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Note on Use of Slope Diffraction Coefficients for Aperture antennas on Finite Ground plane

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CONTENTS

ABSTRACT

The use of slope diffraction coefficients along with regular diffraction coefficients for calculating the radiation patterns of aperture antennas in a finite ground plane is investigated. Explicit expressions for regular diffraction coefficients and slope diffraction coefficients are presented. The expressions for the incident magnetic field in terms of the magnetic current in the aperture are given. The slope of the incident magnetic field is calculated and closed form expressions are presented.

List of Symbols

∇	Del operator
Φ^{sd}	Electric or Magnetic diffracted field for soft polarization
Φ^{i}	Electric or Magnetic incident field for soft polarization
Φ^{td}	Total diffracted field for soft polarization
ψ'	As defined in Figure 1
Ψ^{sd}	Electric or magnetic diffracted field for hard polarization
Ψ^{i}	Electric or Magnetic incident field for hard polarization
Ψ^{td}	Total diffracted field for hard polarization
r, θ, ϕ	Spherical coordinates
D_s	Diffraction coefficient for soft polarization
D_h	Diffraction coefficient for hard polarization
D^i	As defined in equation (16)
D^r	As defined in equation (17)
E_ϕ^{sd}	ϕ-component of diffracted electric field
E_ϕ^{i}	ϕ-component of incident electric field
E_θ^{sd}	θ-component of diffracted electric field
E_θ^{i}	θ-component of incident electric field
F_s	As defined in equation (9)
$g^{\pm}$	As defined in equation (10)
H_θ^{sd}	θ-component of diffracted magnetic field
H_θ^{i}	θ-component of incident magnetic field

H_ϕ^{sd}	ϕ-component of diffracted magnetic field
H_ϕ^{i}	ϕ-component of incident magnetic field
j	$\sqrt{-1}$
k_o	Free space wavenumber
k_x	Fourier transform variable with respect to x
k_y	Fourier transform variable with respect to y
$\vec{M}$	Magnetic current in the aperture
M_x	x-component of magnetic current
M_y	y-component of magnetic current
$\tilde{M}_x$	Fourier transform of M_x (equation (20))
$\tilde{M}_y$	Fourier transform of M_y (equation (21))
$N^{\pm}$	As defined in equation (11)
n	As defined in Figure 1
Q_D	Diffraction point on the ground plane
r, θ, ϕ	Spherical coordinate system
S_1	As defined in equation (30)
S_2	As defined in equation (31)
s'	Distance between the source and the diffraction point
s	Distance between the diffraction point and the observation point
T_1	As defined in equation (28)
T_2	As defined in equation (29)
$\hat{u}$	Unit normal in ψ' direction

1. Introduction

Conformal antennas such as cavity backed aperture antennas are very popular in the aerospace community. These antennas have been analyzed using a number of different techniques[1,2]. The techniques used in these papers include Finite Element Method (FEM), Method of Moments (MoM) and Geometrical Theory of Diffraction (GTD).

In a recent publication, GTD was included to account for the finite termination of a ground plane[2]. The diffraction effects in that paper were accounted for by using only the diffraction which depends on the incident field. However, this indicates that the diffracted field would be zero if the incident field is zero. Physically it is known that the field does not abruptly go to zero. The higher order diffraction effect known as slope diffraction provides a non zero diffracted field, whenever the incident field is zero[3].

In this paper, the slope diffraction coefficient concept is discussed briefly. These slope diffraction coefficients are written explicitly assuming an aperture antenna as the source. The diffracted fields, calculated using the slope diffraction coefficients, are then added to the diffracted fields, which depend on the incident fields to produce the total diffracted fields.

2. Development

For clarity in explaining slope diffraction, the cavity backed aperture is raised above the conducting surface as shown in figure 1. From Balanis[3] with appropriate notation change, the slope diffracted fields from point Q_D are written as

$$\Phi^{sd} = \frac{1}{jk_o}\left[\left.\frac{\partial \Phi^i}{\partial u}\right|_{Q_D}\right]\left(\frac{\partial D_s}{\partial \psi'}\right)\frac{\sqrt{s'}}{s}\exp\left(-jk_o s\right) \tag{1}$$

for soft polarization where

$$\left.\frac{\partial \Phi^i}{\partial u}\right|_{Q_D} = \hat{u} \bullet \nabla \Phi^i = \text{slope of incident field} \tag{2}$$

$$\frac{\partial D_s}{\partial \psi'} = \text{slope diffraction coefficient} \tag{3}$$

and $\Phi^{sd}\left(\Phi^i\right)$ represent either $E_\phi^{sd}\left(E_\phi^i\right)$ or $H_\theta^{sd}\left(H_\theta^i\right)$, respectively. Similarly for hard polarization

$$\Psi^{sd} = \frac{1}{jk_o}\left[\left.\frac{\partial \Psi^i}{\partial u}\right|_{Q_D}\right]\left(\frac{\partial D_h}{\partial \psi'}\right)\frac{\sqrt{s'}}{s}\exp\left(-jk_o s\right) \tag{4}$$

where

$$\left.\frac{\partial \Psi^i}{\partial u}\right|_{Q_D} = \hat{u} \bullet \nabla \Psi^i = \text{slope of incident field} \tag{5}$$

$$\frac{\partial D_h}{\partial \psi'} = \text{slope diffraction coefficient} \tag{6}$$

and $\Psi^{sd}\left(\Psi^i\right)$ represent either $E_\theta^{sd}\left(E_\theta^i\right)$ or $H_\phi^{sd}\left(H_\phi^i\right)$, respectively.

The general diffraction coefficients for the soft and hard polarizations are given in Balanis[3]. These coefficients are written in the notation of the present paper as

$$\frac{\partial D_s}{\partial \psi'} = -\frac{\exp\left(-j\frac{\pi}{4}\right)}{4n^2\sqrt{2\pi k_o}} \left\{ \left[\csc^2\left(\frac{\pi + (\psi - \psi')}{2n}\right) F_s\left[k_o s' g^+ (\psi - \psi')\right] \right.\right.$$

$$\left. -\csc^2\left(\frac{\pi - (\psi - \psi')}{2n}\right) F_s\left[k_o s' g^- (\psi - \psi')\right]\right] \quad +$$

$$\left[\csc^2\left(\frac{\pi + (\psi + \psi')}{2n}\right) F_s\left[k_o s' g^+ (\psi + \psi')\right]\right.$$

$$\left.\left. -\csc^2\left(\frac{\pi - (\psi + \psi')}{2n}\right) F_s\left[k_o s' g^- (\psi + \psi')\right]\right]\right\} \tag{7}$$

$$\frac{\partial D_h}{\partial \psi'} = -\frac{\exp\left(-j\frac{\pi}{4}\right)}{4n^2\sqrt{2\pi k_o}} \left\{ \left[\csc^2\left(\frac{\pi + (\psi - \psi')}{2n}\right) F_s\left[k_o s' g^+ (\psi - \psi')\right] \right.\right.$$

$$\left. -\csc^2\left(\frac{\pi - (\psi - \psi')}{2n}\right) F_s\left[k_o s' g^- (\psi - \psi')\right]\right] \quad -$$

$$\left[\csc^2\left(\frac{\pi + (\psi + \psi')}{2n}\right) F_s\left[k_o s' g^+ (\psi + \psi')\right]\right.$$

$$\left.\left. -\csc^2\left(\frac{\pi - (\psi + \psi')}{2n}\right) F_s\left[k_o s' g^- (\psi + \psi')\right]\right]\right\} \tag{8}$$

where

$$F_s\left[k_o s' g^{\pm} (\psi \pm \psi')\right] = 2j\left|\sqrt{k_o s' g^{\pm} (\psi \pm \psi')}\right| \int_{\sqrt{k_o s' g^{\pm} (\psi \pm \psi')}}^{\infty} \exp(-j\tau)\, d\tau \tag{9}$$

$$g^{\pm} (\psi \pm \psi') = 1 + \cos\left[(\psi \pm \psi') - 2\pi n N^{\pm}\right] \tag{10}$$

$$2\pi n N^{\pm} - (\psi \pm \psi') = \pm\pi \tag{11}$$

$N^{\pm}$ are the integer values which most closely satisfy the equalities.

The total diffracted field from point Q_D is given by

$$\Phi^{td} = \left[\Phi^i D_s + \frac{1}{jk_o} \left[\left. \frac{\partial \Phi^i}{\partial u} \right|_{Q_D} \right] \left(\frac{\partial D_s}{\partial \psi'} \right) \right] \frac{\sqrt{s'}}{s} \exp(-jk_o s) \tag{12}$$

for soft polarization and

$$\Psi^{td} = \left[\Psi^i D_h + \frac{1}{jk_o} \left[\left. \frac{\partial \Psi^i}{\partial u} \right|_{Q_D} \right] \left(\frac{\partial D_h}{\partial \psi'} \right) \right] \frac{\sqrt{s'}}{s} \exp(-jk_o s) \tag{13}$$

for hard polarization. For completeness, the diffraction coefficients D_s and D_h are also given below:

$$D_s = D^i - D^r \tag{14}$$

$$D_h = D^i + D^r \tag{15}$$

where

$$D^i = -\frac{\exp\left(-j\frac{\pi}{4}\right)}{2n\sqrt{2\pi k_o}} \left\{ \cot\left(\frac{\pi + (\psi - \psi')}{2n} \right) F_s\left[k_o s' g^+ (\psi - \psi') \right] + \cot\left(\frac{\pi - (\psi - \psi')}{2n} \right) F_s\left[k_o s' g^- (\psi - \psi') \right] \right\} \tag{16}$$

$$D^r = -\frac{\exp\left(-j\frac{\pi}{4}\right)}{2n\sqrt{2\pi k_o}} \left\{ \cot\left(\frac{\pi + (\psi + \psi')}{2n} \right) F_s\left[k_o s' g^+ (\psi + \psi') \right] + \cot\left(\frac{\pi - (\psi + \psi')}{2n} \right) F_s\left[k_o s' g^- (\psi + \psi') \right] \right\} \tag{17}$$

The remaining task is to find the incident field in terms of either the magnetic field or the electric field. The magnetic incident field is assumed to be the far field at the diffraction point Q_D due to the magnetic current, $\vec{M}(x, y)$ at the aperture and is given as[1]:

$$H^i_\theta(r, \theta, \phi) = -\frac{jk_o}{\eta}\frac{\exp(-jk_o r)}{2\pi r}\cos\theta\left[\tilde{M}_x(k_x, k_y)\cos\phi + \tilde{M}_y(k_x, k_y)\sin\phi\right]\Big|_{Q_D} \tag{18}$$

$$H^i_\phi(r, \theta, \phi) = -\frac{jk_o}{\eta}\frac{\exp(-jk_o r)}{2\pi r}\left[\tilde{M}_y(k_x, k_y)\cos\phi - \tilde{M}_x(k_x, k_y)\sin\phi\right]\Big|_{Q_D} \tag{19}$$

where

$$\tilde{M}_x(k_x, k_y) = \iint_{S_a} M_x(x, y)\exp\{j(k_x x + k_y y)\}\,dxdy \tag{20}$$

and

$$\tilde{M}_y(k_x, k_y) = \iint_{S_a} M_y(x, y)\exp\{j(k_x x + k_y y)\}\,dxdy \tag{21}$$

$$k_x = k_o\cos\phi\sin\theta \tag{22}$$

$$k_y = k_o\sin\phi\sin\theta \tag{23}$$

From figure 1, it may be noted that at the diffraction point Q_D,

$$r = s' = \sqrt{d^2 + D^2}$$

$$\theta = \psi' + 90^o$$

$$\phi = 90^0$$

In equations (18) and (19), image theory is used to account for the aperture in an infinite ground plane.

In terms of the field variable θ, the slope of the incident magnetic field is written as

$$\frac{\partial H_\theta^i}{\partial u} = -\frac{1}{s'}\frac{\partial H_\theta^i}{\partial \theta} \tag{24}$$

for soft polarization and

$$\frac{\partial H_\phi^i}{\partial u} = -\frac{1}{s'}\frac{\partial H_\phi^i}{\partial \theta} \tag{25}$$

for hard polarization. Using the expressions in equations (18) and (19), the partial derivative with respect to θ can be written explicitly as

$$\frac{\partial H_\theta^i}{\partial \theta} = \frac{jk_o}{\eta}\frac{\exp(-jk_o r)}{2\pi r}\left[\,(T_1\cos\phi + T_2\sin\phi)\sin\theta - (S_1\cos\phi + S_2\sin\phi)\cos\theta\right] \tag{26}$$

$$\frac{\partial H_\phi^i}{\partial \theta} = -\frac{jk_o}{\eta}\frac{\exp(-jk_o r)}{2\pi r}\left[S_2\cos\phi - S_1\sin\phi\right] \tag{27}$$

where

$$T_1 = \tilde{M}_x(k_x, k_y) \tag{28}$$

$$T_2 = \tilde{M}_y(k_x, k_y) \tag{29}$$

$$S_1 = \iint_{S_a} M_x(x, y)\;[jk_o(x\cos\phi + y\sin\phi)\cos\theta]\exp[j(k_x x + k_y y)]\,dxdy \tag{30}$$

$$S_2 = \iint_{S_a} M_y(x, y)\;[jk_o(x\cos\phi + y\sin\phi)\cos\theta]\exp[j(k_x x + k_y y)]\,dxdy \tag{31}$$

In writing equations (26) and (27), *Leibnitz's rule* of differentiating under the integral sign is utilized[4].

Allowing the aperture to be lowered to the conducting plane, the slope incident magnetic fields can be computed from equations (26) and (27) with equations (28), (29), (30), (31) for d=0 and $\psi' = 0^o$. The total diffracted magnetic fields can then be computed using either equation (12) or (13) depending on the polarization.

3. Conclusions

Explicit expressions for computing the slope diffraction coefficients are presented. Expressions for the slope of the incident magnetic field are derived and presented. These expressions combined with regular diffraction coefficients can be used to compute the radiation patterns of aperture antennas with a finite ground plane. They are of importance, for predicting the radiation patterns of circularly polarized antennas, where the incident field for one of the polarizations goes to zero at the ground plane.

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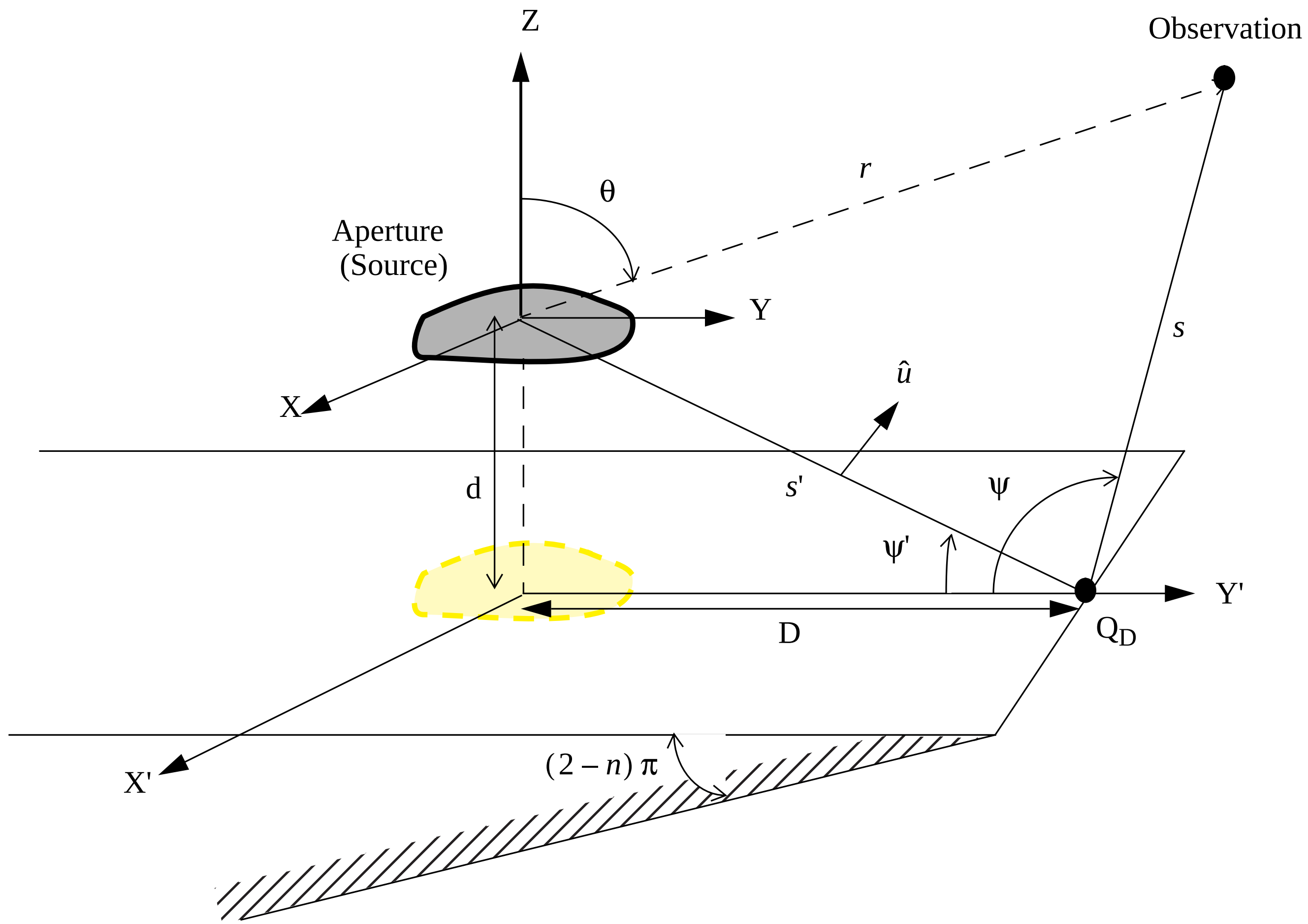

Figure 1. Geometry of the aperture in a finite ground plane.