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**DESIGN PROTOCOLS AND ANALYTICAL STRATEGIES  
THAT INCORPORATE STRUCTURAL RELIABILITY MODELS**

*Final Report*

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## *ATTACHED SUMMARY*

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#### *Section I - Description of the Problem that Motivated the Technology Development*

Historically, most new materials have established viable markets and found commercial success through a linear product development cycle. The materials scientist would develop a new material system, prototypical components were then fabricated and tested, data bases would be established, and design methodologies were developed in a sequential process. This linear product development approach was adequate during the cold-war era when large research and development budgets spawned a number of successful high-technology material systems (e.g., stealth materials, smart materials, the utilization of composite materials in the air frames of jet fighters, etc.). However, as American industry struggles to reinvent itself in the post-cold war era, artifacts such as the linear material development cycle are being systematically discarded. The current political climate, reduced budgets, and the need to develop dual-use technology all demand that economic issues will dominate the direction of materials research and development. The materials community, which includes material scientists and product design engineers (both at the national research labs and within American industry), must form integrated design teams that utilize an assortment of multi-disciplinary skills. In addition, these integrated design teams must involve end-users early in the development cycle to ensure economic viability. A primary goal in forming an integrated design team is a reduction in the material development cycle. The competitiveness of American material suppliers and their product end-users demands that the cycle for product development must be shortened. If a reduction in time-to-market is achieved, the direct results are more American jobs and an improved economic position for American industries in today's global market.

A reduction in the development cycle requires that the new integrated design teams must embrace the concepts of concurrent engineering. Moreover, to establish a concurrent engineering infrastructure for ceramic matrix composites (which is the primary material highlighted in this research proposal) as well as other promising material systems (e.g., inter-metallics, particulate toughened composites, etc.) design guidelines must be established early through codes and standards organizations such as ASTM and ASME. Unless there is a tremendous cost saving or system enhancement (e.g., the use of ceramic matrix composites in jet engines which will lead to a significant

reduction in NO<sub>x</sub> emissions) product engineers will not utilize a new material until they are comfortable knowing that an appropriate design practice has been codified. The reader need only study the commercialization (or lack thereof) of polymer matrix composites and carbon-carbon composites to find the evidence to support this last statement. In addition to the reduction in the development cycle, design engineering has also served as a key to mitigating costs in litigation involving product liability. However, too often the primary industry motivation for executing design studies focuses on product liability, and product enhancements are considered as secondary gains.

Another historical perspective that must be reconsidered is that most programmatic resources are usually committed to processing and coupon testing in the initial stages of the development cycle (the "make and break" mind set). In spite of great improvements in accuracy through the use of computers, design methods, which can be equally critical in establishing the commercial success of a material, have been treated as afterthoughts. Early investment in design and development technologies can easily reduce manufacturing costs later in the product cycle. To avoid lengthy product development times for ceramic composites, funding agencies for materials research must commit resources to support design and development technologies early in the material life cycle. These technologies need not focus on designing the material, rather, the technology must focus on designing with the material, i.e., developing methods to design components fabricated from the new material.

Thus a basic tenet that motivated this research effort is that a persistent need exists for improvements in the analysis of components fabricated from CMC material systems. From an aerospace design engineer's perspective the new generation of ceramic composites offers a significant potential for raising the thrust/weight ratio and reducing NO<sub>x</sub> emissions of gas turbine engines (which was mentioned previously). Continuous ceramic fiber composites exhibit an increase in work of fracture, which allows for "graceful" rather than catastrophic failure. When loaded in the fiber direction, these composites retain substantial strength capacity beyond the initiation of transverse matrix cracking despite the fact that neither of its constituents would exhibit such behavior if tested alone. As additional load is applied beyond first matrix cracking, the matrix tends to break in a series of cracks bridged by the ceramic fibers. Thus any additional load is born increasingly by the fibers until the ultimate strength of the composite is reached. Establishing design protocols that enable the engineer to analyze and predict this type of behavior in ceramic composites was the general goal of this project.

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#### *Section II - Description of the New Technology that was Developed*

The analysis and design of components fabricated from ceramic composite materials require a departure from the usual deterministic design philosophy (i.e., the factor of safety approach) prevalent in the analysis of metallic structural components, which are more tolerant of flaws and material imperfections. Under applied load, large stress concentrations occur at macroscopic as well as at microscopic flaws, which are unavoidably present in the composite as a result of processing or in-service environmental factors. The observed scatter in component strength is caused by a random distribution of these various flaws, and composite fracture occurs when the damage driving force or the effective energy release rate reaches a critical value. Since failure is governed by the scatter in strength (ultimate or microcrack yield), statistical design approaches must be employed. Utilizing structural reliability methods provides a more general accounting of the entire spectrum of values that strength parameters may exhibit. However, the reliability approach demands that the design engineer must tolerate a finite risk of unacceptable performance. This risk of unacceptable performance is identified as a component's probability of failure. The primary concern of the engineer is minimizing this risk in an economical manner. This means that if reliability methods are utilized, appropriate analytical tools needed to quantify uncertainty must be readily available. Past research efforts supported under this grant have produced various quasi-static reliability models for ceramic matrix composites. These models were then incorporated into computer algorithms; T/CARES for whisker-toughened ceramic composites, and C/CARES for continuous fiber-reinforced ceramic composites.

Yet the loss of component integrity under quasi-static loads is not the only failure mode that a product design engineer must contemplate. Consideration must be given to:

- in-service degradation of residual strength;
- in-service degradation of elastic stiffness properties;
- in-service degradation of thermal properties; and
- time-dependent deformation behavior.

All four are time-dependent phenomenon, and will affect the life of a component. Past efforts supported under this cooperative work agreement were directed at modeling the degradation of

residual strength. However, degradation in elastic stiffness properties and thermal properties can be equally important in the design of a component; neither of which directly involves reliability computations. Many authors have rightly pointed out that ceramic material systems should perform better than metal alloys in corrosive service environments that include elevated temperature. If thermal efficiency dominates a component design, then changes in thermal conductivity can easily reduce the service life of a component. Similarly, changes in elastic stiffness properties can lead to unacceptable deformations, or allow stability (i.e., buckling) failure modes to arise. Finally, it must be recognized that excessive deformation is a possible failure mode for a component. Certain classes of ceramic materials are susceptible to creep under sustained loading at elevated temperature (e.g., silicon nitride). Thus time-dependent deformation models (based on concepts found in viscoelasticity or viscoplasticity) are needed, and this issue will be addressed in follow-on efforts.

At the time of the close-out of this grant this effort was not complete. Future work supported by follow-on NASA grants will lead to a C/CARES-Life computer algorithm, analogous to the CARES-Life computer algorithm. However, the CARES-Life code utilizes crack growth models embedded in a theoretical framework based on the principles of fracture mechanics. Using fracture mechanics to predict the life of a component is not a viable alternative when materials with a varying heterogeneous microstructure are utilized. Continuum damage mechanics may provide the necessary theoretical framework to address the degradation of residual strength. However, continuum damage mechanics tends to homogenize the physics of residual strength degradation. Thus any modeling efforts using damage mechanics will produce phenomenological paradigms. This is not a problem until one considers that fracture (and any degradation of residual strength) is an extreme-value event. Thus reliability models based on extreme-value statistics should be utilized. This research avenue has been explored in the past under this grant, and efforts will continue into the future.

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#### ***Section III - Unique and Novel Features of the Technology & Results/Benefits of Application***

The primary objective of the principal investigators efforts is the creation of enabling technologies for the macroanalysis of aerospace components. These enabling technologies will aid in shortening the product development cycle of components fabricated from the new high technology materials. The results/benefits of application are compiled in the following year-by-year summary of this research project.

#### ***ACCOMPLISHMENTS DURING YEAR 1993-1994***

During this year the principal investigator continued his efforts in guiding an ASTM standard practice for estimating Weibull parameters through the ballot process. The standard focuses on the evaluation and subsequent reporting of uniaxial strength data, and the estimation of probability distribution parameters for ceramics which fail in a brittle fashion.

In addition, minor technical and editorial changes have been incorporated. In addition, the principal investigator contributed a chapter to a monograph being published by the Department of Defense (DoD) through CINDAS (Purdue University). This chapter highlights past research efforts related to interactive and noninteractive reliability models, as well as parameter estimation techniques.

A work plan to transfer stochastic design technology (developed by the principal investigator under a previous funding of this grant) to a development group at General Electric Aircraft Engine Business Group (GE/AEBG) was formalized through a NASA Space Act Agreement. Under the Space Act Agreement NASA LeRC and GE/AEBG will conduct research, technology development, and testing efforts in developing structural reliability models for intermetallic single crystal materials. The purpose of this agreement is to help GE/AEBG in characterizing advanced engineering materials, and to develop design methodologies applicable to aircraft gas turbine and various aerospace engines. The research effort outlined in this nonreimbursable Space Act Agreement focuses on single crystal NiAl, an intermetallic material currently under development at GE/AEBG. A proprietary agreement between the principal investigator, Stephen F. Duffy, and GE/AEBG was executed. Jonathan A. Salem (NASA LeRC, Structural Integrity Branch) served as co-principal investigator. The program

plan outlined in the agreement represents only a portion of the future development needed to fabricate gas turbine hot section components (e.g., stator vanes, turbine blades, turbine disks, etc.) from this material, and this program will augment future efforts by focusing attention on the stochastic nature of the material. Specifically, researchers at GE/AEBG have encountered variability in the tensile strength of this material below the ductile-brittle transition temperature. The overall thrust of this work effort involves tailoring analytical methods developed at NASA LeRC such that reliability issues germane to the single crystal NiAl can be addressed. The technical effort was organized as follows:

- Establish a multiaxial, phenomenological failure criterion (i.e., a limit state function) based on experimental data.
- Identify random strength variables associated with the failure criterion, and establish the underlying probability distribution function for each variable (e.g., Weibull, log-normal, etc.).
- Transform the deterministic failure criterion into a stochastic reliability model and embed this reliability model in the reliability algorithm TCARES. The reliability algorithm will allow research engineers at GE/AEBG to establish the probability of failure (and simultaneously account for variations in material strength) of hot section components.
- Tailor the TCARES algorithm to the finite element codes (specifically ANSYS) used at GE/AEBG to perform structural analyses on engine components.
- Establish/generate a multiaxial experimental data base utilizing several stressed volumes (i.e., several sizes of specimen gage section) for single crystal NiAl. Concurrently, the source of failure (i.e., the strength limiting defects) will be ascertained in order to improve component fabrication and processing.

Note that the principal investigator worked closely with research personnel at GE/AEBG so that select individuals had an in-depth working knowledge of the principles that underlie the stochastic approach advocated within this work plan. Note that this Space Act Agreement was instrumental in securing a research and development grant for GE/AEBG from the United States Air Force.

The principal investigator continued his involvement with the NASA Enabling Propulsion Materials (EPM) program. The principal investigator served as a team leader for the Damage Mechanics/Life Models team with research personnel from GE/AEBG, and Pratt & Whitney.

Specifically the team is studying the effects of degradation in elastic stiffness properties as well as degradation of thermal properties, and is attempting to model these effects using continuum damage mechanics. The concepts that underpin damage mechanics enable the design engineer to assemble life prediction methods that assess the evolution of engineering design parameters which influence the response of a component to service load and environment. Note that continuum damage mechanics does not necessarily focus attention on microstructural events, yet it does provide a practical model which macroscopically captures the changes induced by the evolution of voids and defects. This is accomplished without focussing attention on the geometry of discontinuities as is the case with fracture mechanics and fatigue crack growth models. However, comparisons of the continuum and microstructural kinetic equations proposed for monolithic ceramics yields strong resemblances to one another. Thus the persistent opinion that phenomenological damage models have little in common with the underlying physics is sometimes short-sighted, and most times self-serving. Adopting a continuum theory of damage with its attendant phenomenological view would appear to be a logical first approach. An issue studied by the life prediction team within Task A6 (i.e., the combustor) is the degradation of both elastic and thermal properties, and the effect that this type of degradation has on combustor performance. Degradation in stiffness may have an apparent beneficial effect of redistributing stress in components subjected to strain-control. However, at some point a combustor segment may no longer efficiently carry mechanical load. As an example, a secondary failure mode such as localized shell buckling may arise under a sufficient loss in material stiffness. Note that stiffness loss is definitely detrimental under load-control service histories. Alternatively (but no less important), localized damage may reduce the ability to conduct heat away from a given region of the combustor. As heat conduction falls below a predetermined margin, a localized hot region is created that becomes severely overstressed. This tends to increase damage, which leads to a further loss in heat conduction. As might be expected this process may "run away" during service, leading to catastrophic failure.

***Presentations 1993-1994:***

1. ASTM C 28 Advanced Ceramics Committee Mtg (January 5-7, 1994) - Presentation that addressed sub-committee ballots with comments regarding the proposed ASTM standard entitled "Reporting Strength Data and Estimating Weibull Distribution Parameters for Advanced Ceramics," S.F. Duffy (presenter), G.D. Quinn, and C.A. Johnson.



***Awards 1993-1994:***

1. ASTM Award of Appreciation - presented by ASTM Committee C-28 for Advanced Ceramics (January 1994)
2. NASA Lewis Awareness Team Recognition - member of the Brittle Structures Life Prediction Team (January 1994)

**ACCOMPLISHMENTS DURING YEAR 1994-1995**

During the past year the principal investigator guided the development of a design analysis as part of a cooperative agreement with General Electric Aircraft Engines. This effort is focused on modifying the TCARES reliability algorithm for use in the design of engine components fabricated from NiAl. This intermetallic material will be utilized in single crystal form and it exhibits brittle behavior at lower service temperatures. In addition, the brittle strength of this material exhibits stochastic behavior, i.e., the ultimate strength is a random variable. The design approach advocates the following:

- Determination of the statistical nature and source of fracture in a high strength, NiAl single crystal turbine blade material;
- defining a phenomenological failure strength envelope for the material;
- developing a computer algorithm for statistically based reliability models;
- and developing analytical models of turbine blades and vanes for use in test bed engines.

Since the material is brittle and highly anisotropic (Young's modulus varies from 95 to 271 GPa, depending on the direction of crystallization) the statistical nature and source of fracture was studied using flexural test specimens fabricated from small billets (25 x 50 x 100 mm) where the direction of crystallization in each billet was identified prior to specimen preparation. Use of flexural test specimens allowed a maximum number of test specimens to be fabricated from a given region of a billet. This permitted the determination of billet-to-billet, and within billet variation of ultimate strengths. Preliminary flexural strength data indicated a wide dispersion in strength which was characterized using Weibull statistics. Failure origins were identified using fractographic methods in 25 of 27 specimens tested. The attending fractography indicated that failure occurred from coarse Hf-rich or Hf-Ni-Al inclusions.

From these preliminary tests it was concluded that the ultimate strength of this material could be adequately modeled using weakest-link reliability models (often referred to as Weibull analysis in the ceramics literature). The application of reliability theory to this particular material required the modification of current models (which assume isotropic material behavior) to account for anisotropic failure behavior. Several isotropic theories applicable to ceramics and glasses have been incorporated into a computer algorithm entitled Toughened Ceramics Analysis and Reliability Evaluation of Structures (TCARES) developed under previous funding of this grant. This algorithm allows for anisotropic material behavior, however, a reliability model must be tailored to the NiAl material being analyzed in this project. Work began on this aspect of the project in last quarter the grant period and is unfinished at the time of the premature close-out of this cooperative work agreement. Note that GE/AEBG provided researchers supported under this grant with a finite element model of a turbine blade which will be used in a future rig test. This model will be utilized in revamping the TCARES algorithm.

During the past year the principal investigator continued his participation on ASTM Committee C 28 (Advanced Ceramics). The principal investigator has begun work on two new standards. The first standard is entitled "Size Scaling of Uniaxial Strengths Using Weibull Statistics for Advanced Ceramics." This standard practice will detail how to convert a strength measure (e.g., the mean or the Weibull characteristic strength) estimated from data obtained from one test specimen geometry to a strength measure associated with a different specimen geometry. The second standard is entitled "Reporting Strength Data and Estimating Weibull Distribution Parameters for Advanced Ceramics from Pooled Failure Data" This standard practice will extend the methodology in ASTM Standard practice C 1239 (authored by the principal investigator under earlier funding of this grant) so that data from multiple specimen sizes and differing boundary conditions can be assimilated into a single "pool" of data values. Weibull parameter estimates would be calculated from this single data pool. Pooling failure data yields a beneficial reduction of confidence intervals for point estimates of the parameters. During the past year the principal investigator was also elected chair of Section C 28.02.01 (Probabilistic Methods), and received an award of appreciation for his past service to Committee C 28.

The principal investigator also organized a session for the ASME/IGTI Turbo Expo 95 held in Houston, Texas. The session was entitled "Life Prediction Methodologies and Data for Ceramic Materials." In addition the principal investigator presented a paper entitled "Trends in the Design and

Analysis of Components Fabricated From CFCCs" at this conference. Both the paper and the organization of the session represented an effort to provide a forum for the design methods (some of which were developed under this grant) utilized in the analysis of components fabricated from ceramic. The use of the CARES family of computer algorithms were highlighted in a number of papers presented at this conference by authors throughout the gas turbine industry.

**Presentations 1994-1995:**

1. "Current Practice & Future Goals for Probabilistic Strength Standards," ACerS Basic Science Division Fall Meeting , 25 - 28 September 1994, Louisville, Kentucky
2. "Designing Components from Materials that Exhibit Size Effects," S.F. Duffy, J. Hu, R. Martin, and E.H. Baker, The Centers for Composite Materials & Structures and Adhesive & Sealant Science, 4th Combined Technical Conference, 23 - 25 April 1995, Blacksburg, Virginia.
3. "Weibull Parameter Estimation - Extended Tables of Unbiasing Factors & Confidence Bounds," S.F. Duffy, V.J. Tennery, and E.H. Baker, Symposium on Silicon Nitride Relationships - Annual Meeting of the American Ceramic Society, 30 April - 3 May 1995, Cincinnati, Ohio.
4. "Trends in the Design and Analysis of Components Fabricated From CFCCs," S.F. Duffy, J.L. Palko, J.B. Sandifer, C.L. DeBellis, M.J. Edwards, and D.L. Hindman, International Gas Turbine And Aeronautics Congress & Exposition, Houston Texas, 5-8 June 1995.

**Awards 1994-1995:**

1. ASTM C 28 Advanced Ceramics Award (June, 1995)
2. ASME Award of Appreciation (September 1995)

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#### ***Section IV - Utilization of New Technology in Non-Aerospace Applications***

An ongoing informal research collaboration in support of technology transfer from NASA LeRC to researchers at Babcock & Wilcox (B&W) was established under a previous research grant. Specifically, during the tenure of this grant researchers from both organizations conducted a joint feasibility study of an advanced heat exchanger (AHX) in order to ascertain the utility of the C/CARES algorithm. The heat exchanger is a prototype being developed by Babcock & Wilcox under a project partially funded by the Department of Energy (DoE). The AHX is currently being utilized as a critical component of a waste heat recovery system placed in the exhaust path of an industrial furnace that produces highly corrosive flue gases. Due to this corrosive environment an oxide-oxide ceramic composite was selected for the AHX.

Research engineers at Babcock & Wilcox have generally recognized that the scatter in strength associated with ceramic material systems poses a unique design constraint. They have also recognized the need to utilize current technology available in all sectors of the ceramic community. The C/CARES algorithm represents unique design technology that is able to account for variability in material strength by utilizing a stochastic failure criterion which also reflects the anisotropic nature of ceramic composites. The principal investigator was invited to join the project design team in an advisory capacity. This type of partnering allows for an immediate transfer of state-of-the-art technology from government to industry. It also permits federally sponsored researchers to gain valuable insight into key issues that drive commercial application of research concepts. American industries benefit from this technology transfer since they obtain a high level of technical insight from individual researchers who have spent years studying certain aspects of a research concept. In turn, the government receives valuable input regarding applications that either validate or redirect research efforts. Also under certain limited conditions, performance data (some of which is proprietary) is made available to federal researchers who participate in the design project. This type of open interaction (where industry is protected by proprietary and/or Space Act Agreements) has a tendency to shorten the research innovation cycle, and is essential in making the partnership a success.

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#### Section V - Additional Documentation

##### Publications:

1. "Analysis of Whisker-Toughened CMC Structural Components Using an Interactive Reliability Model," S.F. Duffy and J.L. Palko, *AIAA Journal*, Vol. 32, No. 5, pp. 1043-1048, (May, 1994), (also published as NASA CR-190755).
2. "Reliability Analysis of Laminated Ceramic Matrix Composites Using Shell Subelement Techniques," A. Starlinger, D.J. Thomas, S.F. Duffy, and J.P. Gyekenyesi, *AIAA Journal*, Vol. 31, No. 11, pp. 2181-2183, (November, 1993), (also published as NASA TM-105413).
3. "Reporting Strength Data and Estimating Weibull Distribution Parameters for Advanced Ceramics," S.F. Duffy, G.D. Quinn, and C.A. Johnson, Proposed ASTM Standard Practice C 1239-95.
4. "Reliability and Life Prediction of Ceramic Composite Structures at Elevated Temperatures," S.F. Duffy and J.P. Gyekenyesi, in High Temperature Mechanical Behavior of Ceramic Composites, S.V. Nair and K. Jakus, eds., Butterworth-Heinemann, Boston, pp. 471-515, (1995).
5. "High Temperature Life Prediction of Monolithic Silicon Carbide Heat Exchanger Tubes," J.B. Sandifer, M.J. Edwards, T.S. Brown, and S.F. Duffy, in Life Prediction Methodologies and Data for Ceramic Materials, ASTM STP 1201, C.R. Brinkman and S.F. Duffy eds., American Society for Testing and Materials, Philadelphia, pp. 373-389, (1994).
6. A Methodology to Predict Creep Life for Advanced Ceramics Using Continuum Damage Mechanics Concepts," T.-J. Chuang and S.F. Duffy, in Life Prediction Methodologies and Data for Ceramic Materials, ASTM STP 1201, C.R. Brinkman and S.F. Duffy eds., American Society for Testing and Materials, Philadelphia, pp. 207-227, (1994).
7. "Issues Relating to Reliability Based Design Component Design," S.F. Duffy, to appear in Handbook on Whisker and Particulate Reinforced Ceramic Matrix Composites, K. Bowman, S.K. El-Rahaiby, and J. Wachtman eds.; to be published by DoD Ceramics Information Analysis Center.
8. "A Creep Life Prediction Model for Continuous Fiber Reinforced Ceramic Composites," T.-J. Chuang and S.F. Duffy, HITEMP Review 1993, Advanced High Temperature Engine Materials Technology Program, NASA CP-19117, Cleveland, Oh., pp. 77-1 through 77-17, (October, 1993).

9. "Design of a Laminated CMC Heat Exchanger Using the C/CARES Algorithm," J.L. Palko and S.F. Duffy, HITEMP Review 1993, Advanced High Temperature Engine Materials Technology Program, NASA CP-19117, Cleveland, Oh., pp. 75-1 through 75-8, (October, 1993).
10. "Composites Research at NASA Lewis Research Center," S.R. Levine, S.F. Duffy, A. Vary, M.V. Nathal, R.V. Miner, S.M. Arnold, M.G. Castelli, D.A. Hopkins, and M.A. Meador, *Composites Engineering*, Vol. 4, No. 8, pp. 787-810.
11. "Reliability Analysis of Single Crystal NiAl Turbine Blades," J.L. Palko, S.F. Duffy, J.A. Salem, R.D. Noebe, F. Holland, D.R. Wheeler, P.K. Wright, HITEMP Review 1994, Advanced High Temperature Engine Materials Technology Program, NASA CP-10146, Cleveland, Oh., pp. 56-1 through 56-11, (October, 1994).
12. "Trends in the Design and Analysis of Components Fabricated From CFCCs," S.F. Duffy, J.L. Palko, J.B. Sandifer, C.L. DeBellis, M.J. Edwards, and D.L. Hindman, ASME 95-GT-405 (paper has been accepted for publication in the *Transactions of the ASME* ).
13. "An Overview of Engineering Concepts and Current Design Algorithms for Probabilistic Structural Analysis," S.F. Duffy, J. Hu, and D.A. Hopkins, Presented at the 11th Biennial Conference on Reliability, Stress Analysis, and Failure Prevention, Boston MA., 17-21 September 1995.