3]:diff(fxyp,x,1,y,1),
gs[4]:diff(fxym,y,2),gs[5]:diff(fxym,x,2),gs[6]:diff(fxym,x,1,y,1))$

/* the shear tractions q(x)=0 so gtxy=0 for y=0. */
/* According to continuity conditions cc3 and cc4 could be expressed by cc1 */
pp:diff((cc1+cc2*y)*%e^(-ev(r)*abs(s)*y)-(cc3+cc4*y)*%e^(ev(r)*abs(s)*y),y,1)$
ppp:ratsubst(0,y,((cc1+cc2*y)*%e^(-r*abs(s)*y)-(cc3+cc4*y)*%e^(r*abs(s)*y)))$
lls:solve(ppp,cc3)$
cc3:rhs(lls[1])$
ppt:ratsubst(0,'y,pp)$
lls:solve(ppt,'cc4)$
cc4:ev(rhs(lls[1]))$

/* calculate auxilliary function f1(x) */
eqq1:diff((integrate(fxyp,x)),y,2)$
eqq2:diff(fxyp,x,1)$
f1xp:diff((ratsubst(0,y,(a11*eqq1+a12*eqq2))),x,1)$
eqm1:diff((integrate(''fxym,x)),y,2)$
eqm2:diff(''fxym,x,1)$
f1xm:diff((ratsubst(0,y,(a11*eqm1+a12*eqm2))),x,1)$
mi:(ratsimp((f1xp-f1xm)/(2*%pi)))*2*%pi*%e^(%i*s*x)$
lx:solve(mi=f1(t)*%e^(%i*s*t),cc2)$
cc2:rhs(lx[1])$

/* calculate auxilliary function f2(x) */
eqq1:diff((integrate(fxyp,y)),x,2)$
eqq2:diff(fxyp,y,1)$
f2xp:ratsubst(s*abs(s),ks,(ratsubst(ks*abs(s),s^3,(diff((ratsubst(0,y,(a11*eqq1+a12
eqm1:diff((integrate(''fxym,y)),x,2)$
eqm2:diff(''fxym,y,1)$
fm2:ratsubst(0,y,(a11*eqm1+a12*eqm2))$
f2xm:ratsubst(s*abs(s),ks,(ratsubst(ks*abs(s),s^3,(diff(fm2,x,1)))))$
f2x:'integrate((ratsimp((f2xp-f2xm)/(2*%pi))),s,minf,inf)$

/* solve cc1 expressed by the auxiliary function */
mi:(ratsubst(s*abs(s),'integrate(s*abs(s)*%e^(-%i*s*x),s,minf,inf),f2x))*2*%pi$
lx:solve(mi='integrate(f2(t)*%e^(%i*s*t),t,-a,a),cc1)$
cc1:rhs(lx[1])$
cc1:ratsubst(f2(t)*%e^(%i*s*t),'integrate(%e^(%i*s*t)*f2(t),t,-a,a),cc1)$

/* calculate cc2 cc3 and cc4 expressed by the auxiliary function */
cc2:expand(cc2:xthru(ev(cc2)))$
cc21:last(cc2)$
cc2:first(cc2)$
cc21:ratsubst(abs(s),kb,(ratsubst(1,abs(s),(ratsubst(kb,s^2,cc21)))))$
block(cc2:cc2+cc21,cc3:ev(cc3),cc4:ev(cc4))$

/* Get the answer of gsxxp expressed by the auxiliary function */
for j:1 thru 3 do (mwp:expand(ev(gs[j])),
```

```
/* calculate |s| when s>0 */
mip:expand(subst(s,abs(s),mwp)),
/* calculate |s| when s<0 */
mim:expand(ratsubst(-s,s,(subst(-s,abs(s),mwp)))),
/* combine the two terms */
mi:ratsubst(ywm,s*wm,(ratsubst(ywp,s*wp,(subst(wm,%e^(-s*y+%i*s*x-%i*s*t),(subst(wp

if j=3 then ( mi:ratsimp(mi)),
mi:distrib(subst(z,(t-x),((subst((-1/(-y-%i*(t-x))),wm,(subst((-1/(-y+%i*(t-x))),wp

ki:factor(expand(xthru(first(mi)+first(rest(mi,2))))),
kj:factor(expand(xthru(first(rest(mi,1))+last(mi)))),
mi:subst((t-x),z,ki+kj),

/* Get the answer of stresses gs[i] expressed by the auxiliary function */
gs[j]:'integrate(mi,t,-a,a),
LDISPLAY(gs[j]),

ki:ki*4*%pi*a11,
kj:kj*4*%pi*a11,
mi:ki+kj,
gsxxpf[j]:subst((t-x),z,mi)
)$

/***************************************************************************/
/***** calculate four kernels of two cracks *******************************/
/***************************************************************************/
for i:1 thru 3 do (
gsxxpf1t1[i]:subst(t1,t,(subst(x1,x,(subst(y1,y,(subst(1,f1(t),(subst(0,f2(t),(gsxx
gsxxpf1t2[i]:subst(t2,t,(subst(x2,x,(subst(y2,y,(subst(1,f1(t),(subst(0,f2(t),gsxxp
gsxxpf2t1[i]:subst(t1,t,(subst(x1,x,(subst(y1,y,(subst(1,f2(t),(subst(0,f1(t),gsxxp
gsxxpf2t2[i]:subst(t2,t,(subst(x2,x,(subst(y2,y,(subst(1,f2(t),(subst(0,f1(t),gsxxp
)$
kill(wwm)$
gsx2:subst(a2*gt,t2,gsxxpf1t2[1])$
gsy2:subst(a2*gt,t2,gsxxpf1t2[2])$
gtxy2:subst(a2*gt,t2,gsxxpf1t2[3])$
gm21b[1]:-(''gsx2-''gsy2)/2*sin(2*(-gh))+''gtxy2*cos(2*(-gh))$
gm21b[3]:(''gsx2+''gsy2)/2-(''gsx2-''gsy2)/2*cos(2*(-gh))-''gtxy2*sin(2*(-gh))$

gsx2:subst(a2*gt,t2,gsxxpf2t2[1])$
gsy2:subst(a2*gt,t2,gsxxpf2t2[2])$
gtxy2:subst(a2*gt,t2,gsxxpf2t2[3])$
gm21b[2]:-(''gsx2-''gsy2)/2*sin(2*(-gh))+''gtxy2*cos(2*(-gh))$
gm21b[4]:(''gsx2+''gsy2)/2-(''gsx2-''gsy2)/2*cos(2*(-gh))-''gtxy2*sin(2*(-gh))$

/********* calculate the four kernels ************************************/
for i:1 thru 4 do (

gm21[i]:subst(wwm,p3a2a1^2-p4a1^2,(subst(ww,p4a1^2+p3a2a1^2,(subst(p4a1,p4-a1*gc*si

if i=1 then (
gm21[i]:combine(multthru(first(gm21[i]))+multthru(rest(gm21[i],1)))
),
if i=2 then ( gm21[i]:combine(gm21[i])),
if i=3 then (
n:combine((last(gm21[i]))+(multthru(first(gm21[i])))+(multthru(first(rest(gm21[i],1
tt1:first(n),
ts2:part(last(n),1),
tt2:((trigsimp(trigexpand(factor(first(ts2)+first(rest(ts2,1))))))+last(ts2))/ww,
gm21[i]:tt1+tt2
),
if i=4 then (gm21[i]:combine(gm21[i])),

/*  display the four kernels k[i]  after simplification  *******************/
k[i]:(a2/(4*%pi*a11))*gm21[i],
```

```
LDISPLAY(k[i]),
/* where
p3a2a1=-p3+a2*gt-a1*gc*cos(gh)
p4a1=p4-a1*gc*sin(gh)
ww=p4a1^2+p3a2a1^2
wwm=p3a2a1^2-p4a1^2
*/
kill(ww,wwm),

/***  generate FORTRAN code for calculating numerically **********************/
fortran(p3a2a1:-p3+a2*gt(j)-a1*gc(i)*dcos(gh)),
fortran(p4a1:p4-a1*gc(i)*dsin(gh)),
fortran(ww:'p4a1^2+'p3a2a1^2),
fortran(wwm:'p3a2a1^2-'p4a1^2),
fortran(part1:(subst(dsin,sin,(subst(dcos,cos,first(first(gm21[i]))))))),
fortran(part1:'part1/('ww^2)),
fortran(part2:(subst(dsin,sin,(subst(dcos,cos,last(gm21[i])))))),
kill(n1,n2,n3,n,p3a2a1,p4a1,ww,wwm)
)$
```

```
(d3)                                      totallp

(c4) /* solve the ODE get r */

ff:'cc*%e^('r*'y*'s)$

(c5) fq:factor(
diff(ff*%e^(-%i*'s*'x),'x,4)+
2*diff(ff*%e^(-%i*'s*'x),'x,2,'y,2)+
diff(ff*%e^(-%i*'s*'x),'y,4)
);

                                 2          2  4   r s y - %i s x
(d5)                   cc (r - 1)  (r + 1)  s  %e

(c6) remvalue(ff)$

(c7) /* solve the characteristic equation */

q1:solve(fq*%e^(-'r*'y*'s+%i*'s*'x),'r);

(d7)                                [r = - 1, r = 1]

(c8) remvalue(fq)$

(c9) /* it should two identical real roots */

if length(q1)#2 then 'error1$

(c10) if rhs(q1[1])#-rhs(q1[2]) then 'error2$

(c11) r:abs(rhs(q1[1]));

(d11)                                      1

(c12) remvalue(q1)$

(c13) /* the solution of the ODE at y>0 */

fxyp:('cc1+'cc2*'y)*%e^(-''r*abs('s)*'y-%i*'s*'x);

                                         - abs(s) y - %i s x
(d13)                   (cc2 y + cc1) %e

(c14) /* the solution of the ODE at y<0 */

fxym:('cc3+'cc4*'y)*%e^(''r*abs('s)*'y-%i*'s*'x);

                                          abs(s) y - %i s x
(d14)                    (cc4 y + cc3) %e

(c15) /* calculate gsxxp */

gsxxp:diff(fxyp,'y,2);

       2                   - abs(s) y - %i s x
(d15) s  (cc2 y + cc1) %e

                                                          - abs(s) y - %i s x
                                       - 2 cc2 abs(s) %e

(c16) /* calculate gsyyp */

gsyyp:diff(fxyp,'x,2);
```

```
                                  2                      - abs(s) y - %i s x
(d16)                          - s  (cc2 y + cc1) %e

(c17) /* calculate gtxyp */

gtxyp:diff(fxyp,'x,1,'y,1);

                                              - abs(s) y - %i s x
(d17) %i s abs(s) (cc2 y + cc1) %e

                                                                      - abs(s) y - %i s x
                                                         - %i cc2 s %e

(c18) /* calculate gsxxm */

gsxxm:diff(fxym,'y,2);

        2                    abs(s) y - %i s x                     abs(s) y - %i s x
(d18) s  (cc4 y + cc3) %e                     + 2 cc4 abs(s) %e

(c19) /* calculate gsyym */

gsyym:diff(fxym,'x,2);

                                  2                    abs(s) y - %i s x
(d19)                          - s  (cc4 y + cc3) %e

(c20) /* calculate gtxym */

gtxym:diff(fxym,'x,1,'y,1);

                                                abs(s) y - %i s x
(d20) - %i s abs(s) (cc4 y + cc3) %e

                                                                        abs(s) y - %i s x
                                                           - %i cc4 s %e

(c21) /* the shear tractions q(x)=0 so gtxy=0 for y=0.
According to this boundary condition cc2 can be expressed by cc1 */

/*
ui:gtxyp*%e^(%i*'s*'x);
ui:ratsubst(0,'y,ui);
lls:solve(ui,'cc2);
cc2:rhs(lls[1]);
*/

/* Accirding to continuity conditions cc3 and cc4 could be expressed by cc1 */

pp:diff(('cc1+'cc2*'y)*%e^(-''r*abs('s)*'y)-('cc3+'cc4*'y)*%e^(''r*abs('s)*'y),'y,1

                                          abs(s) y           abs(s) y
(d21) - abs(s) (cc4 y + cc3) %e                   - cc4 %e

                                                              - abs(s) y            - abs(s) y
                                   - abs(s) (cc2 y + cc1) %e                + cc2 %e

(c22) ppp:('cc1+'cc2*'y)*%e^(-r*abs(s)*'y)-('cc3+'cc4*'y)*%e^(r*abs('s)*'y);

                                        - abs(s) y                    abs(s) y
(d22)           (cc2 y + cc1) %e                  - (cc4 y + cc3) %e

(c23) ppp:ratsubst(0,'y,ppp);

(d23)                                        cc1 - cc3
```

```
(c24) lls:solve(ppp,'cc3);

(d24)                              [cc3 = cc1]

(c25) remvalue(ppp)$

(c26) cc3:rhs(lls[1]);

(d26)                                  cc1

(c27) ppt:pp;

                                 abs(s) y          abs(s) y
(d27) - abs(s) (cc4 y + cc3) %e          - cc4 %e

                                             - abs(s) y          - abs(s) y
                    - abs(s) (cc2 y + cc1) %e           + cc2 %e

(c28) ppt:ratsubst(0,'y,ppt);

(d28)                    (- cc3 - cc1) abs(s) - cc4 + cc2

(c29) lls:solve(ppt,'cc4);

(d29)                  [cc4 = (- cc3 - cc1) abs(s) + cc2]

(c30) cc4:rhs(lls[1]);

(d30)                      (- cc3 - cc1) abs(s) + cc2

(c31) cc4:''cc4;

(d31)                           cc2 - 2 cc1 abs(s)

(c32) remvalue(lls)$

(c33) remvalue(pp)$

(c34) remvalue(ppt)$

(c35) /* calculate f1(x) */
fbp:('cc1+cc2*'y)*%e^(-''r*abs('s)*'y-%i*'s*'x);

                                     - abs(s) y - %i s x
(d35)                  (cc2 y + cc1) %e

(c36) fbm:(cc3+cc4*'y)*%e^(''r*abs('s)*'y-%i*'s*'x);

                                                abs(s) y - %i s x
(d36)          ((cc2 - 2 cc1 abs(s)) y + cc1) %e

(c37) eqq1:integrate(fbp,'x);

                                        - abs(s) y - %i s x
                       %i (cc2 y + cc1) %e
(d37)                  ------------------------------------
                                            s

(c38) eqq1:diff(eqq1,'y,2);

                                    - abs(s) y - %i s x
(d38) %i s (cc2 y + cc1) %e

                                                                 - abs(s) y - %i s x
                                              2 %i cc2 abs(s) %e
```

```
                                                  - ------------------------------------
                                                                     s

(c39) eqq2:diff(fbp,'x,1);

                                                   - abs(s) y - %i s x
(d39)                - %i s (cc2 y + cc1) %e

(c40) ff1:'a11*eqq1+'a12*eqq2;

                                         - abs(s) y - %i s x
(d40) a11 (%i s (cc2 y + cc1) %e

                               - abs(s) y - %i s x
   2 %i cc2 abs(s) %e
 - ------------------------------------)
                               s

                                    - abs(s) y - %i s x
 - %i a12 s (cc2 y + cc1) %e

(c41) ff1:ratsubst(0,'y,ff1);

                                                                    2     - %i s x
           (2 %i a11 cc2 abs(s) + (%i a12 - %i a11) cc1 s ) %e
(d41)    - -------------------------------------------------------------
                                              s

(c42) f1xp:diff(ff1,'x,1);

                                                                    2     - %i s x
(d42)    %i (2 %i a11 cc2 abs(s) + (%i a12 - %i a11) cc1 s ) %e

(c43) eqm1:integrate(fbm,'x);

                                                              abs(s) y - %i s x
             %i ((cc2 - 2 cc1 abs(s)) y + cc1) %e
(d43)        -------------------------------------------------------
                                              s

(c44) eqm1:diff(eqm1,'y,2);

                                                         abs(s) y - %i s x
(d44) %i s ((cc2 - 2 cc1 abs(s)) y + cc1) %e

                                                                           abs(s) y - %i s x
                                  2 %i abs(s) (cc2 - 2 cc1 abs(s)) %e
                                + -------------------------------------------------------
                                                                    s

(c45) eqm2:diff(fbm,'x,1);

                                                                abs(s) y - %i s x
(d45)       - %i s ((cc2 - 2 cc1 abs(s)) y + cc1) %e

(c46) fm1:'a11*eqm1+'a12*eqm2;

                                                              abs(s) y - %i s x
(d46) a11 (%i s ((cc2 - 2 cc1 abs(s)) y + cc1) %e

                                                   abs(s) y - %i s x
   2 %i abs(s) (cc2 - 2 cc1 abs(s)) %e
 + -------------------------------------------------------)
                                s

                                                         abs(s) y - %i s x
```

```
 - %i a12 s ((cc2 - 2 cc1 abs(s)) y + cc1) %e

(c47) fm1:ratsubst(0,'y,fm1);

                                               2                     2
(d47) - (- 2 %i a11 cc2 abs(s) + (%i a12 - %i a11) cc1 s  + 4 %i a11 cc1 s )

                                                                   - %i s x
                                                                %e          /s

(c48) f1xm:diff(fm1,'x,1);

                                                2                   2
(d48) %i (- 2 %i a11 cc2 abs(s) + (%i a12 - %i a11) cc1 s  + 4 %i a11 cc1 s )

                                                                   - %i s x
                                                                %e

(c49) remvalue(eqq1)$

(c50) remvalue(eqq2)$

(c51) remvalue(ff2)$

(c52) remvalue(eqm1)$

(c53) remvalue(eqm2)$

(c54) remvalue(fm1)$

(c55) remvalue(ui)$

(c56) /* calculate f1(x) */
f1x:(f1xp-f1xm)/(2*%pi);

                                                 2    - %i s x
(d56) (%i (2 %i a11 cc2 abs(s) + (%i a12 - %i a11) cc1 s ) %e

                                             2                    2
 - %i (- 2 %i a11 cc2 abs(s) + (%i a12 - %i a11) cc1 s  + 4 %i a11 cc1 s )

   - %i s x
 %e         )/(2 %pi)

(c57) f1x:ratsimp(f1x);

                                                2    - %i s x
                 (2 a11 cc2 abs(s) - 2 a11 cc1 s ) %e
(d57)          - -------------------------------------------
                                   %pi

(c58) mi:f1x*2*%pi*%e^(%i*s*x);

                                                   2
(d58)               - 2 (2 a11 cc2 abs(s) - 2 a11 cc1 s )

(c59) lx:solve(mi=f1(t)*%e^(%i*'s*'t),'cc2);

                           %i s t                     2
                        %e        f1(t) - 4 a11 cc1 s
(d59)              [cc2 = - -----------------------------]
                                  4 a11 abs(s)

(c60) /*
lx:solve(mi='integrate(f1(t)*%e^(%i*'s*'t),t,-a,a),'cc2);
```

```
*/

cc2:rhs(lx[1]);

                                    %i s t                   2
                                  %e       f1(t) - 4 a11 cc1 s
(d60)                           - -----------------------------
                                          4 a11 abs(s)

(c61) /*cc1:ratsubst(f2(t)*%e^(%i*'s*'t),'integrate('%e^(%i*'s*'t)*f2(t),t,-a,a),cc:

f1x:'integrate(f1x,s,minf,inf);


mi:ratsubst(2*a11*cc2*abs('s)-2*a11*s^2*c11,'integrate(('2*'a11*'cc2*abs('s)-'2*'a1:
*/

/* calculate f2(x) */

eqq1:integrate(fbp,'y);

                           - abs(s) y - %i s x            - abs(s) y - %i s x
      cc2 (abs(s) y + 1) %e                          cc1 %e
(d61) - ---------------------------------------- - -------------------------
                              2                              abs(s)
                             s

(c62) eqq1:diff(eqq1,'x,2);

                                                        2   - abs(s) y - %i s x
                           - abs(s) y - %i s x   cc1 s  %e
(d62) cc2 (abs(s) y + 1) %e                    + ----------------------------
                                                            abs(s)

(c63) eqq2:diff(fbp,'y,1);

                - abs(s) y - %i s x                      - abs(s) y - %i s x
(d63) cc2 %e                      - abs(s) (cc2 y + cc1) %e

(c64) ff2:'a11*eqq1+'a12*eqq2;

                                    - abs(s) y - %i s x
(d64) a11 (cc2 (abs(s) y + 1) %e

          2   - abs(s) y - %i s x
   cc1 s  %e                                  - abs(s) y - %i s x
 + ----------------------------) + a12 (cc2 %e
              abs(s)

                             - abs(s) y - %i s x
 - abs(s) (cc2 y + cc1) %e                        )

(c65) ff2:ratsubst(0,'y,ff2);

                                                    2            2     - %i s x
          ((- a12 - a11) cc2 abs(s) + a12 cc1 s  - a11 cc1 s ) %e
(d65)   - -------------------------------------------------------------------
                                     abs(s)

(c66) f2xp:diff(ff2,'x,1);

                                                      2            2     - %i s x
       %i s ((- a12 - a11) cc2 abs(s) + a12 cc1 s  - a11 cc1 s ) %e
(d66) -----------------------------------------------------------------------
                                     abs(s)
```

```
(c67) f2xp:ratsubst('ks*abs('s),'s^3,f2xp);

                                                                       - %i s x
(d67) - ((%i al2 + %i al1) cc2 s + cc1 (%i al1 ks - %i al2 ks)) %e

(c68) f2xp:ratsubst('s*abs('s),'ks,f2xp);

(d68) - s (cc1 (%i al1 abs(s) - %i al2 abs(s)) + (%i al2 + %i al1) cc2)

                                                                          - %i s x
                                                                        %e

(c69) eqm1:integrate(fbm,'y);

                                      abs(s) y - %i s x
         2 cc1 abs(s) (abs(s) y - 1) %e
(d69) - ---------------------------------------------
                                 2
                                s

                                    abs(s) y - %i s x         abs(s) y - %i s x
                  cc2 (abs(s) y - 1) %e                   cc1 %e
                + ------------------------------------- + -----------------------
                                      2                        abs(s)
                                     s

(c70) eqm1:diff(eqm1,'x,2);

                                    abs(s) y - %i s x
(d70) 2 cc1 abs(s) (abs(s) y - 1) %e

                                                              2   abs(s) y - %i s x
                                 abs(s) y - %i s x    cc1 s   %e
             - cc2 (abs(s) y - 1) %e                - -------------------------
                                                               abs(s)

(c71) eqm2:diff(fbm,'y,1);

                                                abs(s) y - %i s x
(d71) abs(s) ((cc2 - 2 cc1 abs(s)) y + cc1) %e

                                                               abs(s) y - %i s x
                                       + (cc2 - 2 cc1 abs(s)) %e

(c72) fm2:'al1*eqm1+'al2*eqm2;

                                                     abs(s) y - %i s x
(d72) al2 (abs(s) ((cc2 - 2 cc1 abs(s)) y + cc1) %e

                             abs(s) y - %i s x
 + (cc2 - 2 cc1 abs(s)) %e                     )

                                        abs(s) y - %i s x
 + al1 (2 cc1 abs(s) (abs(s) y - 1) %e

                                                      2   abs(s) y - %i s x
                           abs(s) y - %i s x    cc1 s   %e
 - cc2 (abs(s) y - 1) %e                     - -------------------------)
                                                      abs(s)

(c73) fm2:ratsubst(0,'y,fm2);

(d73)

                                                          2              2     - %i s x
      ((- al2 - al1) cc2 abs(s) + (al2 + 2 al1) cc1 s  + al1 cc1 s ) %e
```

```
  - -----------------------------------------------------------------------
                                   abs(s)

(c74) f2xm:diff(fm2,'x,1);

(d74)

                                      2                 2      - %i s x
 %i s ((- a12 - a11) cc2 abs(s) + (a12 + 2 a11) cc1 s  + a11 cc1 s ) %e
 -----------------------------------------------------------------------
                                  abs(s)

(c75) f2xm:ratsubst('ks*abs('s),'s^3,f2xm);

                                                                  - %i s x
(d75) - ((%i a12 + %i a11) cc2 s + cc1 (- %i a12 ks - 3 %i a11 ks)) %e

(c76) f2xm:ratsubst('s*abs('s),'ks,f2xm);

(d76) - s (cc1 (- %i a12 abs(s) - 3 %i a11 abs(s)) + (%i a12 + %i a11) cc2)

                                                                   - %i s x
                                                                 %e

(c77) remvalue(fxyp)$

(c78) remvalue(fxym)$

(c79) remvalue(r)$

(c80) remvalue(fbp)$

(c81) remvalue(fbm)$

(c82) remvalue(eqq1)$

(c83) remvalue(eqq2)$

(c84) remvalue(ff2)$

(c85) remvalue(eqm1)$

(c86) remvalue(eqm2)$

(c87) remvalue(fm2)$

(c88) remvalue(ui)$

(c89) /* calculate f2(x) */
f2x:(f2xp-f2xm)/(2*%pi);

(d89) (s (cc1 (- %i a12 abs(s) - 3 %i a11 abs(s)) + (%i a12 + %i a11) cc2)

   - %i s x
 %e         - s (cc1 (%i a11 abs(s) - %i a12 abs(s)) + (%i a12 + %i a11) cc2)

   - %i s x
 %e         )/(2 %pi)

(c90) f2x:ratsimp(f2x);

                                              - %i s x
                        2 %i a11 cc1 s abs(s) %e
(d90)                 - ---------------------------
                                   %pi
```

```
(c91) f2x:'integrate(f2x,s,minf,inf);

                                  inf
                                 /
                                 [                  - %i s x
                    2 %i all cc1 I     s abs(s) %e           ds
                                 ]
                                 /
                                  minf
(d91)             - ------------------------------------------
                                        %pi

(c92) /* solve cc1 expressed by the auxiliary function */

mi:ratsubst('s*abs('s),'integrate('s*abs('s)*%e^(-%i*s*x),s,minf,inf),f2x);

                                2 %i all cc1 s abs(s)
(d92)                         - ---------------------
                                         %pi

(c93) mi:mi*2*%pi;

(d93)                         - 4 %i all cc1 s abs(s)

(c94) lx:solve(mi='integrate(f2(t)*%e^(%i*'s*'t),t,-a,a),'cc1);

                                    a
                                   /
                                   [      %i s t
                                %i I    %e        f2(t) dt
                                   ]
                                   /
                                    - a
(d94)                    [cc1 = -------------------------]
                                    4 all s abs(s)

(c95) cc1:rhs(lx[1]);

                                 a
                                /
                                [      %i s t
                             %i I    %e        f2(t) dt
                                ]
                                /
                                 - a
(d95)                        -------------------------
                                 4 all s abs(s)

(c96) cc1:ratsubst(f2(t)*%e^(%i*'s*'t),'integrate('%e^(%i*'s*'t)*f2(t),t,-a,a),cc1)

                                     %i s t
                                 %i %e       f2(t)
(d96)                            -----------------
                                  4 all s abs(s)

(c97) /* calculate cc2 cc3 and cc4 expressed by the auxiliary function */

cc2:''cc2;

                                                      %i s t
                         %i s t             %i s %e         f2(t)
                       %e        f1(t) - -------------------
                                                   abs(s)
(d97)                - ---------------------------------------
                                     4 all abs(s)
```

```
(c98) cc2:xthru(cc2);

                             %i s t                 %i s t
                   abs(s) %e        f1(t) - %i s %e       f2(t)
(d98)            - --------------------------------------------
                                           2
                                     4 all s

(c99) cc2:expand(cc2);

                        %i s t                   %i s t
                   %i %e       f2(t)   abs(s) %e       f1(t)
(d99)              ----------------- - ---------------------
                        4 all s                       2
                                                4 all s

(c100) cc2l:last(cc2);

                                     %i s t
                              abs(s) %e       f1(t)
(d100)                      - ---------------------
                                             2
                                       4 all s

(c101) cc2:first(cc2);

                                  %i s t
                              %i %e       f2(t)
(d101)                        -----------------
                                   4 all s

(c102) cc2l:ratsubst(kb,s^2,cc2l);

                                     %i s t
                              abs(s) %e       f1(t)
(d102)                      - ---------------------
                                   4 all kb

(c103) cc2l:ratsubst(1,abs(s),cc2l);

                                  %i s t
                                %e       f1(t)
(d103)                        - --------------
                                   4 all kb

(c104) cc2l:ratsubst(abs(s),kb,cc2l);

                                  %i s t
                                %e       f1(t)
(d104)                        - --------------
                                4 all abs(s)

(c105) cc2:cc2+cc2l;

                        %i s t           %i s t
                   %i %e       f2(t)   %e       f1(t)
(d105)             ----------------- - --------------
                        4 all s        4 all abs(s)

(c106) cc3:''cc3;

                                  %i s t
                              %i %e       f2(t)
(d106)                        -----------------
                              4 all s abs(s)
```

```
(c107) cc4:''cc4;

                                    %i s t          %i s t
                              %i %e       f2(t)   %e       f1(t)
(d107)                      - ---------------- - --------------
                                  4 a11 s         4 a11 abs(s)

(c108) /* Get the answer of gsxxp expressed by the auxiliary function */

mwp:''gsxxp;

                       %i s t           %i s t               %i s t
        2   %i %e          f2(t)   %e          f1(t)      %i %e          f2(t)
(d108) s  ((----------------- - ---------------) y + -----------------)
                4 a11 s          4 a11 abs(s)         4 a11 s abs(s)

                                              %i s t          %i s t
   - abs(s) y - %i s x                   %i %e       f2(t)   %e       f1(t)
 %e                     - 2 abs(s) (---------------- - ---------------)
                                          4 a11 s       4 a11 abs(s)

   - abs(s) y - %i s x
 %e

(c109) mwp:expand(mwp);

                             - abs(s) y - %i s x + %i s t
       %i s f2(t) y %e
(d109) ------------------------------------------
                            4 a11

    2              - abs(s) y - %i s x + %i s t
   s  f1(t) y %e
 - ----------------------------------------
                  4 a11 abs(s)

                          - abs(s) y - %i s x + %i s t
   %i abs(s) f2(t) %e
 - ----------------------------------------------
                         2 a11 s

                   - abs(s) y - %i s x + %i s t
   %i s f2(t) %e
 + ----------------------------------------
                  4 a11 abs(s)

              - abs(s) y - %i s x + %i s t
   f1(t) %e
 + ------------------------------------
                  2 a11

(c110) /* calculate |s| when s>0 */

assume(s>0);

(d110)                           [s > 0]

(c111) mip:''mwp;

                             - s y - %i s x + %i s t
       %i s f2(t) y %e
(d111) --------------------------------------
                           4 a11

              - s y - %i s x + %i s t                   - s y - %i s x + %i s t
```

```
   s f1(t) y %e                                     %i f2(t) %e
 - ------------------------------------------ - ------------------------------------------
                 4 a11                                         4 a11

              - s y - %i s x + %i s t
   f1(t) %e
 + ------------------------------
               2 a11

(c112) mip:expand(mip);

                       - s y - %i s x + %i s t
       %i s f2(t) y %e
(d112) -----------------------------------------
                          4 a11

                - s y - %i s x + %i s t                 - s y - %i s x + %i s t
   s f1(t) y %e                            %i f2(t) %e
 - ------------------------------------ - ------------------------------------
                 4 a11                                   4 a11

              - s y - %i s x + %i s t
   f1(t) %e
 + ------------------------------
               2 a11

(c113) forget(s>0);

(d113)                          [s > 0]

(c114) /* calculate |s| when s<0 */

assume(s<0);

(d114)                          [s < 0]

(c115) mim:''mwp;

                       s y - %i s x + %i s t                  s y - %i s x + %i s t
       %i s f2(t) y %e                          s f1(t) y %e
(d115) ------------------------------------- + ------------------------------------
                        4 a11                                  4 a11

                          s y - %i s x + %i s t           s y - %i s x + %i s t
               %i f2(t) %e                      f1(t) %e
             + -------------------------------- + ------------------------------
                           4 a11                              2 a11

(c116) mim:ratsubst(-s,s,mim);

                                        %i s x                                  %i s x
(d116) - ((%i s f2(t) + s f1(t)) %e           y + (- %i f2(t) - 2 f1(t)) %e           )

                                                            - s y - %i s t
                                                          %e               /(4 a11)

(c117) mim:expand(mim);

                        - s y + %i s x - %i s t
        %i s f2(t) y %e
(d117) - -----------------------------------------
                           4 a11

                - s y + %i s x - %i s t                 - s y + %i s x - %i s t
   s f1(t) y %e                            %i f2(t) %e
 - ------------------------------------ + ------------------------------------
```

```
                 4 a11                                    4 a11

                 - s y + %i s x - %i s t
   f1(t) %e
 + ------------------------------
                 2 a11

(c118) forget(s<0);

(d118)                                [s < 0]

(c119) /* combine the two terms */

mi:mip+mim;

                            - s y + %i s x - %i s t
       %i s f2(t) y %e
(d119) - -------------------------------------
                                4 a11

                      - s y + %i s x - %i s t              - s y + %i s x - %i s t
   s f1(t) y %e                               %i f2(t) %e
 - ---------------------------------- + ---------------------------------
                      4 a11                                    4 a11

                 - s y + %i s x - %i s t                - s y - %i s x + %i s t
   f1(t) %e                              %i s f2(t) y %e
 + ------------------------------ + ------------------------------------
                 2 a11                                    4 a11

                      - s y - %i s x + %i s t              - s y - %i s x + %i s t
   s f1(t) y %e                               %i f2(t) %e
 - ---------------------------------- - ---------------------------------
                      4 a11                                    4 a11

                 - s y - %i s x + %i s t
   f1(t) %e
 + ------------------------------
                 2 a11

(c120) mi:subst(wp,%e^(-s*y-%i*s*x+%i*s*t),mi);

                            - s y + %i s x - %i s t
       %i s f2(t) y %e
(d120) - -------------------------------------
                                4 a11

                      - s y + %i s x - %i s t              - s y + %i s x - %i s t
   s f1(t) y %e                               %i f2(t) %e
 - ---------------------------------- + ---------------------------------
                      4 a11                                    4 a11

                 - s y + %i s x - %i s t
   f1(t) %e                              %i s f2(t) wp y   s f1(t) wp y
 + ------------------------------ + -------------- - ------------
                 2 a11                    4 a11            4 a11

   %i f2(t) wp    f1(t) wp
 - ----------- + --------
      4 a11        2 a11

(c121) mi:subst(wm,%e^(-s*y+%i*s*x-%i*s*t),mi);

       %i s f2(t) wp y   s f1(t) wp y   %i s f2(t) wm y   s f1(t) wm y
(d121) --------------- - ------------ - --------------- - ------------
            4 a11            4 a11          4 a11            4 a11
```

```
                                        %i f2(t) wp    f1(t) wp    %i f2(t) wm    f1(t) wm
                                      - ----------- + -------- + ----------- + --------
                                           4 a11        2 a11       4 a11        2 a11

(c122) mi:ratsubst(ywp,s*wp,mi);

(d122) - (y (- %i f2(t) ywp + f1(t) ywp + (%i s f2(t) + s f1(t)) wm)

             + (%i f2(t) - 2 f1(t)) wp + (- %i f2(t) - 2 f1(t)) wm)/(4 a11)

(c123) mi:ratsubst(ywm,s*wm,mi);

(d123) ((%i f2(t) - f1(t)) y ywp + y (- %i f2(t) ywm - f1(t) ywm)

              + (2 f1(t) - %i f2(t)) wp + (%i f2(t) + 2 f1(t)) wm)/(4 a11)

(c124) ywp:(1/((-y+%i*(t-x))^2));

                                   1
(d124)                     ----------------
                                          2
                           (%i (t - x) - y)

(c125) mi:mi;

(d125) ((%i f2(t) - f1(t)) y ywp + y (- %i f2(t) ywm - f1(t) ywm)

              + (2 f1(t) - %i f2(t)) wp + (%i f2(t) + 2 f1(t)) wm)/(4 a11)

(c126) ywm:(1/((-y-%i*(t-x))^2));

                                    1
(d126)                    ------------------
                                             2
                          (- y - %i (t - x))

(c127) mi:''mi;

        (%i f2(t) - f1(t)) y            %i f2(t)                f1(t)
(d127) (-------------------- + (- ------------------ - ------------------) y
                           2                        2                    2
         (%i (t - x) - y)         (- y - %i (t - x))    (- y - %i (t - x))

              + (2 f1(t) - %i f2(t)) wp + (%i f2(t) + 2 f1(t)) wm)/(4 a11)

(c128) wp:(-1/(-y+%i*(t-x)));

                                    1
(d128)                     - --------------
                             %i (t - x) - y

(c129) mi:mi;

        (%i f2(t) - f1(t)) y            %i f2(t)                f1(t)
(d129) (-------------------- + (- ------------------ - ------------------) y
                           2                        2                    2
         (%i (t - x) - y)         (- y - %i (t - x))    (- y - %i (t - x))

              + (2 f1(t) - %i f2(t)) wp + (%i f2(t) + 2 f1(t)) wm)/(4 a11)

(c130) wm:(-1/(-y-%i*(t-x)));

                                   1
(d130)                    - ----------------
                            - y - %i (t - x)
```

```
(c131) mi:''mi;

          (%i f2(t) - f1(t)) y                %i f2(t)                 f1(t)
(d131) (-------------------- + (- ------------------- - -------------------) y
                           2                        2                    2
          (%i (t - x) - y)       (- y - %i (t - x))    (- y - %i (t - x))

                                      2 f1(t) - %i f2(t)   %i f2(t) + 2 f1(t)
                                    - ------------------ - ------------------)/(4 all)
                                       %i (t - x) - y      - y - %i (t - x)

(c132) mi:mi/(2*%pi);

          (%i f2(t) - f1(t)) y                %i f2(t)                 f1(t)
(d132) (-------------------- + (- ------------------- - -------------------) y
                           2                        2                    2
          (%i (t - x) - y)       (- y - %i (t - x))    (- y - %i (t - x))

                                 2 f1(t) - %i f2(t)   %i f2(t) + 2 f1(t)
                               - ------------------ - ------------------)/(8 %pi all)
                                  %i (t - x) - y      - y - %i (t - x)

(c133) mi:subst(z,(t-x),mi);

                 %i f2(t)          f1(t)          2 f1(t) - %i f2(t)
(d133) (y (- ------------- - -------------) - ------------------
                          2                2
             (- %i z - y)    (- %i z - y)            %i z - y

                              (%i f2(t) - f1(t)) y   %i f2(t) + 2 f1(t)
                            + -------------------- - ------------------)/(8 %pi all)
                                               2
                                     (%i z - y)         - %i z - y

(c134) mi:distrib(mi);

/usr/local/macsyma_309/mrg/scs.o being loaded.

           %i f2(t)          f1(t)
       y (- ------------- - -------------)
                         2               2
           (- %i z - y)    (- %i z - y)       2 f1(t) - %i f2(t)
(d134) ------------------------------------------ - --------------------
                   8 %pi all                    8 %pi all (%i z - y)

                                  (%i f2(t) - f1(t)) y     %i f2(t) + 2 f1(t)
                                + -------------------- - ----------------------
                                                    2    8 %pi all (- %i z - y)
                                  8 %pi all (%i z - y)

(c135) ki:first(mi);

                                  %i f2(t)          f1(t)
                             y (- ------------- - -------------)
                                               2               2
                                  (- %i z - y)    (- %i z - y)
(d135)                       ----------------------------------
                                         8 %pi all

(c136) ki1:rest(mi,2);

                     (%i f2(t) - f1(t)) y     %i f2(t) + 2 f1(t)
(d136)               -------------------- - ----------------------
                                       2    8 %pi all (- %i z - y)
                     8 %pi all (%i z - y)
```

```
(c137) ki:ki+first(ki1);

                   %i f2(t)             f1(t)
          y (- ------------ - -------------)
                             2                 2
                (- %i z - y)    (- %i z - y)     (%i f2(t) - f1(t)) y
(d137)    ---------------------------------- + --------------------
                       8 %pi a11                                   2
                                               8 %pi a11 (%i z - y)

(c138) ki:xthru(ki);

                                            2                                        2
       (- %i f2(t) - f1(t)) y (%i z - y)  + (%i f2(t) - f1(t)) y (- %i z - y)
(d138) ----------------------------------------------------------------------
                                           2            2
                      8 %pi a11 (- %i z - y)  (%i z - y)

(c139) ki:expand(ki);

                                          2
                             2 f1(t) y z
(d139) ----------------------------------------------
                     4                2  2              4
       8 %pi a11 z  + 16 %pi a11 y  z  + 8 %pi a11 y

                                2
                        4 f2(t) y  z
  - ----------------------------------------------
                   4                2  2              4
     8 %pi a11 z  + 16 %pi a11 y  z  + 8 %pi a11 y

                                  3
                         2 f1(t) y
  - ----------------------------------------------
                   4                2  2              4
     8 %pi a11 z  + 16 %pi a11 y  z  + 8 %pi a11 y

(c140) ki:factor(ki);

                                    2                                 2
                      y (f1(t) z  - 2 f2(t) y z - f1(t) y )
(d140)                ---------------------------------------
                                                  2     2 2
                                   4 %pi a11 (z  + y )

(c141) kj:rest(mi,1);

          2 f1(t) - %i f2(t)      (%i f2(t) - f1(t)) y        %i f2(t) + 2 f1(t)
(d141) - -------------------- + -------------------- - ----------------------
          8 %pi a11 (%i z - y)                         2   8 %pi a11 (- %i z - y)
                                  8 %pi a11 (%i z - y)

(c142) kj:first(kj);

                                       2 f1(t) - %i f2(t)
(d142)                              - --------------------
                                       8 %pi a11 (%i z - y)

(c143) kj:kj+last(mi);

                   2 f1(t) - %i f2(t)      %i f2(t) + 2 f1(t)
(d143)          - -------------------- - ----------------------
                   8 %pi a11 (%i z - y)   8 %pi a11 (- %i z - y)
```

```
(c144) kj:xthru(kj);

        (- %i f2(t) - 2 f1(t)) (%i z - y) + (%i f2(t) - 2 f1(t)) (- %i z - y)
(d144) -----------------------------------------------------------------------
                         8 %pi a11 (- %i z - y) (%i z - y)

(c145) kj:expand(kj);

                 2 f2(t) z                         4 f1(t) y
(d145)    -------------------------- + --------------------------
                        2              2              2              2
           8 %pi a11 z  + 8 %pi a11 y    8 %pi a11 z  + 8 %pi a11 y

(c146) kj:factor(kj);

                                  f2(t) z + 2 f1(t) y
(d146)                            -------------------
                                            2    2
                                  4 %pi a11 (z  + y )

(c147) mi:ki+kj;

                    2                        2
         y (f1(t) z  - 2 f2(t) y z - f1(t) y )    f2(t) z + 2 f1(t) y
(d147)   ------------------------------------- + -------------------
                        2    2 2                            2    2
               4 %pi a11 (z  + y )               4 %pi a11 (z  + y )

(c148) mi:subst((t-x),z,mi);

                    2                                         2
       y (- f1(t) y  - 2 f2(t) (t - x) y + f1(t) (t - x) )
(d148) ---------------------------------------------------
                                2          2 2
                   4 %pi a11 (y  + (t - x) )

                                                    2 f1(t) y + f2(t) (t - x)
                                                  + -------------------------
                                                                2          2
                                                    4 %pi a11 (y  + (t - x) )

(c149) /* Get the answer of gsxxp expressed by the auxiliary function */
gsxxp:'integrate(mi,t,-a,a);

        a
       /                 2                                        2
       [     y (- f1(t) y  - 2 f2(t) (t - x) y + f1(t) (t - x) )
(d149) I     (---------------------------------------------------
       ]                             2          2 2
       /                4 %pi a11 (y  + (t - x) )
        - a

                                                 2 f1(t) y + f2(t) (t - x)
                                               + -------------------------) dt
                                                             2          2
                                                 4 %pi a11 (y  + (t - x) )

(c150) ki:ki*4*%pi*a11;

                               2                        2
                    y (f1(t) z  - 2 f2(t) y z - f1(t) y )
(d150)              -------------------------------------
                                2    2 2
                              (z  + y )
```

```
(c151) kj:kj*4*%pi*all;

                                   f2(t) z + 2 f1(t) y
(d151)                             -------------------
                                          2    2
                                         z  + y

(c152) mi:ki+kj;

                    2                           2
         y (f1(t) z  - 2 f2(t) y z - f1(t) y )    f2(t) z + 2 f1(t) y
(d152)   -------------------------------------- + -------------------
                       2    2 2                          2    2
                     (z  + y )                          z  + y

(c153) gsxxpf:subst((t-x),z,mi);

                  2                                       2
       y (- f1(t) y  - 2 f2(t) (t - x) y + f1(t) (t - x) )
(d153) ----------------------------------------------------
                          2          2 2
                        (y  + (t - x) )

                                                       2 f1(t) y + f2(t) (t - x)
                                                     + -------------------------
                                                            2          2
                                                           y  + (t - x)

(c154) /* calculate the gsyyp and make it simplified */

mwp:''gsyyp;

                     %i s t           %i s t                 %i s t
           2    %i %e        f2(t)  %e       f1(t)      %i %e        f2(t)
(d154) - s  ((------------------ - ---------------) y + ------------------)
                   4 all s           4 all abs(s)        4 all s abs(s)

                                                          - abs(s) y - %i s x
                                                        %e

(c155) mwp:expand(mwp);

                               - abs(s) y - %i s x + %i s t
          %i s f2(t) y %e
(d155) - ------------------------------------------
                            4 all

    2           - abs(s) y - %i s x + %i s t
   s  f1(t) y %e
 + ------------------------------------------
                4 all abs(s)

                - abs(s) y - %i s x + %i s t
   %i s f2(t) %e
 - ------------------------------------------
                4 all abs(s)

(c156) /* calculate |s| when s>0 */

assume(s>0);

(d156)                             [s > 0]

(c157) mip:''mwp;

                          - s y - %i s x + %i s t
```

```
                                                    
        %i s f2(t) y %e
(d157) - ----------------------------------------
                          4 a11

                         - s y - %i s x + %i s t                          - s y - %i s x + %i s t
        s f1(t) y %e                                %i f2(t) %e
     + ------------------------------------- - ------------------------------------
                      4 a11                                       4 a11

(c158) mip:expand(mip);

                              - s y - %i s x + %i s t
        %i s f2(t) y %e
(d158) - ----------------------------------------
                          4 a11

                         - s y - %i s x + %i s t                          - s y - %i s x + %i s t
        s f1(t) y %e                                %i f2(t) %e
     + ------------------------------------- - ------------------------------------
                      4 a11                                       4 a11

(c159) forget(s>0);

(d159)                               [s > 0]

(c160) /* calculate |s| when s<0 */
assume(s<0);

(d160)                               [s < 0]

(c161) mim:''mwp;

                              s y - %i s x + %i s t
        %i s f2(t) y %e
(d161) - -------------------------------------
                         4 a11

                           s y - %i s x + %i s t                    s y - %i s x + %i s t
          s f1(t) y %e                             %i f2(t) %e
        - ----------------------------------- + ---------------------------------
                         4 a11                                 4 a11

(c162) mim:ratsubst(-s,s,mim);

                                       %i s x                    %i s x     - s y - %i s t
        ((%i s f2(t) + s f1(t)) %e             y + %i f2(t) %e           ) %e
(d162) ---------------------------------------------------------------------------
                                         4 a11

(c163) mim:expand(mim);

                             - s y + %i s x - %i s t
        %i s f2(t) y %e
(d163) ----------------------------------------
                        4 a11

                        - s y + %i s x - %i s t                         - s y + %i s x - %i s t
        s f1(t) y %e                               %i f2(t) %e
     + ------------------------------------- + ------------------------------------
                      4 a11                                      4 a11

(c164) forget(s<0);

(d164)                               [s < 0]
```

```
(c165) /* combine the two terms */

mi:mip+mim;

                                 - s y + %i s x - %i s t
       %i s f2(t) y %e
(d165) ----------------------------------------
                          4 a11

                          - s y + %i s x - %i s t                - s y + %i s x - %i s t
   s f1(t) y %e                                      %i f2(t) %e
 + ------------------------------------- + ------------------------------------
                     4 a11                                 4 a11

                             - s y - %i s x + %i s t                 - s y - %i s x + %i s t
   %i s f2(t) y %e                                    s f1(t) y %e
 - ---------------------------------------- + -------------------------------------
                      4 a11                                   4 a11

                        - s y - %i s x + %i s t
   %i f2(t) %e
 - ------------------------------------
                     4 a11

(c166) mi:subst(wwp,%e^(-s*y-%i*s*x+%i*s*t),mi);

                                 - s y + %i s x - %i s t
       %i s f2(t) y %e
(d166) ----------------------------------------
                          4 a11

                          - s y + %i s x - %i s t                - s y + %i s x - %i s t
   s f1(t) y %e                                      %i f2(t) %e
 + ------------------------------------- + ------------------------------------
                     4 a11                                 4 a11

   %i s f2(t) wwp y   s f1(t) wwp y   %i f2(t) wwp
 - ---------------- + ------------- - ------------
        4 a11             4 a11          4 a11

(c167) mi:subst(wwm,%e^(-s*y+%i*s*x-%i*s*t),mi);

         %i s f2(t) wwp y   s f1(t) wwp y   %i s f2(t) wwm y   s f1(t) wwm y
(d167) - ---------------- + ------------- + ---------------- + -------------
              4 a11             4 a11            4 a11              4 a11

                                                              %i f2(t) wwp   %i f2(t) wwm
                                                            - ------------ + ------------
                                                                 4 a11          4 a11

(c168) mi:ratsubst(ywwp,s*wwp,mi);

(d168) (y (- %i f2(t) ywwp + f1(t) ywwp + (%i s f2(t) + s f1(t)) wwm)

                                              - %i f2(t) wwp + %i f2(t) wwm)/(4 a11)

(c169) mi:ratsubst(ywwm,s*wwm,mi);

(d169) - ((%i f2(t) - f1(t)) y ywwp + y (- %i f2(t) ywwm - f1(t) ywwm)

                                              + %i f2(t) wwp - %i f2(t) wwm)/(4 a11)

(c170) ywwp:(1/((-y+%i*(t-x))^2));

                                  1
(d170)                     ----------------
```

```
                                              2
                             (%i (t - x) - y)

(c171) mi:mi;

(d171) - ((%i f2(t) - f1(t)) y ywwp + y (- %i f2(t) ywwm - f1(t) ywwm)

                                     + %i f2(t) wwp - %i f2(t) wwm)/(4 a11)

(c172) ywwm:(1/((-y-%i*(t-x))^2));

                                   1
(d172)                     ------------------
                                           2
                           (- y - %i (t - x))

(c173) mi:''mi;

           (%i f2(t) - f1(t)) y           %i f2(t)                 f1(t)
(d173) - (-------------------- + (- ------------------ - ------------------)
                           2                        2                    2
           (%i (t - x) - y)         (- y - %i (t - x))   (- y - %i (t - x))

                                      y + %i f2(t) wwp - %i f2(t) wwm)/(4 a11)

(c174) wwp:(-1/(-y+%i*(t-x)));

                                    1
(d174)                     - --------------
                              %i (t - x) - y

(c175) mi:mi;

           (%i f2(t) - f1(t)) y           %i f2(t)                 f1(t)
(d175) - (-------------------- + (- ------------------ - ------------------)
                           2                        2                    2
           (%i (t - x) - y)         (- y - %i (t - x))   (- y - %i (t - x))

                                      y + %i f2(t) wwp - %i f2(t) wwm)/(4 a11)

(c176) wwm:(-1/(-y-%i*(t-x)));

                                    1
(d176)                    - ----------------
                             - y - %i (t - x)

(c177) mi:''mi;

           (%i f2(t) - f1(t)) y           %i f2(t)                 f1(t)
(d177) - (-------------------- + (- ------------------ - ------------------)
                           2                        2                    2
           (%i (t - x) - y)         (- y - %i (t - x))   (- y - %i (t - x))

                                  %i f2(t)           %i f2(t)
                           y - -------------- + ----------------)/(4 a11)
                               %i (t - x) - y   - y - %i (t - x)

(c178) mi:mi/(2*%pi);

           (%i f2(t) - f1(t)) y           %i f2(t)                 f1(t)
(d178) - (-------------------- + (- ------------------ - ------------------)
                           2                        2                    2
           (%i (t - x) - y)         (- y - %i (t - x))   (- y - %i (t - x))

                              %i f2(t)           %i f2(t)
                       y - -------------- + ----------------)/(8 %pi a11)
```

```
                                 %i (t - x) - y    - y - %i (t - x)

(c179) mi:subst(z,(t-x),mi);

                    %i f2(t)            f1(t)          %i f2(t)    (%i f2(t) - f1(t)) y
(d179) - (y (- ------------- - -------------) - -------- + --------------------
                             2                2   %i z - y                      2
               (- %i z - y)     (- %i z - y)                        (%i z - y)

                                                                     %i f2(t)
                                                             + ----------)/(8 %pi a11)
                                                               - %i z - y

(c180) mi:distrib(mi);

                   %i f2(t)            f1(t)
           y (- ------------- - -------------)
                              2               2
                (- %i z - y)     (- %i z - y)              %i f2(t)
(d180) - ---------------------------------- + --------------------
                        8 %pi a11                 8 %pi a11 (%i z - y)

                                   (%i f2(t) - f1(t)) y           %i f2(t)
                                 - -------------------- - ----------------------
                                                        2   8 %pi a11 (- %i z - y)
                                    8 %pi a11 (%i z - y)

(c181) ki:first(mi);

                                    %i f2(t)            f1(t)
                            y (- ------------- - -------------)
                                               2               2
                                 (- %i z - y)     (- %i z - y)
(d181)                    - ----------------------------------
                                         8 %pi a11

(c182) ki1:rest(mi,2);

                 (%i f2(t) - f1(t)) y           %i f2(t)
(d182)         - -------------------- - ----------------------
                                      2   8 %pi a11 (- %i z - y)
                  8 %pi a11 (%i z - y)

(c183) ki:ki+first(ki1);

                 %i f2(t)            f1(t)
         y (- ------------- - -------------)
                            2               2
              (- %i z - y)     (- %i z - y)      (%i f2(t) - f1(t)) y
(d183)   - ---------------------------------- - --------------------
                      8 %pi a11                                     2
                                                 8 %pi a11 (%i z - y)

(c184) ki:xthru(ki);

(d184)

                                                2                                        2
      - (- %i f2(t) - f1(t)) y (%i z - y)  - (%i f2(t) - f1(t)) y (- %i z - y)
      ---------------------------------------------------------------------------
                                                  2            2
                         8 %pi a11 (- %i z - y)  (%i z - y)

(c185) ki:expand(ki);
```

(d185)
$$-\frac{2\,f1(t)\,y\,z}{8\,\%pi\,a11\,z^{4}+16\,\%pi\,a11\,y^{2}\,z^{2}+8\,\%pi\,a11\,y^{4}}$$
$$+\frac{4\,f2(t)\,y^{2}\,z}{8\,\%pi\,a11\,z^{4}+16\,\%pi\,a11\,y^{2}\,z^{2}+8\,\%pi\,a11\,y^{4}}$$
$$+\frac{2\,f1(t)\,y^{3}}{8\,\%pi\,a11\,z^{4}+16\,\%pi\,a11\,y^{2}\,z^{2}+8\,\%pi\,a11\,y^{4}}$$

(c186) ki:factor(ki);

(d186)
$$-\frac{y\,(f1(t)\,z^{2}-2\,f2(t)\,y\,z-f1(t)\,y^{2})}{4\,\%pi\,a11\,(z^{2}+y^{2})^{2}}$$

(c187) kj:rest(mi,1);

(d187)
$$\frac{\%i\,f2(t)}{8\,\%pi\,a11\,(\%i\,z-y)}-\frac{(\%i\,f2(t)-f1(t))\,y}{8\,\%pi\,a11\,(\%i\,z-y)^{2}}-\frac{\%i\,f2(t)}{8\,\%pi\,a11\,(-\%i\,z-y)}$$

(c188) kj:first(kj);

(d188)
$$\frac{\%i\,f2(t)}{8\,\%pi\,a11\,(\%i\,z-y)}$$

(c189) kj:kj+last(mi);

(d189)
$$\frac{\%i\,f2(t)}{8\,\%pi\,a11\,(\%i\,z-y)}-\frac{\%i\,f2(t)}{8\,\%pi\,a11\,(-\%i\,z-y)}$$

(c190) kj:xthru(kj);

(d190)
$$\frac{\%i\,f2(t)\,(-\%i\,z-y)-\%i\,f2(t)\,(\%i\,z-y)}{8\,\%pi\,a11\,(-\%i\,z-y)\,(\%i\,z-y)}$$

(c191) kj:expand(kj);

(d191)
$$\frac{f2(t)\,z}{4\,\%pi\,a11\,z^{2}+4\,\%pi\,a11\,y^{2}}$$

(c192) kj:factor(kj);

(d192)
$$\frac{f2(t)\,z}{4\,\%pi\,a11\,(z^{2}+y^{2})}$$

(c193) mi:ki+kj;

(d193) $$\frac{f2(t)\ z}{4\ \%pi\ al1\ (z^2 + y^2)} - \frac{y\ (f1(t)\ z^2 - 2\ f2(t)\ y\ z - f1(t)\ y^2)}{4\ \%pi\ al1\ (z^2 + y^2)^2}$$

```
(c194) mi:subst((t-x),z,mi);
```

(d194) $$\frac{f2(t)\ (t - x)}{4\ \%pi\ al1\ (y^2 + (t - x)^2)} - \frac{y\ (- f1(t)\ y^2 - 2\ f2(t)\ (t - x)\ y + f1(t)\ (t - x)^2)}{4\ \%pi\ al1\ (y^2 + (t - x)^2)^2}$$

```
(c195) /* Get the answer of gsyyp expressed by the auxiliary function */
gsyyp:'integrate(mi,t,-a,a);
```

(d195) $$\int_{-a}^{a} \left(\frac{f2(t)\ (t - x)}{4\ \%pi\ al1\ (y^2 + (t - x)^2)} - \frac{y\ (- f1(t)\ y^2 - 2\ f2(t)\ (t - x)\ y + f1(t)\ (t - x)^2)}{4\ \%pi\ al1\ (y^2 + (t - x)^2)^2}\right) dt$$

```
(c196) ki:ki*4*%pi*al1;
```

(d196) $$- \frac{y\ (f1(t)\ z^2 - 2\ f2(t)\ y\ z - f1(t)\ y^2)}{(z^2 + y^2)^2}$$

```
(c197) kj:kj*4*%pi*al1;
```

(d197) $$\frac{f2(t)\ z}{z^2 + y^2}$$

```
(c198) mi:ki+kj;
```

(d198) $$\frac{f2(t)\ z}{z^2 + y^2} - \frac{y\ (f1(t)\ z^2 - 2\ f2(t)\ y\ z - f1(t)\ y^2)}{(z^2 + y^2)^2}$$

```
(c199) gsyypf:subst((t-x),z,mi);
```

(d199) $$\frac{f2(t)\ (t - x)}{{}^{2} \qquad {}^{2}} - \frac{y\ (- f1(t)\ y^2 - 2\ f2(t)\ (t - x)\ y + f1(t)\ (t - x)^2)}{{}^{2} \qquad {}^{2\ 2}}$$

```
      y  + (t - x)                        (y  + (t - x) )

(c200) /* calculate the gtxyp and make it simplified */

mwp:''gtxyp;

                                 %i s t            %i s t                 %i s t
                             %i %e        f2(t)   %e         f1(t)     %i %e        f2(t)
(d200) %i s abs(s) ((----------------- - --------------) y + -----------------)
                                4 all s          4 all abs(s)         4 all s abs(s)

                                             %i s t           %i s t
   - abs(s) y - %i s x                 %i %e        f2(t)   %e         f1(t)
 %e                        - %i s (----------------- - --------------)
                                          4 all s           4 all abs(s)

   - abs(s) y - %i s x
 %e

(c201) mwp:expand(mwp);

                                  - abs(s) y - %i s x + %i s t
         abs(s) f2(t) y %e
(d201) - ----------------------------------------------
                                   4 all

                            - abs(s) y - %i s x + %i s t
   %i s f1(t) y %e
 - -------------------------------------------
                              4 all

                         - abs(s) y - %i s x + %i s t
   %i s f1(t) %e
 + ----------------------------------------
                        4 all abs(s)

(c202) /* calculate |s| when s>0 */

assume(s>0);

(d202)                              [s > 0]

(c203) mip:''mwp;

                              - s y - %i s x + %i s t
         s f2(t) y %e
(d203) - -----------------------------------
                                4 all

                           - s y - %i s x + %i s t                     - s y - %i s x + %i s t
   %i s f1(t) y %e                               %i f1(t) %e
 - ------------------------------------- + ----------------------------------
                        4 all                                    4 all

(c204) mip:expand(mip);

                              - s y - %i s x + %i s t
         s f2(t) y %e
(d204) - -----------------------------------
                                4 all

                           - s y - %i s x + %i s t                     - s y - %i s x + %i s t
   %i s f1(t) y %e                               %i f1(t) %e
 - ------------------------------------- + ----------------------------------
                        4 all                                    4 all
```

```
(c205) forget(s>0);

(d205)                           [s > 0]

(c206) /* calculate |s| when s<0 */

assume(s<0);

(d206)                           [s < 0]

(c207) mim:''mwp;

                     s y - %i s x + %i s t                      s y - %i s x + %i s t
         s f2(t) y %e                          %i s f1(t) y %e
(d207)   ---------------------------------- - ------------------------------------
                     4 a11                                    4 a11

                                                                s y - %i s x + %i s t
                                                    %i f1(t) %e
                                                  - ---------------------------------
                                                                4 a11

(c208) mim:ratsubst(-s,s,mim);

(d208)

                                       %i s x                  %i s x    - s y - %i s t
       ((s f2(t) - %i s f1(t)) %e          y + %i f1(t) %e          ) %e
     - -------------------------------------------------------------------------
                                          4 a11

(c209) mim:expand(mim);

                       - s y + %i s x - %i s t
           s f2(t) y %e
(d209) - ------------------------------------
                       4 a11

                         - s y + %i s x - %i s t                - s y + %i s x - %i s t
       %i s f1(t) y %e                              %i f1(t) %e
     + ------------------------------------- - ----------------------------------
                       4 a11                                    4 a11

(c210) forget(s<0);

(d210)                           [s < 0]

(c211) /* combine the two terms */

mi:mip+mim;

                       - s y + %i s x - %i s t
           s f2(t) y %e
(d211) - ------------------------------------
                       4 a11

                         - s y + %i s x - %i s t                - s y + %i s x - %i s t
       %i s f1(t) y %e                              %i f1(t) %e
     + ------------------------------------- - ----------------------------------
                       4 a11                                    4 a11

                   - s y - %i s x + %i s t                    - s y - %i s x + %i s t
       s f2(t) y %e                           %i s f1(t) y %e
     - ---------------------------------- - ------------------------------------
                   4 a11                                      4 a11
```

```
                  - s y - %i s x + %i s t
   %i f1(t) %e
 + ---------------------------------
                  4 a11

(c212) mi:subst(twp,%e^(-s*y-%i*s*x+%i*s*t),mi);

                         - s y + %i s x - %i s t
         s f2(t) y %e
(d212) - ----------------------------------
                         4 a11

                 - s y + %i s x - %i s t                  - s y + %i s x - %i s t
   %i s f1(t) y %e                           %i f1(t) %e
 + ------------------------------------- - ---------------------------------
                  4 a11                                  4 a11

   s f2(t) twp y   %i s f1(t) twp y   %i f1(t) twp
 - ------------- - ---------------- + ------------
      4 a11             4 a11            4 a11

(c213) mi:subst(twm,%e^(-s*y+%i*s*x-%i*s*t),mi);

         s f2(t) twp y   %i s f1(t) twp y   s f2(t) twm y   %i s f1(t) twm y
(d213) - ------------- - ---------------- - ------------- + ----------------
            4 a11             4 a11             4 a11            4 a11

                                                         %i f1(t) twp   %i f1(t) twm
                                                       + ------------ - ------------
                                                            4 a11          4 a11

(c214) mi:ratsubst(twwp,s*twp,mi);

(d214) - ((f2(t) twwp + %i f1(t) twwp + (s f2(t) - %i s f1(t)) twm) y

                                            - %i f1(t) twp + %i f1(t) twm)/(4 a11)

(c215) mi:ratsubst(twwm,s*twm,mi);

(d215) - (((f2(t) + %i f1(t)) twwp + f2(t) twwm - %i f1(t) twwm) y

                                            - %i f1(t) twp + %i f1(t) twm)/(4 a11)

(c216) twwp:(1/((-y+%i*(t-x))^2));

                                    1
(d216)                      -----------------
                                            2
                            (%i (t - x) - y)

(c217) mi:mi;

(d217) - (((f2(t) + %i f1(t)) twwp + f2(t) twwm - %i f1(t) twwm) y

                                            - %i f1(t) twp + %i f1(t) twm)/(4 a11)

(c218) twwm:(1/((-y-%i*(t-x))^2));

                                    1
(d218)                     ------------------
                                             2
                           (- y - %i (t - x))

(c219) mi:''mi;

          f2(t) + %i f1(t)            f2(t)                  %i f1(t)
```

```
(d219) - ((------------------ + ------------------- - -------------------) y
                          2                       2                       2
           (%i (t - x) - y)     (- y - %i (t - x))    (- y - %i (t - x))

                                        - %i f1(t) twp + %i f1(t) twm)/(4 a11)

(c220) twp:(-1/(-y+%i*(t-x)));

                                         1
(d220)                           - --------------
                                   %i (t - x) - y

(c221) mi:mi;

           f2(t) + %i f1(t)            f2(t)                %i f1(t)
(d221) - ((------------------ + ------------------- - -------------------) y
                          2                       2                       2
           (%i (t - x) - y)     (- y - %i (t - x))    (- y - %i (t - x))

                                        - %i f1(t) twp + %i f1(t) twm)/(4 a11)

(c222) twm:(-1/(-y-%i*(t-x)));

                                         1
(d222)                          - ----------------
                                  - y - %i (t - x)

(c223) mi:''mi;

           f2(t) + %i f1(t)            f2(t)                %i f1(t)
(d223) - ((------------------ + ------------------- - -------------------) y
                          2                       2                       2
           (%i (t - x) - y)     (- y - %i (t - x))    (- y - %i (t - x))

                                      %i f1(t)          %i f1(t)
                                + -------------- - ----------------)/(4 a11)
                                  %i (t - x) - y   - y - %i (t - x)

(c224) mi:mi/(2*%pi);

           f2(t) + %i f1(t)            f2(t)                %i f1(t)
(d224) - ((------------------ + ------------------- - -------------------) y
                          2                       2                       2
           (%i (t - x) - y)     (- y - %i (t - x))    (- y - %i (t - x))

                                  %i f1(t)          %i f1(t)
                            + -------------- - ----------------)/(8 %pi a11)
                              %i (t - x) - y   - y - %i (t - x)

(c225) mi:subst(z,(t-x),mi);

(d225)

      f2(t) + %i f1(t)       f2(t)          %i f1(t)       %i f1(t)    %i f1(t)
    y (---------------- + ------------ - -------------) + -------- - ----------
                   2                2               2     %i z - y   - %i z - y
         (%i z - y)       (- %i z - y)   (- %i z - y)
- ---------------------------------------------------------------------------
                                  8 %pi a11

(c226) mi:distrib(mi);

      f2(t) + %i f1(t)       f2(t)         %i f1(t)
    y (---------------- + ------------ - -------------)
                   2                2               2
         (%i z - y)       (- %i z - y)   (- %i z - y)
```

```
(d226) - --------------------------------------------------------------
                                  8 %pi a11

                                        %i f1(t)                 %i f1(t)
                              - ------------------- + ----------------------
                                8 %pi a11 (%i z - y)   8 %pi a11 (- %i z - y)

(c227) ki:first(mi);

                  f2(t) + %i f1(t)        f2(t)          %i f1(t)
               y (----------------- + ------------- - -------------)
                              2                  2                2
                    (%i z - y)       (- %i z - y)     (- %i z - y)
(d227)      - -------------------------------------------------------------
                                       8 %pi a11

(c228) ki:xthru(ki);

                                            2                                     2
          y ((f2(t) - %i f1(t)) (%i z - y)  + (f2(t) + %i f1(t)) (- %i z - y) )
(d228) - ---------------------------------------------------------------------------
                                                2            2
                              8 %pi a11 (- %i z - y)  (%i z - y)

(c229) ki:expand(ki);

                                          2
                          2 f2(t) y z
(d229) -------------------------------------------------------
                      4                  2  2                  4
        8 %pi a11 z  + 16 %pi a11 y  z  + 8 %pi a11 y

                                    2
                          4 f1(t) y  z
  + -------------------------------------------------------
                      4                  2  2                  4
     8 %pi a11 z  + 16 %pi a11 y  z  + 8 %pi a11 y

                                    3
                           2 f2(t) y
  - -------------------------------------------------------
                      4                  2  2                  4
     8 %pi a11 z  + 16 %pi a11 y  z  + 8 %pi a11 y

(c230) ki:factor(ki);

                                  2                             2
                    y (f2(t) z  + 2 f1(t) y z - f2(t) y )
(d230)              ---------------------------------------------
                                         2    2 2
                            4 %pi a11 (z  + y )

(c231) kj:rest(mi,1);

                       %i f1(t)                  %i f1(t)
(d231)           ---------------------- - ---------------------
                 8 %pi a11 (- %i z - y)   8 %pi a11 (%i z - y)

(c232) kj:xthru(kj);

                  %i f1(t) (%i z - y) - %i f1(t) (- %i z - y)
(d232)            -------------------------------------------------
                        8 %pi a11 (- %i z - y) (%i z - y)

(c233) kj:expand(kj);
```

```
                                  f1(t) z
(d233)                  - --------------------------
                                     2              2
                          4 %pi a11 z  + 4 %pi a11 y

(c234) kj:factor(kj);

                                  f1(t) z
(d234)                     - -------------------
                                        2    2
                             4 %pi a11 (z  + y )

(c235) mi:ki+kj;

                        2                     2
     y (f2(t) z  + 2 f1(t) y z - f2(t) y )          f1(t) z
(d235) ------------------------------------ - -------------------
                             2    2 2                    2    2
             4 %pi a11 (z  + y )            4 %pi a11 (z  + y )

(c236) mi:subst((t-x),z,mi);

                        2                                       2
   y (- f2(t) y  + 2 f1(t) (t - x) y + f2(t) (t - x) )
(d236) -----------------------------------------------------
                              2            2 2
               4 %pi a11 (y  + (t - x) )

                                                  f1(t) (t - x)
                                         - -------------------------
                                                      2            2
                                           4 %pi a11 (y  + (t - x) )

(c237) /* Get the answer of gsyyp expressed by the auxiliary function */
gtxyp:'integrate(mi,t,-a,a);

        a
       /                  2                                       2
       [     y (- f2(t) y  + 2 f1(t) (t - x) y + f2(t) (t - x) )
(d237) I    (-----------------------------------------------------
       ]                             2            2 2
       /             4 %pi a11 (y  + (t - x) )
        - a

                                                f1(t) (t - x)
                                       - -------------------------) dt
                                                    2            2
                                         4 %pi a11 (y  + (t - x) )

(c238) ki:ki*4*%pi*a11;

                               2                     2
                   y (f2(t) z  + 2 f1(t) y z - f2(t) y )
(d238)             ------------------------------------
                                 2    2 2
                               (z  + y )

(c239) kj:kj*4*%pi*a11;

                                   f1(t) z
(d239)                           - -------
                                    2    2
                                   z  + y

(c240) mi:ki+kj;
```

```
                                2                              2
             y (f2(t) z  + 2 f1(t) y z - f2(t) y )    f1(t) z
(d240)       ------------------------------------- - -------
                          2    2 2                     2    2
                        (z  + y )                     z  + y

(c241) gsxypf:subst((t-x),z,mi);

                        2                                              2
       y (- f2(t) y  + 2 f1(t) (t - x) y + f2(t) (t - x) )   f1(t) (t - x)
(d241) -------------------------------------------------- - -------------
                            2          2 2                     2          2
                          (y  + (t - x) )                     y  + (t - x)

(c242) y:0;

(d242)                                0

(c243) ev(gtxyp);

                                a
                               /
                               [     f1(t)
                               I     ----- dt
                               ]     t - x
                               /
                                - a
(d243)                       - ------------
                                4 %pi all

(c244) kill(y);

(d244)                              done

(c245) /*************************************************************/
f1(t):=1;

(d245)                          f1(t) := 1

(c246) f2(t):=0;

(d246)                          f2(t) := 0

(c247) ''gsxxpf;

                                                    2     2
                          2 y             y ((t - x)  - y )
(d247)              ------------- + -----------------
                      2          2     2          2 2
                     y  + (t - x)    (y  + (t - x) )

(c248) subst(y1,y,%);

                                                     2      2
                         2 y1            y1 ((t - x)  - y1 )
(d248)             -------------- + ------------------
                     2          2      2          2 2
                   y1  + (t - x)    (y1  + (t - x) )

(c249) subst(x1,x,%);

                                                      2      2
                        2 y1             y1 ((t - x1)  - y1 )
(d249)            -------------- + -------------------
```

$$y1^2 + (t - x1)^2 \qquad (y1^2 + (t - x1)^2)^2$$

(c250) gsxxpf1t1:subst(t1,t,%);

(d250)
$$\frac{2\ y1}{y1^2 + (t1 - x1)^2} + \frac{y1\ ((t1 - x1)^2 - y1^2)}{(y1^2 + (t1 - x1)^2)^2}$$

(c251) ''gsxxpf;

(d251)
$$\frac{2\ y}{y^2 + (t - x)^2} + \frac{y\ ((t - x)^2 - y^2)}{(y^2 + (t - x)^2)^2}$$

(c252) subst(y2,y,%);

(d252)
$$\frac{2\ y2}{y2^2 + (t - x)^2} + \frac{y2\ ((t - x)^2 - y2^2)}{(y2^2 + (t - x)^2)^2}$$

(c253) subst(x2,x,%);

(d253)
$$\frac{2\ y2}{y2^2 + (t - x2)^2} + \frac{y2\ ((t - x2)^2 - y2^2)}{(y2^2 + (t - x2)^2)^2}$$

(c254) gsxxpf1t2:subst(t2,t,%);

(d254)
$$\frac{2\ y2}{y2^2 + (t2 - x2)^2} + \frac{y2\ ((t2 - x2)^2 - y2^2)}{(y2^2 + (t2 - x2)^2)^2}$$

(c255) kill(f1);

(d255) done

(c256) kill(f2);

(d256) done

(c257) f1(t):=0;

(d257) f1(t) := 0

(c258) f2(t):=1;

(d258) f2(t) := 1

(c259) ''gsxxpf;

(d259)
$$\frac{t - x}{y^2 + (t - x)^2} - \frac{2\ (t - x)\ y^2}{(y^2 + (t - x)^2)^2}$$

```
(c260) subst(y1,y,%);

                                                          2
                          t - x              2 (t - x) y1
(d260)               -------------- - ------------------
                       2          2     2          2 2
                     y1  + (t - x)    (y1  + (t - x) )

(c261) subst(x1,x,%);

                                                           2
                         t - x1             2 (t - x1) y1
(d261)              --------------- - -------------------
                      2           2     2           2 2
                    y1  + (t - x1)    (y1  + (t - x1) )

(c262) gsxxpf2t1:subst(t1,t,%);

                                                             2
                        t1 - x1             2 (t1 - x1) y1
(d262)             ---------------- - --------------------
                     2            2     2            2 2
                   y1  + (t1 - x1)    (y1  + (t1 - x1) )

(c263) ''gsxxpf;

                                                        2
                           t - x             2 (t - x) y
(d263)                ------------- - -----------------
                       2          2     2          2 2
                      y  + (t - x)    (y  + (t - x) )

(c264) subst(y2,y,%);

                                                          2
                          t - x              2 (t - x) y2
(d264)               -------------- - ------------------
                       2          2     2          2 2
                     y2  + (t - x)    (y2  + (t - x) )

(c265) subst(x2,x,%);

                                                           2
                         t - x2             2 (t - x2) y2
(d265)              --------------- - -------------------
                      2           2     2           2 2
                    y2  + (t - x2)    (y2  + (t - x2) )

(c266) gsxxpf2t2:subst(t2,t,%);

                                                             2
                        t2 - x2             2 (t2 - x2) y2
(d266)             ---------------- - --------------------
                     2            2     2            2 2
                   y2  + (t2 - x2)    (y2  + (t2 - x2) )

(c267) kill(f1);

(d267)                            done

(c268) kill(f2);

(d268)                            done

(c269) /************************************************************/
```

```
f1(t):=1;

(d269)                              f1(t) := 1

(c270) f2(t):=0;

(d270)                              f2(t) := 0

(c271) ''gsyypf;

                                            2     2
                                   y ((t - x)  - y )
(d271)                           - -----------------
                                      2          2 2
                                    (y  + (t - x) )

(c272) subst(y1,y,%);

                                             2      2
                                  y1 ((t - x)  - y1 )
(d272)                          - -------------------
                                       2          2 2
                                   (y1  + (t - x) )

(c273) subst(x1,x,%);

                                             2       2
                                 y1 ((t - x1)  - y1 )
(d273)                         - --------------------
                                      2           2 2
                                  (y1  + (t - x1) )

(c274) gsyypf1t1:subst(t1,t,%);

                                              2       2
                                 y1 ((t1 - x1)  - y1 )
(d274)                         - ---------------------
                                      2            2 2
                                  (y1  + (t1 - x1) )

(c275) ''gsyypf;

                                            2     2
                                   y ((t - x)  - y )
(d275)                           - -----------------
                                      2          2 2
                                    (y  + (t - x) )

(c276) subst(y2,y,%);

                                             2      2
                                  y2 ((t - x)  - y2 )
(d276)                          - -------------------
                                       2          2 2
                                   (y2  + (t - x) )

(c277) subst(x2,x,%);

                                             2       2
                                 y2 ((t - x2)  - y2 )
(d277)                         - --------------------
                                      2           2 2
                                  (y2  + (t - x2) )

(c278) gsyypf1t2:subst(t2,t,%);
```

(d278)
$$- \frac{y2\ ((t2 - x2)^2 - y2^2)}{(y2^2 + (t2 - x2)^2)^2}$$

(c279) kill(f1);

(d279) done

(c280) kill(f2);

(d280) done

(c281) f1(t):=0;

(d281) f1(t) := 0

(c282) f2(t):=1;

(d282) f2(t) := 1

(c283) ''gsyypf;

(d283)
$$\frac{t - x}{y^2 + (t - x)^2} + \frac{2\ (t - x)\ y^2}{(y^2 + (t - x)^2)^2}$$

(c284) subst(y1,y,%);

(d284)
$$\frac{t - x}{y1^2 + (t - x)^2} + \frac{2\ (t - x)\ y1^2}{(y1^2 + (t - x)^2)^2}$$

(c285) subst(x1,x,%);

(d285)
$$\frac{t - x1}{y1^2 + (t - x1)^2} + \frac{2\ (t - x1)\ y1^2}{(y1^2 + (t - x1)^2)^2}$$

(c286) gsyypf2t1:subst(t1,t,%);

(d286)
$$\frac{t1 - x1}{y1^2 + (t1 - x1)^2} + \frac{2\ (t1 - x1)\ y1^2}{(y1^2 + (t1 - x1)^2)^2}$$

(c287) ''gsyypf;

(d287)
$$\frac{t - x}{y^2 + (t - x)^2} + \frac{2\ (t - x)\ y^2}{(y^2 + (t - x)^2)^2}$$

(c288) subst(y2,y,%);

2

(d288)
$$\frac{t - x}{y2^2 + (t - x)^2} + \frac{2 (t - x) y2}{(y2^2 + (t - x)^2)^2}$$

```
(c289) subst(x2,x,%);
```

(d289)
$$\frac{t - x2}{y2^2 + (t - x2)^2} + \frac{2 (t - x2) y2^2}{(y2^2 + (t - x2)^2)^2}$$

```
(c290) gsyypf2t2:subst(t2,t,%);
```

(d290)
$$\frac{t2 - x2}{y2^2 + (t2 - x2)^2} + \frac{2 (t2 - x2) y2^2}{(y2^2 + (t2 - x2)^2)^2}$$

```
(c291) kill(f1);

(d291)                              done

(c292) kill(f2);

(d292)                              done

(c293) /***********************************************************/
f1(t):=1;

(d293)                          f1(t) := 1

(c294) f2(t):=0;

(d294)                          f2(t) := 0

(c295) ''gsxypf;
```

(d295)
$$\frac{2 (t - x) y^2}{(y^2 + (t - x)^2)^2} - \frac{t - x}{y^2 + (t - x)^2}$$

```
(c296) subst(y1,y,%);
```

(d296)
$$\frac{2 (t - x) y1^2}{(y1^2 + (t - x)^2)^2} - \frac{t - x}{y1^2 + (t - x)^2}$$

```
(c297) subst(x1,x,%);
```

(d297)
$$\frac{2 (t - x1) y1^2}{(y1^2 + (t - x1)^2)^2} - \frac{t - x1}{y1^2 + (t - x1)^2}$$

```
(c298) gsxypf1t1:subst(t1,t,%);
```

(d298) $$\frac{2\ (t1 - x1)\ y1}{(y1^{2} + (t1 - x1)^{2})^{2}} - \frac{t1 - x1}{y1^{2} + (t1 - x1)^{2}}$$

(c299) ''gsxypf;

(d299) $$\frac{2\ (t - x)\ y^{2}}{(y^{2} + (t - x)^{2})^{2}} - \frac{t - x}{y^{2} + (t - x)^{2}}$$

(c300) subst(y2,y,%);

(d300) $$\frac{2\ (t - x)\ y2^{2}}{(y2^{2} + (t - x)^{2})^{2}} - \frac{t - x}{y2^{2} + (t - x)^{2}}$$

(c301) subst(x2,x,%);

(d301) $$\frac{2\ (t - x2)\ y2^{2}}{(y2^{2} + (t - x2)^{2})^{2}} - \frac{t - x2}{y2^{2} + (t - x2)^{2}}$$

(c302) gsxypf1t2:subst(t2,t,%);

(d302) $$\frac{2\ (t2 - x2)\ y2^{2}}{(y2^{2} + (t2 - x2)^{2})^{2}} - \frac{t2 - x2}{y2^{2} + (t2 - x2)^{2}}$$

(c303) kill(f1);

(d303) done

(c304) kill(f2);

(d304) done

(c305) f1(t):=0;

(d305) f1(t) := 0

(c306) f2(t):=1;

(d306) f2(t) := 1

(c307) ''gsxypf;

(d307) $$\frac{y\ ((t - x)^{2} - y^{2})}{(y^{2} + (t - x)^{2})^{2}}$$

(c308) subst(y1,y,%);

(d308) $$\frac{y1\ ((t - x)^{2} - y1^{2})}{}$$

$$(y1^2 + (t - x)^2)^2$$

(c309) subst(x1,x,%);

(d309) $$\frac{y1\,((t - x1)^2 - y1^2)}{(y1^2 + (t - x1)^2)^2}$$

(c310) gsxypf2t1:subst(t1,t,%);

(d310) $$\frac{y1\,((t1 - x1)^2 - y1^2)}{(y1^2 + (t1 - x1)^2)^2}$$

(c311) ''gsxypf;

(d311) $$\frac{y\,((t - x)^2 - y^2)}{(y^2 + (t - x)^2)^2}$$

(c312) subst(y2,y,%);

(d312) $$\frac{y2\,((t - x)^2 - y2^2)}{(y2^2 + (t - x)^2)^2}$$

(c313) subst(x2,x,%);

(d313) $$\frac{y2\,((t - x2)^2 - y2^2)}{(y2^2 + (t - x2)^2)^2}$$

(c314) gsxypf2t2:subst(t2,t,%);

(d314) $$\frac{y2\,((t2 - x2)^2 - y2^2)}{(y2^2 + (t2 - x2)^2)^2}$$

(c315) kill(f1);

(d315) done

(c316) kill(f2);

(d316) done

(c317) /***** calculate kernel of two cracks **

kill(wwm);

(d317) done

(c318) x1:a1*gc;

```
(d318)                            a1 gc

(c319) t1:a1*gt;

(d319)                            a1 gt

(c320) t2:a2*gt;

(d320)                            a2 gt

(c321) gsx2:''gsxxpf1t2;

                                                            2      2
                   2 y2             y2 ((a2 gt - x2)  - y2 )
(d321)       ------------------ + -----------------------
               2               2        2                 2 2
             y2  + (a2 gt - x2)     (y2  + (a2 gt - x2) )

(c322) gsy2:''gsyypf1t2;

                                             2      2
                           y2 ((a2 gt - x2)  - y2 )
(d322)                   - -----------------------
                                2                 2 2
                            (y2  + (a2 gt - x2) )

(c323) gtxy2:''gsxypf1t2;

                                      2
               2 (a2 gt - x2) y2            a2 gt - x2
(d323)        ---------------------- - ------------------
                2                 2 2     2               2
              (y2  + (a2 gt - x2) )    y2  + (a2 gt - x2)

(c324) x2:p3+x1*cos(-gh)-y1*sin(-gh);

(d324)                   sin(gh) y1 + p3 + a1 gc cos(gh)

(c325) y2:p4+x1*sin(-gh)+y1*cos(-gh);

(d325)                   cos(gh) y1 + p4 - a1 gc sin(gh)

(c326) /* 1 * k(11,21) 2 to 1 term f21 gtxy *** c21f21xy *************************
gtxy21b:-('gsx2-'gsy2)/2*sin(2*(-gh))+'gtxy2*cos(2*(-gh));

                                        sin(2 gh) (gsx2 - gsy2)
(d326)               cos(2 gh) gtxy2 + -----------------------
                                                   2

(c327) gtxy21:-(''gsx2-''gsy2)/2*sin(2*(-gh))+''gtxy2*cos(2*(-gh));

                                                                 2
(d327) cos(2 gh) (2 (cos(gh) y1 + p4 - a1 gc sin(gh))

 (- sin(gh) y1 - p3 + a2 gt - a1 gc cos(gh))

                                                              2
/expt((- sin(gh) y1 - p3 + a2 gt - a1 gc cos(gh))

                                              2
 + (cos(gh) y1 + p4 - a1 gc sin(gh)) , 2)

 - (- sin(gh) y1 - p3 + a2 gt - a1 gc cos(gh))
```

```
                                                    2
/((- sin(gh) y1 - p3 + a2 gt - a1 gc cos(gh))

                                           2
 + (cos(gh) y1 + p4 - a1 gc sin(gh))  ))

 + sin(2 gh) (2 (cos(gh) y1 + p4 - a1 gc sin(gh))

                                                    2
/((- sin(gh) y1 - p3 + a2 gt - a1 gc cos(gh))

                                           2
 + (cos(gh) y1 + p4 - a1 gc sin(gh))  ) + 2 (cos(gh) y1 + p4 - a1 gc sin(gh))

                                                    2
 ((- sin(gh) y1 - p3 + a2 gt - a1 gc cos(gh))

                                           2
 - (cos(gh) y1 + p4 - a1 gc sin(gh))  )

                                                        2
/expt((- sin(gh) y1 - p3 + a2 gt - a1 gc cos(gh))

                                           2
 + (cos(gh) y1 + p4 - a1 gc sin(gh))  , 2))/2
(c328) y1:0;
(d328)                                0
(c329) ev(gtxy21);
                                        2 (p4 - a1 gc sin(gh))
(d329) sin(2 gh) (-------------------------------------------------------
                                     2                                  2
                  (p4 - a1 gc sin(gh))  + (- p3 + a2 gt - a1 gc cos(gh))

                                                                2
 + 2 (p4 - a1 gc sin(gh)) ((- p3 + a2 gt - a1 gc cos(gh))

                         2
 - (p4 - a1 gc sin(gh))  )

                          2                                  2 2
/((p4 - a1 gc sin(gh))  + (- p3 + a2 gt - a1 gc cos(gh))  ) )/2

                                                                          2
                      2 (- p3 + a2 gt - a1 gc cos(gh)) (p4 - a1 gc sin(gh))
 + cos(2 gh) (----------------------------------------------------------
                                  2                                  2 2
              ((p4 - a1 gc sin(gh))  + (- p3 + a2 gt - a1 gc cos(gh))  )

                 - p3 + a2 gt - a1 gc cos(gh)
 - -------------------------------------------------------)
                          2                                  2
   (p4 - a1 gc sin(gh))  + (- p3 + a2 gt - a1 gc cos(gh))
(c330) subst(p3a2a1,-p3+a2*gt-a1*gc*cos(gh),%);
                            2 (p4 - a1 gc sin(gh))
(d330) sin(2 gh) (-----------------------------
                                     2         2
                  (p4 - a1 gc sin(gh))  + p3a2a1

                                      2                        2
   2 (p4 - a1 gc sin(gh)) (p3a2a1  - (p4 - a1 gc sin(gh))  )
```

$$+ \frac{}{((p4 - a1\ gc\ \sin(gh))^2 + p3a2a1^2)^2})/2$$

$$+ \cos(2\ gh)\ (\frac{2\ p3a2a1\ (p4 - a1\ gc\ \sin(gh))^2}{((p4 - a1\ gc\ \sin(gh))^2 + p3a2a1^2)^2}$$

$$- \frac{p3a2a1}{(p4 - a1\ gc\ \sin(gh))^2 + p3a2a1^2})$$

```
(c331) subst(p4a1,p4-a1*gc*sin(gh),%);
```

$$(d331)\quad \frac{\sin(2\ gh)\ (\frac{2\ p4a1}{p4a1^2 + p3a2a1^2} + \frac{2\ p4a1\ (p3a2a1^2 - p4a1^2)}{(p4a1^2 + p3a2a1^2)^2})}{2}$$

$$+ \cos(2\ gh)\ (\frac{2\ p3a2a1\ p4a1^2}{(p4a1^2 + p3a2a1^2)^2} - \frac{p3a2a1}{p4a1^2 + p3a2a1^2})$$

```
(c332) subst(ww,p4a1^2+p3a2a1^2,%);
```

$$(d332)\quad \frac{\sin(2\ gh)\ (\frac{2\ p4a1}{ww} + \frac{2\ p4a1\ (p3a2a1^2 - p4a1^2)}{ww^2})}{2}$$

$$+ \cos(2\ gh)\ (\frac{2\ p3a2a1\ p4a1^2}{ww^2} - \frac{p3a2a1}{ww})$$

```
(c333) fn:subst(wwm,p3a2a1^2-p4a1^2,%);
```

$$(d333)\quad \frac{\sin(2\ gh)\ (\frac{2\ p4a1\ wwm}{ww^2} + \frac{2\ p4a1}{ww})}{2} + \cos(2\ gh)\ (\frac{2\ p3a2a1\ p4a1^2}{ww^2} - \frac{p3a2a1}{ww})$$

```
(c334) first(fn);
```

$$(d334)\quad \frac{\sin(2\ gh)\ (\frac{2\ p4a1\ wwm}{ww^2} + \frac{2\ p4a1}{ww})}{2}$$

```
(c335) n2:multthru(%);

                           sin(2 gh) p4a1 wwm   sin(2 gh) p4a1
(d335)                     ------------------ + --------------
                                   2                 ww
                                 ww

(c336) rest(fn,1);

                                                    2
                                  2 p3a2a1 p4a1    p3a2a1
(d336)                     cos(2 gh) (-------------- - ------)
                                           2            ww
                                         ww

(c337) n3:multthru(%);

                                                 2
                       2 cos(2 gh) p3a2a1 p4a1    cos(2 gh) p3a2a1
(d337)                 ------------------------ - ----------------
                                  2                      ww
                                ww

(c338) n:n2+n3;
       sin(2 gh) p4a1 wwm   sin(2 gh) p4a1   cos(2 gh) p3a2a1
(d338) ------------------ + -------------- - ----------------
               2                  ww                ww
             ww

                                                                   2
                                              2 cos(2 gh) p3a2a1 p4a1
                                            + ------------------------
                                                         2
                                                       ww

(c339) fn:combine(n);

                                                            2
       sin(2 gh) p4a1 wwm + 2 cos(2 gh) p3a2a1 p4a1
(d339) ---------------------------------------------
                              2
                            ww

                                    sin(2 gh) p4a1 - cos(2 gh) p3a2a1
                                  + ---------------------------------
                                                   ww

(c340) /* 1 * k(11,21) 2 to 1 term f21 gtxy *** c21f21xy *************************
k1121:(a2/(4*%pi*a11))*fn;

                                                            2
           sin(2 gh) p4a1 wwm + 2 cos(2 gh) p3a2a1 p4a1
(d340) a2 (---------------------------------------------
                                 2
                               ww

                         sin(2 gh) p4a1 - cos(2 gh) p3a2a1
                       + ---------------------------------)/(4 %pi a11)
                                        ww

(c341) /* where
p3a2a1=-p3+a2*gt-a1*gc*cos(gh)

p4a1=p4-a1*gc*sin(gh)
```

```
ww=p4a1^2+p3a2a1^2

wwm=p3a2a1^2-p4a1^2
*/

fortran(p3a2a1:-p3+a2*gt(j)-a1*gc(i)*dcos(gh))$
      p3a2a1 = -p3+a2*gt(j)-a1*dcos(gh)*gc(i)

(c342) fortran(p4a1:p4-a1*gc(i)*dsin(gh))$
      p4a1 = p4-a1*dsin(gh)*gc(i)

(c343) fortran(ww:'p4a1^2+'p3a2a1^2)$
      ww = p4a1**2+p3a2a1**2

(c344) fortran(wwm:'p3a2a1^2-'p4a1^2)$
      wwm = p3a2a1**2-p4a1**2

(c345) subst(dcos,cos,first(first(fn)))$

(c346) subst(dsin,sin,%)$

(c347) fortran(part1:%)$
      part1 = dsin(2*gh)*p4a1*wwm+2*dcos(2*gh)*p3a2a1*p4a1**2

(c348) fortran(part1:'part1/('ww^2))$
      part1 = part1/ww**2

(c349) subst(dcos,cos,last(fn))$

(c350) subst(dsin,sin,%)$

(c351) fortran(part2:%)$
      part2 = (dsin(2*gh)*p4a1-dcos(2*gh)*p3a2a1)/ww

(c352) kill(x1)$

(c353) kill(x2)$

(c354) kill(y1)$

(c355) kill(y2)$

(c356) kill(t1)$

(c357) kill(t2)$

(c358) kill(p3a2a1)$

(c359) kill(p4a1)$

(c360) kill(ww)$

(c361) kill(wwm)$

(c362) x1:a1*gc;

(d362)                                  a1 gc

(c363) t1:a1*gt;

(d363)                                  a1 gt

(c364) t2:a2*gt;

(d364)                                  a2 gt
```

```
(c365) gsx2:''gsxxpf2t2;

                                                       2
                  a2 gt - x2          2 (a2 gt - x2) y2
(d365)        ------------------ - ----------------------
                2              2     2               2 2
              y2  + (a2 gt - x2)   (y2  + (a2 gt - x2) )

(c366) gsy2:''gsyypf2t2;

                                                       2
                  a2 gt - x2          2 (a2 gt - x2) y2
(d366)        ------------------ + ----------------------
                2              2     2               2 2
              y2  + (a2 gt - x2)   (y2  + (a2 gt - x2) )

(c367) gtxy2:''gsxypf2t2;

                                            2     2
                             y2 ((a2 gt - x2)  - y2 )
(d367)                       ------------------------
                                2               2 2
                              (y2  + (a2 gt - x2) )

(c368) x2:p3+x1*cos(-gh)-y1*sin(-gh);

(d368)                  sin(gh) y1 + p3 + a1 gc cos(gh)

(c369) y2:p4+x1*sin(-gh)+y1*cos(-gh);

(d369)                  cos(gh) y1 + p4 - a1 gc sin(gh)

(c370) /* 2 * k(11,22) 2 to 1 term f22 gtxy *** c21f22xy ************************
gtxy21b:-('gsx2-'gsy2)/2*sin(2*(-gh))+'gtxy2*cos(2*(-gh));

                                          sin(2 gh) (gsx2 - gsy2)
(d370)              cos(2 gh) gtxy2 + -----------------------
                                                   2

(c371) gtxy21:-(''gsx2-''gsy2)/2*sin(2*(-gh))+''gtxy2*cos(2*(-gh));

(d371) cos(2 gh) (cos(gh) y1 + p4 - a1 gc sin(gh))

                                                  2
 ((- sin(gh) y1 - p3 + a2 gt - a1 gc cos(gh))

                                          2
 - (cos(gh) y1 + p4 - a1 gc sin(gh)) )

                                                      2
/expt((- sin(gh) y1 - p3 + a2 gt - a1 gc cos(gh))

                                          2
 + (cos(gh) y1 + p4 - a1 gc sin(gh)) , 2)

                                                     2
 - 2 sin(2 gh) (cos(gh) y1 + p4 - a1 gc sin(gh))

 (- sin(gh) y1 - p3 + a2 gt - a1 gc cos(gh))

                                                      2
/expt((- sin(gh) y1 - p3 + a2 gt - a1 gc cos(gh))

                                         2
```

```
 + (cos(gh) y1 + p4 - a1 gc sin(gh)) , 2)

(c372) y1:0;

(d372)                          0

(c373) ev(gtxy21);

                                                                      2
(d373) cos(2 gh) (p4 - a1 gc sin(gh)) ((- p3 + a2 gt - a1 gc cos(gh))

                          2
 - (p4 - a1 gc sin(gh))  )

                         2                                   2 2
/((p4 - a1 gc sin(gh))  + (- p3 + a2 gt - a1 gc cos(gh))  )

                                                                     2
   2 sin(2 gh) (- p3 + a2 gt - a1 gc cos(gh)) (p4 - a1 gc sin(gh))
 - -----------------------------------------------------------------
                             2                                 2 2
      ((p4 - a1 gc sin(gh))  + (- p3 + a2 gt - a1 gc cos(gh))  )

(c374) subst(p3a2a1,-p3+a2*gt-a1*gc*cos(gh),%);

                                               2                        2
       cos(2 gh) (p4 - a1 gc sin(gh)) (p3a2a1  - (p4 - a1 gc sin(gh))  )
(d374) ------------------------------------------------------------------
                                           2          2 2
                       ((p4 - a1 gc sin(gh))  + p3a2a1  )

                                                                          2
                                        2 sin(2 gh) p3a2a1 (p4 - a1 gc sin(gh))
                                      - ---------------------------------------
                                                          2           2 2
                                          ((p4 - a1 gc sin(gh))  + p3a2a1  )

(c375) subst(p4a1,p4-a1*gc*sin(gh),%);

                                  2       2                               2
          cos(2 gh) p4a1 (p3a2a1  - p4a1 )   2 sin(2 gh) p3a2a1 p4a1
(d375)    -------------------------------- - ------------------------
                      2          2 2                   2          2 2
                 (p4a1  + p3a2a1  )               (p4a1  + p3a2a1  )

(c376) subst(ww,p4a1^2+p3a2a1^2,%);

                                  2       2                               2
          cos(2 gh) p4a1 (p3a2a1  - p4a1 )   2 sin(2 gh) p3a2a1 p4a1
(d376)    -------------------------------- - ------------------------
                          2                            2
                        ww                           ww

(c377) fn:subst(wwm,p3a2a1^2-p4a1^2,%);

                                                         2
                cos(2 gh) p4a1 wwm   2 sin(2 gh) p3a2a1 p4a1
(d377)          ------------------ - ------------------------
                          2                      2
                        ww                     ww

(c378) fn:combine(fn);

                                                         2
                cos(2 gh) p4a1 wwm - 2 sin(2 gh) p3a2a1 p4a1
(d378)          ---------------------------------------------
```

```
                                   2
                                 ww

(c379) /* 2 * k(11,22) 2 to 1 term f22 gtxy *** c21f22xy *************************

k1122:(a2/(4*%pi*a11))*fn;

                                                                            2
               a2 (cos(2 gh) p4a1 wwm - 2 sin(2 gh) p3a2a1 p4a1 )
(d379)         ---------------------------------------------------
                                                      2
                                         4 %pi a11 ww

(c380) /* where
p3a2a1=-p3+a2*gt-a1*gc*cos(gh)

p4a1=p4-a1*gc*sin(gh)

ww=p4a1^2+p3a2a1^2

wwm=p3a2a1^2-p4a1^2
*/

length(fn);

(d380)                                2

(c381) fortran(p3a2a1:-p3+a2*gt(j)-a1*gc(i)*dcos(gh))$
      p3a2a1 = -p3+a2*gt(j)-a1*dcos(gh)*gc(i)

(c382) fortran(p4a1:p4-a1*gc(i)*dsin(gh))$
      p4a1 = p4-a1*dsin(gh)*gc(i)

(c383) fortran(ww:'p4a1^2+'p3a2a1^2)$
      ww = p4a1**2+p3a2a1**2

(c384) fortran(wwm:'p3a2a1^2-'p4a1^2)$
      wwm = p3a2a1**2-p4a1**2

(c385) subst(dcos,cos,first(fn))$

(c386) subst(dsin,sin,%)$

(c387) fortran(part1:%)$
      part1 = dcos(2*gh)*p4a1*wwm-2*dsin(2*gh)*p3a2a1*p4a1**2

(c388) fortran(part1:'part1/('ww^2))$
      part1 = part1/ww**2

(c389) kill(x1)$

(c390) kill(x2)$

(c391) kill(y1)$

(c392) kill(y2)$

(c393) kill(t1)$

(c394) kill(t2)$

(c395) kill(p2a2a1)$

(c396) kill(p4a1)$

(c397) kill(ww)$
```

```
(c398) kill(wwm)$

(c399) x1:a1*gc;

(d399)                             a1 gc

(c400) t1:a1*gt;

(d400)                             a1 gt

(c401) t2:a2*gt;

(d401)                             a2 gt

(c402) gsx2:''gsxxpf1t2;

                                                       2     2
                        2 y2                y2 ((a2 gt - x2)  - y2 )
(d402)           ------------------ + ------------------------
                   2              2        2              2 2
                 y2  + (a2 gt - x2)     (y2  + (a2 gt - x2) )

(c403) gsy2:''gsyypf1t2;

                                         2     2
                            y2 ((a2 gt - x2)  - y2 )
(d403)                    - ------------------------
                                  2              2 2
                              (y2  + (a2 gt - x2) )

(c404) gtxy2:''gsxypf1t2;

                                     2
                   2 (a2 gt - x2) y2          a2 gt - x2
(d404)            ---------------------- - ------------------
                     2              2 2      2              2
                  (y2  + (a2 gt - x2) )    y2  + (a2 gt - x2)

(c405) x2:p3+x1*cos(-gh)-y1*sin(-gh);

(d405)                   sin(gh) y1 + p3 + a1 gc cos(gh)

(c406) y2:p4+x1*sin(-gh)+y1*cos(-gh);

(d406)                   cos(gh) y1 + p4 - a1 gc sin(gh)

(c407) /* 3 * k(12,21) 2 to 1 term f21 gsyy *** c21f21y ***************************
gsy21b:('gsx2+'gsy2)/2-('gsx2-'gsy2)/2*cos(2*(-gh))-'gtxy2*sin(2*(-gh));

                                  gsy2 + gsx2   cos(2 gh) (gsx2 - gsy2)
(d407)       sin(2 gh) gtxy2 + ----------- - -----------------------
                                       2                  2

(c408) gsy21:(''gsx2+''gsy2)/2-(''gsx2-''gsy2)/2*cos(2*(-gh))-''gtxy2*sin(2*(-gh));

                                                             2
(d408) sin(2 gh) (2 (cos(gh) y1 + p4 - a1 gc sin(gh))
 (- sin(gh) y1 - p3 + a2 gt - a1 gc cos(gh))

                                                    2
/expt((- sin(gh) y1 - p3 + a2 gt - a1 gc cos(gh))

                                       2
```

```
 + (cos(gh) y1 + p4 - a1 gc sin(gh)) , 2)
 - (- sin(gh) y1 - p3 + a2 gt - a1 gc cos(gh))
                                               2
/((- sin(gh) y1 - p3 + a2 gt - a1 gc cos(gh))
                                      2
 + (cos(gh) y1 + p4 - a1 gc sin(gh)) ))
 - cos(2 gh) (2 (cos(gh) y1 + p4 - a1 gc sin(gh))
                                               2
/((- sin(gh) y1 - p3 + a2 gt - a1 gc cos(gh))
                                      2
 + (cos(gh) y1 + p4 - a1 gc sin(gh)) ) + 2 (cos(gh) y1 + p4 - a1 gc sin(gh))
                                               2
 ((- sin(gh) y1 - p3 + a2 gt - a1 gc cos(gh))
                                      2
 - (cos(gh) y1 + p4 - a1 gc sin(gh)) )
                                                   2
/expt((- sin(gh) y1 - p3 + a2 gt - a1 gc cos(gh))
                                      2
 + (cos(gh) y1 + p4 - a1 gc sin(gh)) , 2))/2
 + (cos(gh) y1 + p4 - a1 gc sin(gh))
                                               2
/((- sin(gh) y1 - p3 + a2 gt - a1 gc cos(gh))
                                      2
 + (cos(gh) y1 + p4 - a1 gc sin(gh)) )
(c409) y1:0;
(d409)                                 0
(c410) ev(gsy21);
                                     2 (p4 - a1 gc sin(gh))
(d410) - cos(2 gh) (--------------------------------------------------------
                                             2                                2
                    (p4 - a1 gc sin(gh))  + (- p3 + a2 gt - a1 gc cos(gh))
                                                              2
 + 2 (p4 - a1 gc sin(gh)) ((- p3 + a2 gt - a1 gc cos(gh))
                         2
 - (p4 - a1 gc sin(gh)) )
                         2                                2 2
/((p4 - a1 gc sin(gh))  + (- p3 + a2 gt - a1 gc cos(gh)) ) )/2
                                                                   2
                 2 (- p3 + a2 gt - a1 gc cos(gh)) (p4 - a1 gc sin(gh))
 + sin(2 gh) (----------------------------------------------------------
                                      2                                2 2
              ((p4 - a1 gc sin(gh))  + (- p3 + a2 gt - a1 gc cos(gh)) )
                 - p3 + a2 gt - a1 gc cos(gh)
 - --------------------------------------------------)
```

```
                            2                              2
  (p4 - a1 gc sin(gh))  + (- p3 + a2 gt - a1 gc cos(gh))
                   p4 - a1 gc sin(gh)
+ --------------------------------------------------------
                            2                              2
  (p4 - a1 gc sin(gh))  + (- p3 + a2 gt - a1 gc cos(gh))

(c411) subst(p3a2a1,-p3+a2*gt-a1*gc*cos(gh),%);

(d411) - cos(2 gh)

                      2 (p4 - a1 gc sin(gh))
(--------------------------------------------------------------
                       2                                         2
  (p4 - a1 gc sin(gh))  + (- p3 + a2 gt(j) - a1 dcos(gh) gc(i))

                                                                  2
 + 2 (p4 - a1 gc sin(gh)) ((- p3 + a2 gt(j) - a1 dcos(gh) gc(i))

                        2
 - (p4 - a1 gc sin(gh))  )

                        2                                         2 2
/((p4 - a1 gc sin(gh))  + (- p3 + a2 gt(j) - a1 dcos(gh) gc(i))  ) )/2

                                                                                  2
                 2 (- p3 + a2 gt(j) - a1 dcos(gh) gc(i)) (p4 - a1 gc sin(gh))
 + sin(2 gh) (--------------------------------------------------------------------
                                     2                                         2 2
               ((p4 - a1 gc sin(gh))  + (- p3 + a2 gt(j) - a1 dcos(gh) gc(i))  )
                   - p3 + a2 gt(j) - a1 dcos(gh) gc(i)
 - --------------------------------------------------------------)
                          2                                         2
   (p4 - a1 gc sin(gh))  + (- p3 + a2 gt(j) - a1 dcos(gh) gc(i))
                       p4 - a1 gc sin(gh)
 + --------------------------------------------------------------
                          2                                         2
   (p4 - a1 gc sin(gh))  + (- p3 + a2 gt(j) - a1 dcos(gh) gc(i))

(c412) subst(p4a1,p4-a1*gc*sin(gh),%);

                                           2 p4a1
(d412) - cos(2 gh) (------------------------------------------------
                          2                                         2
                      p4a1  + (- p3 + a2 gt(j) - a1 dcos(gh) gc(i))

                                                          2        2
   2 p4a1 ((- p3 + a2 gt(j) - a1 dcos(gh) gc(i))  - p4a1  )
 + --------------------------------------------------------)/2
             2                                         2 2
      (p4a1  + (- p3 + a2 gt(j) - a1 dcos(gh) gc(i))  )

                                                               2
                 2 (- p3 + a2 gt(j) - a1 dcos(gh) gc(i)) p4a1
 + sin(2 gh) (---------------------------------------------------
                      2                                         2 2
               (p4a1  + (- p3 + a2 gt(j) - a1 dcos(gh) gc(i))  )
          - p3 + a2 gt(j) - a1 dcos(gh) gc(i)
 - ----------------------------------------------)
          2                                         2
   p4a1  + (- p3 + a2 gt(j) - a1 dcos(gh) gc(i))
```

```
                                  p4a1
 + ---------------------------------------------
            2                                  2
   p4a1  + (- p3 + a2 gt(j) - a1 dcos(gh) gc(i))

(c413) subst(ww,p4a1^2+p3a2a1^2,%);

(d413)

                                                                     2        2
  cos(2 gh)  (2 p4a1    2 p4a1 ((- p3 + a2 gt(j) - a1 dcos(gh) gc(i))  - p4a1 )
             (------ + ---------------------------------------------------------)
               ww                                        2
                                                       ww
- ---------------------------------------------------------------------------
                                      2

                                                              2
                   2 (- p3 + a2 gt(j) - a1 dcos(gh) gc(i)) p4a1
 + sin(2 gh) (--------------------------------------------
                                      2
                                    ww

   - p3 + a2 gt(j) - a1 dcos(gh) gc(i)      p4a1
 - -----------------------------------) + ----
                  ww                        ww

(c414) fn:subst(wwm,p3a2a1^2-p4a1^2,%);

                     2 p4a1 wwm    2 p4a1
          cos(2 gh)  (---------- + ------)
                          2          ww
                        ww
(d414) - -----------------------------
                         2

                                                              2
                   2 (- p3 + a2 gt(j) - a1 dcos(gh) gc(i)) p4a1
 + sin(2 gh) (--------------------------------------------
                                      2
                                    ww

   - p3 + a2 gt(j) - a1 dcos(gh) gc(i)      p4a1
 - -----------------------------------) + ----
                  ww                        ww

(c415) n1:last(fn);

                                  p4a1
(d415)                            ----
                                   ww

(c416) first(fn);

                                 2 p4a1 wwm    2 p4a1
                      cos(2 gh)  (---------- + ------)
                                      2          ww
                                    ww
(d416)               - -----------------------------
                                     2

(c417) n2:multthru(%);

                    cos(2 gh) p4a1 wwm   cos(2 gh) p4a1
(d417)            - ------------------ - --------------
                            2                  ww
```

```
                                  ww

(c418) rest(fn,1);

                                                                  2
                   2 (- p3 + a2 gt(j) - a1 dcos(gh) gc(i)) p4a1
(d418) sin(2 gh) (----------------------------------------------
                                         2
                                       ww

                                     - p3 + a2 gt(j) - a1 dcos(gh) gc(i)    p4a1
                                   - ---------------------------------) + ----
                                                    ww                     ww

(c419) first(%);

                                                                  2
                   2 (- p3 + a2 gt(j) - a1 dcos(gh) gc(i)) p4a1
(d419) sin(2 gh) (----------------------------------------------
                                         2
                                       ww

                                          - p3 + a2 gt(j) - a1 dcos(gh) gc(i)
                                        - ---------------------------------)
                                                         ww

(c420) n3:multthru(%);

                                                                       2
        2 sin(2 gh) (- p3 + a2 gt(j) - a1 dcos(gh) gc(i)) p4a1
(d420) -------------------------------------------------------
                                  2
                                ww

                                 sin(2 gh) (- p3 + a2 gt(j) - a1 dcos(gh) gc(i))
                               - -----------------------------------------------
                                                       ww

(c421) n:n1+n2+n3;

          cos(2 gh) p4a1 wwm   cos(2 gh) p4a1   p4a1
(d421) - ------------------ - -------------- + ----
                   2                ww          ww
                 ww

   sin(2 gh) (- p3 + a2 gt(j) - a1 dcos(gh) gc(i))
 - -----------------------------------------------
                         ww

                                                              2
   2 sin(2 gh) (- p3 + a2 gt(j) - a1 dcos(gh) gc(i)) p4a1
 + -------------------------------------------------------
                              2
                            ww

(c422) n:combine(n);

(d422)

                                                        2
2 sin(2 gh) (- p3 + a2 gt(j) - a1 dcos(gh) gc(i)) p4a1   - cos(2 gh) p4a1 wwm
------------------------------------------------------------------------------
                                      2
                                    ww

   - cos(2 gh) p4a1 + p4a1 - sin(2 gh) (- p3 + a2 gt(j) - a1 dcos(gh) gc(i))
```

```
+ -------------------------------------------------------------------------
                                   ww

(c423) t1:first(%);
(d423)
                                                                    2
  2 sin(2 gh) (- p3 + a2 gt(j) - a1 dcos(gh) gc(i)) p4a1  - cos(2 gh) p4a1 wwm
  ---------------------------------------------------------------------------
                                         2
                                       ww

(c424) last(n);
(d424)
     - cos(2 gh) p4a1 + p4a1 - sin(2 gh) (- p3 + a2 gt(j) - a1 dcos(gh) gc(i))
     ------------------------------------------------------------------------
                                    ww

(c425) t2:part(%,1);
(d425) - cos(2 gh) p4a1 + p4a1 - sin(2 gh)
                                       (- p3 + a2 gt(j) - a1 dcos(gh) gc(i))

(c426) first(%)+first(rest(%,1));
(d426)                        p4a1 - cos(2 gh) p4a1

(c427) factor(%);
(d427)                       - (cos(2 gh) - 1) p4a1

(c428) trigexpand(%);
                                   2           2
(d428)                    - (- sin (gh) + cos (gh) - 1) p4a1

(c429) t2:(trigsimp(%)+last(t2))/ww;
              2
       2 sin (gh) p4a1 - sin(2 gh) (- p3 + a2 gt(j) - a1 dcos(gh) gc(i))
(d429) ----------------------------------------------------------------
                                   ww

(c430) /* 3 * k(12,21) 2 to 1 term f21 gsyy *** c21f21y ***************************
k1221:(a2/(4*%pi*a11))*(t1+t2);
(d430) a2
                                                                    2
  2 sin(2 gh) (- p3 + a2 gt(j) - a1 dcos(gh) gc(i)) p4a1  - cos(2 gh) p4a1 wwm
 (---------------------------------------------------------------------------
                                         2
                                       ww

         2
   2 sin (gh) p4a1 - sin(2 gh) (- p3 + a2 gt(j) - a1 dcos(gh) gc(i))
 + -----------------------------------------------------------------)
                                  ww

/(4 %pi a11)
```

```
(c431) /* where
p3a2a1=-p3+a2*gt-a1*gc*cos(gh)

p4a1=p4-a1*gc*sin(gh)

ww=p4a1^2+p3a2a1^2

wwm=p3a2a1^2-p4a1^2
*/

fortran(p3a2a1:-p3+a2*gt(j)-a1*gc(i)*dcos(gh))$
      p3a2a1 = -p3+a2*gt(j)-a1*dcos(gh)*gc(i)

(c432) fortran(p4a1:p4-a1*gc(i)*dsin(gh))$
      p4a1 = p4-a1*dsin(gh)*gc(i)

(c433) fortran(ww:'p4a1^2+'p3a2a1^2)$
      ww = p4a1**2+p3a2a1**2

(c434) fortran(wwm:'p3a2a1^2-'p4a1^2)$
      wwm = p3a2a1**2-p4a1**2

(c435) subst(dcos,cos,first(t1))$

(c436) subst(dsin,sin,%)$

(c437) fortran(part1:%)$
      part1 = 2*dsin(2*gh)*(-p3+a2*gt(j)-a1*dcos(gh)*gc(i))*p4a1**2-dcos
     1   (2*gh)*p4a1*wwm

(c438) fortran(part1:'part1/('ww^2))$
      part1 = part1/ww**2

(c439) subst(dcos,cos,t2)$

(c440) subst(dsin,sin,%)$

(c441) fortran(part2:%)$
      part2 = (2*dsin(gh)**2*p4a1-dsin(2*gh)*(-p3+a2*gt(j)-a1*dcos(gh)*g
     1   c(i)))/ww

(c442) kill(x1)$

(c443) kill(x2)$

(c444) kill(y1)$

(c445) kill(y2)$

(c446) kill(t1)$

(c447) kill(t2)$

(c448) kill(p3a2a1)$

(c449) kill(p4a1)$

(c450) kill(ww)$

(c451) kill(wwm)$

(c452) x1:a1*gc;

(d452)                                    a1 gc

(c453) t1:a1*gt;
```

```
(d453)                              a1 gt

(c454) t2:a2*gt;

(d454)                              a2 gt

(c455) gsx2:''gsxxpf2t2;

                                                                   2
                      a2 gt - x2             2 (a2 gt - x2) y2
(d455)           ------------------ - ----------------------
                   2               2      2                  2 2
                 y2  + (a2 gt - x2)    (y2  + (a2 gt - x2) )

(c456) gsy2:''gsyypf2t2;

                                                                   2
                      a2 gt - x2             2 (a2 gt - x2) y2
(d456)           ------------------ + ----------------------
                   2               2      2                  2 2
                 y2  + (a2 gt - x2)    (y2  + (a2 gt - x2) )

(c457) gtxy2:''gsxypf2t2;

                                                    2     2
                             y2 ((a2 gt - x2)  - y2 )
(d457)                       ------------------------
                                  2                2 2
                              (y2  + (a2 gt - x2) )

(c458) x2:p3+x1*cos(-gh)-y1*sin(-gh);

(d458)                  sin(gh) y1 + p3 + a1 gc cos(gh)

(c459) y2:p4+x1*sin(-gh)+y1*cos(-gh);

(d459)                  cos(gh) y1 + p4 - a1 gc sin(gh)

(c460) /* 4 * k(12,22) 2 to 1 term f22 gsyy *** c21f22y ***************************

gsy21b:('gsx2+'gsy2)/2-('gsx2-'gsy2)/2*cos(2*(-gh))-'gtxy2*sin(2*(-gh));

                                   gsy2 + gsx2    cos(2 gh) (gsx2 - gsy2)
(d460)      sin(2 gh) gtxy2 + ----------- - -----------------------
                                        2                    2

(c461) gsy21:(''gsx2+''gsy2)/2-(''gsx2-''gsy2)/2*cos(2*(-gh))-''gtxy2*sin(2*(-gh));

(d461) (- sin(gh) y1 - p3 + a2 gt - a1 gc cos(gh))

                                                  2
/((- sin(gh) y1 - p3 + a2 gt - a1 gc cos(gh))

                                           2
 + (cos(gh) y1 + p4 - a1 gc sin(gh))  ) + sin(2 gh)

 (cos(gh) y1 + p4 - a1 gc sin(gh))

                                                  2
 ((- sin(gh) y1 - p3 + a2 gt - a1 gc cos(gh))

                                          2
 - (cos(gh) y1 + p4 - a1 gc sin(gh))  )

                                                     2
```

```
/expt((- sin(gh) y1 - p3 + a2 gt - a1 gc cos(gh))

                                       2
 + (cos(gh) y1 + p4 - a1 gc sin(gh)) , 2)

                                                 2
 + 2 cos(2 gh) (cos(gh) y1 + p4 - a1 gc sin(gh))
 (- sin(gh) y1 - p3 + a2 gt - a1 gc cos(gh))

                                                2
/expt((- sin(gh) y1 - p3 + a2 gt - a1 gc cos(gh))

                                       2
 + (cos(gh) y1 + p4 - a1 gc sin(gh)) , 2)
(c462) y1:0;
(d462)                                   0
(c463) ev(gsy21);
                      - p3 + a2 gt - a1 gc cos(gh)
(d463) -----------------------------------------------------
                              2                              2
       (p4 - a1 gc sin(gh))  + (- p3 + a2 gt - a1 gc cos(gh))

                                                                     2
 + sin(2 gh) (p4 - a1 gc sin(gh)) ((- p3 + a2 gt - a1 gc cos(gh))

                          2
 - (p4 - a1 gc sin(gh)) )

                      2                              2 2
/((p4 - a1 gc sin(gh))  + (- p3 + a2 gt - a1 gc cos(gh)) )

                                                                      2
   2 cos(2 gh) (- p3 + a2 gt - a1 gc cos(gh)) (p4 - a1 gc sin(gh))
 + ----------------------------------------------------------------
                                2                              2 2
      ((p4 - a1 gc sin(gh))  + (- p3 + a2 gt - a1 gc cos(gh)) )
(c464) subst(p3a2a1,-p3+a2*gt-a1*gc*cos(gh),%);
                     p3a2a1
(d464) -------------------------------
                              2          2
       (p4 - a1 gc sin(gh))  + p3a2a1

                                                2                        2
   sin(2 gh) (p4 - a1 gc sin(gh)) (p3a2a1  - (p4 - a1 gc sin(gh)) )
 + -----------------------------------------------------------------
                                            2           2 2
                       ((p4 - a1 gc sin(gh))  + p3a2a1 )

                                                 2
   2 cos(2 gh) p3a2a1 (p4 - a1 gc sin(gh))
 + -----------------------------------------
                               2           2 2
      ((p4 - a1 gc sin(gh))  + p3a2a1 )
(c465) subst(p4a1,p4-a1*gc*sin(gh),%);

                                                    2
            p3a2a1           2 cos(2 gh) p3a2a1 p4a1
(d465) --------------- + -----------------------
```

```
        2          2             2          2 2
   p4a1  + p3a2a1        (p4a1  + p3a2a1 )

                                                                       2         2
                                                 sin(2 gh) p4a1 (p3a2a1  - p4a1 )
                                               + --------------------------------
                                                              2          2 2
                                                        (p4a1  + p3a2a1 )

(c466) subst(ww,p4a1^2+p3a2a1^2,%);

                                             2                           2         2
        p3a2a1   2 cos(2 gh) p3a2a1 p4a1     sin(2 gh) p4a1 (p3a2a1  - p4a1 )
(d466)  ------ + ------------------------ + --------------------------------
          ww                 2                              2
                           ww                             ww

(c467) fn:subst(wwm,p3a2a1^2-p4a1^2,%);

                                                                      2
             sin(2 gh) p4a1 wwm   p3a2a1   2 cos(2 gh) p3a2a1 p4a1
(d467)       ------------------ + ------ + ------------------------
                      2             ww                 2
                    ww                               ww

(c468) fn:combine(fn);

                                                                   2
             sin(2 gh) p4a1 wwm + 2 cos(2 gh) p3a2a1 p4a1     p3a2a1
(d468)       --------------------------------------------- + ------
                                      2                         ww
                                    ww

(c469) /* 4 * k(12,22) 2 to 1 term f22 gsyy *** c21f22y **************************

k1222:(a2/(4*%pi*a11))*fn;

                                                                     2
               sin(2 gh) p4a1 wwm + 2 cos(2 gh) p3a2a1 p4a1     p3a2a1
          a2 (--------------------------------------------- + ------)
                                       2                         ww
                                     ww
(d469)    ------------------------------------------------------------
                                 4 %pi a11

(c470) /* where
p3a2a1=-p3+a2*gt-a1*gc*cos(gh)

p4a1=p4-a1*gc*sin(gh)

ww=p4a1^2+p3a2a1^2

wwm=p3a2a1^2-p4a1^2
*/

fortran(p3a2a1:-p3+a2*gt(j)-a1*gc(i)*dcos(gh))$
      p3a2a1 = -p3+a2*gt(j)-a1*dcos(gh)*gc(i)

(c471) fortran(p4a1:p4-a1*gc(i)*dsin(gh))$
      p4a1 = p4-a1*dsin(gh)*gc(i)

(c472) fortran(ww:'p4a1^2+'p3a2a1^2)$
      ww = p4a1**2+p3a2a1**2

(c473) fortran(wwm:'p3a2a1^2-'p4a1^2)$
      wwm = p3a2a1**2-p4a1**2
```

```
(c474) subst(dcos,cos,first(first(fn)))$

(c475) subst(dsin,sin,%)$

(c476) fortran(part1:%)$
      part1 = dsin(2*gh)*p4a1*wwm+2*dcos(2*gh)*p3a2a1*p4a1**2

(c477) fortran(part1:'part1/('ww^2))$
      part1 = part1/ww**2

(c478) subst(dcos,cos,last(fn))$

(c479) subst(dsin,sin,%)$

(c480) fortran(part2:%)$
      part2 = p3a2a1/ww

(c481) kill(x1)$

(c482) kill(x2)$

(c483) kill(y1)$

(c484) kill(y2)$

(c485) kill(t1)$

(c486) kill(t2)$

(c487) kill(p3a2a1)$

(c488) kill(p4a1)$

(c489) kill(ww)$

(c490) kill(wwm)$

(c491) x2:a2*gc;

(d491)                                a2 gc

(c492) t1:a1*gt;

(d492)                                a1 gt

(c493) t2:a2*gt;

(d493)                                a2 gt

(c494) gsx1:''gsxxpf1t1;

                                                            2      2
                     2 y1                 y1 ((a1 gt - x1)  - y1 )
(d494)        ------------------ + ------------------------
                2                  2      2              2 2
              y1  + (a1 gt - x1)     (y1  + (a1 gt - x1) )

(c495) gsy1:''gsyypf1t1;

                                         2      2
                            y1 ((a1 gt - x1)  - y1 )
(d495)                   - ------------------------
                                2              2 2
                             (y1  + (a1 gt - x1) )
```

```
(c496) gtxy1:''gsxypf1t1;

                                              2
                       2 (a1 gt - x1) y1              a1 gt - x1
(d496)               ---------------------- - -------------------
                          2                2 2     2                2
                      (y1  + (a1 gt - x1) )     y1  + (a1 gt - x1)

(c497) x1:p1+x2*cos(gh)-y2*sin(gh);

(d497)                  - sin(gh) y2 + p1 + a2 gc cos(gh)

(c498) y1:p2+x2*sin(gh)+y2*cos(gh);

(d498)                   cos(gh) y2 + p2 + a2 gc sin(gh)

(c499) /* 5 * k(21,11) 1 to 2 term f11 gtxy *** c12f11xy ************************
gtxy12b:-('gsx1-'gsy1)/2*sin(2*gh)+'gtxy1*cos(2*gh);

                                        sin(2 gh) (gsx1 - gsy1)
(d499)              cos(2 gh) gtxy1 - -----------------------
                                                   2

(c500) gtxy12:-(''gsx1-''gsy1)/2*sin(2*gh)+''gtxy1*cos(2*gh);

                                                              2
(d500) cos(2 gh) (2 (cos(gh) y2 + p2 + a2 gc sin(gh))
 (sin(gh) y2 - p1 + a1 gt - a2 gc cos(gh))

                                                         2
/expt((sin(gh) y2 - p1 + a1 gt - a2 gc cos(gh))

                                          2
 + (cos(gh) y2 + p2 + a2 gc sin(gh)) , 2)
 - (sin(gh) y2 - p1 + a1 gt - a2 gc cos(gh))

                                                     2
/((sin(gh) y2 - p1 + a1 gt - a2 gc cos(gh))

                                          2
 + (cos(gh) y2 + p2 + a2 gc sin(gh)) ))
 - sin(2 gh) (2 (cos(gh) y2 + p2 + a2 gc sin(gh))

                                                     2
/((sin(gh) y2 - p1 + a1 gt - a2 gc cos(gh))

                                          2
 + (cos(gh) y2 + p2 + a2 gc sin(gh)) ) + 2 (cos(gh) y2 + p2 + a2 gc sin(gh))

                                                     2
 ((sin(gh) y2 - p1 + a1 gt - a2 gc cos(gh))

                                          2
 - (cos(gh) y2 + p2 + a2 gc sin(gh)) )

                                                         2
/expt((sin(gh) y2 - p1 + a1 gt - a2 gc cos(gh))

                                          2
 + (cos(gh) y2 + p2 + a2 gc sin(gh)) , 2))/2

(c501) y2:0;
```

```
(d501)                                0

(c502) ev(gtxy12);

                                                                              2
                      2 (- p1 + a1 gt - a2 gc cos(gh)) (p2 + a2 gc sin(gh))
(d502) cos(2 gh) (------------------------------------------------------------
                                          2                                  2 2
                   ((p2 + a2 gc sin(gh))  + (- p1 + a1 gt - a2 gc cos(gh))  )

                  - p1 + a1 gt - a2 gc cos(gh)
 - -------------------------------------------------------)
                            2                                2
   (p2 + a2 gc sin(gh))  + (- p1 + a1 gt - a2 gc cos(gh))

                                   2 (p2 + a2 gc sin(gh))
 - sin(2 gh) (--------------------------------------------------------
                                   2                                 2
               (p2 + a2 gc sin(gh))  + (- p1 + a1 gt - a2 gc cos(gh))

                                                              2
 + 2 (p2 + a2 gc sin(gh)) ((- p1 + a1 gt - a2 gc cos(gh))

                          2
 - (p2 + a2 gc sin(gh))  )

                        2                                 2 2
/((p2 + a2 gc sin(gh))  + (- p1 + a1 gt - a2 gc cos(gh))  ) )/2

(c503) subst(p1a1a2,-p1+a1*gt-a2*gc*cos(gh),%);

                                                    2
                      2 p1a1a2 (p2 + a2 gc sin(gh))
(d503) cos(2 gh) (----------------------------------
                                          2          2 2
                   ((p2 + a2 gc sin(gh))  + p1a1a2  )

                p1a1a2
 - ------------------------------) - sin(2 gh)
                            2          2
   (p2 + a2 gc sin(gh))  + p1a1a2

      2 (p2 + a2 gc sin(gh))
 (------------------------------
                          2          2
  (p2 + a2 gc sin(gh))  + p1a1a2

                                         2                        2
   2 (p2 + a2 gc sin(gh)) (p1a1a2  - (p2 + a2 gc sin(gh))  )
 + -------------------------------------------------------------)/2
                                   2          2 2
              ((p2 + a2 gc sin(gh))  + p1a1a2  )

(c504) subst(p2a2,p2+a2*gc*sin(gh),%);

                                       2
                     2 p1a1a2 p2a2               p1a1a2
(d504) cos(2 gh) (----------------- - ---------------)
                          2          2 2        2          2
                   (p2a2  + p1a1a2  )    p2a2  + p1a1a2

                                                                 2         2
                                     2 p2a2         2 p2a2 (p1a1a2  - p2a2  )
                        sin(2 gh) (--------------- + ------------------------)
                                        2            2          2            2 2
```

```
                                   p2a2   + p1a1a2        (p2a2   + p1a1a2 )
                        - ---------------------------------------------------
                                                    2

(c505) subst(ww,p2a2^2+p1a1a2^2,%);

                                  2
                2 p1a1a2 p2a2    p1a1a2
(d505) cos(2 gh) (-------------- - ------)
                         2          ww
                       ww

                                                          2       2
                                   2 p2a2   2 p2a2 (p1a1a2  - p2a2 )
                     sin(2 gh) (------ + -----------------------)
                                    ww               2
                                                   ww
                   - ---------------------------------------------
                                           2

(c506) fn:subst(wwm,p1a1a2^2-p2a2^2,%);

                                                   2 p2a2 wwm   2 p2a2
                                       sin(2 gh) (---------- + ------)
                                  2                    2         ww
                2 p1a1a2 p2a2    p1a1a2              ww
(d506) cos(2 gh) (-------------- - ------) - ------------------------------
                         2          ww                    2
                       ww

(c507) first(fn);

                                            2
                           2 p1a1a2 p2a2    p1a1a2
(d507)            cos(2 gh) (-------------- - ------)
                                    2          ww
                                  ww

(c508) n2:multthru(%);

                                          2
              2 cos(2 gh) p1a1a2 p2a2    cos(2 gh) p1a1a2
(d508)        ------------------------ - ----------------
                           2                    ww
                         ww

(c509) rest(fn,1);

                               2 p2a2 wwm   2 p2a2
                    sin(2 gh) (---------- + ------)
                                   2         ww
                                 ww
(d509)            - -----------------------------
                                 2

(c510) n3:multthru(%);

                sin(2 gh) p2a2 wwm   sin(2 gh) p2a2
(d510)        - ------------------ - --------------
                         2                ww
                       ww

(c511) n:n2+n3;

          sin(2 gh) p2a2 wwm   sin(2 gh) p2a2   cos(2 gh) p1a1a2
(d511) - ------------------ - -------------- - ----------------
```

```
                      2                        ww                 ww
                    ww

                                                                                   2
                                                            2 cos(2 gh) p1a1a2 p2a2
                                                          + -----------------------
                                                                      2
                                                                    ww

(c512) fn:combine(n);

                                       2
       2 cos(2 gh) p1a1a2 p2a2  - sin(2 gh) p2a2 wwm
(d512) --------------------------------------------
                                       2
                                     ww

                                                  - sin(2 gh) p2a2 - cos(2 gh) p1a1a2
                                                + -----------------------------------
                                                                  ww

(c513) /* 5 * k(21,11) 1 to 2 term f11 gtxy *** c12f11xy *************************
k2111:(a1/(4*%pi*a11))*fn;

                                           2
           2 cos(2 gh) p1a1a2 p2a2  - sin(2 gh) p2a2 wwm
(d513) a1 (--------------------------------------------
                                           2
                                         ww

                                    - sin(2 gh) p2a2 - cos(2 gh) p1a1a2
                                  + -----------------------------------)/(4 %pi a11)
                                                     ww

(c514) /* where
p1a1a2=-p1+a1*gt-a2*gc*cos(gh)

p2a2=p2+a2*gc*sin(gh)

ww=p2a2^2+p1a1a2^2

wwm=p1a1a2^2-p2a2^2
*/

fortran(p1a1a2:-p1+a1*gt(j)-a2*gc(i)*dcos(gh))$
      p1a1a2 = -p1+a1*gt(j)-a2*dcos(gh)*gc(i)

(c515) fortran(p2a2:p2+a2*gc(i)*dsin(gh))$
      p2a2 = p2+a2*dsin(gh)*gc(i)

(c516) fortran(ww:'p2a2^2+'p1a1a2^2)$
      ww = p2a2**2+p1a1a2**2

(c517) fortran(wwm:'p1a1a2^2-'p2a2^2)$
      wwm = p1a1a2**2-p2a2**2

(c518) subst(dcos,cos,first(first(fn)))$

(c519) subst(dsin,sin,%)$

(c520) fortran(part1:%)$
      part1 = 2*dcos(2*gh)*p1a1a2*p2a2**2-dsin(2*gh)*p2a2*wwm

(c521) fortran(part1:'part1/('ww^2))$
      part1 = part1/ww**2
```

```
(c522) subst(dcos,cos,last(fn))$

(c523) subst(dsin,sin,%)$

(c524) fortran(part2:%)$
      part2 = (-dsin(2*gh)*p2a2-dcos(2*gh)*p1a1a2)/ww

(c525) kill(x1)$

(c526) kill(x2)$

(c527) kill(y1)$

(c528) kill(y2)$

(c529) kill(t1)$

(c530) kill(t2)$

(c531) kill(p1a1a2)$

(c532) kill(p2a2)$

(c533) kill(ww)$

(c534) kill(wwm)$

(c535) x2:a2*gc;

(d535)                              a2 gc

(c536) t1:a1*gt;

(d536)                              a1 gt

(c537) t2:a2*gt;

(d537)                              a2 gt

(c538) gsx1:''gsxxpf2t1;

                                                                   2
                     a1 gt - x1            2 (a1 gt - x1) y1
(d538)          ------------------ - ----------------------
                  2                2     2                 2 2
                y1  + (a1 gt - x1)    (y1  + (a1 gt - x1) )

(c539) gsy1:''gsyypf2t1;

                                                                   2
                     a1 gt - x1            2 (a1 gt - x1) y1
(d539)          ------------------ + ----------------------
                  2                2     2                 2 2
                y1  + (a1 gt - x1)    (y1  + (a1 gt - x1) )

(c540) gtxy1:''gsxypf2t1;

                                                2     2
                           y1 ((a1 gt - x1)  - y1 )
(d540)                     ----------------------
                               2                 2 2
                            (y1  + (a1 gt - x1) )

(c541) x1:p1+x2*cos(gh)-y2*sin(gh);
```

```
(d541)                    - sin(gh) y2 + p1 + a2 gc cos(gh)

(c542) y1:p2+x2*sin(gh)+y2*cos(gh);

(d542)                     cos(gh) y2 + p2 + a2 gc sin(gh)

(c543) /* 6 * k(21,12) 1 to 2 term f12 gtxy *** c12f12xy *************************
gtxy12b:-('gsx1-'gsy1)/2*sin(2*gh)+'gtxy1*cos(2*gh);

                                       sin(2 gh) (gsx1 - gsy1)
(d543)                cos(2 gh) gtxy1 - -----------------------
                                                  2

(c544) gtxy12:-(''gsx1-''gsy1)/2*sin(2*gh)+''gtxy1*cos(2*gh);

(d544) cos(2 gh) (cos(gh) y2 + p2 + a2 gc sin(gh))

                                                 2
 ((sin(gh) y2 - p1 + a1 gt - a2 gc cos(gh))

                                         2
 - (cos(gh) y2 + p2 + a2 gc sin(gh)) )

                                                       2
/expt((sin(gh) y2 - p1 + a1 gt - a2 gc cos(gh))

                                         2
 + (cos(gh) y2 + p2 + a2 gc sin(gh)) , 2)

                                                          2
 + 2 sin(2 gh) (cos(gh) y2 + p2 + a2 gc sin(gh))
 (sin(gh) y2 - p1 + a1 gt - a2 gc cos(gh))

                                                       2
/expt((sin(gh) y2 - p1 + a1 gt - a2 gc cos(gh))

                                         2
 + (cos(gh) y2 + p2 + a2 gc sin(gh)) , 2)

(c545) y2:0;

(d545)                                    0

(c546) ev(gtxy12);

                                                                          2
(d546) cos(2 gh) (p2 + a2 gc sin(gh)) ((- p1 + a1 gt - a2 gc cos(gh))

                           2
 - (p2 + a2 gc sin(gh)) )

                           2                                  2 2
/((p2 + a2 gc sin(gh))  + (- p1 + a1 gt - a2 gc cos(gh)) )

                                                                          2
   2 sin(2 gh) (- p1 + a1 gt - a2 gc cos(gh)) (p2 + a2 gc sin(gh))
 + ----------------------------------------------------------------
                             2                                  2 2
       ((p2 + a2 gc sin(gh))  + (- p1 + a1 gt - a2 gc cos(gh)) )

(c547) subst(p1a1a2,-p1+a1*gt-a2*gc*cos(gh),%);

                                                  2                        2
      cos(2 gh) (p2 + a2 gc sin(gh)) (p1a1a2  - (p2 + a2 gc sin(gh)) )
```

```
(d547) ----------------------------------------------------------------
                                                2          2 2
                         ((p2 + a2 gc sin(gh))   + p1a1a2 )

                                                                          2
                                   2 sin(2 gh) p1a1a2 (p2 + a2 gc sin(gh))
                                 + ---------------------------------------
                                                       2          2 2
                                      ((p2 + a2 gc sin(gh))   + p1a1a2 )

(c548) subst(p2a2,p2+a2*gc*sin(gh),%);

                                      2                           2        2
        2 sin(2 gh) p1a1a2 p2a2     cos(2 gh) p2a2 (p1a1a2   - p2a2 )
(d548)  ------------------------ + -------------------------------
                  2         2 2                   2         2 2
           (p2a2   + p1a1a2 )             (p2a2   + p1a1a2 )

(c549) subst(ww,p2a2^2+p1a1a2^2,%);

                                      2                           2        2
        2 sin(2 gh) p1a1a2 p2a2     cos(2 gh) p2a2 (p1a1a2   - p2a2 )
(d549)  ------------------------ + -------------------------------
                     2                               2
                   ww                              ww

(c550) fn:subst(wwm,p1a1a2^2-p2a2^2,%);

                                                             2
               cos(2 gh) p2a2 wwm   2 sin(2 gh) p1a1a2 p2a2
(d550)         ------------------ + ------------------------
                        2                       2
                      ww                      ww

(c551) fn:combine(fn);

                                                               2
               cos(2 gh) p2a2 wwm + 2 sin(2 gh) p1a1a2 p2a2
(d551)         ---------------------------------------------
                                     2
                                   ww

(c552) /* 6 * k(21,12) 1 to 2 term f12 gtxy *** c12f12xy *************************
k2112:(a1/(4*%pi*a11))*fn;

                                                                   2
             a1 (cos(2 gh) p2a2 wwm + 2 sin(2 gh) p1a1a2 p2a2 )
(d552)       ---------------------------------------------------
                                              2
                               4 %pi a11 ww

(c553) /* where
p1a1a2=-p1+a1*gt-a2*gc*cos(gh)

p2a2=p2+a2*gc*sin(gh)

ww=p2a2^2+p1a1a2^2

wwm=p1a1a2^2-p2a2^2
*/

fortran(p1a1a2:-p1+a1*gt(j)-a2*gc(i)*dcos(gh))$
      p1a1a2 = -p1+a1*gt(j)-a2*dcos(gh)*gc(i)

(c554) fortran(p2a2:p2+a2*gc(i)*dsin(gh))$
```

```
      p2a2 = p2+a2*dsin(gh)*gc(i)

(c555) fortran(ww:'p2a2^2+'p1a1a2^2)$
      ww = p2a2**2+p1a1a2**2

(c556) fortran(wwm:'p1a1a2^2-'p2a2^2)$
      wwm = p1a1a2**2-p2a2**2

(c557) subst(dcos,cos,first(fn))$

(c558) subst(dsin,sin,%)$

(c559) fortran(part1:%)$
      part1 = dcos(2*gh)*p2a2*wwm+2*dsin(2*gh)*p1a1a2*p2a2**2

(c560) fortran(part1:'part1/('ww^2))$
      part1 = part1/ww**2

(c561) kill(x1)$

(c562) kill(x2)$

(c563) kill(y1)$

(c564) kill(y2)$

(c565) kill(t1)$

(c566) kill(t2)$

(c567) kill(p1a1a2)$

(c568) kill(p2a2)$

(c569) kill(ww)$

(c570) kill(wwm)$

(c571) x2:a2*gc;

(d571)                          a2 gc

(c572) t1:a1*gt;

(d572)                          a1 gt

(c573) t2:a2*gt;

(d573)                          a2 gt

(c574) gsx1:''gsxxpf1t1;

                                                      2      2
                     2 y1               y1 ((a1 gt - x1)  - y1 )
(d574)        ------------------- + ------------------------
                2               2       2                2 2
              y1  + (a1 gt - x1)     (y1  + (a1 gt - x1) )

(c575) gsy1:''gsyypf1t1;

                                            2      2
                           y1 ((a1 gt - x1)  - y1 )
(d575)                   - ------------------------
                              2                2 2
                           (y1  + (a1 gt - x1) )
```

```
(c576) gtxy1:''gsxypf1t1;

                                     2
                   2 (a1 gt - x1) y1           a1 gt - x1
(d576)            ---------------------- - -------------------
                     2                2 2     2                 2
                  (y1  + (a1 gt - x1) )     y1  + (a1 gt - x1)

(c577) x1:p1+x2*cos(gh)-y2*sin(gh);

(d577)                    - sin(gh) y2 + p1 + a2 gc cos(gh)

(c578) y1:p2+x2*sin(gh)+y2*cos(gh);

(d578)                     cos(gh) y2 + p2 + a2 gc sin(gh)

(c579) /* 7  * k(22,11) 1 to 2 term f11 gsyy *** c12f11y *************************
gsy12b:('gsx1+'gsy1)/2-('gsx1-'gsy1)/2*cos(2*gh)-'gtxy1*sin(2*gh);

                                  gsy1 + gsx1   cos(2 gh) (gsx1 - gsy1)
(d579)      - sin(2 gh) gtxy1 + ----------- - -----------------------
                                       2                  2

(c580) gsy12:(''gsx1+''gsy1)/2-(''gsx1-''gsy1)/2*cos(2*gh)-''gtxy1*sin(2*gh);

                                                                   2
(d580) - sin(2 gh) (2 (cos(gh) y2 + p2 + a2 gc sin(gh))
 (sin(gh) y2 - p1 + a1 gt - a2 gc cos(gh))

                                                         2
/expt((sin(gh) y2 - p1 + a1 gt - a2 gc cos(gh))

                                       2
 + (cos(gh) y2 + p2 + a2 gc sin(gh)) , 2)
 - (sin(gh) y2 - p1 + a1 gt - a2 gc cos(gh))

                                                     2
/((sin(gh) y2 - p1 + a1 gt - a2 gc cos(gh))

                                       2
 + (cos(gh) y2 + p2 + a2 gc sin(gh)) ))
 - cos(2 gh) (2 (cos(gh) y2 + p2 + a2 gc sin(gh))

                                                     2
/((sin(gh) y2 - p1 + a1 gt - a2 gc cos(gh))

                                       2
 + (cos(gh) y2 + p2 + a2 gc sin(gh)) ) + 2 (cos(gh) y2 + p2 + a2 gc sin(gh))

                                                     2
 ((sin(gh) y2 - p1 + a1 gt - a2 gc cos(gh))

                                       2
 - (cos(gh) y2 + p2 + a2 gc sin(gh)) )

                                                         2
/expt((sin(gh) y2 - p1 + a1 gt - a2 gc cos(gh))

                                       2
 + (cos(gh) y2 + p2 + a2 gc sin(gh)) , 2))/2
 + (cos(gh) y2 + p2 + a2 gc sin(gh))
```

```
                                                 2
/((sin(gh) y2 - p1 + a1 gt - a2 gc cos(gh))

                                           2
 + (cos(gh) y2 + p2 + a2 gc sin(gh)) )

(c581) y2:0;

(d581)                          0

(c582) ev(gsy12);

                                      2 (p2 + a2 gc sin(gh))
(d582) - cos(2 gh) (-----------------------------------------------------
                                       2                               2
                    (p2 + a2 gc sin(gh))  + (- p1 + a1 gt - a2 gc cos(gh))

                                                              2
 + 2 (p2 + a2 gc sin(gh)) ((- p1 + a1 gt - a2 gc cos(gh))

                         2
 - (p2 + a2 gc sin(gh)) )

                        2                               2 2
/((p2 + a2 gc sin(gh))  + (- p1 + a1 gt - a2 gc cos(gh)) ) )/2

                                                                            2
                 2 (- p1 + a1 gt - a2 gc cos(gh)) (p2 + a2 gc sin(gh))
 - sin(2 gh) (-----------------------------------------------------
                                          2                               2 2
              ((p2 + a2 gc sin(gh))  + (- p1 + a1 gt - a2 gc cos(gh)) )
                 - p1 + a1 gt - a2 gc cos(gh)
 - ----------------------------------------------------)
                         2                               2
   (p2 + a2 gc sin(gh))  + (- p1 + a1 gt - a2 gc cos(gh))
                         p2 + a2 gc sin(gh)
 + ----------------------------------------------------
                         2                               2
   (p2 + a2 gc sin(gh))  + (- p1 + a1 gt - a2 gc cos(gh))

(c583) subst(p1a1a2,-p1+a1*gt-a2*gc*cos(gh),%);

                               2 (p2 + a2 gc sin(gh))
(d583) - cos(2 gh) (------------------------------
                                          2             2
                    (p2 + a2 gc sin(gh))  + p1a1a2

                                         2                          2
   2 (p2 + a2 gc sin(gh)) (p1a1a2  - (p2 + a2 gc sin(gh)) )
 + ---------------------------------------------------------)/2
                                       2             2 2
              ((p2 + a2 gc sin(gh))  + p1a1a2 )

                                                  2
                 2 p1a1a2 (p2 + a2 gc sin(gh))
 - sin(2 gh) (------------------------------
                                       2             2 2
              ((p2 + a2 gc sin(gh))  + p1a1a2 )

                p1a1a2                                    p2 + a2 gc sin(gh)
 - -------------------------------) + -------------------------------
                           2             2                           2             2
   (p2 + a2 gc sin(gh))  + p1a1a2     (p2 + a2 gc sin(gh))  + p1a1a2
```

```
(c584) subst(p2a2,p2+a2*gc*sin(gh),%);

                                                   2          2
                    2 p2a2          2 p2a2 (p1a1a2  - p2a2 )
        cos(2 gh) (--------------- + -----------------------)
                       2           2            2          2 2
                   p2a2  + p1a1a2       (p2a2  + p1a1a2 )
(d584) - ----------------------------------------------------
                                   2

                                         2
                       2 p1a1a2 p2a2              p1a1a2                 p2a2
          - sin(2 gh) (----------------- - ---------------) + ---------------
                            2          2 2     2          2      2          2
                       (p2a2  + p1a1a2 )   p2a2  + p1a1a2    p2a2  + p1a1a2

(c585) subst(ww,p2a2^2+p1a1a2^2,%);

                                          2         2
                    2 p2a2   2 p2a2 (p1a1a2  - p2a2 )
        cos(2 gh) (------ + -----------------------)
                     ww               2
                                    ww
(d585) - --------------------------------------------
                              2

                                                   2
                                   2 p1a1a2 p2a2      p1a1a2     p2a2
                      - sin(2 gh) (-------------- - ------) + ----
                                          2           ww        ww
                                        ww

(c586) fn:subst(wwm,p1a1a2^2-p2a2^2,%);

                   2 p2a2 wwm   2 p2a2
       cos(2 gh) (---------- + ------)
                       2         ww
                     ww                                      2
                                                2 p1a1a2 p2a2      p1a1a2
(d586) - ------------------------------ - sin(2 gh) (-------------- - ------)
                       2                                  2           ww
                                                        ww

                                                                       p2a2
                                                                     + ----
                                                                        ww

(c587) n1:last(fn);

                                  p2a2
(d587)                            ----
                                   ww

(c588) first(fn);

                               2 p2a2 wwm   2 p2a2
                   cos(2 gh) (---------- + ------)
                                   2          ww
                                 ww
(d588)             - ------------------------------
                                   2

(c589) n2:multthru(%);

                   cos(2 gh) p2a2 wwm   cos(2 gh) p2a2
(d589)             - ------------------ - --------------
```

```
                                2                         ww
                             ww

(c590) rest(fn,1);

                                                   2
               p2a2                2 p1a1a2 p2a2      p1a1a2
(d590)         ---- - sin(2 gh) (-------------- - ------)
                ww                        2          ww
                                        ww

(c591) last(%);

                                                 2
                             2 p1a1a2 p2a2      p1a1a2
(d591)         - sin(2 gh) (-------------- - ------)
                                    2          ww
                                  ww

(c592) n3:multthru(%);

                                                                2
              sin(2 gh) p1a1a2   2 sin(2 gh) p1a1a2 p2a2
(d592)        ---------------- - ------------------------
                     ww                      2
                                           ww

(c593) n:n1+n2+n3;

        cos(2 gh) p2a2 wwm   cos(2 gh) p2a2   p2a2   sin(2 gh) p1a1a2
(d593) - ------------------ - -------------- + ---- + ----------------
                 2                  ww          ww          ww
               ww

                                                                   2
                                              2 sin(2 gh) p1a1a2 p2a2
                                            - ------------------------
                                                          2
                                                        ww

(c594) n:combine(n);

                                                          2
        - cos(2 gh) p2a2 wwm - 2 sin(2 gh) p1a1a2 p2a2
(d594) ------------------------------------------------
                                2
                              ww

                       - cos(2 gh) p2a2 + p2a2 + sin(2 gh) p1a1a2
                     + ------------------------------------------
                                          ww

(c595) t1:first(%);

                                                               2
              - cos(2 gh) p2a2 wwm - 2 sin(2 gh) p1a1a2 p2a2
(d595)        ------------------------------------------------
                                      2
                                    ww

(c596) last(n);

               - cos(2 gh) p2a2 + p2a2 + sin(2 gh) p1a1a2
(d596)         ------------------------------------------
                                  ww
```

```
(c597) t2:part(%,1);

(d597)              - cos(2 gh) p2a2 + p2a2 + sin(2 gh) p1a1a2

(c598) first(%)+first(rest(%,1));

(d598)                          p2a2 - cos(2 gh) p2a2

(c599) factor(%);

(d599)                         - (cos(2 gh) - 1) p2a2

(c600) trigexpand(%);

                                   2            2
(d600)                    - (- sin (gh) + cos (gh) - 1) p2a2

(c601) t2:(trigsimp(%)+last(t2))/ww;

                                 2
                          2 sin (gh) p2a2 + sin(2 gh) p1a1a2
(d601)                    ----------------------------------
                                         ww

(c602) /* 7 * k(22,11) 1 to 2 term f11 gsyy *** c12f11y **************************
k2211:(a1/(4*%pi*a11))*(t1+t2);

                                                                2
        - cos(2 gh) p2a2 wwm - 2 sin(2 gh) p1a1a2 p2a2
(d602) a1 (-----------------------------------------------
                                     2
                                   ww

                                          2
                                   2 sin (gh) p2a2 + sin(2 gh) p1a1a2
                                 + ----------------------------------)/(4 %pi a11)
                                                  ww

(c603) /* where
p1a1a2=-p1+a1*gt-a2*gc*cos(gh)

p2a2=p2+a2*gc*sin(gh)

ww=p2a2^2+p1a1a2^2

wwm=p1a1a2^2-p2a2^2
*/

fortran(p1a1a2:-p1+a1*gt(j)-a2*gc(i)*dcos(gh))$
      p1a1a2 = -p1+a1*gt(j)-a2*dcos(gh)*gc(i)

(c604) fortran(p2a2:p2+a2*gc(i)*dsin(gh))$
      p2a2 = p2+a2*dsin(gh)*gc(i)

(c605) fortran(ww:'p2a2^2+'p1a1a2^2)$
      ww = p2a2**2+p1a1a2**2

(c606) fortran(wwm:'p1a1a2^2-'p2a2^2)$
      wwm = p1a1a2**2-p2a2**2

(c607) subst(dcos,cos,first(t1))$

(c608) subst(dsin,sin,%)$

(c609) fortran(part1:%)$
```

```
      part1 = -dcos(2*gh)*p2a2*wwm-2*dsin(2*gh)*p1a1a2*p2a2**2

(c610) fortran(part1:'part1/('ww^2))$
      part1 = part1/ww**2

(c611) subst(dcos,cos,t2)$

(c612) subst(dsin,sin,%)$

(c613) fortran(part2:%)$
      part2 = (2*dsin(gh)**2*p2a2+dsin(2*gh)*p1a1a2)/ww

(c614) kill(x1)$

(c615) kill(x2)$

(c616) kill(y1)$

(c617) kill(y2)$

(c618) kill(t1)$

(c619) kill(t2)$

(c620) kill(p1a1a2)$

(c621) kill(p2a2)$

(c622) kill(ww)$

(c623) kill(wwm)$

(c624) x2:a2*gc;

(d624)                              a2 gc

(c625) t1:a1*gt;

(d625)                              a1 gt

(c626) t2:a2*gt;

(d626)                              a2 gt

(c627) gsx1:''gsxxpf2t1;

                                                           2
                     a1 gt - x1            2 (a1 gt - x1) y1
(d627)           ------------------ - ----------------------
                   2               2     2                2 2
                 y1  + (a1 gt - x1)    (y1  + (a1 gt - x1) )

(c628) gsy1:''gsyypf2t1;

                                                           2
                     a1 gt - x1            2 (a1 gt - x1) y1
(d628)           ------------------ + ----------------------
                   2               2     2                2 2
                 y1  + (a1 gt - x1)    (y1  + (a1 gt - x1) )

(c629) gtxy1:''gsxypf2t1;

                                          2     2
                         y1 ((a1 gt - x1)  - y1 )
(d629)                   ------------------------
                            2                2 2
```

```
                           (y1  + (a1 gt - x1) )

(c630) x1:p1+x2*cos(gh)-y2*sin(gh);

(d630)                  - sin(gh) y2 + p1 + a2 gc cos(gh)

(c631) y1:p2+x2*sin(gh)+y2*cos(gh);

(d631)                   cos(gh) y2 + p2 + a2 gc sin(gh)

(c632) /* 8 * k(22,12) 1 to 2 term f12 gsyy *** c12f12y ***************************

gsy12b:('gsx1+'gsy1)/2-('gsx1-'gsy1)/2*cos(2*gh)-'gtxy1*sin(2*gh);

                                    gsy1 + gsx1   cos(2 gh) (gsx1 - gsy1)
(d632)      - sin(2 gh) gtxy1 + ----------- - -----------------------
                                         2                  2

(c633) gsy12:(''gsx1+''gsy1)/2-(''gsx1-''gsy1)/2*cos(2*gh)-''gtxy1*sin(2*gh);

(d633) (sin(gh) y2 - p1 + a1 gt - a2 gc cos(gh))

                                            2
/((sin(gh) y2 - p1 + a1 gt - a2 gc cos(gh))

                                       2
 + (cos(gh) y2 + p2 + a2 gc sin(gh)) ) - sin(2 gh)

                                                                             2
 (cos(gh) y2 + p2 + a2 gc sin(gh)) ((sin(gh) y2 - p1 + a1 gt - a2 gc cos(gh))

                                       2
 - (cos(gh) y2 + p2 + a2 gc sin(gh)) )

                                                 2
/expt((sin(gh) y2 - p1 + a1 gt - a2 gc cos(gh))

                                       2
 + (cos(gh) y2 + p2 + a2 gc sin(gh)) , 2)

                                                   2
 + 2 cos(2 gh) (cos(gh) y2 + p2 + a2 gc sin(gh))
 (sin(gh) y2 - p1 + a1 gt - a2 gc cos(gh))

                                                 2
/expt((sin(gh) y2 - p1 + a1 gt - a2 gc cos(gh))

                                       2
 + (cos(gh) y2 + p2 + a2 gc sin(gh)) , 2)

(c634) y2:0;

(d634)                                    0

(c635) ev(gsy12);

                          - p1 + a1 gt - a2 gc cos(gh)
(d635) ---------------------------------------------------------
                                   2                               2
        (p2 + a2 gc sin(gh))  + (- p1 + a1 gt - a2 gc cos(gh))

                                                                     2
 - sin(2 gh) (p2 + a2 gc sin(gh)) ((- p1 + a1 gt - a2 gc cos(gh))

                         2
```

```
 - (p2 + a2 gc sin(gh)) )

                          2                                 2 2
/((p2 + a2 gc sin(gh))  + (- p1 + a1 gt - a2 gc cos(gh))  )

                                                                  2
   2 cos(2 gh) (- p1 + a1 gt - a2 gc cos(gh)) (p2 + a2 gc sin(gh))
 + ----------------------------------------------------------------
                              2                                 2 2
      ((p2 + a2 gc sin(gh))  + (- p1 + a1 gt - a2 gc cos(gh))  )

(c636) subst(p1a1a2,-p1+a1*gt-a2*gc*cos(gh),%);

                     p1a1a2
(d636) ------------------------------
                              2         2
        (p2 + a2 gc sin(gh))  + p1a1a2

                                             2                       2
   sin(2 gh) (p2 + a2 gc sin(gh)) (p1a1a2  - (p2 + a2 gc sin(gh))  )
 - ----------------------------------------------------------------
                                          2          2 2
                     ((p2 + a2 gc sin(gh))  + p1a1a2  )

                                                   2
   2 cos(2 gh) p1a1a2 (p2 + a2 gc sin(gh))
 + ---------------------------------------
                              2         2 2
      ((p2 + a2 gc sin(gh))  + p1a1a2  )

(c637) subst(p2a2,p2+a2*gc*sin(gh),%);

                                                          2
           p1a1a2           2 cos(2 gh) p1a1a2 p2a2
(d637) --------------- + ------------------------
             2          2              2          2 2
       p2a2  + p1a1a2         (p2a2  + p1a1a2  )

                                                     2           2
                                    sin(2 gh) p2a2 (p1a1a2  - p2a2  )
                                  - ---------------------------------
                                                  2          2 2
                                             (p2a2  + p1a1a2  )

(c638) subst(ww,p2a2^2+p1a1a2^2,%);

                                              2                           2         2
       p1a1a2   2 cos(2 gh) p1a1a2 p2a2     sin(2 gh) p2a2 (p1a1a2  - p2a2  )
(d638) ------ + ------------------------ - ---------------------------------
         ww                  2                                  2
                           ww                                 ww

(c639) fn:subst(wwm,p1a1a2^2-p2a2^2,%);

                                                                      2
          sin(2 gh) p2a2 wwm     p1a1a2     2 cos(2 gh) p1a1a2 p2a2
(d639)  - ------------------ + ------ + ------------------------
                    2            ww                  2
                  ww                               ww

(c640) fn:combine(fn);

                                          2
         2 cos(2 gh) p1a1a2 p2a2  - sin(2 gh) p2a2 wwm    p1a1a2
(d640)   ------------------------------------------- + ------
                               2                         ww
```

```
                                  ww

(c641) /* 8 * k(22,12) 1 to 2 term f12 gsyy *** c12f12y ***************************

k2212:(a1/(4*%pi*a11))*fn;

                                            2
             2 cos(2 gh) p1a1a2 p2a2  - sin(2 gh) p2a2 wwm    p1a1a2
          a1 (------------------------------------------ + ------)
                                            2                  ww
                                          ww
(d641)    ----------------------------------------------------------
                                   4 %pi a11

(c642) /* where
p1a1a2=-p1+a1*gt-a2*gc*cos(gh)

p2a2=p2+a2*gc*sin(gh)

ww=p2a2^2+p1a1a2^2

wwm=p1a1a2^2-p2a2^2
*/

fortran(p1a1a2:-p1+a1*gt(j)-a2*gc(i)*dcos(gh))$
      p1a1a2 = -p1+a1*gt(j)-a2*dcos(gh)*gc(i)

(c643) fortran(p2a2:p2+a2*gc(i)*dsin(gh))$
      p2a2 = p2+a2*dsin(gh)*gc(i)

(c644) fortran(ww:'p2a2^2+'p1a1a2^2)$
      ww = p2a2**2+p1a1a2**2

(c645) fortran(wwm:'p1a1a2^2-'p2a2^2)$
      wwm = p1a1a2**2-p2a2**2

(c646) subst(dcos,cos,first(first(fn)))$

(c647) subst(dsin,sin,%)$

(c648) fortran(part1:%)$
      part1 = 2*dcos(2*gh)*p1a1a2*p2a2**2-dsin(2*gh)*p2a2*wwm

(c649) fortran(part1:'part1/('ww^2))$
      part1 = part1/ww**2

(c650) subst(dcos,cos,last(fn))$

(c651) subst(dsin,sin,%)$

(c652) fortran(part2:%)$
      part2 = p1a1a2/ww

(c653) kill(x1)$

(c654) kill(x2)$

(c655) kill(y1)$

(c656) kill(y2)$

(c657) kill(t1)$

(c658) kill(t2)$

(c659) kill(p1a1a2)$
```

```
(c660) kill(p2a2)$

(c661) kill(ww)$

(c662) kill(wwm)$

(c663) closefile(totallp);
```