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Simulated Dynamic Response of a Multi-Stage Compressor with Variable Molecular Weight Flow Medium

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Contents

Nomenclature

A	area
A^*	minimum flow area
CL	effective cylinder length
C_p	molar average constant pressure specific heat
C_v	molar average constant volume specific heat
err	error between process and set point
E	energy
F	molar flow
g_c	conversion constant
ic	initial condition
I_{cmd}	integral command
K	shock loss coefficient
K_p	controller proportional gain
K_i	controller integral gain
M_{wt}	molecular weight
M_a	mach number
N	moles
P	pressure
P_{cmd}	proportional command
PI	proportional integral
Q	heat transfer
R	universal gas constant
strk	piston stroke
t	time
T	temperature
tau	valve time constant
V	volume
w	compressor angular speed
W	work
x	displacement
X	concentration heavy gas in mixture
Y	valve position
Y_{cmd}	command
Y_{dot}	valve velocity

γ	molar average specific heat ratio
ρ	density
Ø	phase angle between cylinders

subscripts

air	air
cond	condensed molar flow
cv	control volume
heat	heat transferred
hg	heavy gas
in	inlet conditions
out	outlet conditions
set	set point
t	stagnation conditions
1, 2	states 1 and 2

Abstract

A mathematical model of a multi-stage compressor with variable molecular weight flow medium is derived. The modeled system consists of a five stage, six cylinder, double acting, piston type compressor. Each stage is followed by a water cooled heat exchanger which serves to transfer the heat of compression from the gas. A high molecular weight gas (CFC-12) mixed with air in varying proportions is introduced to the suction of the compressor. Condensation of the heavy gas may occur in the upper stage heat exchangers. The state equations for the system are integrated using the Advanced Continuous Simulation Language (ACSL) for determining the system's dynamic and steady state characteristics under varying operating conditions.

1.0 Introduction

The Transonic Dynamics Tunnel (TDT) is a large scale wind tunnel located at NASA Langley Research Center. This tunnel utilizes a high molecular weight gas for improved testing characteristics such as higher Reynolds number, lower speed of sound as well as lower power requirements. Currently the tunnel uses CFC-12 (R12) as the test medium. A major project is under way to convert the heavy gas medium from R12, production of which will cease December 1995, to HFC-134a (R134a). However, it was recognized that R134a is combustible under certain conditions of pressure, temperature, and concentration with air.[1]

During down times or model changes, the tunnel and plenum medium may consist of pure air, pure R12, or any ratio of the two gases. The R12 is reclaimed by compressing and cooling the mixture which condenses the heavy gas for storage and future use. During the process of compression, the temperature of the gas mixture increases. Situations may occur during the operation of the compressor which result in discharge temperatures exceeding 500^{o}F, the lower limit auto-ignition temperature for R134a/air mixtures.[1]

A control strategy has been devised to limit the compression ratio across any stage of the compressor, thereby limiting the maximum discharge temperature.

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[1]Babcock, D. A., Bruce, R. A., "Combustibility Tests of 1,1,1,2-tetrafluoroethane in a Simulated Compressor Cylinder." Unplublished Report, 1995.

In order to evaluate the control strategy and compressor operation and performance, a mathematical model of the system was developed. The system was modeled using the Advanced Continuous Simulation Language (ACSL) (Reference 1) computer code to solve the system of nonlinear time dependent ordinary differential equations. The model utilizes a lumped volume approach, solving the continuity of mass and energy equations. Friction loss and restrictions are represented by flow resistance coefficients. Spatial variations are neglected which is a suitable assumption for low velocity systems.

ACSL is a FORTRAN based simulation language convenient for modeling dynamic systems. Variables may be changed at run time to evaluate their impact on system dynamics. A choice of integration routines is available. A fourth order Runga-Kutta routine with fixed time step is currently used. The time step is based on the lowest characteristic time constant of the system.

In addition to control strategy and compressor operation evaluation, ignition of the R134a/air mixture in the compressor cylinders was simulated. This simulation included the ignition of the gaseous mixture due to high cylinder pressure ratios. Relief valve dynamics and system interaction were evaluated to assess resultant pressures.

Model dynamic and steady state accuracy were evaluated by comparing compressor data with simulations. The model was then modified for simulating R134a use and other modifications to the system.

2.0 Model Description

Figure 1 is a schematic of the modeled system. The system consists of four control valves, 7 fixed volumes, 12 volumes with moving boundaries and 32 branch flows. A gaseous mixture flows into the suction volume where the pressure is regulated by a control valve. Flow enters the first stage of the machine which consists of two, double acting cylinders. Each cylinder volume has a suction valve which acts essentially as a check valve limiting flow to one direction. The gas is then compressed and discharged through discharge valves in the cylinder to a common manifold. The gas then enters a water cooled heat exchanger which removes the heat added during the compression cycle. The remaining stages of the machine operate similarly to the first stage but are single cylinders. Typically the pressure in the third stage heat exchanger is high enough to condense heavy gas. A condensation line is shown leaving the third stage heat exchanger. In order to keep high pressure in the third stage, gas is recirculated from the fifth stage discharge to the third stage discharge via a pressure control bypass valve. Flow proceeds to the fourth and fifth stages of the machine where condensation may also occur. Discharge pressure is maintained by a PI pressure control valve at the fifth stage discharge. An overall bypass valve may unload the machine by routing discharge flow to machine inlet. This reduces overall machine pressure ratio and individual stage pressure ratios such that stage discharge temperatures remain in a safe regime.

2.1 Volume Analysis

A schematic of a system with moving boundaries is shown in Figure 2. On its downward stroke, the piston pulls a charge of gaseous mixture through the suction valve into the control volume. The piston then compresses and discharges the gas through the discharge valve into the piping system. Assuming adiabatic walls, the conservation equations of mass and energy are written and manipulated such that the system state variables are pressure and temperature.

Piston position for any time is given by:

$$x = \left(\frac{strk}{2}\right)\sin\left(wt + \phi\right)$$

Instantaneous volume is given by:

$$V_{cv} = (CL - x)\,A_{cv} = \left(CL - \left(\frac{strk}{2}\right)\sin\left(wt + \phi\right)\right)A_{cv}$$

With cylinder length:

$$CL = \left(\frac{1}{2} + \frac{\%clearance}{100}\right)strk$$

Rate of molar storage for the control volume:

$$1)\quad \frac{dN_{cv}}{dt} = \sum F_{in} - \sum F_{out}$$

with total number of moles, $N_{cv} = \int dN_{cv} + N_{cv_{ic}}$

Similarly, the rate of heavy gas molar storage is:

$$\frac{dNhg_{cv}}{dt} = \sum F_{in} X_{in} - \sum F_{out} X_{cv}$$

with total moles of heavy gas given as: $Nhg_{cv} = \int dNhg_{cv} + Nhg_{cv_{ic}}$

The heavy gas concentration in the control volume is then given as:

$$X_{cv} = \frac{Nhg_{cv}}{N_{cv}}$$

Rate of energy storage for the control volume:

$$2)\quad \frac{dE_{cv}}{dt} = \sum C_{p_{in}} F_{in} T_{in} - \sum C_{p_{cv}} F_{out} T_{out} + Q_{cv} - W_{cv}$$

$$\text{with,}\quad W_{cv} = P_{cv} \frac{dV_{cv}}{dt}$$

The derivative of the control volume is evaluated as:

$$3)\quad \frac{dV_{cv}}{dt} = -\left(\frac{strk}{2}\right) w \cos\left(wt + \phi\right) A_{cv}$$

It is assumed the gas behaves as an ideal gas with:

$$P_{cv} = N_{cv} \frac{RT}{V_{cv}}$$

Differentiating:

$$\text{4)} \quad \frac{dP_{cv}}{dt} = \frac{R}{V_{cv}} \left(N_{cv} \frac{dT_{cv}}{dt} + T_{cv} \frac{dN_{cv}}{dt} - \frac{N_{cv} T_{cv}}{V_{cv}} \frac{dV_{cv}}{dt} \right)$$

Differentiate the energy equation, $E_{CV} = N_{CV}\ Cv\ T_{CV}$, and rearrange to get:

$$\text{5)} \quad \frac{dT_{cv}}{dt} = \left(\frac{1}{C_v} \frac{dE_{cv}}{dt} - T_{cv} \frac{dN_{cv}}{dt} \right) \frac{1}{N_{cv}}$$

Equations 1, 2 and 3 along with the equation of state are substituted into Equations 4 and 5 which are than integrated to determine the system temperature and pressure with time. For fixed boundary volumes, dVcv/dt is set to 0.

2.2 Branch Flow Analysis

Flow through a restriction or frictional loss is represented by an incompressible resistance equation of the form:

$$\Delta P = K \rho V^2, \qquad V = \text{gas velocity}$$

Which can be written as:

$$\Delta P = K_1 \frac{T}{P} M_{wt} F^2$$

2.3 Evaluation of Heavy Gas Condensate Flow

Molar air flow through the heat exchanger remains constant while heavy gas flow in the process stream is reduced by the amount of material condensed. For a constant temperature heat exchanger, condensate flow may be determined from inlet and outlet concentrations which are known. Figure 3 shows a schematic of the heat exchanger.

Heavy gas concentration is defined as:

$$x = \frac{F_{hg}}{\left(F_{air} + F_{hg}\right)}, \quad \text{with } F_{air} \text{ being constant for the process}$$

Entering molar flow of Heavy gas is given as:

$$F_{hg1} = F_{air} \frac{X_1}{\left(1 - X_1\right)}$$

Similarly, exiting molar flow of heavy gas is given as:

$$F_{hg2} = F_{air} \frac{X_2}{\left(1 - X_2\right)}$$

Condensate flow is the difference in inlet and outlet heavy gas molar flow:

$$F_{cond} = F_{hg1} - F_{hg2} = F_{air} \left[\frac{X_1\left(1 - X_2\right) - X_2\left(1 - X_1\right)}{\left(1 - X_1\right)\left(1 - X_2\right)} \right]$$

2.4 Valve Dynamics and Control

Control valves in the system are pneumatically positioned. The positioner and valve dynamics together are taken as a first order system. The time constant for the valve dynamic system was either measured or assumed.

Valve response is given by: $$Y_{dot} = \frac{\left(Y_{cmd} - Y\right)}{Tau}$$

This equation is integrated to give valve position:

$$Y = \int Y_{dot}\,dt + ic \qquad \text{limit } Y\Big|_0^1$$

The control system is simple proportional integral (PI) control where the error between the control variable and process variable is: $err = P - P_{set}$

Proportional and integral commands are:

$$P_{cmd} = K_p \, err$$

$$I_{cmd} = \int K_i err \, dt + ic \quad \text{limit } I_{cmd} \Big|_0^1$$ for anti-wind up integral control

with command to the valve given as: $Y_{cmd} = P_{cmd} + I_{cmd}$

2.5 Relief Valve Analysis

The compressor system has relief valves located approximately at the heat exchanger for each stage. These relief valves protect the heat exchangers, which have the lowest rated design pressure in the system. The model evaluates system overpressure due to volume/relief valve interaction.

For the relief valves, compressible nozzle flow analysis is used with mass flow given as:

$$\dot{m} = \frac{P}{RT} A^* M_a \sqrt{\gamma g_c R T M_{wt}}$$

For choked flow, $M_a = 1$, $\frac{A^*}{A} = 1$

For an ideal gas, the ratio of static to stagnation temperature and pressure can be related by specific heat ratios:

$$\frac{P}{P_t} = \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma}{\gamma - 1}}, \qquad \frac{T}{T_t} = \frac{2}{\gamma + 1}$$

Substituting into the flow equation and solving for molar flow gives:

$$F = P_t A \sqrt{\frac{\gamma g_c}{R T_t M_{wt}} \left(\frac{2}{\gamma + 1}\right)^{\left(\frac{\gamma+1}{2\gamma-2}\right)}}$$

This equation agrees very well with the manufacturer's published data of flow versus pressure for these relief valves.

3.0 Method of Solution

A computer model of the above equations (Section 7.0) was generated using the ACSL code. ACSL is written to facilitate the solution of a system of nth order, nonlinear ordinary differential equations which describe a physical system. Special functions are available in addition to standard FORTRAN commands to provide flexibility in the problem description. The code sorts the commands into an executable sequence such that a variable is calculated before it is used, allowing the system of equations to be inputted in a logical sequence. Nine integration algorithms are available (default is fourth order Runga-Kutta) in addition to a user supplied routine. The code assigns lower order derivatives a variable name, then simultaneously solves the system of first order equations and determines the system states. The code allows input of user defined transfer functions for simplified plant or controller simulation. ACSL has extensive graphical capability and allows plotting of data from external files.

4.0 Simulations

Simulations of the compressor system were performed to validate the mathematical model and to evaluate compressor system operating conditions. Pressure transducers located in the suction and discharge side of each stage provide a pressure history of the compressor. Data was compiled for various compressor operations and compared with predicted data.

The operator of the compressor system controls the suction pressure to the machine, discharge pressure of the machine, and 3rd stage discharge pressure. A concentration analyzer monitors the fraction of heavy gas at the machine suction. The pressure and temperature data of the machine and heavy gas analyzer data is recorded on a PC at a rate of six data sets per minute.

4.1 Steady State Analysis

Simulated steady state conditions were initially compared with Dresser-Rand (Reference 2) calculated data and compressor measured data. Several cases of air mode calculated data from Reference 2 were compared with simulated data and were found to agree within 5%. Compressor data collected March 9, 1995 was compared with calculated data. The data from this run includes a steady state condition with air mode, then a transition in inlet concentration and discharge pressure. The measured steady state air mode pressure data was compared with calculated data and is shown in Table 1. Slight adjustments in the interstage pressure loss coefficient were made for better agreement. As can be seen from the table, there is good agreement between calculated and measured steady state data. In addition to the above condition, several steady state cases were run. Table 2 shows calculated conditions typical of nominal compressor operation including effects of inlet concentration variation. The results show good agreement with observed operation. Note that for high R12 concentrations, condensation in the 4th stage goes to zero. This occurs due to the bypass flow from the 5th stage being at 21% F12 mixed with the 66% concentration of the 3rd stage discharge. These two gaseous streams mix for a concentration of 30% which is less than the R12 partial pressure at the 4th stage condensing temperature (43% partial pressure). Also note from the table that the inlet molar flow remains fairly constant with inlet concentration. However, the inlet mass flow increases with molecular weight which is reflected in higher interstage pressure drop.

Figure 4 was constructed to show the effects of increased inlet pressure for a constant discharge pressure of 415 psia. Note that above about 6 psia suction pressure, the 4th stage discharge pressure exceeds the 5th stage discharge pressure by the amount of interstage pressure drop.

Table 1. Steady State Data Comparison: Air Mode Data of 3/9/95

Location	measured (press psia)	calculated (press psia)	difference (%)	measured (temp °F)	calculated (temp °F)	difference* (%)
1st stage suction	2.75	2.75	code input	constant ~ 80 °F		
1st stage discharge	10.12	10.17	0.49	293.30	316.04	3.02
2nd stage suction	7.43	7.59	2.15	constant ~ 80 °F		
2nd stage discharge	36.39	34.92	4.04	382.09	362.84	2.29
3rd stage suction	32.60	31.11	4.57	constant ~80 °F		
3rd stage discharge	204.47	204.47	code input	462.06	450.75	1.23
4th stage suction	190.86	191.11	0.13	constant ~80 °F		
4th stage discharge	380.26	383.27	0.79	204.38	191.54	1.93
5th stage suciton	368.49	371.29	0.76	constant ~80 °F		
5th stage discharge	465.61	465.61	code input	126.47	112.76	2.34

* Base on absolute (Rankine) scale

Table 2. Calculated Steady State Conditions for the Clark Compressor

inlet concentration	0.	21%	33%	50%	66%	95%
P1suc	5.5	5.5	5.5	5.5	5.5	8.00
P1dis	22.07	23.33	23.93	24.59	25.13	34.24
P2suc	17.27	16.90	16.79	16.61	16.47	20.74
P2dis	69.76	72.13	73.89	75.17	76.30	97.25
P3suc	52.32	48.95	48.11	46.51	45.35	52.33
P3dis	200.14	194.39	200.00	200.00	200.00	200.00
P4suc	179.11	164.67	164.83	158.33	152.61	143.90
P4dis	396.2	385.67	390.43	351.45	312.82	375.93
P5suc	375.47	355.46	354.42	314.96	278.89	320.30
P5dis	615.02	615.00	615.01	615.00	615.00	615.01
suction flow	0.129	0.1138	0.1075	0.1006	0.0953	0.1119
F_{cond3}	0.	0.	0.	0.	0.0002	0.1024
F_{cond4}	0.	0.	0.	0.0177	0.0343	0.
F_{cond5}	0.	0.	0.0156	0.0183	0.0187	0.0095
bypas flow	0.	0.	0.00402	0.0088	0.0126	0.07732
X_{in}	0.	0.21	0.33	0.50	0.66	0.95
X_{3rd}	0.	0.21	0.33	0.50	0.659	0.6593
$X_{4th\ inlet}$	0.	0.21	0.325	0.476	0.606	0.2964
X_{4th}	0.	0.21	0.325	0.375	0.4215	0.2964
X_{5th}	0.	0.21	0.214	0.2144	0.2144	0.2144
discharge flow	0.127	0.1136	0.0915	0.0636	0.0406	0.007857

All pressures in psia, all flows in moles/sec.
Fcond3,4,5 - flows from collection pot.
bypas flow - flow from 5th stage discharge to 4th stage suction
When comparing with measured data, suction pressures should be the same for valid comparison.
M_{wt} air = 28.97
M_{wt} F12 = 120.93
Mass flow of collection pots = (moles/sec)*M_{wtF12} = lbm/sec
Mass flow suction, discharge, and bypass = (moles/sec)*(X*M_{wtF12}+(1-X)*M_{wtair}) = lbm/sec

4.2 Transient Analysis

A comparison was made of measured data versus calculated for an operational run conducted March 9, 1995. For this run, approximate steady conditions were achieved with very low inlet concentration of about 1% R12, suction pressure of 2.74 psia, 3rd stage discharge of 205 psia, and a machine discharge pressure of 470 psia. From the steady conditions, inlet pressure was reduced, then increased, held steady, and then increased again. During this period, inlet concentration varied from about 1% up to 92% R12. The 3rd stage discharge pressure and machine discharge pressure were also varied during this period. This transition period covered about $4^{1}/_{2}$ minutes during the run of March 9, 1995. It should be noted that the suction pressure, 3rd stage discharge pressure, and machine back pressure set points are controlled manually by the operator from the control panel. These set points are dialed in by the operator to achieve conditions deemed optimum. The set points are not recorded; however, the resultant process variable is. In order to assess the math model transient performance, the input process variables of inlet concentration, suction pressure, 3rd stage discharge pressure, and machine back pressure were used as set points to the math model. These set points were compared to process variables and the error used to generate a command to the control valve as discussed in Section 2. Figures 5 through 15 compare calculated versus measured data for inlet concentration, stage suction and stage discharge pressures. Recorded data is shown as triangles and calculated data as solid lines.

Comparison of the calculated data with measured data shows agreement within about 20% for variable magnitudes. Some error between the calculated and measured data is due to the method of introducing control set points. Figures 5, 6, 11, and 15 show the control variables. The math model process variables are a result of using measured process variables as set points as opposed to the actual set points; therefore, the differences in control gain modeling can manifest in differences in process variables between measured and calculated. It is, however, concluded that the math model transient response is representative of the actual machine and, therefore, is suitable for use in the prediction of machine response to transient conditions.

4.3 Relief Valve Dynamic Analysis

An analysis of the relief valves and their dynamic interaction with the compressor system was performed for several cases. Case 1 evaluated the relief valve capacity relative to worse case pressure condition resulting from a failure of the discharge valve to the closed position. From steady state conditions of 5.5 psia suction and 615 psia discharge, the back pressure valve was commanded shut at 80 seconds into the simulation. Figure 16 shows valve position with time. The valve completely shuts in about 4 seconds. The discharge pressure, shown in Figure 17, increases as flow continues to fill the discharge volume. Approximately 7 seconds into the event, the 5th stage pressure reached the relief valve cracking pressure of 720 psia and flow through the relief valve was established as shown in Figure 18. The results of this simulation show that the relief valve capacity is adequate for this type of failure.

As stated previously, R134a refrigerant is combustible under certain conditions. From the combustibility tests, a molar heat of combustion for a stoichiometric mixture and approximate flame velocity were determined for a gas/air mixture.[1] The burning dynamics were modeled with ignition occurring at the cylinder discharge. Five simulations were evaluated for a worst case ignition scenario for each stage. From a steady state condition of 5.5 psia suction and 615 psia discharge, the stage flow entering the adjacent upstream stage, or discharge valve for the 5th stage, was set to zero. Simultaneously ignition was initiated at the cylinder discharge. Pressure rises in the trapped gas as combustion proceeds and exit flow is held to zero. Results for the 4th stage analysis are shown in Figures 19 and 20. From steady state conditions, ignition was initiated at 50 seconds, simultaneously setting volume discharge flow to zero. Figure 19 shows 4th stage discharge pressure increase due to combustion and blocked flow to reach approximate steady conditions. The relief valve cracks about $3^1/_2$ seconds into the event. It's molar flow is shown in Figure 20. Results for all cases are shown in Table 3.

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[1]Babcock, D. A., Bruce, R. A., "Combustibility Tests of 1,1,1,2-tetrafluoroethane in a Simulated Compressor Cylinder." Unplublished Report, 1995.

Table 3. Relief Valve Capacity and Stage Overpressure Due To Combustion

stage	1st	2nd	3rd	4th	5th
crack press					
(psig)	35	105	350	600	705
(psia)	49.7	119.7	364.7	614.7	719.7
capacity*					
(SCFM)	24643	3537	13089	22094	
25686					
peak pressure due to comb. (psia)	52	169	450	652	780
max allowable pressure** (psia)	290	620	1620	1960	1960
peak relief valve flow (moles/sec) due to comb.	0.175	0.29	0.30	0.50	0.55

*capacity at 60 deg F and 10% overpressure
**internal pressure at ultimate

4.4 Control System Evaluation

The control system consists of operator set PI pressure control for the suction pressure, the third stage discharge pressure, and machine discharge pressure. Consideration is being given to scheduling third stage pressure and discharge pressure set points as a function of machine suction pressure and inlet concentration. A scheme is implemented to maintain a pressure ratio of 6 or less across any individual stage of the compressor system. That scheme will be considered here.

As discussed, the lower limit for auto-ignition of a R134a/air mixture is about 500 ^{o}F.[1] Such a condition may occur if the isentropic compression ratio for any cylinder exceeds 6.6 with an inlet temperature of 100 ^{o}F and specific heat ratio of 1.4 (air). It should be noted that as the concentration of heavy gas increases, specific heat ratio decreases, greatly increasing required pressure ratio for a 500 ^{o}F discharge temperature. It was decided, however, to use a pressure ratio set point of 6.0 for each cylinder regardless of concentration. Referring to Figure 1, the pressure ratio for each cylinder is calculated and compared to the set point. If the cylinder pressure ratio is less than the set point, the bypass valve from 5th stage discharge to machine suction remains closed. If the cylinder pressure ratio exceeds the set point, the PI controller generates a command to the valve which diverts flow from the machine discharge to suction, proportional to the error. This method of control raises the suction pressure and lowers the discharge pressure such that machine and individual stage pressure ratios are reduced only by that required to maintain a ratio of 6 or less. As the transient or abnormal operating condition is alleviated, normal control and operation is restablished.

An example using the model to predict compressor performance is presented for a scenario in which the suction valve is commanded shut from a normal steady state operating condition. With the simulation at steady state conditions of 5.5 psia suction and 615 psia discharge, the suction pressure set point is set to zero 65 seconds into the simulation. Such an action could occur due to operator error or failure of the suction control valve. Figure 21 shows the action

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[1]Babcock, D. A., Bruce, R. A., "Combustibility Tests of 1,1,1,2-tetrafluoroethane in a Simulated Compressor Cylinder." Unplublished Report, 1995.

of the suction control valve. Figure 22 shows the decrease with time of the suction pressure. Due to the decrease in suction pressure, the pressure ratio (pressure ratio = discharge/suction) across each stage increases with time. The pressure ratio across the 2nd stage is shown in Figure 23. The 2nd stage pressure ratio reaches a value of 6 about 15 seconds into the event and reaches this value before the other stages.

This same simulation was then run with a pressure ratio set point of 6 for any stage. The second stage reaches the limiting ratio first and regulation begins. Figure 24 shows the 2nd stage pressure ratio versus time. As before, the upset begins 65 seconds into the simulation. At about 80 seconds into the simulation, the 2nd stage pressure ratio reaches 6 and regulation begins. From Figure 24, it is seen there is some overshoot and then oscillation in stage pressure ratio as regulation begins. Molar flow from the discharge to the suction side of the machine is shown in Figure 25. The flow appears to oscillate at about .02 moles per second. This flow serves to raise the suction pressure and, therefore, reduce the pressure ratio across each stage. Figure 26 shows machine suction pressure for the controlled case and is compared to Figure 22 which is the uncontrolled suction pressure.

Results of this simulation show that such a control scheme is viable. The model allows controller gain evaluation and simulation of various failure or abnormal operating scenarios.

5.0 Conclusion

A mathematical model of a multi-stage compressor with variable molecular weight flow medium was derived. The equations of state were solved using the Advanced Continuous Simulation Language (ACSL). Results of simulated operation were compared with measured data. The code has input variables and constants defined initially and can be changed interactively to easily assess the impact of hardware changes or changes in control scheme. The code is in a general format making application to other multi-stage compressor systems simple.

6.0 References

1. Mitchell and Gauthier Associates, "Advanced Continous Simulation Language," 10th ed., Concord, Mass, 1991.

2. Dresser Rand Corp, "Report on Performance of CBA-6 Compressor", Sverdrup Technology, Inc., 1993.

7.0 Bibliography

1. Babcock, D. A., "Simulated Dynamic Response of a Servovalve Controlled Hydraulic Actuator", NASA Technical Memorandum 101644, June 1990.

2. Franklin, G. F., Powell, J. D., "Feedback Control of Dynamic Systems", Addison-Wesley Company, Menlo Park, California, 1986.

3. Sonntag, R. E., Van Wylen, G. J., "Introduction to Thermodynamics: Classical and Statistical", John Wiley & Sons, Inc., New York, NY, 1971.

8.0 Code Listing

```
program clark compressor
"23456789012345678921234567893123456789412345678951234567896123456789 7"
"--------------------------------------------------------------------"
"                   TDT CLARK COMPRESSOR PERFORMANCE                 "
"                              ANALYSIS                              "
"                   D. A. BABCOCK  JUNE 1995                         "
"--------------------------------------------------------------------"
"                            nomenclature                            "
" f      flow                                    moles/sec           "
" fdt    rate of change of flow                  moles/sec**2        "
"                                                                    "
" n      total number of moles                       n               "
" ndt    rate of change of moles                   n/sec             "
"                                                                    "
" e      energy                                     btu              "
" edt    rate of change of energy                  btu/sec           "
"                                                                    "
" tc     temperature                               deg R             "
" tcdt   rate of change of temperature             deg R/sec         "
"                                                                    "
" p      pressure                                   psi              "
" pdt    rate of change of pressure                psi/sec           "
"--------------------------------------------------------------------"
"                         constants for air                          "
"                                                                    "
constant cpa =  7.1846     $"btu/(lbmole-deg r), const pres spec ht  "
constant cva =  5.1277     $"btu/(lbmole-deg r), const vol spec ht   "
constant r   =  1545.0     $"ft-lbf/(lbmole-deg r), ideal gas const  "
constant ma  =  28.97      $"lbm/lbmole, molecular wt air            "
"                                                                    "
"--------------------------------------------------------------------"
"                         constants for R-12                         "
"                                                                    "
constant cphg = 24.68      $"btu/(lbmole-deg r), const pres spec ht  "
constant cvhg = 22.69      $"btu/(lbmole-deg r), const vol spec ht   "
constant mhg  = 120.93     $"lbm/lbmole, molecular wt R-12           "
constant vp100 = 131.86    $"psia, sat vapor at 100 deg F            "
constant hliq = 3761.0     $"btu/lbmole, liquid enthalpy             "
"                                                                    "
"--------------------------------------------------------------------"
"                          geometry constants                        "
"                                                                    "
constant a1s1  =  7.0686   $"ft**2, cross section area stage 1 cyl   "
constant a1s2  =  7.0195   $"stage 1 cross section less rod area     "
constant a1s3  =  7.0686   $"ft**2, cross section area stage 1 cyl   "
constant a1s4  =  7.0195   $"stage 1 cross section less rod area     "
constant a2s1  =  4.2761   $"ft**2, cross section area stage 2 cyl   "
constant a2s2  =  4.2270   $"stage 2 cross section less rod area     "
constant a3s1  =  1.3104   $"ft**2, cross section area stage 3 cyl   "
constant a3s2  =  1.2613   $"stage 3 cross section less rod area     "
constant a4s1  =  0.3712   $"ft**2, cross section area stage 4 cyl   "
constant a4s2  =  0.3221   $"stage 4 cross section less rod area     "
constant a5s1  =  0.1963   $"ft**2, cross section area stage 5 cyl   "
constant a5s2  =  0.1473   $"stage 5 cross section less rod area     "
"                                                                    "
```

```
constant strk = 1.4167      $"ft, piston stroke                                   "
constant vef1 = 1.13        $"compression eff                                     "
constant vef2 = 1.1         $"compression eff                                     "
constant vef3 = 1.06        $"compression eff                                     "
constant vef4 = 1.03        $"compression eff                                     "
constant vef5 = 1.02        $"compression eff                                     "
"                                                                                 "
constant v0  =  3284.0      $"ft**3, suction volume                               "
constant v1  =  315.0       $"ft**3, dwn stream first stage                       "
constant v2  =  303.0       $"ft**3, dwn stream second stage                      "
constant v3  =  208.0       $"ft**3, dwn stream third stage                       "
constant v4  =  146.0       $"ft**3, dwn stream fourth stage                      "
constant v5  =  63.0        $"ft**3, dwn stream fifth stage                       "
constant v3m =  10.0        $"ft**3, dwn stream fifth stage                       "
"                                                                                 "
"                 other constants                                                 "
"                                                                                 "
constant pcb1=415.0         $"psia, pressure dwnstrm of back pres vlv             "
constant cg=66.0            $"back pressure valve flow coeficient                 "
constant tstrt = 0.0, tstp = 1.0
"               controller set points                                             "
constant set   = 615.0      $"psia, pressure set point machine disch              "
constant set0  = 5.5        $"psia, pressure set point suction press              "
constant prset = 20.0       $"psia, pressure set point pratio set point"
constant setb  = 0.0        $"psia, pressure set point 3rd stg dis                "
"                 discharge controller                                            "
constant kp = 0.05          $"proportional constant                               "
constant ki = .01           $"integral constant                                   "
constant tau = 0.95         $"sec, valve time constant                            "
"                ratio controller                                                 "
constant kp1 = 0.1          $"proportional constant                               "
constant ki1 = .05          $"integral constant                                   "
constant tau1 = 1.0         $"sec, inlet valve time constant                      "
constant vcnst1 = 12.0      $"psi, constant for flow cntrl valve                  "
"               suction controller                                                "
constant kp0 = 0.13         $"proportional constant                               "
constant ki0 = .055         $"integral constant                                   "
constant tau0 = 0.75        $"sec, inlet valve time constant                      "
constant vcnst = 800.0      $"psi, constant for flow cntrl valve                  "
"               3rd stage pressure controller                                     "
constant kpb = 0.05         $"proportional constant                               "
constant kib = .001         $"integral constant                                   "
constant taub = 1.00        $"sec, inlet valve time constant                      "
constant vcnstb= 800.0      $"psi, constant for flow cntrl valve                  "
"                                                                                 "
"----------------  other constants  ---------------------------------"
"                                                                                 "
constant j = 778.16         $"ft-lbf/btu, conversion factor                       "
constant omega = 31.415927  $"rad/sec, 300 rpm machine                            "
constant pi = 3.1415927     $"rad, constant                                       "
"                                                                                 "
"              relief valve constants                                             "
"                                                                                 "
constant a1 = 25.25 , a2=10.734,a3=1.784,a4=1.784,a5=1.784
" these are the choked areas for the relief valves                                "
"                                                                                 "
constant cp1=49.7   ,fp1=53.2
constant cp2=164.7  ,fp2=179.7
constant cp3=364.7  ,fp3=399.7
constant cp4=614.7  ,fp4=674.7
```

```
constant cp5=719.7   ,fp5=790.2
constant cnst=63.69
" cp is the crack pressure for the relief valve, fp is the full flow  "
" pressure of the relief valve                                         "
"                                                                     "
"    following are the loss coefficients for the compressor valves    "
constant kv0=.15588
constant k1s1up=1.5,k1s2up=1.5,k1s3up=1.5,k1s4up=1.5
constant k1s1dn=1.5,k1s2dn=1.5,k1s3dn=1.5,k1s4dn=1.5
constant k2s1up=2.03,k2s2up=2.03
constant k2s1dn=2.03,k2s2dn=2.03
constant k3s1up=2.9,k3s2up=2.9
constant k3s1dn=2.9,k3s2dn=2.9
constant k4s1up=2.9,k4s2up=2.9
constant k4s1dn=2.9,k4s2dn=2.9
constant k5s1up=2.9,k5s2up=2.9
constant k5s1dn=2.9,k5s2dn=2.9
"---------------------------------------------------------------------"
initial $"initial conditions of state variables                        "
"                                                                     "
los12 = 0.23
los23 = 2.86
los34 = 2.1014
los45 = 19.71
kv3m =  7.434     $"loss coef from vol 3 to vol3 mix chamber           "
"                                                                     "
"       initial temperatures and pressures of fixed volumes            "
"                                                                     "
tv3mic = 560.0  $  tv3m = tv3mic $ pv3mic = 14.7 $ pv3m=pv3mic
tsuc =  560.0   $   psuc = 14.7  $ ycmd0=.5      $ y0=.5
tv0ic = 560.0   $   tv0 = tv0ic  $ pv0ic = 14.7  $ pv0 = pv0ic
tv1ic = 560.0   $   tv1 = tv1ic  $ pv1ic = 14.7  $ pv1 = pv1ic
tv2ic = 560.0   $   tv2 = tv2ic  $ pv2ic = 14.7  $ pv2 = pv2ic
tv3ic = 560.0   $   tv3 = tv3ic  $ pv3ic = 14.7  $ pv3 = pv3ic
tv4ic = 560.0   $   tv4 = tv4ic  $ pv4ic = 14.7  $ pv4 = pv4ic
tv5ic = 560.0   $   tv5 = tv5ic  $ pv5ic = 14.7  $ pv5 = pv5ic
"                                                                     "
pfs1ic=pv0ic $ pfs2ic=pv1ic $ pfs3ic=pv2ic $ pfs4ic=pv3ic
pfs5ic=pv4ic $ pfd1ic=pv1ic $ pfd2ic=pv2ic $ pfd3ic=pv3ic
pfd4ic=pv4ic $ pfd5ic=pv5ic
tfd1ic=tv1ic $ tfd2ic=tv2ic $ tfd3ic=tv3ic $ tfd4ic=tv4ic
tfd5ic=tv5ic
"---------------------------------------------------------------------"
"   initial temperatures and pressures of cylinder volumes            "
" clark 1st stage                                                     "
ts1ic=100. $ ts2ic=100. $ ts3ic=100. $ ts4ic=100. $ ts5ic=100.
ts1=ts1ic $ ts2=ts2ic $ ts3=ts3ic $ ts4=ts4ic $ ts5=ts5ic
t1=ts1 $ t2=ts2 $ t3=ts3 $ t4=ts4 $ t5=ts5
t1s1ic = 560.0 $ t1s1 = t1s1ic $ p1s1ic = 14.7 $ p1s1 = p1s1ic
t1s2ic = 560.0 $ t1s2 = t1s2ic $ p1s2ic = 14.7 $ p1s2 = p1s2ic
t1s3ic = 560.0 $ t1s3 = t1s3ic $ p1s3ic = 14.7 $ p1s3 = p1s3ic
t1s4ic = 560.0 $ t1s4 = t1s4ic $ p1s4ic = 14.7 $ p1s4 = p1s4ic
" clark 2nd stage                                                     "
t2s1ic = 560.0 $ t2s1 = t2s1ic $ p2s1ic = 14.7 $ p2s1 = p2s1ic
t2s2ic = 560.0 $ t2s2 = t2s2ic $ p2s2ic = 14.7 $ p2s2 = p2s2ic
" clark 3rd stage                                                     "
t3s1ic = 560.0 $ t3s1 = t3s1ic $ p3s1ic = 14.7 $ p3s1 = p3s1ic
t3s2ic = 560.0 $ t3s2 = t3s2ic $ p3s2ic = 14.7 $ p3s2 = p3s2ic
" clark 4th stage                                                     "
t4s1ic = 560.0 $ t4s1 = t4s1ic $ p4s1ic = 14.7 $ p4s1 = p4s1ic
```

```
t4s2ic = 560.0 $ t4s2 = t4s2ic $ p4s2ic = 14.7 $ p4s2 = p4s2ic
" clark 5th stage                                                      "
t5s1ic = 560.0 $ t5s1 = t5s1ic $ p5s1ic = 14.7 $ p5s1 = p5s1ic
t5s2ic = 560.0 $ t5s2 = t5s2ic $ p5s2ic = 14.7 $ p5s2 = p5s2ic
"                                                                      "
tin = 540.0  $"all cylinder inlet temperatures are 100 deg f           "
xin = 0.0    $"entering concentration of HG, hg/air moler basis        "
"--------------------- initial moles of HG ------------------------"
nhg0ic = pv0ic*144.*v0/(r*tv0ic)*x0
nhg1ic = pv1ic*144.*v1/(r*tv1ic)*x1
nhg2ic = pv2ic*144.*v2/(r*tv2ic)*x2
nhg3ic = pv3ic*144.*v3/(r*tv3ic)*x3v
nhg4ic = pv4ic*144.*v4/(r*tv4ic)*x4v
nhg5ic = pv4ic*144.*v5/(r*tv5ic)*x5v
"------------------------------------------------------------------"
"                        compressor efficiency                      "
cls1=strk*(.5+.078*vef1)
cls2=strk*(.5+.076*vef2)
cls3=strk*(.5+.070*vef3)
cls4=strk*(.5+.126*vef4)
cls5=strk*(.5+.340*vef5)
end $"of initial section                                               "
dynamic
derivative
"                                                                      "
"------------------- volume 0 analysis ----------------------------"
"                                                                      "
procedural
cp0 = x0*cphg + (1.-x0)*cpa
cv0 = x0*cvhg + (1.-x0)*cva
m0  = x0*mhg  + (1.-x0)*ma
cpin = xin*cphg + (1.-xin)*cpa
end
"                                                                      "
f0 = sqrt(abs(psuc-pv0))*(psuc/tsuc)/kv0*sign(1.0,psuc-pv0)*y0
ydot0=(ycmd0-y0) * tau0
y0 = limint (ydot0,0.0,0.0,1.0)
err0 = -(pv0-set0)
pcmd0=kp0*err0
icmd0=limint(ki0*err0,0.0,0.0,1.0)
ycmd0=bound(0.0,1.0,pcmd0+icmd0)
nv0dt = f0 - (f1s1up+f1s2up+f1s3up+f1s4up) +f5to0
nv0   = pv0*144.0*v0/(r*tv0)
nhg0dt=f0*xin+f5to0*x5-(f1s1up+f1s2up+f1s3up+f1s4up)*x0
nhg0 = integ(nhg0dt,nhg0ic)
x0=nhg0.nv0
tv0dt = (ev0dt/cv0-nv0dt*tv0)/nv0
tv0 = integ(tv0dt,tv0ic)
pv0dt = r/(v0*144.)*(nv0*tv0dt+tv0*nv0dt)
pv0 = integ(pv0dt,pv0ic)
ev0dt = cp0*(f0*tsuc-(f1s1up+f1s2up+f1s3up+f1s4up)*tv0)+cp5*f5to0*tin
"                                                                      "
"--------------------- clark 1st stage analysis -------------------"
"                                                                      "
f1s1up=rsw(psuc1-p1s1.le.0.,0.,sqrt(psuc1-p1s1)/k1s1up)
f1s2up=rsw(psuc1-p1s2.le.0.,0.,sqrt(psuc1-p1s2)/k1s2up)
f1s3up=rsw(psuc1-p1s3.le.0.,0.,sqrt(psuc1-p1s3)/k1s3up)
f1s4up=rsw(psuc1-p1s4.le.0.,0.,sqrt(psuc1-p1s4)/k1s4up)
n1s1dt = f1s1up-f1s1dn
n1s2dt = f1s2up-f1s2dn
```

```
n1s3dt = f1s3up-f1s3dn
n1s4dt = f1s4up-f1s4dn
n1s1 = p1s1*144.*v1s1/(r*t1s1)
n1s2 = p1s2*144.*v1s2/(r*t1s2)
n1s3 = p1s3*144.*v1s3/(r*t1s3)
n1s4 = p1s4*144.*v1s4/(r*t1s4)
t1s1dt=(e1s1dt/cv0-n1s1dt*t1s1)/n1s1
t1s2dt=(e1s2dt/cv0-n1s2dt*t1s2)/n1s2
t1s3dt=(e1s3dt/cv0-n1s3dt*t1s3)/n1s3
t1s4dt=(e1s4dt/cv0-n1s4dt*t1s4)/n1s4
t1s1 = integ(t1s1dt,t1s1ic)
t1s2 = integ(t1s2dt,t1s2ic)
t1s3 = integ(t1s3dt,t1s3ic)
t1s4 = integ(t1s4dt,t1s4ic)
p1s1dt=r/v1s1*(n1s1*t1s1dt+t1s1*n1s1dt-n1s1*t1s1/v1s1*v1s1dt)/144.
p1s2dt=r/v1s2*(n1s2*t1s2dt+t1s2*n1s2dt-n1s2*t1s2/v1s2*v1s2dt)/144.
p1s3dt=r/v1s3*(n1s3*t1s3dt+t1s3*n1s3dt-n1s3*t1s3/v1s3*v1s3dt)/144.
p1s4dt=r/v1s4*(n1s4*t1s4dt+t1s4*n1s4dt-n1s4*t1s4/v1s4*v1s4dt)/144.
p1s1 = integ(p1s1dt,p1s1ic)
p1s2 = integ(p1s2dt,p1s2ic)
p1s3 = integ(p1s3dt,p1s3ic)
p1s4 = integ(p1s4dt,p1s4ic)
"                                                                    "
v1s1=(c1s1*vef1-strk/2.*sin(omega*t))*a1s1
v1s2=(c1s1*vef1-strk/2.*sin(omega*t+pi))*a1s2
v1s3=(c1s1*vef1-strk/2.*sin(omega*t+pi))*a1s3
v1s4=(c1s1*vef1-strk/2.*sin(omega*t))*a1s4
"                                                                    "
v1s1dt=-a1s1*strk/2.*omega*cos(omega*t)
v1s2dt=-a1s2*strk/2.*omega*cos(omega*t+pi)
v1s3dt=-a1s3*strk/2.*omega*cos(omega*t+pi)
v1s4dt=-a1s4*strk/2.*omega*cos(omega*t)
"                                                                    "
e1s1dt=cp0*(f1s1up*tin-f1s1dn*t1s1)-p1s1*v1s1dt*144./j
e1s2dt=cp0*(f1s2up*tin-f1s2dn*t1s2)-p1s2*v1s2dt*144./j
e1s3dt=cp0*(f1s3up*tin-f1s3dn*t1s3)-p1s3*v1s3dt*144./j
e1s4dt=cp0*(f1s4up*tin-f1s4dn*t1s4)-p1s4*v1s4dt*144./j
"                                                                    "
t1=rsw(f1s1dn.gt.0.,t1s1-460.,t1)
"                                                                    "
"------------------------- volume 1 analysis ------------------------"
"                                                                    "
f1s1dn=rsw(p1s1-pv1.le.0.,0.,sqrt(p1s1-pv1)/k1s1dn)
f1s2dn=rsw(p1s2-pv1.le.0.,0.,sqrt(p1s2-pv1)/k1s2dn)
f1s3dn=rsw(p1s3-pv1.le.0.,0.,sqrt(p1s3-pv1)/k1s3dn)
f1s4dn=rsw(p1s4-pv1.le.0.,0.,sqrt(p1s4-pv1)/k1s4dn)
"                                                                    "
nv1dt = (f1s1dn+f1s2dn+f1s3dn+f1s4dn)-(f2s1up+f2s2up)
nv1 = pv1*144.*v1/(r*tv1)
nhg1dt = f1in*x0 - f1out*x1
nhg1 = integ(nhg1dt,nhg1ic)
x1 = nhg1/nv1
tv1dt = (ev1dt/cv0-nv1dt*tv1)/nv1
tv1 = integ(tv1dt,tv1ic)
pv1dt = r/(v1*144.)*(nv1*tv1dt+tv1*nv1dt)
pv1 = integ(pv1dt,pv1ic)
"                                                                    "
ev1dt=cp*(f1s1dn*t1s1+f1s2dn*t1s2+f1s3dn*t1s3+f1s4dn*t1s4)+enrout
enrout = -(f2s1up+f2s2up+flow1)*tv1*cp + qburn1
"                                                                    "
```

```
tgo1 = rsw(t1.gt.500.0.or.tgo1.eq.1.0,1.0,0.0)
qburn1=hcomb*(f1s1dn+f1s2dn+f1s3dn+f1s4dn+.00054*pv1)*tgo1
ratio1=(pv1-cp1)/(fp1-cp1)
full1=rsw(pv1-cp1.le.0.,0.,fp1*a1/sqrt(tv1)/cnst)
flow1=rsw(ratio1.lt.1.0,ratio1*full1,pv1/fp1*full1)
"                                                                      "
"                                                                      "
"----------------------- clark 2nd stage analysis ----------------"
"                                                                      "
f2s1up=rsw(psuc2-p2s1.le.0.,0.,sqrt(psuc2-p2s1)/k2s1up)
f2s2up=rsw(psuc2-p2s2.le.0.,0.,sqrt(psuc2-p2s2)/k2s2up)
n2s1dt = f2s1up-f2s1dn
n2s2dt = f2s2up-f2s2dn
n2s1 = p2s1*144.*v2s1/(r*t2s1)
n2s2 = p2s2*144.*v2s2/(r*t2s2)
t2s1dt=(e2s1dt/cv0-n2s1dt*t2s1)/n2s1
t2s2dt=(e2s2dt/cv0-n2s2dt*t2s2)/n2s2
t2s1 = integ(t2s1dt,t2s1ic)
t2s2 = integ(t2s2dt,t2s2ic)
p2s1dt=r/v2s1*(n2s1*t2s1dt+t2s1*n2s1dt-n2s1*t2s1/v2s1*v2s1dt)/144.
p2s2dt=r/v2s2*(n2s2*t2s2dt+t2s2*n2s2dt-n2s2*t2s2/v2s2*v2s2dt)/144.
p2s1 = integ(p2s1dt,p2s1ic)
p2s2 = integ(p2s2dt,p2s2ic)
"                                                                      "
v2s1=(cls2*vef2-strk/2.*sin(omega*t))*a2s1
v2s2=(cls2*vef2-strk/2.*sin(omega*t+pi))*a2s2
"                                                                      "
v2s1dt=-a2s1*strk/2.*omega*cos(omega*t)
v2s2dt=-a2s2*strk/2.*omega*cos(omega*t+pi)
"                                                                      "
e2s1dt=cp0*(f2s1up*tin-f2s1dn*t2s1)-p2s1*v2s1dt*144./j
e2s2dt=cp0*(f2s2up*tin-f2s2dn*t2s2)-p2s2*v2s2dt*144./j
"                                                                      "
t2=rsw(f2s1dn.gt.0.,t2s1-460.,t2)
"                                                                      "
"------------------------ volume 2 analysis ------------------------"
"                                                                      "
f2s1dn=rsw(p2s1-pv2.le.0.,0.,sqrt(p2s1-pv2)/k2s1dn)
f2s2dn=rsw(p2s2-pv2.le.0.,0.,sqrt(p2s2-pv2)/k2s2dn)
"                                                                      "
nv2dt = (f2s1dn+f2s2dn)-(f3s1up+f3s2up)
nv2 = pv2*144.*v2/(r*tv2)
nhg2dt = f2in*x0 - f2out*x2
nhg2 = integ(nhg2dt,nhg2ic)
x2 = nhg2/nv2
tv2dt = (ev2dt/cv0-nv2dt*tv2)/nv2
tv2 = integ(tv2dt,tv2ic)
pv2dt = r/(v2*144.)*(nv2*tv2dt+tv2*nv2dt)
pv2 = integ(pv2dt,pv2ic)
"                                                                      "
ev2dt=cp*(f2s1dn*t2s1+f2s2dn*t2s2-(f3s1up+f3s2up+flow2)*tv2)+qburn2
"                                                                      "
tgo2 = rsw(t2.gt.500.0.or.tgo2.eq.1.0,1.0,0.0)
qburn2=hcomb*(f2s1dn+f2s2dn+.00032*pv2)*tgo2
ratio2=(pv2-cp2)/(fp2-cp2)
full2=rsw(pv2-cp2.le.0.,0.,fp2*a2/sqrt(tv2)/cnst)
flow2=rsw(ratio2.lt.1.0,ratio2*full2,pv2/fp2*full2)
"                                                                      "
"----------------------- clark 3rd stage analysis ----------------"
"                                                                      "
```

```
f3s1up=rsw(psuc3-p3s1.le.0.,0.,sqrt(psuc3-p3s1)/k3s1up)
f3s2up=rsw(psuc3-p3s2.le.0.,0.,sqrt(psuc3-p3s2)/k3s2up)
n3s1dt = f3s1up-f3s1dn
n3s2dt = f3s2up-f3s2dn
n3s1 = p3s1*144.*v3s1/(r*t3s1)
n3s2 = p3s2*144.*v3s2/(r*t3s2)
t3s1dt=(e3s1dt/cv0-n3s1dt*t3s1)/n3s1
t3s2dt=(e3s2dt/cv0-n3s2dt*t3s2)/n3s2
t3s1 = integ(t3s1dt,t3s1ic)
t3s2 = integ(t3s2dt,t3s2ic)
p3s1dt=r/v3s1*(n3s1*t3s1dt+t3s1*n3s1dt-n3s1*t3s1/v3s1*v3s1dt)/144.
p3s2dt=r/v3s2*(n3s2*t3s2dt+t3s2*n3s2dt-n3s2*t3s2/v3s2*v3s2dt)/144.
p3s1 = integ(p3s1dt,p3s1ic)
p3s2 = integ(p3s2dt,p3s2ic)
"                                                                    "
v3s1=(cls3*vef3-strk/2.*sin(omega*t+pi))*a3s1
v3s2=(cls3*vef3-strk/2.*sin(omega*t))*a3s2
"                                                                    "
v3s1dt=-a3s1*strk/2.*omega*cos(omega*t+pi)
v3s2dt=-a3s2*strk/2.*omega*cos(omega*t)
"                                                                    "
e3s1dt=cp0*(f3s1up*tin-f3s1dn*t3s1)-p3s1*v3s1dt*144./j
e3s2dt=cp0*(f3s2up*tin-f3s2dn*t3s2)-p3s2*v3s2dt*144./j
"                                                                    "
t3=rsw(f3s1dn.gt.0.,t3s1-460.,t3)
"                                                                     "
"------------------------ volume 3 analysis ------------------------"
"                                                                     "
f3s1dn=rsw(p3s1-pv3.le.0.,0.,sqrt(p3s1-pv3)/k3s1dn)
f3s2dn=rsw(p3s2-pv3.le.0.,0.,sqrt(p3s2-pv3)/k3s2dn)
"                                                                     "
nv3dt = (f3s1dn+f3s2dn)-(f3out)
nv3 = pv3*144.*v3/(r*tv3)
nhg2dt = f2in*x0 - f2Out*x2
nhg2 = integ(nhg2dt,nhg2ic)
x2 = nhg2/nv2
tv3dt = (ev3dt/cv0-nv3dt*tv3)/nv3
tv3 = integ(tv3dt,tv3ic)
pv3dt = r/(v3*144.)*(nv3*tv3dt+tv3*nv3dt)
pv3 = integ(pv3dt,pv3ic)
"                                                                    "
ev3dt=cp0*((f3s1dn*t3s1+f3s2dn*t3s2)-(f3out)*tv3)+qburn3
"                                                                    "
tgo3 = rsw(t3.gt.500.0.or.tgo3.eq.1.0,1.0,0.0)
qburn3=hcomb*(f3s1dn+f3s2dn+.000116*pv3)*tgo3
ratio3=(pv3-cp3)/(fp3-cp3)
full3=rsw(pv3-cp3.le.0.,0.,fp3*a3/sqrt(tv3)/cnst)
flow3=rsw(ratio3.lt.1.0,ratio3*full3,pv3/fp3*full3)
"                                                                    "
"                                                                    "
"------------ conditions leaving 3rd stage after cooler ------------"
"                                                                    "
x3calc=vp100/pv3
x3=rsw(pv3.gt.vp100.and.x3calc.lt.x0,x3calc,x0)
dp3m=pv3-pv3m
f3out=realpl(0.5,sqrt(abs(dp3m)*pv3/(tv3*kv3m*m0))*sign(1.,dp3m),0.0)
f3air=f3out*(1.-x0)
fcond3=f3air*(x0*(1.-x3)-x3*(1.-x0))/(1.-x0)/(1.-x3)
f3mix= f3out - fcond3
"                                                                    "
```

```
"----------5th stage discharge to 4th stage suction valve-----------"
"                                                                    "
"            valve dynamics and controller, assume 1st order         "
"                                                                    "
ydotb = (ycmdb-yb)*taub
yb=bound(0.,1.,limint(ydotb,0.0,0.0,1.0))
fbypas = yb*(pv5-pv3m)/vcnstb
errb=rsw(pv5.lt.pv3m,-pv3,setb-pv3)
pcmdb=kpb*errb
icmdb=limint(kib*errb,0.0,0.0,1.0)
ycmdb=bound(0.0,1.0,pcmdb+icmdb)
"                                                                    "
"----------------  volume 3 mixing chamber  ------------------------"
"                                                                    "
procedural
x3m=rsw(f3mix+fbypas.le.0.,0.,(f3mix*x3+fbypas*x5)/(f3mix+fbypas))
cp3m=x3m*cphg + (1.-x3m)*cpa
cv3m=x3m*cvhg + (1.-x3m)*cva
m3m=x3m*mhg + (1.-x3m)*ma
end
"                                                                    "
nv3mdt = f3mix - (f4s1up+f4s2up) + fbypas
nv3m = pv3m*144.*v3m/(r*tv3m)
tv3mdt=(ev3mdt/cv3m-nv3mdt*tv3m)/nv3m
tv3m=integ(tv3mdt,tv3mic)
pv3mdt = r/(v3m*144.)*(nv3m*tv3mdt+tv3m*nv3mdt)
pv3m = integ(pv3mdt,pv3mic)
ev3mdt=cp3m*((f3mix+fbypas)*tin-(f4s1up+f4s2up)*tv3m)
"                                                                    "
"------------------------ clark 4th stage analysis -----------------"
"                                                                    "
f4s1up=rsw(psuc4-p4s1.le.0.,0.,sqrt(psuc4-p4s1)/k4s1up)
f4s2up=rsw(psuc4-p4s2.le.0.,0.,sqrt(psuc4-p4s2)/k4s2up)
n4s1dt = f4s1up-f4s1dn
n4s2dt = f4s2up-f4s2dn
n4s1 = p4s1*144.*v4s1/(r*t4s1)
n4s2 = p4s2*144.*v4s2/(r*t4s2)
t4s1dt=(e4s1dt/cv3m-n4s1dt*t4s1)/n4s1
t4s2dt=(e4s2dt/cv3m-n4s2dt*t4s2)/n4s2
t4s1 = integ(t4s1dt,t4s1ic)
t4s2 = integ(t4s2dt,t4s2ic)
p4s1dt=r/v4s1*(n4s1*t4s1dt+t4s1*n4s1dt-n4s1*t4s1/v4s1*v4s1dt)/144.
p4s2dt=r/v4s2*(n4s2*t4s2dt+t4s2*n4s2dt-n4s2*t4s2/v4s2*v4s2dt)/144.
p4s1 = integ(p4s1dt,p4s1ic)
p4s2 = integ(p4s2dt,p4s2ic)
"                                                                    "
v4s1=(cls4*vef4-strk/2.*sin(omega*t))*a4s1
v4s2=(cls4*vef4-strk/2.*sin(omega*t+pi))*a4s2
"                                                                    "
v4s1dt=-a4s1*strk/2.*omega*cos(omega*t)
v4s2dt=-a4s2*strk/2.*omega*cos(omega*t+pi)
"                                                                    "
e4s1dt=cp3m*(f4s1up*tin-f4s1dn*t4s1)-p4s1*v4s1dt*144./j
e4s2dt=cp3m*(f4s2up*tin-f4s2dn*t4s2)-p4s2*v4s2dt*144./j
"                                                                    "
t4=rsw(f4s1dn.gt.0.,t4s1-460.,t4)
"                                                                    "
"------------------------- volume 4 analysis -----------------------"
"                                                                    "
f4s1dn=rsw(p4s1-pv4.le.0.,0.,sqrt(p4s1-pv4)/k4s1dn)
```

```
f4s2dn=rsw(p4s2-pv4.le.0.,0.,sqrt(p4s2-pv4)/k4s2dn)
"                                                                          "
nv4dt = (f4s1dn+f4s2dn)-(f5s1up+f5s2up) - fcond4
nv4 = pv4*144.*v4/(r*tv4)
nhg4dt = f4in*x0 - f4Out*x4
nhg4 = integ(nhg4dt,nhg4ic)
x4 = nhg4/nv4
tv4dt = (ev4dt/cv3m-nv4dt*tv4)/nv4
tv4 = integ(tv4dt,tv4ic)
pv4dt = r/(v4*144.)*(nv4*tv4dt+tv4*nv4dt)
pv4 = integ(pv4dt,pv4ic)
"                                                                         "
ev4dt=cp3m*(f4s1dn*t4s1+f4s2dn*t4s2-(f5s1up+f5s2up+fcond4)*tv4)+qburn4
tgo4 = rsw(t4.gt.500.0.or.tgo4.eq.1.0,1.0,0.0)
qburn4=hcomb*(f4s1dn+f4s2dn+.000116*pv4)*tgo4
ratio4=(pv4-cp4)/(fp4-cp4)
full4=rsw(pv4-cp4.le.0.,0.,fp4*a4/sqrt(tv4)/cnst)
flow4=rsw(ratio4.lt.1.0,ratio4*full4,pv4/fp4*full4)
"                                                                         "
"-------------- 4th stage condensate flow and concentration               "
"                                                                         "
x4calc=vp100/pv4
x4=rsw(pv4.gt.vp100.and.x4calc.lt.x3m,x4calc,x3m)
cp4 = x4*cphg +(1.-x4)*cpa
cv4 = x4*cvhg +(1.-x4)*cva
m4 = x4*mhg +(1.-x4)*ma
f4air = (f4s1dn+f4s2dn)*(1.-x3m)
fcond4=f4air*(x3m*(1.-x4)-x4*(1.-x3m))/(1.-x3m)/(1.-x4)
"                                                                         "
"------------------------ clark 5th stage analysis -----------------"
"                                                                         "
f5s1up=rsw(psuc5-p5s1.le.0.,0.,sqrt(psuc5-p5s1)/k5s1up)
f5s2up=rsw(psuc5-p5s2.le.0.,0.,sqrt(psuc5-p5s2)/k5s2up)
n5s1dt = f5s1up-f5s1dn
n5s2dt = f5s2up-f5s2dn
n5s1 = p5s1*144.*v5s1/(r*t5s1)
n5s2 = p5s2*144.*v5s2/(r*t5s2)
t5s1dt=(e5s1dt/cv4-n5s1dt*t5s1)/n5s1
t5s2dt=(e5s2dt/cv4-n5s2dt*t5s2)/n5s2
t5s1 = integ(t5s1dt,t5s1ic)
t5s2 = integ(t5s2dt,t5s2ic)
p5s1dt=r/v5s1*(n5s1*t5s1dt+t5s1*n5s1dt-n5s1*t5s1/v5s1*v5s1dt)/144.
p5s2dt=r/v5s2*(n5s2*t5s2dt+t5s2*n5s2dt-n5s2*t5s2/v5s2*v5s2dt)/144.
p5s1 = integ(p5s1dt,p5s1ic)
p5s2 = integ(p5s2dt,p5s2ic)
"                                                                          "
v5s1=(cls5*vef5-strk/2.*sin(omega*t+pi))*a5s1
v5s2=(cls5*vef5-strk/2.*sin(omega*t))*a5s2
"                                                                          "
v5s1dt=-a5s1*strk/2.*omega*cos(omega*t+pi)
v5s2dt=-a5s2*strk/2.*omega*cos(omega*t)
"                                                                          "
e5s1dt=cp4*(f5s1up*tin-f5s1dn*t5s1)-p5s1*v5s1dt*144./j
e5s2dt=cp4*(f5s2up*tin-f5s2dn*t5s2)-p5s2*v5s2dt*144./j
"                                                                          "
t5=rsw(f5s1dn.gt.0.,t5s1-460.,t5)
"                                                                          "
"------------------------ volume 5 analysis ------------------------"
"                                                                          "
f5s1dn=rsw(p5s1-pv5.le.0.,0.,sqrt(p5s1-pv5)/k5s1dn)
```

```
f5s2dn=rsw(p5s2-pv5.le.0.,0.,sqrt(p5s2-pv5)/k5s2dn)
"                                                                          "
nv5dt =  (f5s1dn+f5s2dn) - (fvlv+f5to0+fbypas+fcond5)
nv5 = pv5*144.*v5/(r*tv5)
nhg5dt = f5in*x4 - f5out*x5
nhg5 = integ(nhg5dt,nhg5ic)
x5 = nhg5/nv5
tv5dt = (ev5dt/cv4-nv5dt*tv5)/nv5
tv5 = integ(tv5dt,tv5ic)
pv5dt = r/(v5*144.)*(nv5*tv5dt+tv5*nv5dt)
pv5 = integ(pv5dt,pv5ic)
"                                                                          "
ev5dt=cp4*(f5s1dn*t5s1+f5s2dn*t5s2-(fvlv+f5to0+fbypas+fcond5)*tv5)...
+qburn5
tgo5 = rsw(t5.gt.500.0.or.tgo5.eq.1.0,1.0,0.0)
qburn5=hcomb*(f5s1dn+f5s2dn+.00006*pv5)*tgo5
ratio5=(pv5-cp5)/(fp5-cp5)
full5=rsw(pv5-cp5.le.0.,0.,fp5*a5/sqrt(tv5)/cnst)
flow5=rsw(ratio5.lt.1.0,ratio5*full5,pv5/fp5*full5)
"                                                                          "
"--------------------  volume 5 discharge conditions ---------------"
"                                                                          "
x5calc = vp100/pv5
x5=rsw(pv5.gt.vp100.and.x5calc.lt.x4,x5calc,x4)
f5air=(f5s1dn+f5s2dn)*(1.-x4)
fcond5=f5air*(x4*(1.-x5)-x5*(1.-x4))/(1.-x4)/(1.-x5)
cp5 = x5*cphg + (1.-x5)*cpa
"------------------determine suction pressures-----------------------"
f12=realpl(1.,((f2s1up+f2s2up)**2.*los12*(tin/pv1)*m1),0.)
f23=realpl(1.,((f3s1up+f3s2up)**2.*los23*(tin/pv2)*m2),0.)
f34=realpl(1.,((f4s1up+f4s2up)**2.*los34*(tin/pv3m)*m3m),0.)
f45=realpl(1.,((f5s1up+f5s2up)**2.*los45*(tin/pv4)*m4),0.)
ts1=realpl(1.,t1,ts1ic)
ts2=realpl(1.,t2,ts2ic)
ts3=realpl(1.,t3,ts3ic)
ts4=realpl(1.,t4,ts4ic)
ts5=realpl(1.,t5,ts5ic)
liq3=realpl(1.,fcond3,0.)
liq4=realpl(1.,fcond4,0.)
liq5=realpl(1.,fcond5,0.)
con3=x3
con3m=x3m
con4=x4
con5=x5
pr1=pfd1/pfs1
pr2=pfd2/pfs2
pr3=pfd3/pfs3
pr4=pfd4/pfs4
pr5=pfd5/pfs5
psuc1 = pv0
psuc2 = pv1-f12
psuc3 = pv2-f23
psuc4 = pv3m-f34
psuc5 = pv4-f45
pfs1=realpl(1.,pv0,pfs1ic)
pfs2=realpl(1.,psuc2,pfs2ic)
pfs3=realpl(1.,psuc3,pfs3ic)
pfs4=realpl(1.,psuc4,pfs4ic)
pfs5=realpl(1.,psuc5,pfs5ic)
pfd1=realpl(1.,pv1,pfd1ic)
```

```
pfd2=realpl(1.,pv2,pfd2ic)
pfd3=realpl(1.,pv3,pfd3ic)
pfd4=realpl(1.,pv4,pfd4ic)
pfd5=realpl(1.,pv5,pfd5ic)
tfd1=realpl(1.,tv1,tfd1ic)-460.
tfd2=realpl(1.,tv2,tfd2ic)-460.
tfd3=realpl(1.,tv3,tfd3ic)-460.
tfd4=realpl(1.,tv4,tfd4ic)-460.
tfd5=realpl(1.,tv5,tfd5ic)-460.
"---------------------- back pressure control valve -----------------"
"          valve dynamics and controller, assume 1st order            "
ydot = (ycmd - y) * tau
y = limint(ydot,0.0, 0.0, 1.0)
open=.4881*y-1.2738*y**2.+1.7857*y**3.
dp=pv5-pcb1
yex=1.0-0.75*dp/pv5
fvlv=rsw(dp.le.0.,0.0,cg*yex*.000227*sqrt(dp*pv5/m0)*open)
err = pv5-set
pcmd=kp*err
icmd=limint(ki*err,0.0,0.0,1.0)
ycmd=bound(0.0,1.0,pcmd+icmd)
"                                                                     "
"---------------- discharge to suction bypass valve -----------------"
"          valve dynamics and controller, assume 1st order            "
ydot1 = (ycmd1 - y1) * tau1
y1 = limint(ydot1,0.0, 0.0, 1.0)
open1=.4881*y1-1.2738*y1**2.+1.7857*y1**3.
f5to0 = (pv5)/vcnst1/sqrt(tv5)*open1
err1 = rsw(pr1-pr2.ge.0.,pr1-prset,pr2-prset)
pcmd1=kp1*err1
icmd1=limint(ki1*err1,0.0,0.0,1.0)
ycmd1=bound(0.0,1.0,pcmd1+icmd1)
"                                                                     "
termt(t.ge.tstp)
end $"of derivative"
end $"of dynamic"
end $"of program"
```

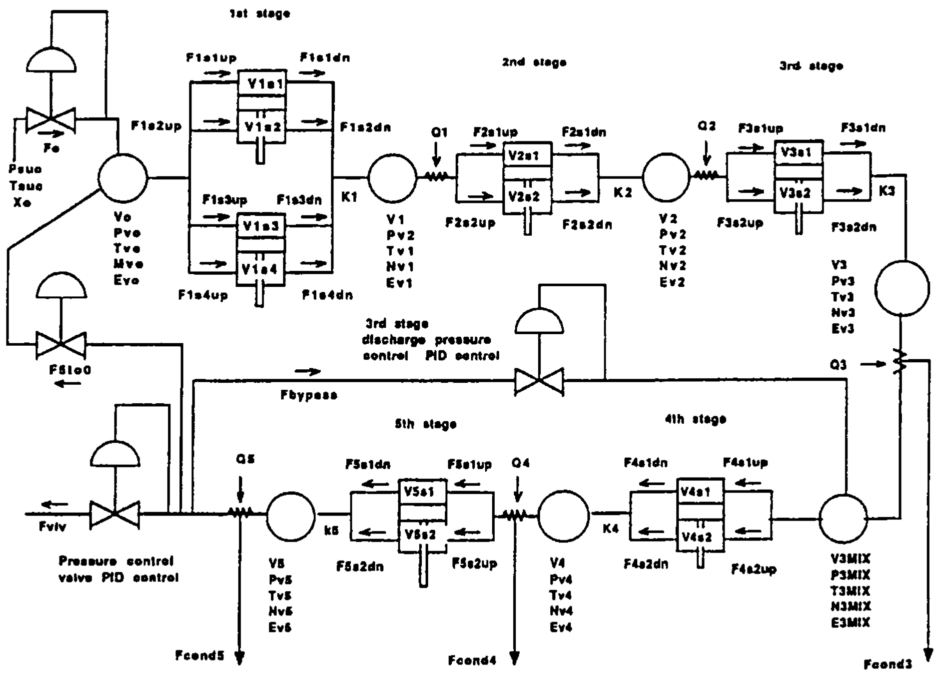

Figure 1. Model schematic

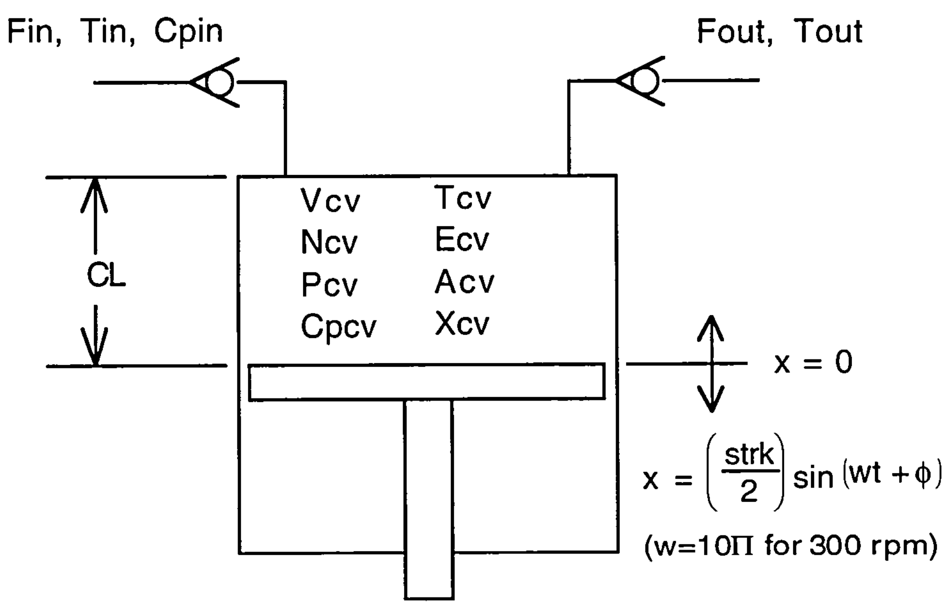

Figure 2. Moving boundary analysis

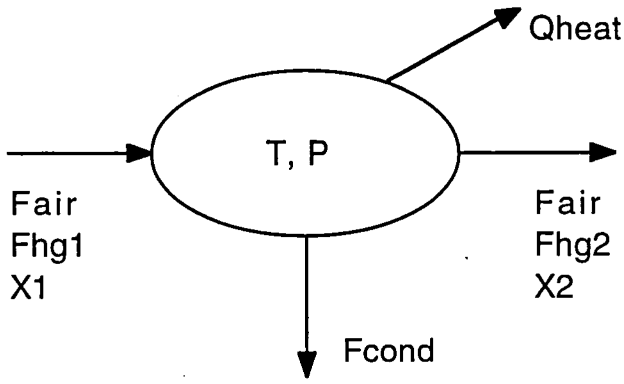

Figure 3. Heat exchanger schematic.

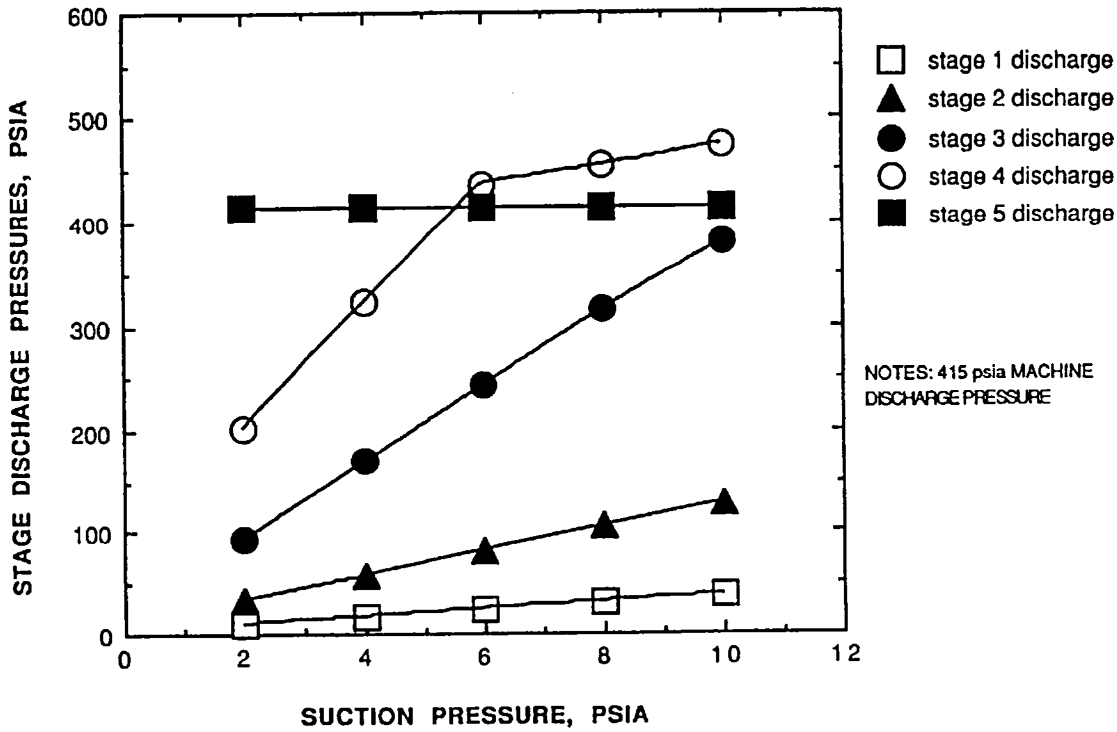

Figure 4. Calculated steady state stage discharge pressures vs. suction pressure: air mode.

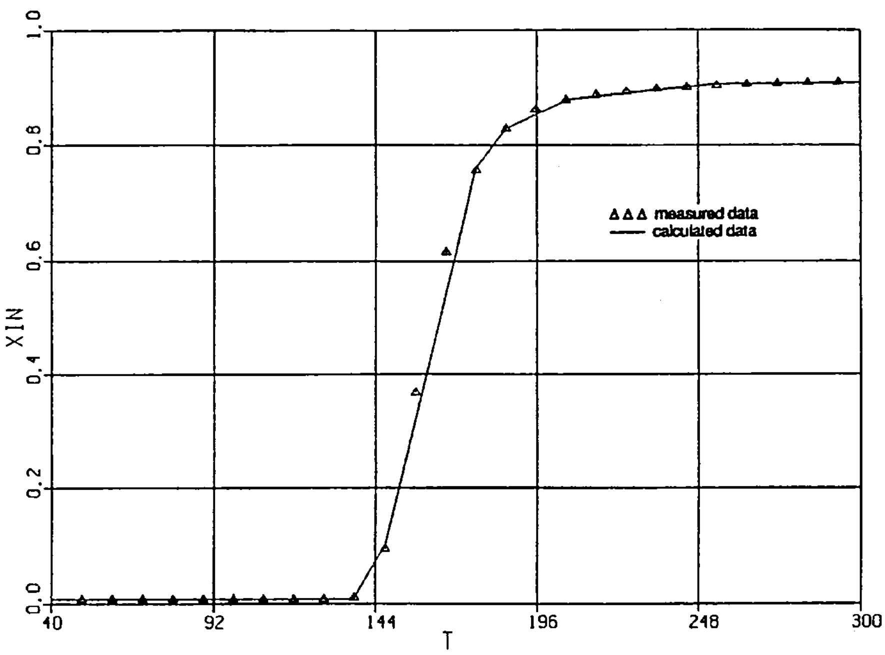

Figure 5. Machine inlet concentration vs. time.

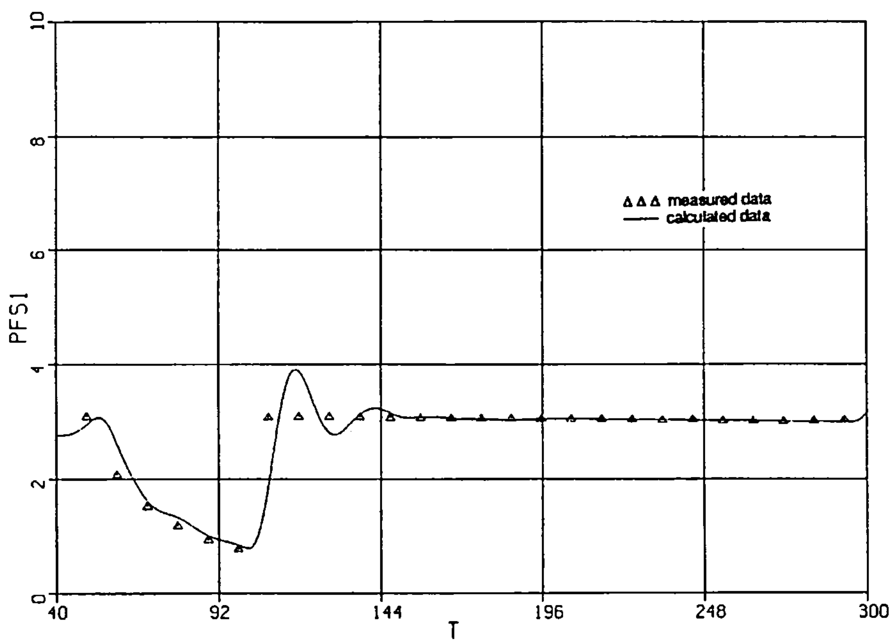

Figure 6. Machine suction pressure vs. time.

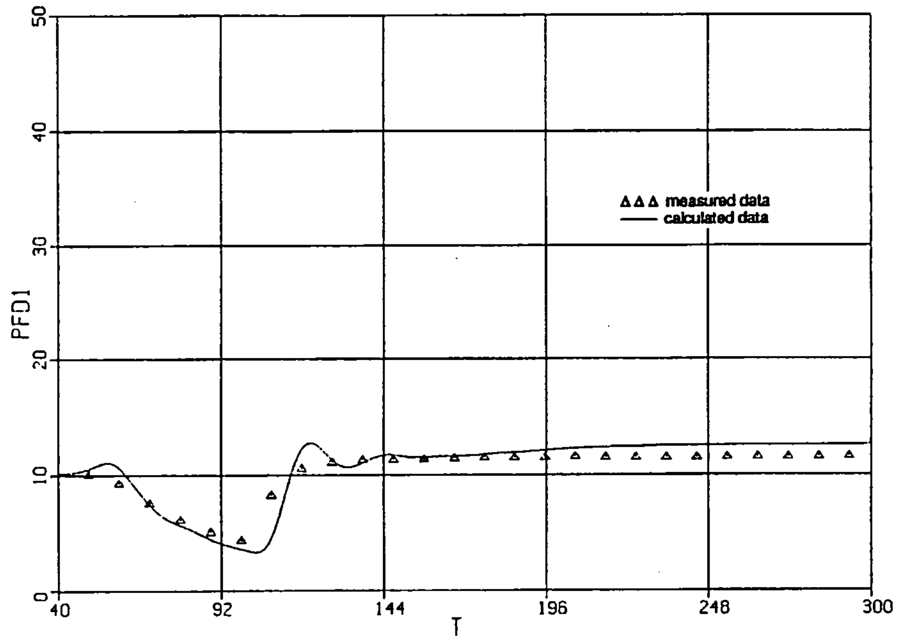

Figure 7. First stage discharge pressure vs. time.

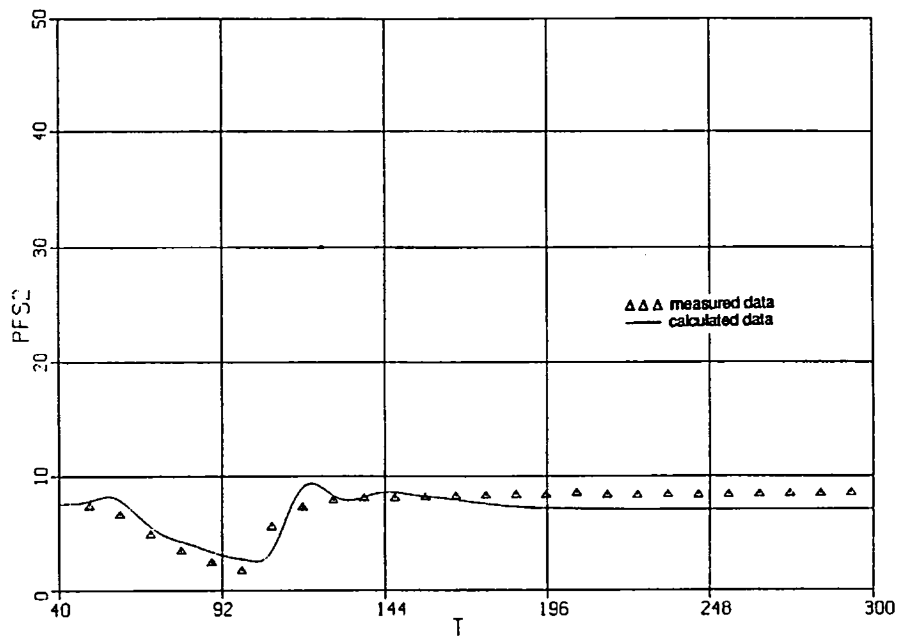

Figure 8. Second stage suction pressure vs. time.

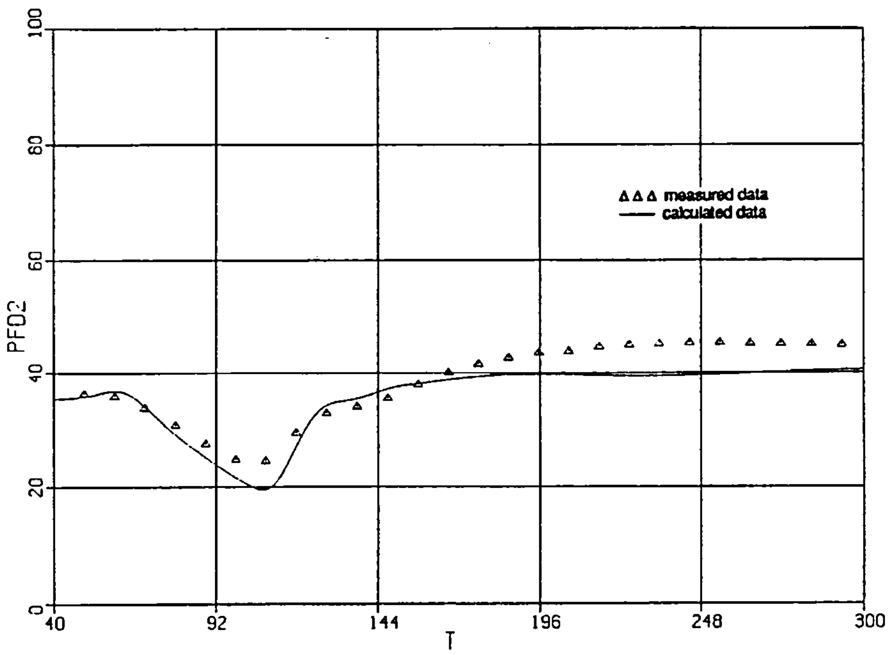

Figure 9. Second stage discharge pressure vs. time.

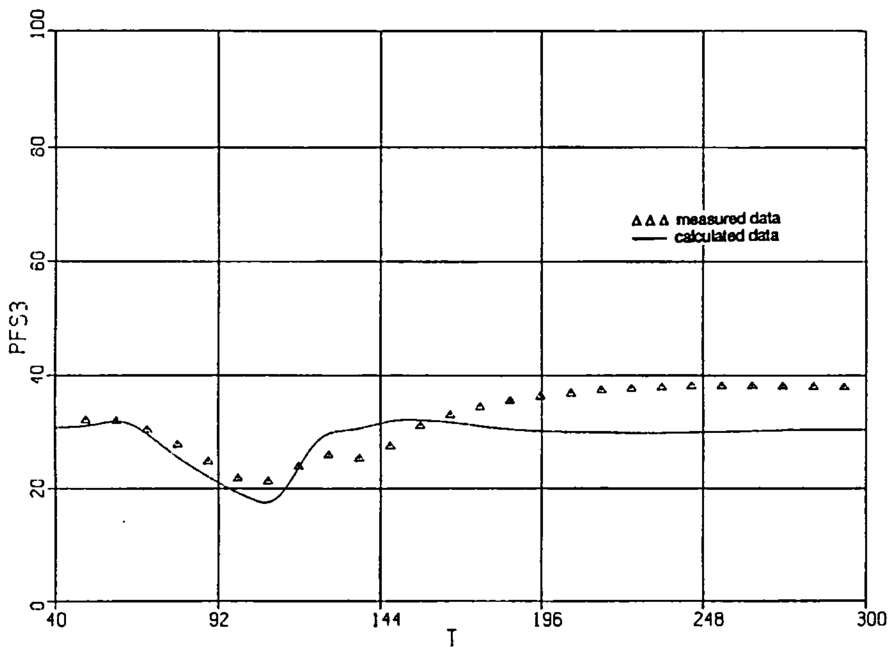

Figure 10. Third stage suction pressure vs. time.

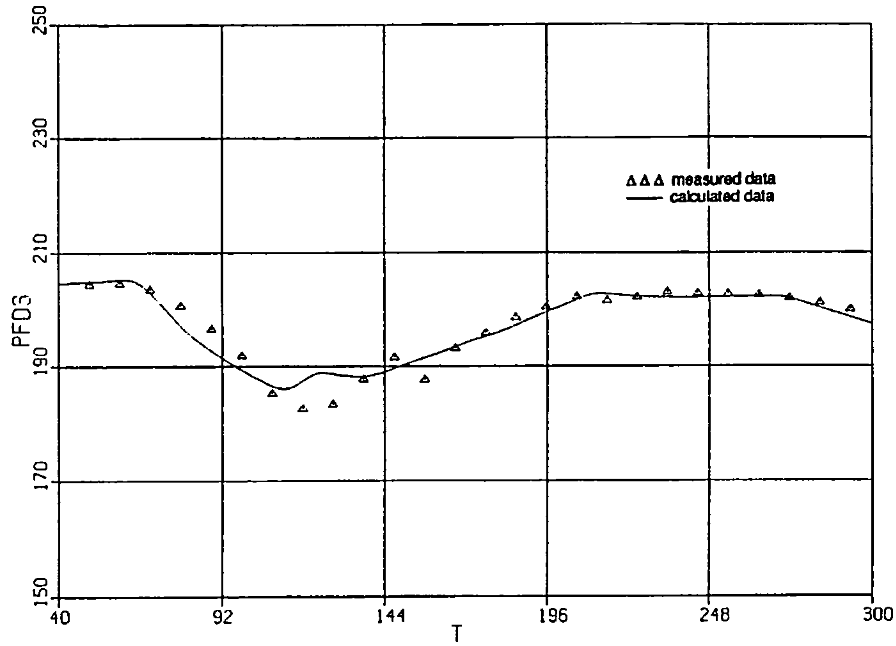

Figure 11. Third stage discharge pressure vs. time.

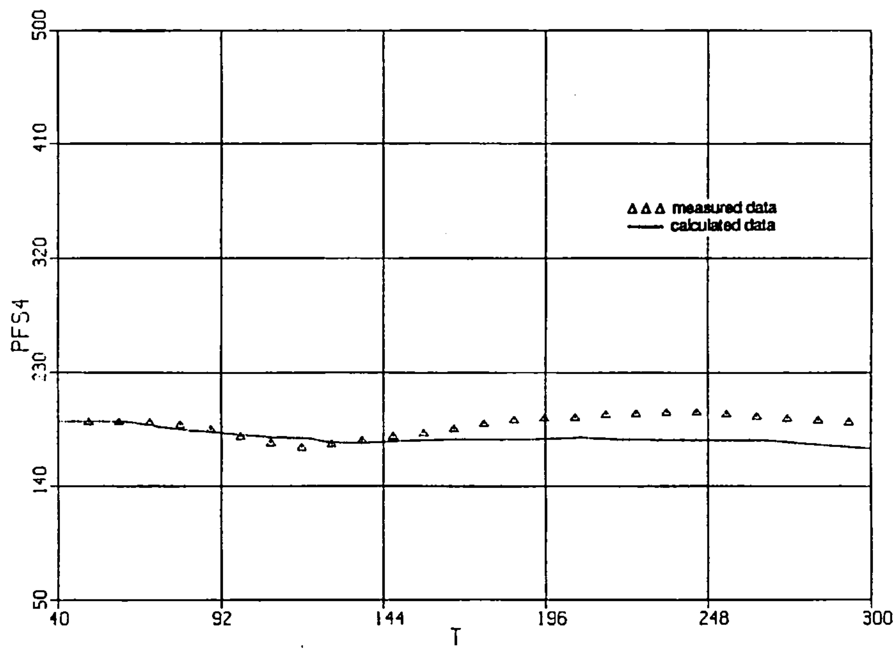

Figure 12. Fourth stage suction pressure vs. time.

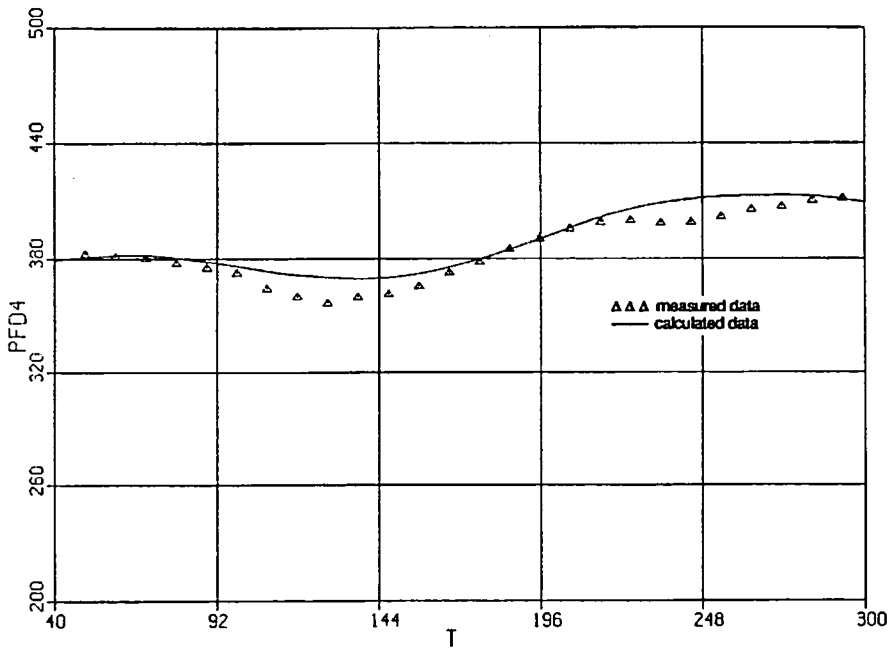

Figure 13. Fourth stage discharge pressure vs. time.

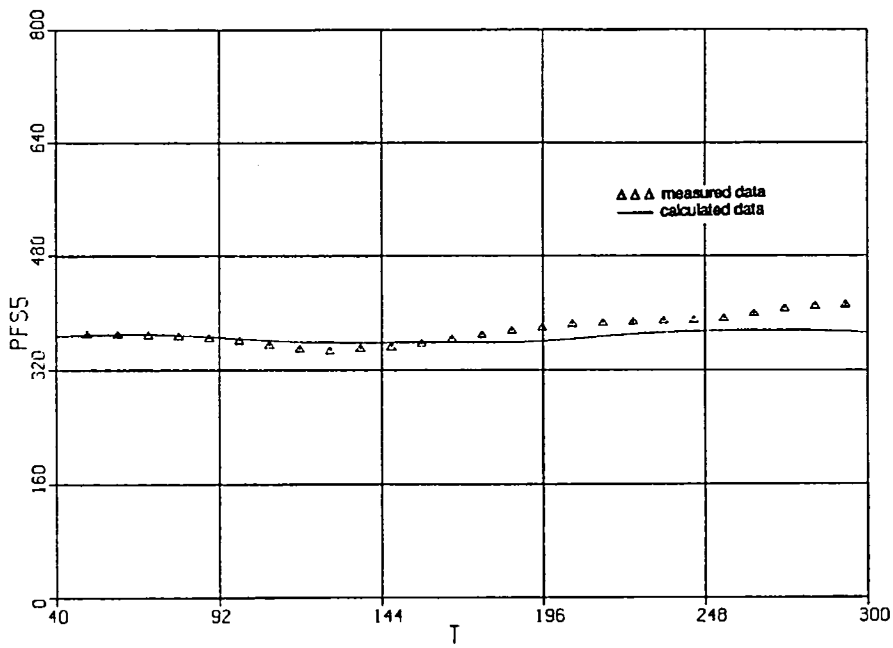

Figure 14. Fifth stage suction pressure vs. time.

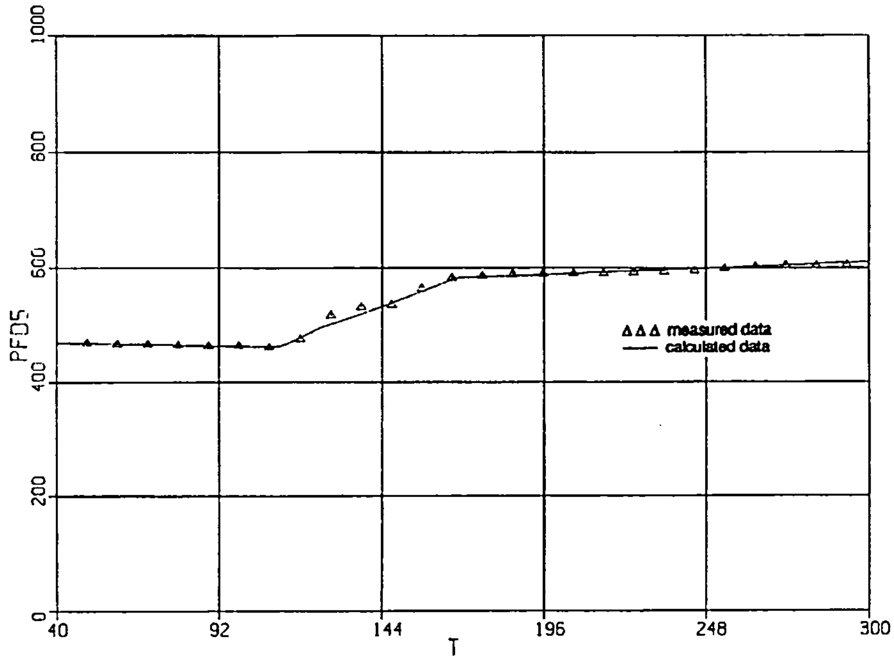

Figure 15. Fifth stage discharge pressure vs. time.

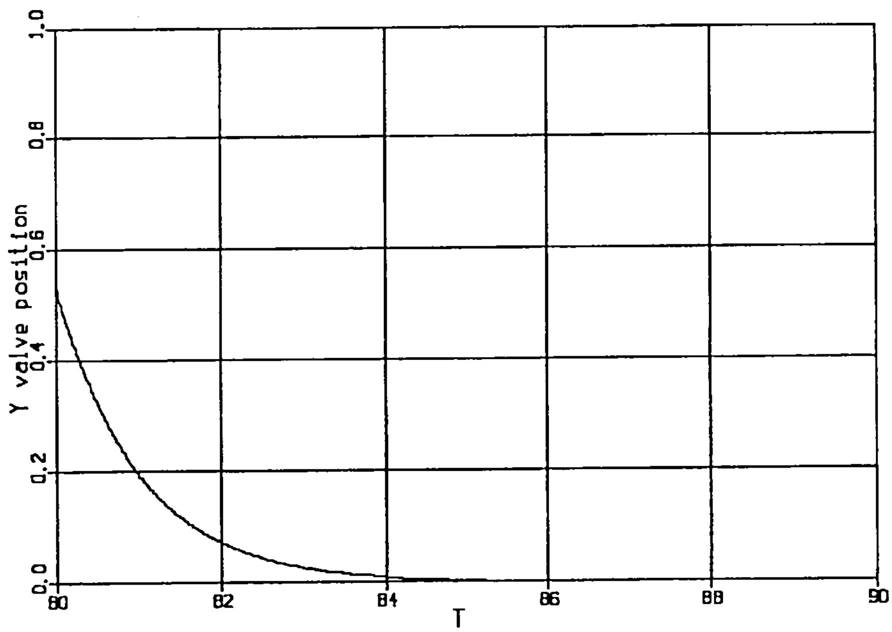

Figure 16. Discharge valve position vs. time.

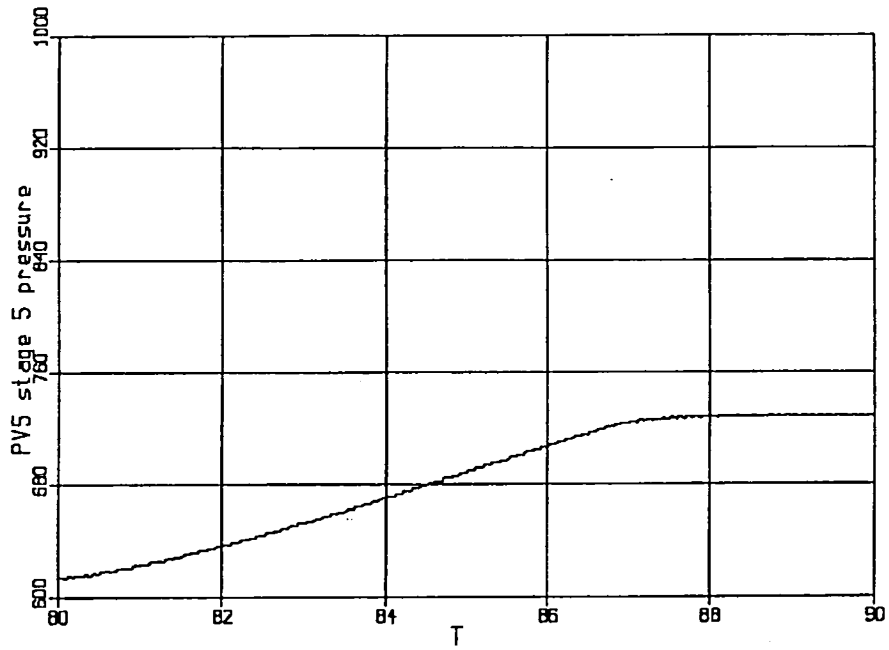

Figure 17. Fifth stage discharge pressure vs. time.

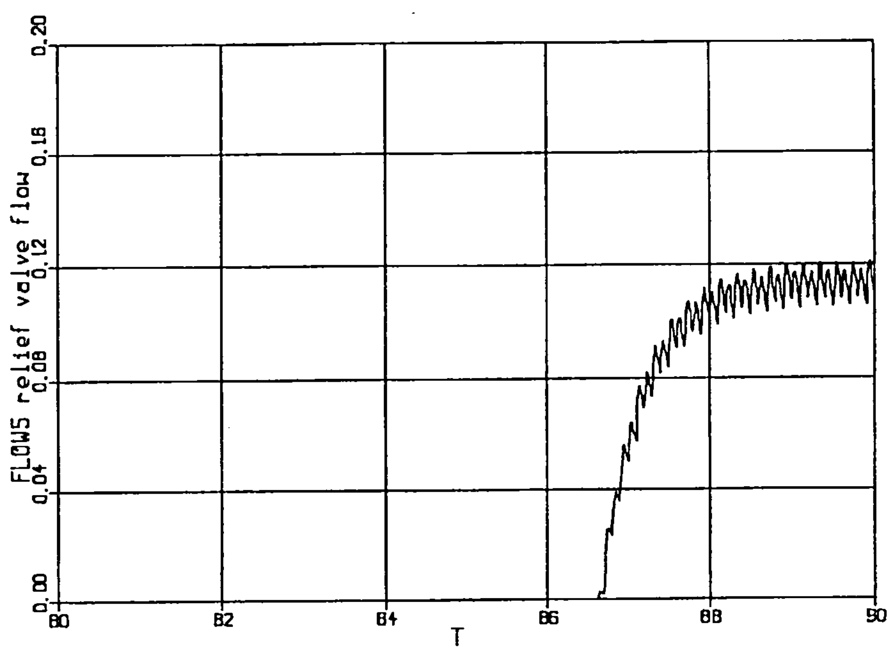

Figure 18. Fifth stage relief valve molar flow vs. time.

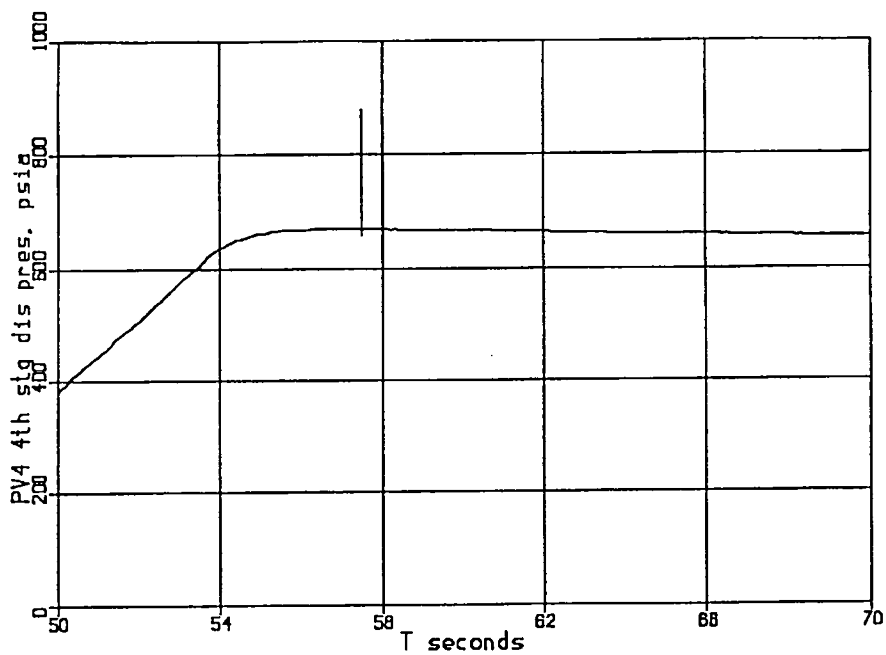

Figure 19. Fourth stage discharge pressure vs. time.

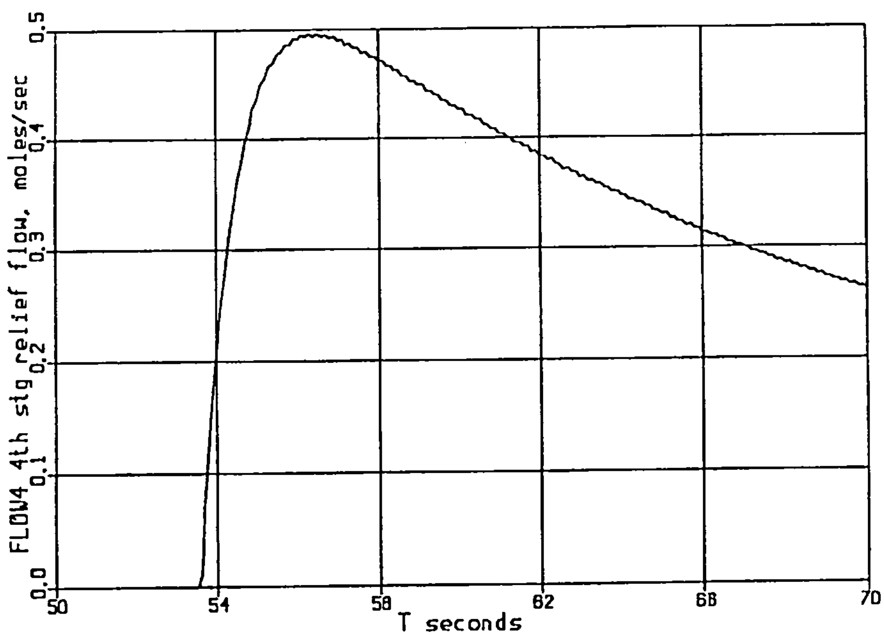

Figure 20. Fourth stage relief valve molar flow vs. time.

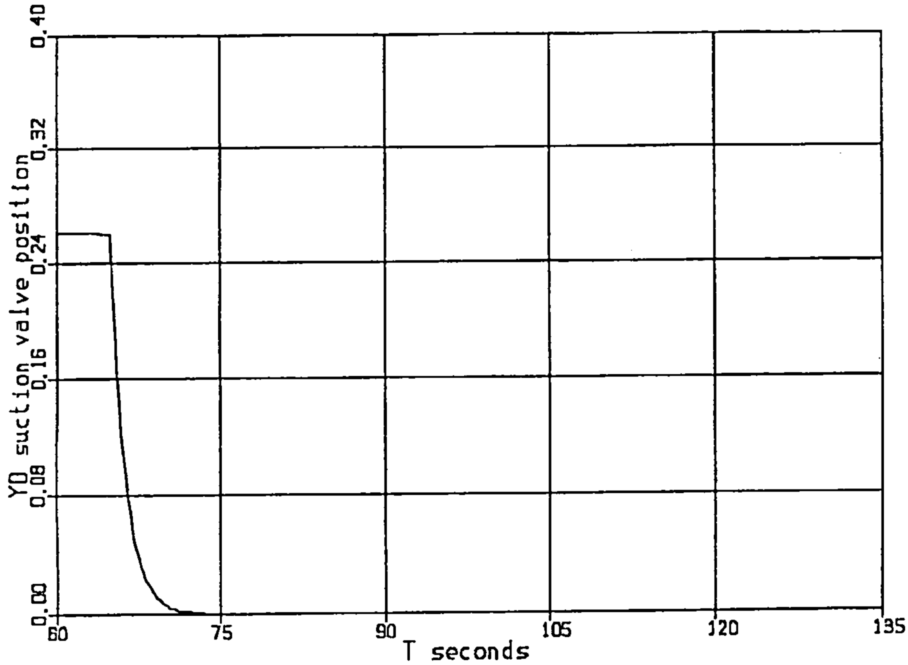

Figure 21. Suction valve position vs. time.

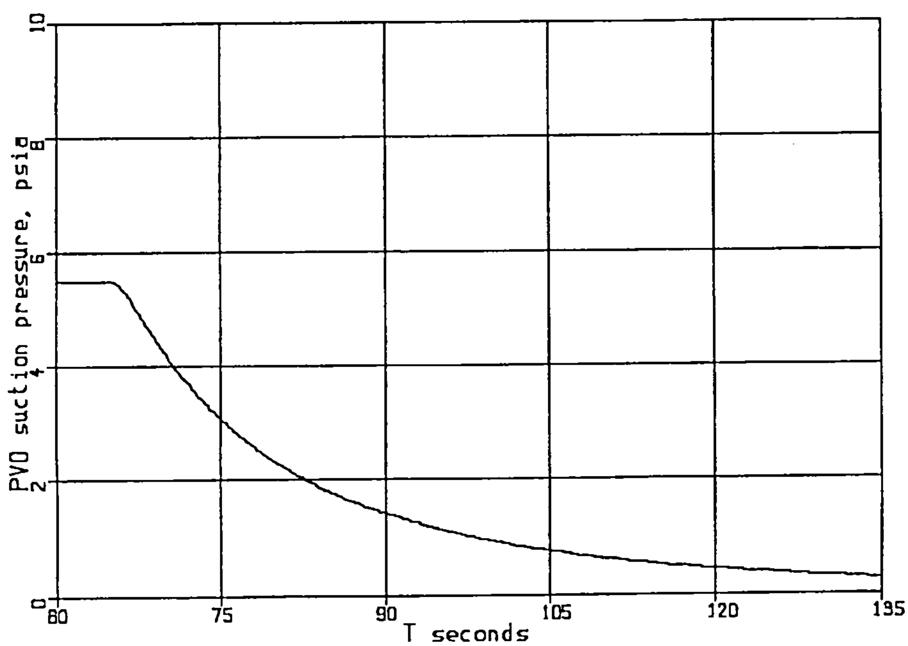

Figure 22. First stage uncontrolled suction pressure vs. time.

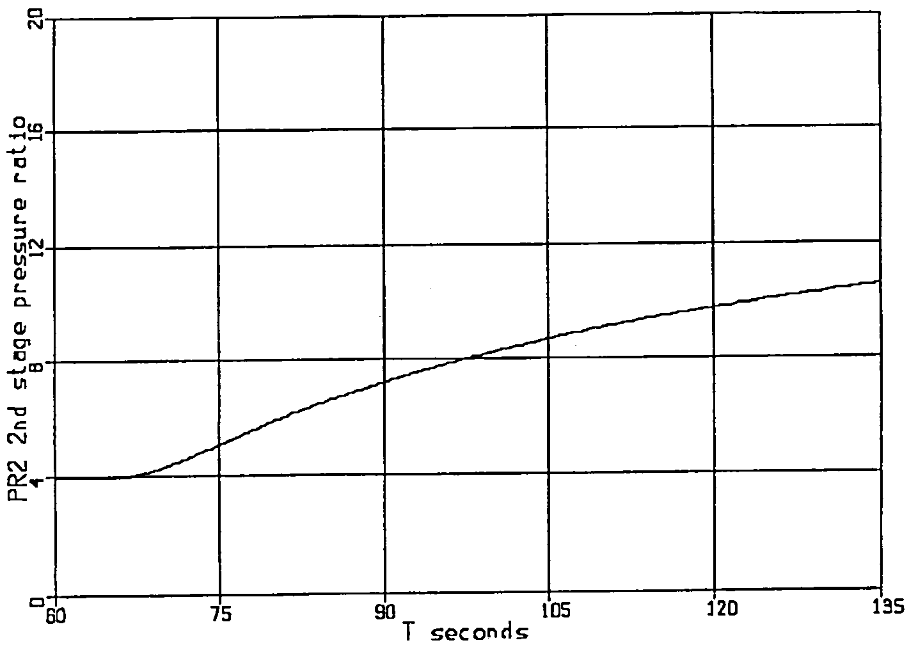

Figure 23. Second stage uncontrolled pressure ratio vs. time.

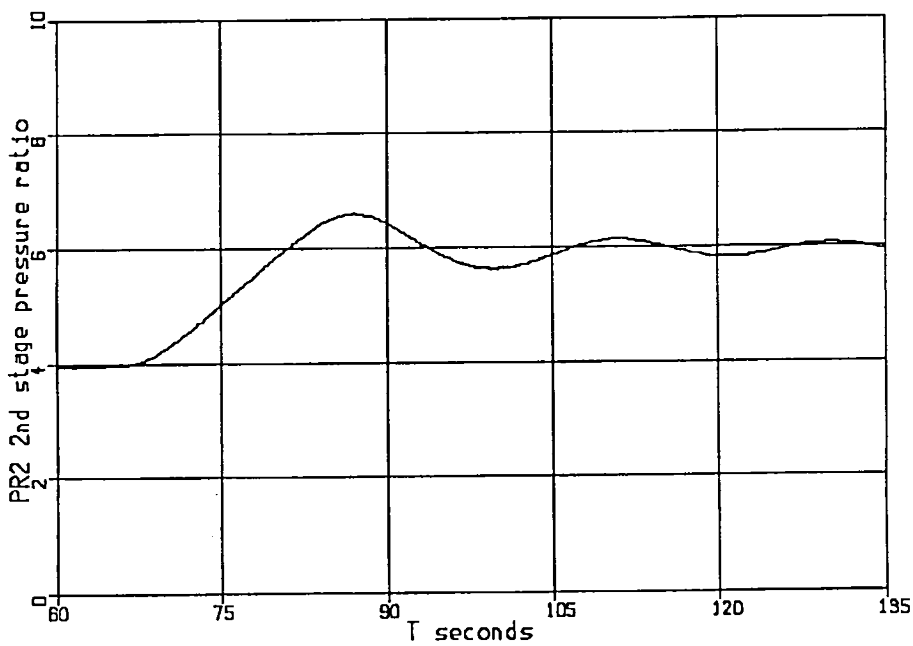

Figure 24. Second stage controlled pressure ratio vs. time.

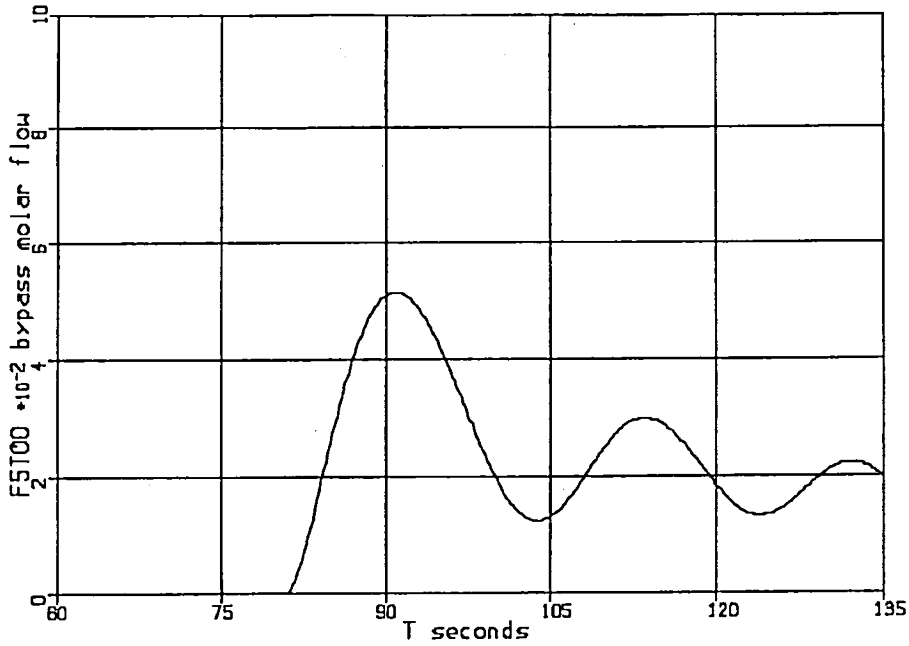

Figure 25. Bypass molar flow vs. time.

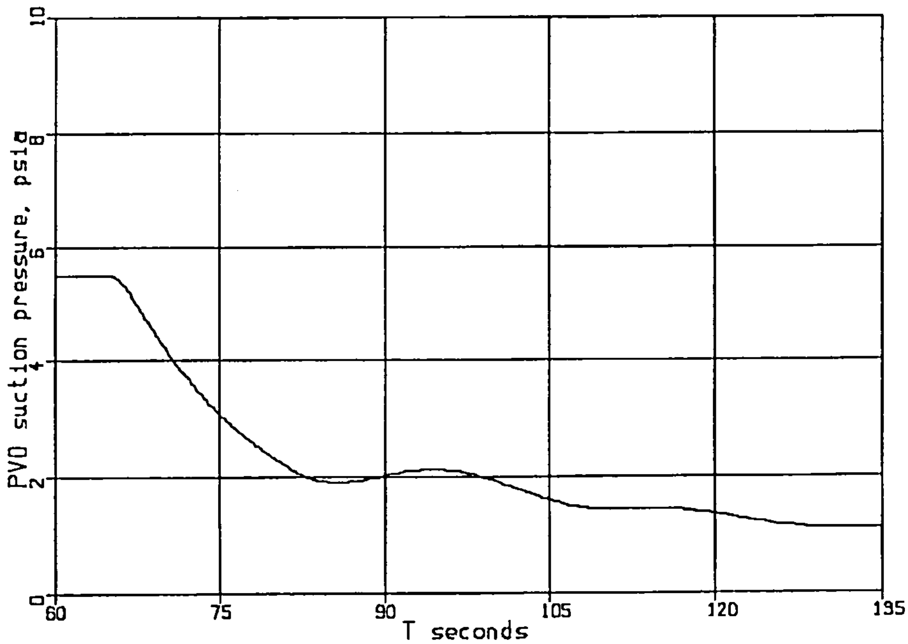

Figure 26. First stage controlled suction pressure vs. time.

REPORT DOCUMENTATION PAGE

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2. REPORT DATE
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Simulated Dynamic Response of a Multi-Stage Compressor with Variable Molecular Weight Flow Medium

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A mathematical model of a multi-stage compressor with variable molecular weight flow medium is derived. The modeled system consists of a five stage, six cylinder, double acting, piston type compressor. Each stage is followed by a water cooled heat exchanger which serves to transfer the heat of compression from the gas. A high molecular weight gas (CFC-12) mixed with air in varying proportions is introduced to the suction of the compressor. Condensation of the heavy gas may occur in the upper stage heat exchangers. The state equations for the system are integrated using the Advanced Continuous Simulation Language (ACSL) for determining the system's dynamic and steady state characteristics under varying operating conditions.

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